IJCTA, 9(39), 2016, pp. 225-232

Ascent Trajectory Optimiztion for the First Stage of a two – Stage – to – Orbit Reusable Launch Vehicle with Load Factor Constraint During re – entry

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Abstract : The main objective of using reusable launch vehicle is to minimize the operational cost of launching satellites into outer-space. This is achieved by flying back the launch vehicle back to launch station or to any designated landing site, thus the vehicle can be recovered and reused. This paper discusses about optimizing the steering angle (θ_2) at burnout during ascent phase for the first stage of a TSTO launch vehicle and thereby to maximize the velocity at zero flight path angle, subjected to the constraint involved in re-entry. Since optimization of ascent phase is performed by considering the re-entry constraint, the optimization is carried out in multi-disciplinary optimization framework.

Keywords : RLV, Trajectory optimizaton, Load factor Constraint.

1. INTRODUCTION

Reusable launch vehicles are one of the best solution to minimize the launch cost associated with satellite launches and space travel missions. By flying back the launch vehicle back to a designated site, may be to launch station itself or by performing parachute assisted water landing, the vehicle can be recovered, refurbished and reused. Thereby, the cost of launching can be brought down substantially [1][2].

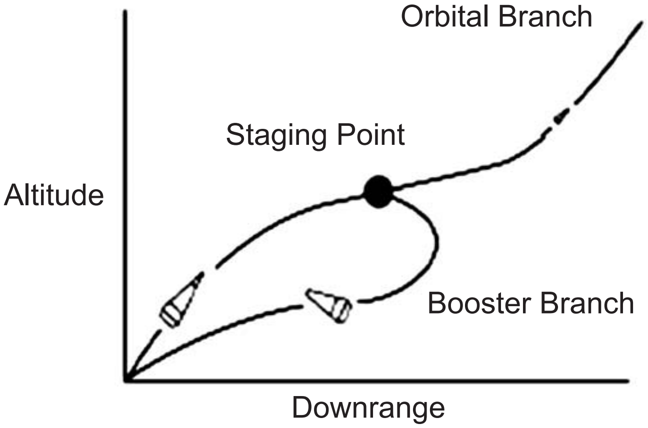

Figure 1: Two-stage-to-orbit launch vehicle

A two – stage – to – orbit launch vehicle, as shown in figure 1, contains two stages and the objective of first stage is to take the vehicle from launch site to sub-orbit altitude and second stage of the vechile carries the payload to desired orbit. This paper aims at maximizing the velocity of first stage of vehicle with the load factor constraint during re-entry.

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The re-entry constraints associated during fly-back have significant effects on the structural strength of the vehicle. During fly-back, if the constraints are violated, it may cause damage to the vehicle and it cannot be reused. These fly-back constraints will depend on parameters associated during the cruise climb. The load factor acting on vehicle structure is one such re-entry constraint [3]which depends on the terminal states of ascent trajectory from which the vehicle is re-entering. This paper aims at maximizing the velocity of first stage of the vehicle by optimizing the steering angle of the vehicle with dynamic pressure constraint for ascent phase and load factor constraint during re-entry.

2. FLIGHT PROFILE

The first stage of the launch vehicle, a test demonstrator, should be flown to an altitude between 60 km to 80 km and the maximum downrange for entire flight is around 600 km. Test demonstrator is launched using a booster mechanism and once the booster burns out it separates from the demonstrator and vehicle continues the unpowered ascent, enter into a transition stage and starts the re-entry phase. The flight profile for the experiment is shown in figure 2[4].

