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A Novel 3-D Jerk Chaotic System with Two Quadratic Nonlinearities and its Adaptive Backstepping Control

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Abstract: First, this paper announces a seven-term novel 3-D jerk chaotic system with two quadratic nonlinearities. The phase portraits of the novel jerk chaotic system are displayed and the mathematical properties are discussed. The proposed novel jerk chaotic system has two equilibrium points along the x_1 axis, which are both unstable. The Lyapunov exponents of the novel jerk chaotic system are obtained as $L_1 = 0.11245$, $L_2 = 0$ and $L_3 = -0.55363$. Thus, the Maximal Lyapunov Exponent (MLE) of the novel jerk chaotic system is derived as$L_1 = 0.11245$. Since the sum of the Lyapunov exponents of the novel jerk chaotic system is dissipative, the novel jerk chaotic system is dissipative. Also, the Kaplan-Yorke dimension of the novel jerk chaotic system is derived as $D_{KY} = 2.20311$. Next, an adaptive controller is designed via backstepping control method to globally stabilize the novel jerk chaotic system with unknown parameters. Moreover, an adaptive controller is also designed via backstepping control method to achieve global and exponential synchronization of the identical novel jerk chaotic systems with unknown parameters. The main adaptive backstepping control results for stabilization and synchronization of the novel jerk chaotic system are established using Lyapunov stability theory.

Keywords: Chaos, chaotic systems, jerk systems, chaos control, chaos synchronization, backstepping control, stability theory.

1. INTRODUCTION

Chaos theory describes the qualitative study of unstable aperiodic behaviour in deterministic nonlinear dynamical systems. A dynamical system is called *chaotic* if it satisfies the three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1]. Chaos theory has applications in several areas in Science and Engineering.

A significant development in chaos theory occurred when Lorenz discovered a 3-D chaotic system of a weather model [2]. Subsequently, Rössler found a 3-D chaotic system [3], which is algebraically simpler than the Lorenz system. Indeed, Lorenz’s system is a seven-term chaotic system with two quadratic nonlinearities, while Rössler’s system is a seven-term chaotic system with just one quadratic nonlinearity.

Some well-known paradigms of 3-D chaotic systems are Arneodo system [4], Sprott systems [5], Chen system [6], Lü-Chen system [7], Liu system [8], Cai system [9], Tigan system [10], etc.

In the last two decades, many new chaotic systems have been also discovered like Li system [11], Sundarapandian systems [12-13], Vaidyanathan systems [14-33], Pehlivan systems [34-35], Pham systems [36-37], Jafari system [38], etc.

Hyperchaotic systems are the chaotic systems with more than one positive Lyapunov exponent. They have important applications in control and communication engineering. Some recently discovered 4-D hyperchaotic systems are hyperchaotic Vaidyanathan systems [39-40], hyperchaotic Vaidyanathan-Azar system [41], etc. A 5-D hyperchaotic system with three positive Lyapunov exponents was also recently found [42].

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Chaos theory has several applications in a variety of fields such as oscillators [43-44], chemical reactors [45-58], biology [59-80], ecology [81-82], neural networks [83-84], robotics [85-86], memristors [87-89], fuzzy systems [90-91], etc.

The problem of control of a chaotic system is to find a state feedback control law to stabilize a chaotic system around its unstable equilibrium [92-93]. Some popular methods for chaos control are active control [94-98], adaptive control [99-100], sliding mode control [101-103], etc.

Chaos synchronization problem can be stated as follows. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of the synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically.

The synchronization of chaotic systems has applications in secure communications [104-107], cryptosystems [108-109], encryption [110-111], etc.

The chaos synchronization problem has been paid great attention in the literature and a variety of impressive approaches have been proposed. Since the pioneering work by Pecora and Carroll [112-113] for the chaos synchronization problem, many different methods have been proposed in the control literature such as active control method [114-132], adaptive control method [133-149], sampled-data feedback control method [150-151], time-delay feedback approach [152], backstepping method [153-164], sliding mode control method [165-173], etc.