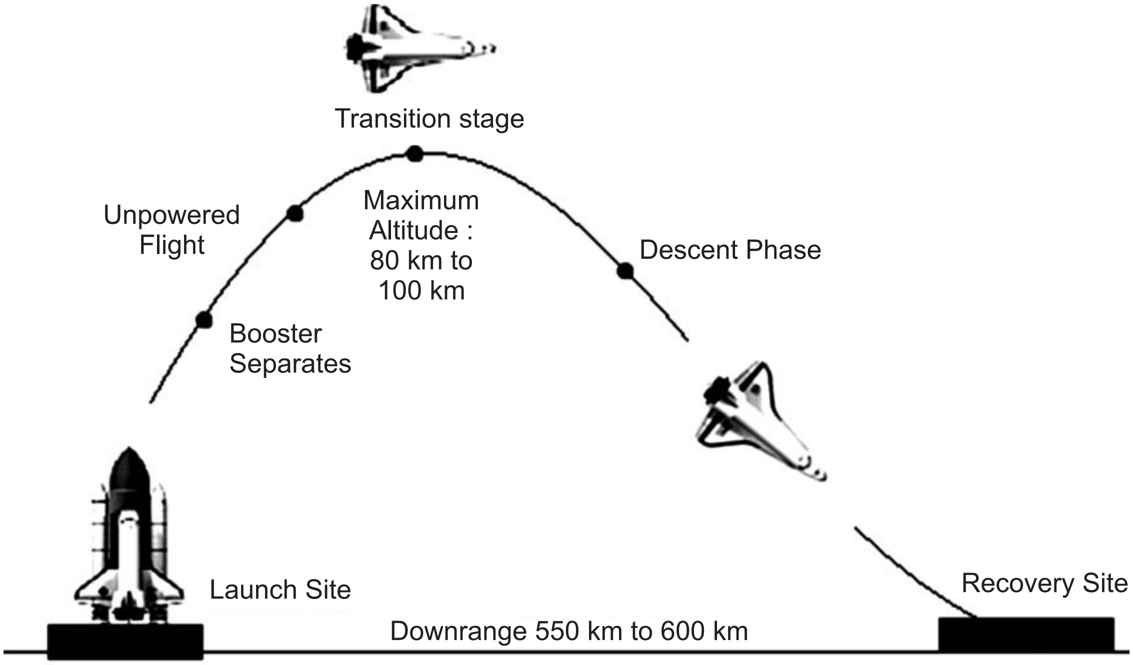

Figure 2: Flight Profile for first stage of RLV – Test Demostrator

3. MATHEMATICAL MODEL FOR LAUNCH VEHICLE FLIGHT

Flight of a launch vehicle mainly consist of two phases, (*i*) Ascent Phase and (*ii*) Descent phase. The phase of flight from launch station to zero flight path angle condition is considered as ascent phase. This stage of vehicle at zero flight angle path is called transition stage, from this stage, flying back to designated point/ recovery site is considered as descent phase. Ascent phase and descent phase of flight are governed by equations (1) to (7) [5][6].

Ascent phase equations are as follows,

$$\dot{r} = v \sin\gamma \tag{1}$$

$$\dot{x} = v \cos\gamma \tag{2}$$

$$\dot{v} = \frac{\mathrm{T} \cos(\theta - \gamma) + \mathrm{D}}{m} - g \sin\gamma \tag{3}$$

$$\dot{\gamma} = \frac{\mathrm{T} \sin(\theta - \gamma) + \mathrm{L}}{mv} - \left(\frac{v}{r} - \frac{g}{v}\right)\cos\gamma \tag{4}$$

where, v is the velocity(m/sec), γ is the flight path angle(deg), θ is the steering angle(deg), r is the cross range(m), x is the downrange(m), g is the gravitational acceleration, m is the mass, L is the lift and D is the drag.

Descent phase equations are as follows,

$$\dot{r} = v \sin\gamma \tag{5}$$

$$\dot{v} = \frac{-\mathrm{D}}{m} - g \sin \gamma \tag{6}$$

$$v\dot{\gamma} = \left(\frac{v}{\gamma} - g\right)\cos \gamma + \frac{\mathrm{D}}{\mathrm{M}}u \tag{7}$$

where, control u is the vertical or in-plane component of lift to drag ratio.

Descent phase equations are written on the assumption that the side slip angle of the vehicle on re-entry is zero and also symmetric flight condition exsist.