In the recent decades, there is some good interest in finding novel chaotic systems, which can be expressed by an explicit third order differential equation describing the time evolution of the single scalar variable x given by

$$\dddot{x} = J(x, \dot{x}, \ddot{x}) \tag{1}$$

The differential equation (1) is called "jerk system" because the third order time derivative in mechanical systems is called *jerk*. Thus, in order to study different aspects of chaos, the ODE (1) can be considered instead of a 3-D system.

In this paper, we announce a seven-term novel 3-D jerk chaotic system with two quadratic nonlinearities. The phase portraits of the novel jerk chaotic system are displayed and mathematical properties are discussed. The novel jerk chaotic system has two equilibrium points on the x_1–axis, which are both unstable. The Lyapunov exponents of the novel system are obtained as $L_1 = 0.11245$, $L_2 = 0$ and $L_3 = -0.55363$. The Kaplan-Yorke dimension of the novel jerk system is derived as $D_{KY} = 2.20311$.

Next, using backstepping control method, we derive an adaptive control law that stabilizes the novel chaotic system, when the system parameters are unknown. Using backstepping control method, we also derive an adaptive control law that achieves global chaos synchronization of the identical novel jerk chaotic systems with unknown parameters. The backstepping control method is a recursive procedure that links the choice of a Lyapunov function with the design of a controller and guarantees global asymptotic stability of strict feedback systems.

Next, this paper derives an adaptive control law that stabilizes the novel jerk chaotic system with unknown system parameters. This paper also derives an adaptive control law that achieves global chaos synchronization of identical jerk chaotic systems with unknown parameters.

This paper is organized as follows. In Section 2, we describe the novel jerk chaotic system with two quadratic nonlinearities. In Section 3, we describe the qualitative properties of the novel jerk chaotic system. In Section 4, we detail the adaptive backstepping control design for the global chaos stabilization of the novel jerk chaotic system with unknown parameters. In Section 5, we detail the adaptive backstepping control design for the global and exponential synchronization of the identical novel jerk chaotic systems. In Section 6, we give a summary of the main results obtained in this research work.

2. A NOVEL 3-D JERK CHAOTIC SYSTEM

In this section, we describe a seven-term novel jerk chaotic system with two quadratic nonlinearities, which is modeled by the 3-D dynamics

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -ax_3 - bx_2 + cx_2^2 - x_1(1+x_1) \end{cases} \tag{2}$$

where x_1, x_2, x_3 are state variables and a, b, c are constant, positive, parameters of the system.

The novel jerk system (2) exhibits a *strange chaotic attractor* for the values

$$a = 0.44, \quad b = 1.1, \quad c = 0.06 \tag{3}$$

For numerical simulations, we take the initial conditions of the state as

$$x_1(0) = 0.2, \quad x_2(0) = 0.2, \quad x_3(0) = 0.2 \tag{4}$$

The Lyapunov exponents of the jerk chaotic system (2) for the parameter values (3) and the initial conditions (4) are numerically calculated as

$$L_1 = 0.11245, \quad L_2 = 0, \quad L_3 = -0.55363 \tag{5}$$

Figure 1 shows the 3-D phase portrait of the jerk chaotic system (2). Figures 2-4 show the 2-D projection of the jerk chaotic system (2) on the (x_1, x_2), (x_2, x_3), and (x_1, x_3) planes, respectively.

3. PROPERTIES OF THE NOVEL JERK CHAOTIC SYSTEM

In this section, we shall discuss the qualitative properties of the jerk chaotic system (2) introduced in Section 2. We suppose that the parameter values of the jerk system (2) are as in the chaotic case (3), i.e. a = 0.44, b = 1.1 and c = 0.06.

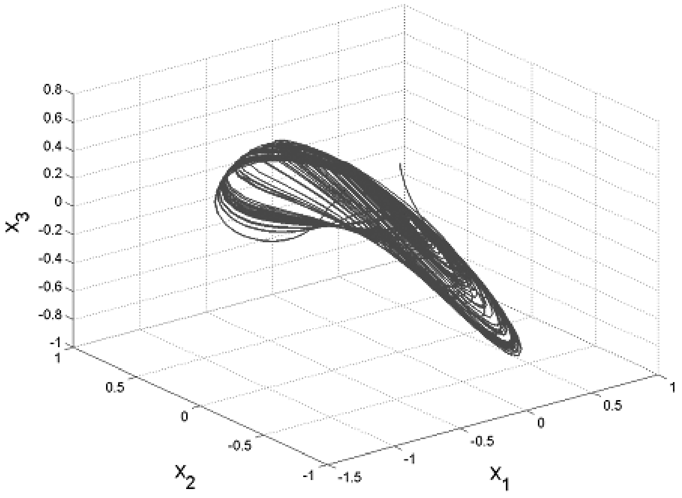

Figure 1: Phase portrait of the jerk chaotic system

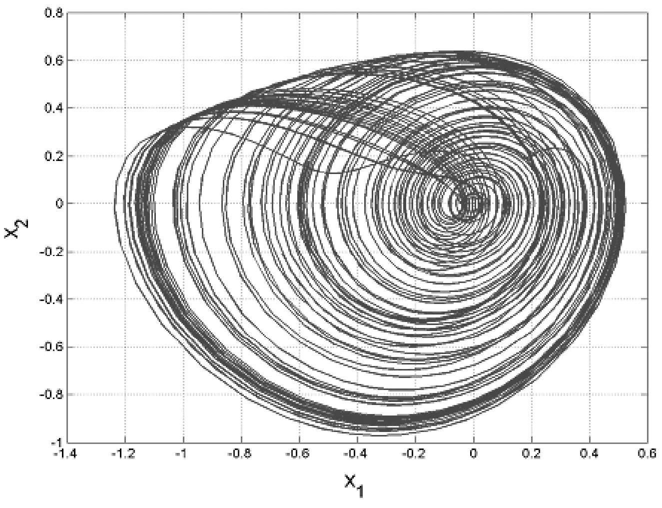

Figure 2: 2-D projection of the jerk chaotic system on the (x_1, x_2) plane

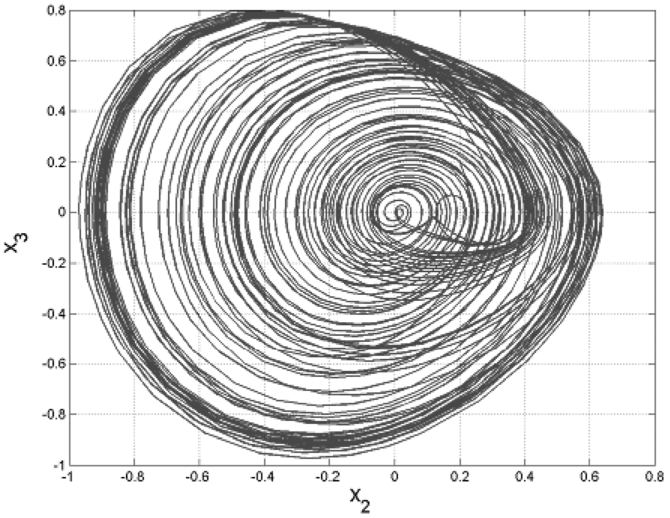

Figure 3: 2-D projection of the jerk chaotic system on the (x_2, x_3) plane

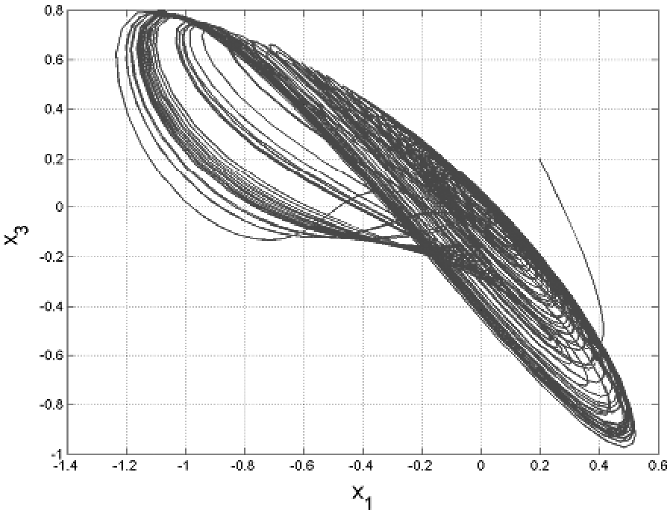

Figure 4: 2-D projection of the jerk chaotic system on the (x_1, x_3) plane

3.1. Dissipativity of the Flow

In vector notation, we may express the system (2) as

$$\dot{x} = f(x) = \begin{bmatrix} f_1(x_1, x_2, x_3) \\ f_2(x_1, x_2, x_3) \\ f_3(x_1, x_2, x_3) \end{bmatrix} \tag{6}$$