4. TRAJECTORY GENERATION

Flight trajectory of vehicle can be generated by integrating the equations of motion (1) – (4) and (5) – (7) with required initial conditions. Initial conditions for ascent trajectory are as follows: Initial Mass of vehicle, m = 13750 kg; Altitude, h = 0 km; Flight path angle, γ = 90 degree, Downrange, x = 0 km.

For the ascent phase the trajectory constraint imposed is the dynamic pressure which should not exceed 60kPa. Booster used for ascent phase is 9.14 ton which burns out completely in 96 seconds. The thrust output provided by the booster is according to table 1[7].

Table 1

Thrust ouput of booster

Time(Sec)	*Thrust(N)*	*Fuel Consumed(kg)*
0	0	0
10	283310	935
20	279580	2190
30	218090	3235
40	213790	4070
50	241360	4967
60	264750	5960
70	287140	7042
80	305780	8084
90	114420	9065
95	0	9140

For descent trajectory, the initial conditions such as altitude, flight path angle and velocity are obtained from the state vector of the vehicle at zero flight path angle. The terminal conditions for re-entry trajectory are as follows:

Altitude, h = 5km;
Velocity, V = 450 m/sec
and Downrange = 550km.

5. OPTIMIZATION AND SIMULATION RESULTS

The objective is to optimize the parameter steering angle at burnout such that the velocity of vehicle at FPA = 0 can be maximized, without violating ascent trajectory constraint of dynamic pressure and the load factor constraint of re-entry trajectory. The dynamic pressure during ascent phase should not exceed 60kPa and load factor during re-entry should not exceed 3g.

The optimization problem can be formulated as,

$$\begin{aligned} &\text{Maximize} && \text{Velocity (m/sec)} \\ &\text{Subject to:} && q_{\max} \leq 60 \text{ kPa (Ascent trajectory)} \\ & && n_{\max} \leq 1.5\text{g (Descent trajectory)} \end{aligned}$$

Since the optimization is done for parameters of ascent trajectory subjected to re-entry constraints, the optimization is done in MDO framework. The optimization is performed using MATLAB optimization tool fmincon. Different types of MDO techniques such as, *i*. fixed point iteration (FPI), *ii*. optimization based decomposition (OBD), *iii*. collaborative method are discussed in literature. This paper makes use of FPI technique because, FPI has an advantage that it will find the true system optimum without conflicting objectives from the re-entry phase[8]. The FPI structure is shown in figure 3[8].

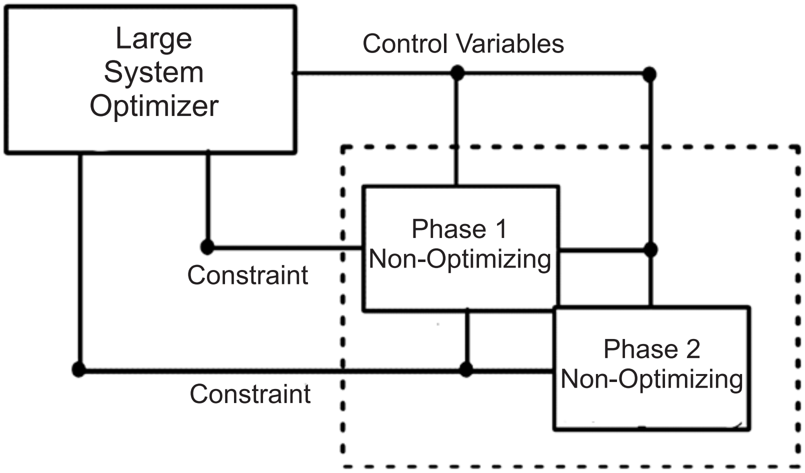

Figure 3: Fixed Point Iteration Structure

The dynamic pressure constraint involved in ascent phase and load factor constraint involved in re-entry phase are functions of density and velocity. Dynamic pressure is given by the equation (8) and load factor is given by equation (9)

$$\text{Dynamic Pressure, } q = \frac{1}{2}\rho v^2 \tag{8}$$

$$\text{Load factor, } n = \frac{\mathrm{C_L}\cos(\alpha) + \mathrm{C}_d \sin(\alpha)}{m} \times q \times \mathrm{S}_{ref} \tag{9}$$

The velocity attained by the vehicle at zero FPA will depend on the steering angle (θ^2) of the vehicle at booster burn-out. Therefore, the steering angle at 96 seconds need to be optimized such that the vehicle will attain maximum velocity without violating the constraints.