where

$$\begin{cases} f_1(x_1, x_2, x_3) = x_2 \\ f_2(x_1, x_2, x_3) = x_3 \\ f_3(x_1, x_2, x_3) = -ax_3 - bx_2 + cx_2^2 - x_1(1 + x_1) \end{cases} \tag{7}$$

Let Ω be any region in R^3 with a smooth boundary and also $\Omega(t) = \Phi_t(\Omega)$, where Φ_t is the flow of the vector field f. Furthermore, let $V(t)$ denote the volume of $\Omega(t)$.

By Liouville's theorem, we have

$$\dot{V} = \int\limits_{\Omega(t)} \left(\nabla \cdot f\right) dx_1\ dx_2\ dx_3 \tag{8}$$

The divergence of the novel chaotic system (2) is easily found as

$$\nabla \cdot f = \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} + \frac{\partial f_3}{\partial x_3} = 0 + 0 - a = -a \tag{9}$$

Substituting (9) into (8), we obtain the first order ODE

$$\dot{V} = \int\limits_{\Omega(t)} \left(-a\right) dx_1\ dx_2\ dx_3 = -aV \tag{10}$$

Integrating (10), we obtain the unique solution as

$$V(t) = \exp(-at)\ V(0) \text{ for all } t \geq 0 \tag{11}$$

Since $a > 0$, we conclude from Eq. (11) that $V(t) \to 0$ exponentially as $(t) \to \infty$.

This shows that the novel 3-D jerk chaotic system (2) is dissipative. Hence, the system limit sets are ultimately confined into a specific limit set of zero volume, and the asymptotic motion of the novel jerk chaotic system (2) settles onto a strange attractor of the system.

3.2. Equilibrium Points

We take the values of the parameters as in the chaotic case (3), *i.e.* $a = 0.44$, $b = 1.1$ and $c = 0.06$.

The equilibrium points of the jerk system (2) are obtained by solving the system of equations

$$x_2 = 0 \tag{12a}$$

$$x_3 = 0 \tag{12b}$$

$$-ax_3 - bx_2 + cx_2^2 - x_1(1 + x_1) = 0 \tag{12c}$$

Solving the system (12), we obtain two equilibrium points given by

$$E_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad E_1 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \tag{13}$$

The Jacobian of the jerk chaotic system (2) at any point $x \in R^3$ is given by

$$J(x) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1-2x_1 & -1.1+0.12x_2 & -0.44 \end{bmatrix} \tag{14}$$

The Jacobian of the jerk chaotic system (2) at E_0 is obtained as

$$J_0 = J(E_0) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1.1 & -0.44 \end{bmatrix} \tag{15}$$

The eigenvalues of J_0 are numerically obtained as

$$\lambda_1 = -0.7503, \quad \lambda_{2,3} = 0.1551 \pm 1.1440i \tag{16}$$

This shows that E_0 is a saddle-focus, which is unstable.

Next, the Jacobian of the jerk chaotic system (2) at E_1 is obtained as

$$J_1 = J(E_1) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1.1 & -0.44 \end{bmatrix} \tag{17}$$

The eigenvalues of the matrix J_1 are numerically obtained as

$$\lambda_1 = 0.5872, \quad \lambda_{2,3} = -0.5136 \pm 1.1997i \tag{18}$$

This shows that E_1 is a saddle-focus, which is unstable.

3.4. Lyapunov Exponents and Kaplan-Yorke Dimension

We take the parameter values of the novel system (2) as in the chaotic case (3), *i.e.*

$$a = 0.44, \quad b = 1.1, \quad c = 0.06 \tag{19}$$

We choose the initial values of the novel system (2) as

$$x_1(0) = 0.2, \quad x_2(0) = 0.2, \quad x_3(0) = 0.2 \tag{20}$$

Then we obtain the Lyapunov exponents of the system (2) as

$$L_1 = 0.11245, \quad L_2 = 0, \quad L_3 = -0.55363 \tag{21}$$

Figure 5 shows the Lyapunov exponents of the system (2) as determined by MATLAB.

We note that the sum of the Lyapunov exponents of the system (2) is negative, which shows that the novel jerk system (2) is dissipative.

Also, the Maximal Lyapunov Exponent (MLE) of the jerk chaotic system (2) is $L_1 = 0.11245$.

The Kaplan-Yorke dimension of the jerk chaotic system (2) is derived as

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.20311 \tag{22}$$

4. ADAPTIVE BACKSTEPPING CONTROL DESIGN FOR THE STABILIZATION OF THE NOVEL JERK CHAOTIC SYSTEM

In this section, we consider the novel jerk system with a single control given by

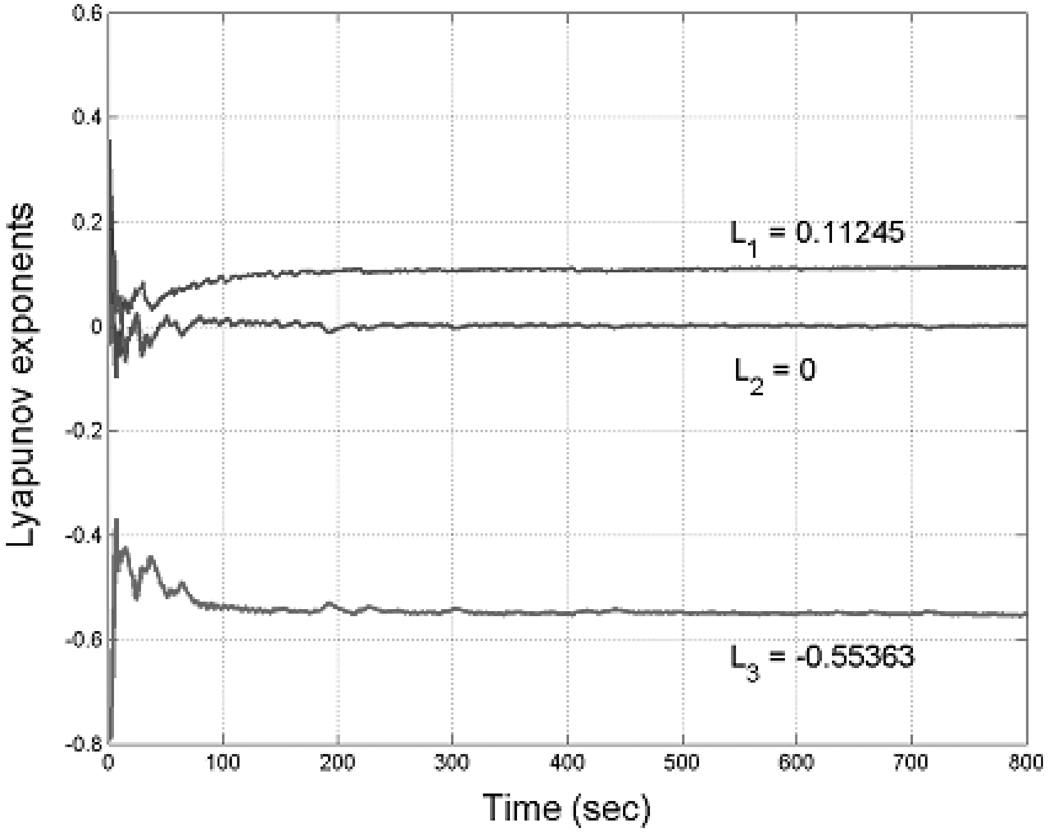

Figure 5: Lyapunov exponents of the jerk chaotic system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -ax_3 - bx_2 + cx_2^2 - x_1(1 + x_1) + u \end{cases} \tag{23}$$

In (23), x_1, x_2, x_3 are the states, a, b, c are unknown constant parameters, and u is a backstepping control law to be determined using estimates $\hat{a}(t)$, $\hat{b}(t)$, $\hat{c}(t)$ of the unknown parameters a, b, c, respectively.