Optimization is done for two cases, in first case, the ascent trajectory is optimized by considering only the dynamic pressure constraint and the θ^2 is obtained as 14.5 degree. When the re-entry trajectory is simulated with the state vector at zero FPA as initial condition, the load factor constraint is violated.

For the second case, where both ascent and re-entry constraints are considered, the optimization is done in MDO framework such that optimization of steering angle can be performed by subjecting to load factor constraint. The lift of steering angle (θ_1) is taken as 90 degree and (θ_2) at burnout is optimized.

The optimum θ_2 value obtained is 8.7 degree and the state vector at zero FPA for this steering angle is shown in table 3 and terminal conditions obtained for re-entry trajectory is shown in table 4. Maximum dynamic pressure and load factor obtained for two cases are shown in table 5. The trajectory profiles for case *ii* are shown in figures 4 – 7. Dynamic pressure and load factor variations are shown in figures 8 and 9.

Table 2

State Vector at FPA = 0 degree

Parameters	*Values at FPA = 0degree*
Altitude	61660 m
Velocity	1619 m/sec
Mach	5.3
Down Range	163000 m

Table 3

Terminal Conditions obtained for re-entry trajectory

Parameters	*Values*
Altitude	14040 m
Velocity	504 m/sec
Down Range	484100 m

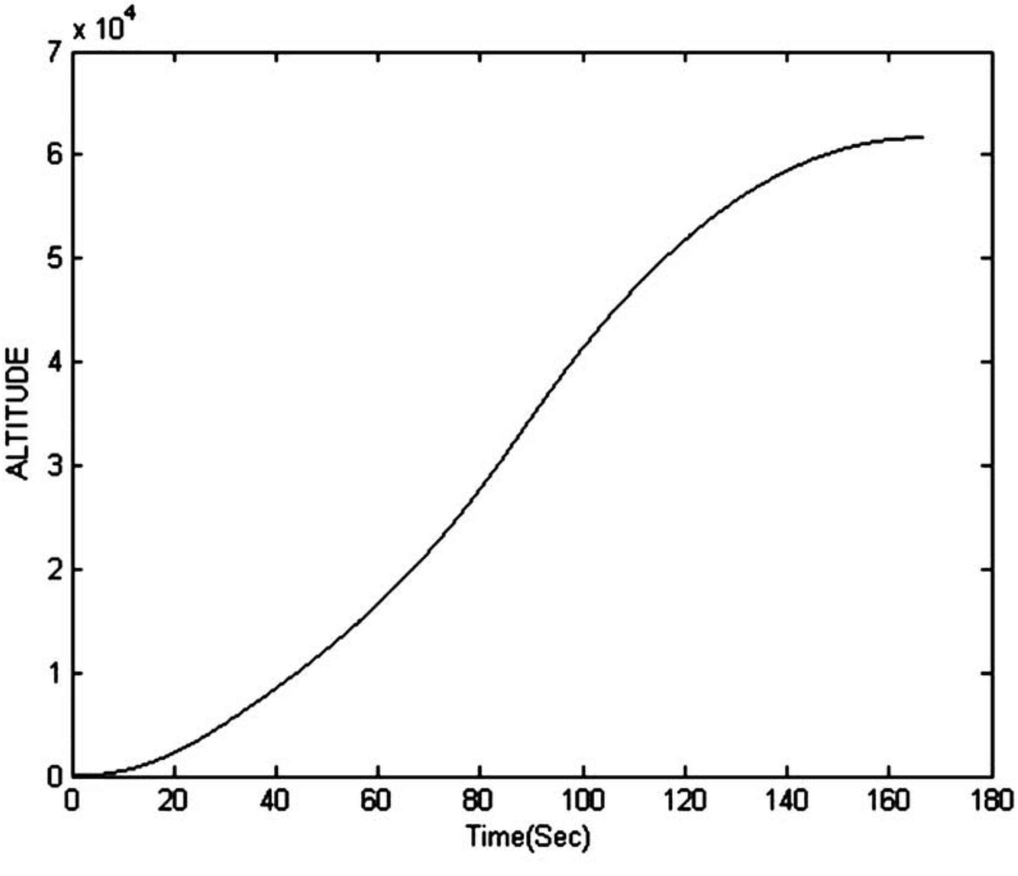

Figure 4: Altitude(m) - Ascent phase

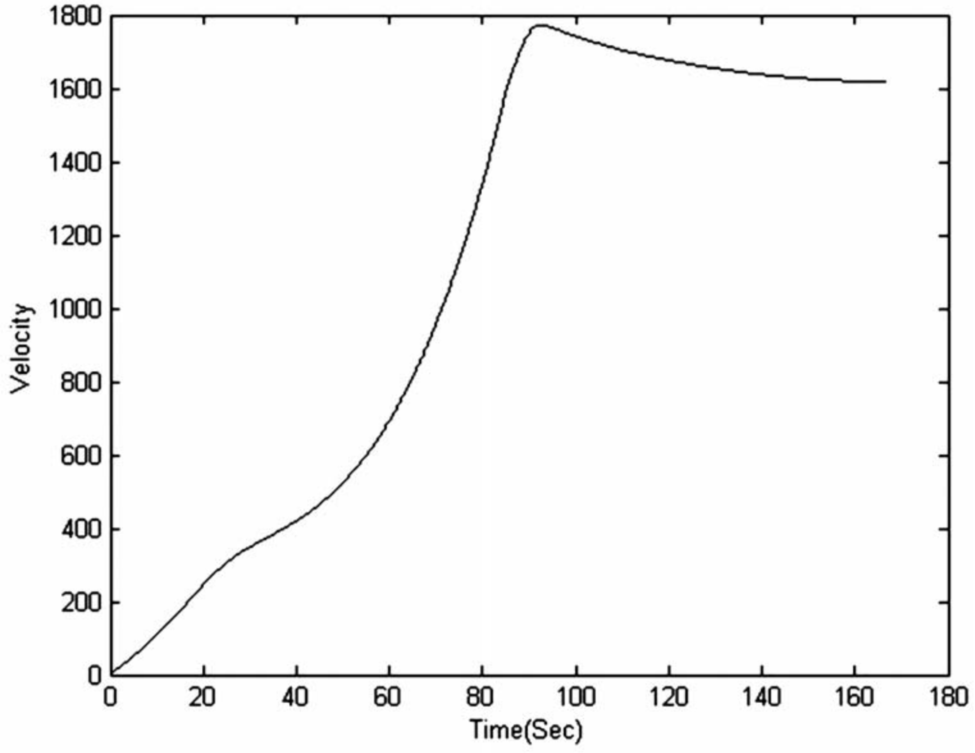

Figure 5: Velocity (m/sec) – Ascent Phase

Table 4

Maximum dynamic pressure and Load factor for case 1 and case 2

Steering Angle (Degree)	*Max. dynamic pressure(kPa)*	*Max. Load Factor*
Case (*i*) $\theta_1 = 14.5$	43.33	1.7 g
Case (*ii*) $\theta_2 = 8.7$	45.18	1.3 g