The parameter estimation errors are defined as follows:

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_b(t) = b - \hat{b}(t) \\ e_c(t) = c - \hat{c}(t) \end{cases} \tag{24}$$

Differentiating (24) with respect to t, we obtain

$$\begin{cases} \dot{e}_a(t) = -\dot{\hat{a}}(t) \\ \dot{e}_b(t) = -\dot{\hat{b}}(t) \\ \dot{e}_c(t) = -\dot{\hat{c}}(t) \end{cases} \tag{25}$$

Next, we shall state and prove the main result of this section.

Theorem 1. The 3-D novel jerk chaotic system (23) with unknown parameters is globally and exponentially stabilized by the adaptive feedback control law

$$u = -2x_1 - \left[5 - \hat{b}(t)\right] x_2 - \left[3 - \hat{a}(t)\right] x_3 + x_1^2 - \hat{c}(t) x_2^2 - kz_3 \tag{26}$$

where $k > 0$ is a gain constant, with

$$z_3 = 2x_1 + 2x_2 + x_3 \tag{27}$$

and the parameter update law is given by

$$\begin{cases} \dot{\hat{a}} = -x_3 z_3 \\ \dot{\hat{b}} = -x_2 z_3 \\ \dot{\hat{c}} = x_2^2 z_3 \end{cases} \tag{28}$$

Proof. We prove this result via backstepping control method and Lyapunov stability theory [174].

First, we define a quadratic Lyapunov function

$$V_1(z_1) = \frac{1}{2} z_1^2 \tag{29}$$

where

$$z_1 = x_1 \tag{30}$$

Differentiating V_1 along the dynamics (23), we obtain

$$\dot{V}_1 = x_1 x_2 = -z_1^2 + z_1(x_1 + x_2) \tag{31}$$

Now we define

$$z_2 = x_1 + x_2 \tag{32}$$

Using (32), we can simplify (31) as

$$\dot{V}_1 = -z_1^2 + z_1 z_2 \tag{33}$$

Next, we define a quadratic Lyapunov function

$$V_2(z_1, z_2) = V_1(z_1) + \frac{1}{2} z_2^2 = \frac{1}{2}\left(z_1^2 + z_2^2\right) \tag{34}$$

Differentiating V_2 along the dynamics (23), we obtain

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2(2x_1 + 2x_2 + x_3) \tag{35}$$

Now, we define

$$z_3 = 2x_1 + 2x_2 + x_3 \tag{36}$$

Using (36), we can simplify (35) as

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2 z_3 \tag{37}$$

Finally, we define a quadratic Lyapunov function

$$V(\mathbf{z}, e_a, e_b, e_c) = V_2(z_1, z_2) + \frac{1}{2} z_3^2 + \frac{1}{2}(e_a^2 + e_b^2 + e_c^2) \tag{38}$$

From (38), it is clear that V is a positive definite function on R^6.

Differentiating V along the dynamics (23) and (28), we obtain

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 + z_3 S - e_a \dot{\hat{a}} - e_b \dot{\hat{b}} - e_c \dot{\hat{c}} \tag{39}$$

In (39), S is given by

$$S = z_3 + z_2 + \dot{z}_2 = z_3 + z_2 + 2\dot{x}_1 + 2\dot{x}_2 + \dot{x}_3 \tag{40}$$

Simplifying the equation (40), we obtain

$$S = 2x_1 + (5-b)x_2 + (3-a)x_3 - x_1^2 + cx_2^2 + u \tag{41}$$

Substituting the control law (26) into (41), we get

$$S = -[b - \hat{b}(t)]x_2 - [a - \hat{a}(t)]x_3 + [c - \hat{c}(t)]x_2^2 - kz_3 \tag{42}$$

Using the definitions in (24), we can simplify the equation (42) as

$$S = -e_b x_2 - e_a x_3 + e_c x_2^2 - kz_3 \tag{43}$$

Substituting (43) into (39), we obtain

$$\dot{V} = -z_1^2 - z_2^2 - (1+k)z_3^2 + e_a\left[-x_3 z_3 - \dot{\hat{a}}\right] + e_b\left[-x_2 z_3 - \dot{\hat{b}}\right] + e_c\left[x_2^2 z_3 - \dot{\hat{c}}\right] \tag{44}$$

Substituting the parameter update law (28) into (44), we obtain

$$\dot{V} = -z_1^2 - z_2^2 - (1+k)z_3^2 \tag{45}$$

Thus, it is clear that $\dot{V}$ is a negative semi-definite function on R^6.

From (45), it is clear that the vector $z(t) = (z_1(t), z_2(t), z_3(t))$ and the parameter estimation error $(e_a(t), e_b(t), e_c(t))$ are globally bounded, *i.e.*

$$\begin{bmatrix} z_1(t) & z_2(t) & z_3(t) & e_a(t) & e_b(t) & e_c(t) \end{bmatrix}^T \in \boldsymbol{L}_\infty \tag{46}$$

Also, it follows from (45) that

$$\dot{V} \le -z_1^2 - z_2^2 - z_3^2 = -\|\boldsymbol{z}\|^2 \tag{47}$$

or

$$\|\boldsymbol{z}(t)\|^2 \le -\dot{V}(t) \tag{48}$$

Integrating the inequality (48) from 0 to t, we get

$$\int_0^t \|z(\tau)\|^2 d\tau \le V(0) - V(t) \tag{49}$$

From (49), it follows that $z(t) \in L_2$.

From (23), it can be deduced that $z(t) \in L_\infty$.

Thus, using Barbalat's lemma [174], we can conclude that $z(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $z(0) \in R^3$.

Hence, it is immediate that $x(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $x(0) \in R^3$.

This completes the proof. ■

For numerical simulations, the classical fourth-order Runge-Kutta method with step size $h = 10^{-8}$ is used to solve the system of differential equations (23) and (28), when the adaptive controller (26) is implemented.

The parameter values of the novel 3-D jerk chaotic system (23) are taken as in the chaotic case, *i.e.*

$$a = 0.44, \quad b = 1.1, \quad c = 0.06 \tag{50}$$

The positive gain constant k is taken as $k = 10$.

The initial conditions of the novel jerk system (23) are taken as

$$x_1(0) = 15.4, \quad x_2(0) = 7.3, \quad x_3(0) = -6.8 \tag{51}$$

The initial conditions of the parameter estimates are taken as

$$\hat{a}(0) = 6.4, \quad \hat{b}(0) = 15.7, \quad \hat{c}(0) = 8.2 \tag{52}$$

Figure 6 shows the time-history of the controlled states $x_1(t)$, $x_2(t)$, $x_3(t)$.

5. ADAPTIVE BACKSTEPPING CONTROL DESIGN FOR THE GLOBAL CHAOS SYNCHRONIZATION OF THE IDENTICAL NOVEL JERK CHAOTIC SYSTEMS

In this section, we use backstepping control method to derive an adaptive feedback control law for globally synchronizing identical 3-D novel jerk chaotic systems with unknown parameters.