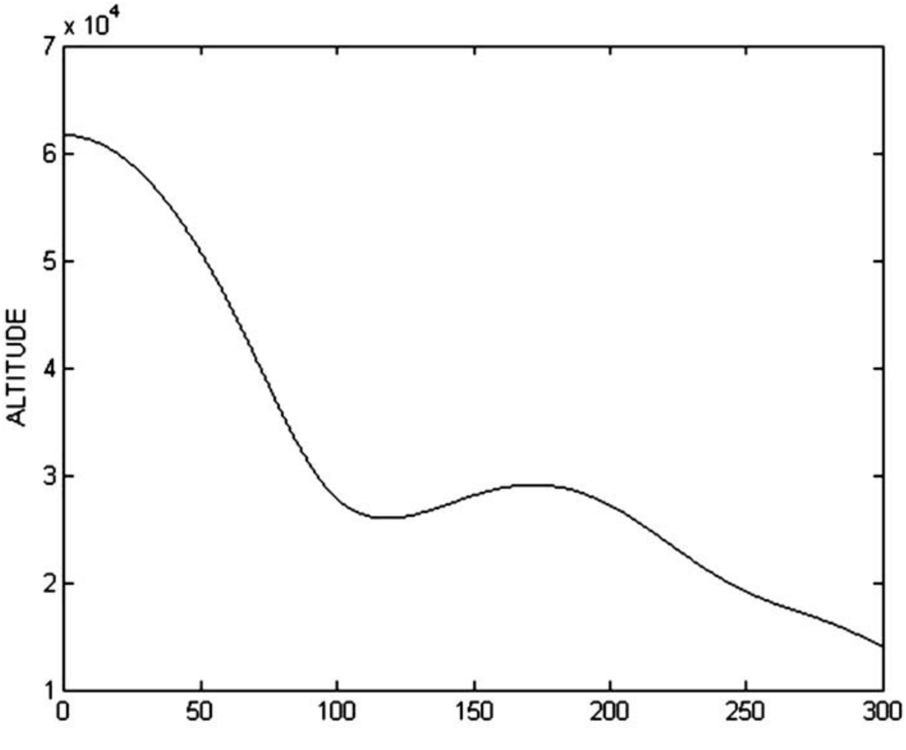

Figure 6: Altitude(m) - Descent phase

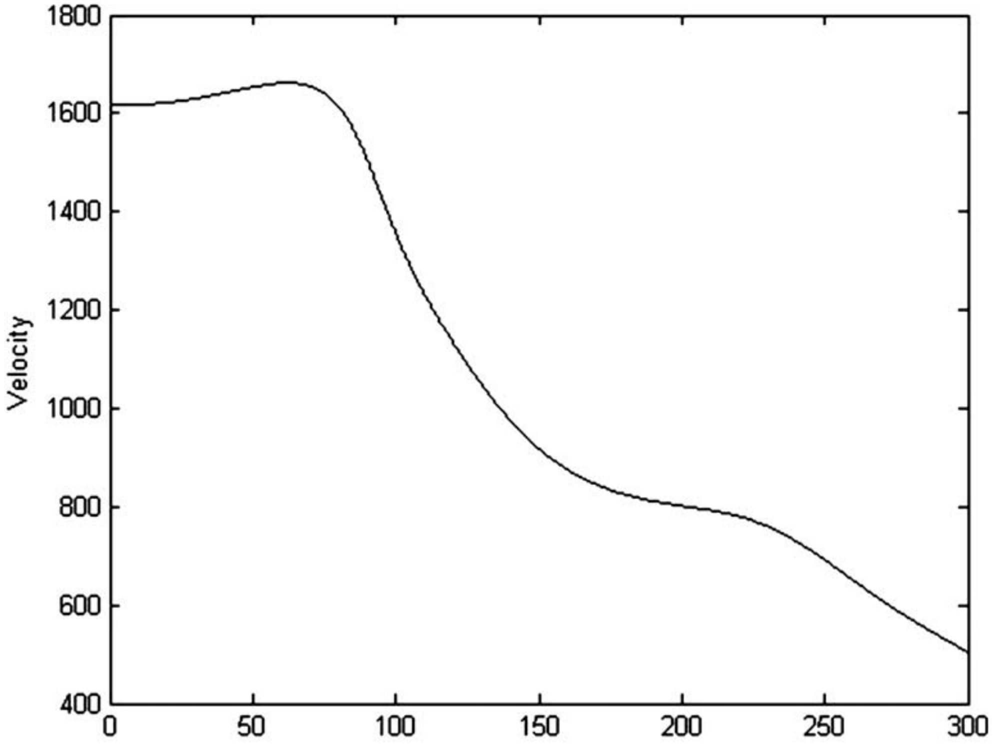

Figure 7: Velocity(m/sec) - Descent phase

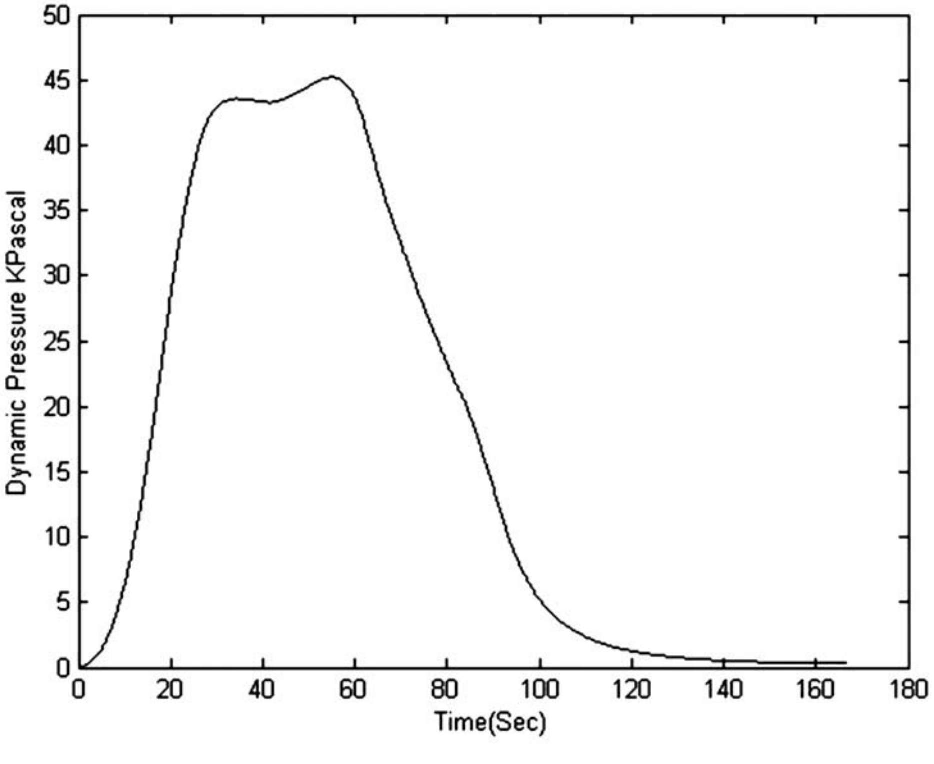

Figure 8: Dynamic Pressure (Ascent)

6. CONCLUSION

The steering angle for first stage of a TSTO re-usable launch vehicle is optimized for maximizing the velocity of the vehicle at zero flight path angle, subjected to dynamic pressure and load factor constraint. Since re-entry constraints are present, the optimization is done in MDO framework so that the optimization of steering angle for ascent phase can be done simultaneously along with checking for constraints in re-entry phase. Steering angle (θ_2) at burn-out instant is obtained as 8.7 degree.

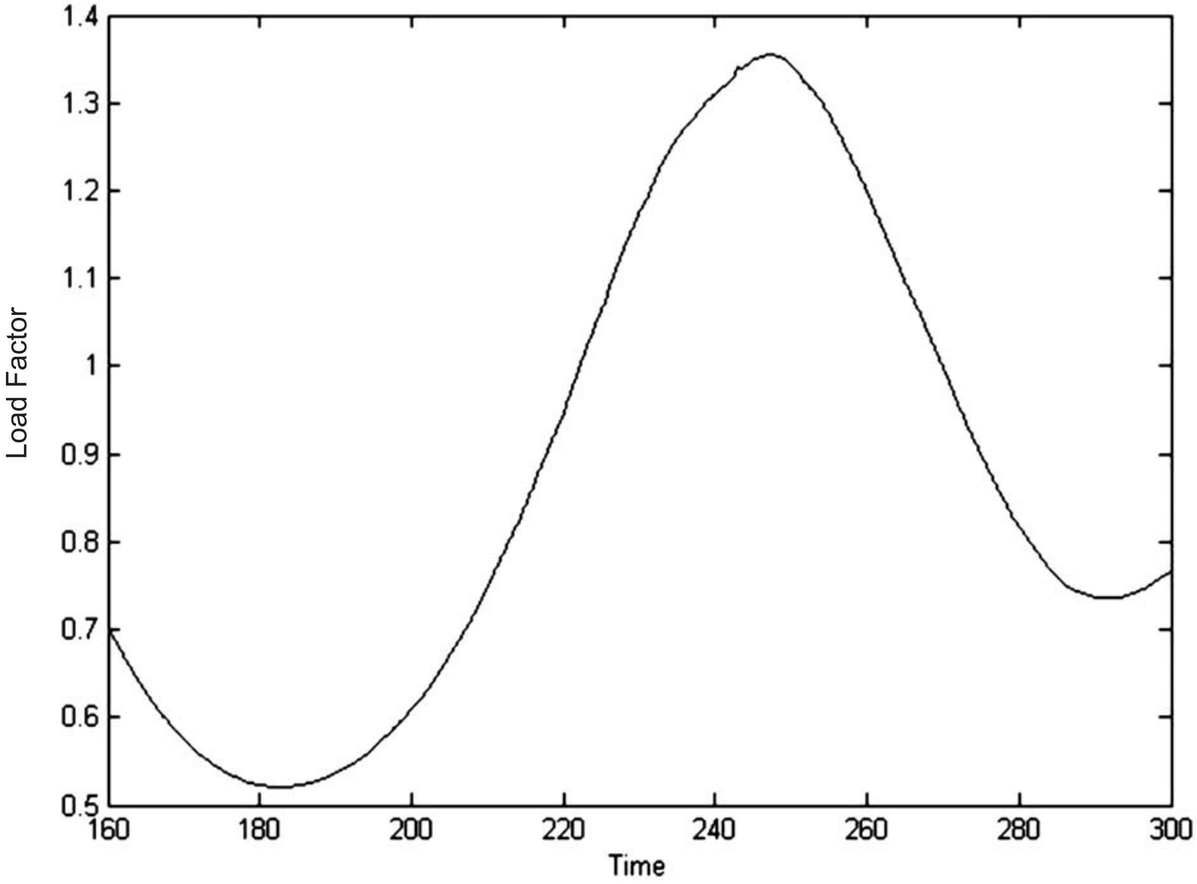

Figure 9: Load Factor (Descent)

7. REFERENCES

1. Freeman, D.C. et al., "The Future Space transportation (FSTS) study," *Astronautics & Aeronautics*, Vol. 21, No.6, 1983, pp. 36-57
2. Talay, T.A., and Morris, W.D., "Advanced Manned launch systems ," *European Aerospace Conference*, Paper' 89-17, May 1989
3. Ping Lu, "Entry Guidance and Trajectory Control for Reusable Launch Vehicle", *Journal of Guidance, Control, and Dynamics*. 1997. 20(1), pp 143-149
4. Pamadi, Bandu N. et al., "Simulation and Analyses of Stage Separation of Two-stage Reusable Launch Vehicles", *Journal of Spacecraft and Rockets*, Vol. 44, No. 1, 2007, pp 66-80.
5. P.F. Gath and A.J. Calise, "Optimization of Launch Vehicle Ascent Trajectories with Path Constraints and Coast Arcs," *Journal of Guidance, Control, and Dynamics*, Vol.24, 2001, pp. 296-304.
6. Jiang, Z. and Ordonez, R., "Trajectory generation on approach and landing for RLVs using motion Primitives and Neighboring optimal Control", *American control conference*, Paper' 1091-1096, July 11-13, 2007, USA
7. Jaison Joseph et al., "Trajectory optimization for suborbital Re-entry technology demonstration mission" *2nd European Conference for Aerospace Sciences*, 2-6 July, 2007, Brussels, Belgium.
8. Old, John. R and Ledsinger, Laura. A., "Multidisciplinary Optimization Techniques for Branching Trajectories," *36th Aerospace Sciences Meeting and Exhibit*, January 12-15, 1998/Reno, NV.