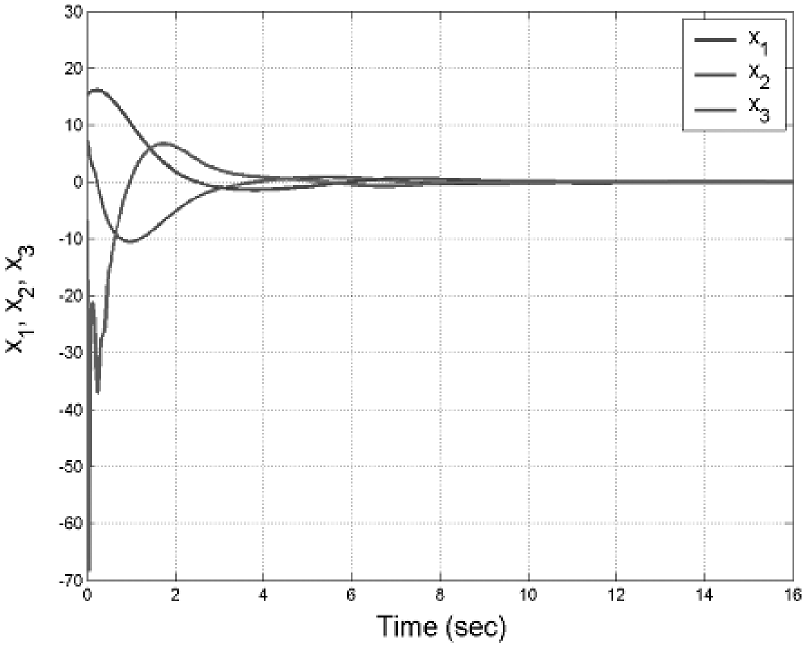

Figure 6: Time-history of the controlled state trajectories $x_1(t)$, $x_2(t)$, $x_3(t)$

As the master system, we consider the novel jerk system given by

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -ax_3 - bx_2 + cx_2^2 - x_1(1 + x_1) \end{cases} \tag{53}$$

In (53), x_1, x_2, x_3 are the states and a, b, c are unknown system parameters.

As the slave system, we consider the controlled novel jerk system given by

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = y_3 \\ \dot{y}_3 = -ay_3 - by_2 + cy_2^2 - y_1(1 + y_1) + u \end{cases} \tag{54}$$

In (54), y_1, y_2, y_3 are the states and u is the adaptive control to be determined using estimates of the unknown system parameters.

The complete synchronization error between the systems (53) and (54) is defined by

$$\begin{cases} e_1 = y_1 - x_1 \\ e_2 = y_2 - x_2 \\ e_3 = y_3 - x_3 \end{cases} \tag{55}$$

Then the synchronization error dynamics is obtained as

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = e_3 \\ \dot{e}_3 = -e_1 - be_2 - ae_3 + c(y_2^2 - x_2^2) - y_1^2 + x_1^2 + u \end{cases} \tag{56}$$

The parameter estimation errors are defined as follows:

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_b(t) = b - \hat{b}(t) \\ e_c(t) = c - \hat{c}(t) \end{cases} \tag{57}$$

Differentiating (57) with respect to *t*, we obtain

$$\begin{cases} \dot{e}_a(t) = -\dot{\hat{a}}(t) \\ \dot{e}_b(t) = -\dot{\hat{b}}(t) \\ \dot{e}_c(t) = -\dot{\hat{c}}(t) \end{cases} \tag{58}$$

Next, we shall state and prove the main result of this section.

Theorem 2. The 3-D novel jerk chaotic systems (53) and (54) with unknown parameters is globally and exponentially synchronized by the adaptive feedback control law

$$u = -2e_1 - \left[5 - \hat{b}(t)\right]e_2 - \left[3 - \hat{a}(t)\right]e_3 + y_1^2 - x_1^2 - \hat{c}(t)(y_2^2 - x_2^2) - kz_3 \tag{59}$$

where $k > 0$ is a gain constant, with

$$z_3 = 2e_1 + 2e_2 + e_3 \tag{60}$$

and the parameter update law is given by

$$\begin{cases} \dot{\hat{a}} = -e_3 z_3 \\ \dot{\hat{b}} = -e_2 z_3 \\ \dot{\hat{c}} = (y_2^2 - x_2^2) z_3 \end{cases} \tag{61}$$

Proof. We prove this result via backstepping control method and Lyapunov stability theory [174].

First, we define a quadratic Lyapunov function

$$V_1(z_1) = \frac{1}{2} z_1^2 \tag{62}$$

where

$$z_1 = e_1 \tag{63}$$

Differentiating V_1 along the dynamics (56), we obtain

$$\dot{V}_1 = e_1 e_2 = -z_1^2 + z_1(e_1 + e_2) \tag{64}$$

Now we define

$$z_2 = e_1 + e_2 \tag{65}$$

Using (65), we can simplify (64) as

$$\dot{V}_1 = -z_1^2 + z_1 z_2 \tag{66}$$

Next, we define a quadratic Lyapunov function

$$V_2(z_1, z_2) = V_1(z_1) + \frac{1}{2} z_2^2 = \frac{1}{2}\left(z_1^2 + z_2^2\right) \tag{67}$$

Differentiating V_2 along the dynamics (56), we obtain

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2(2e_1 + 2e_2 + e_3) \tag{68}$$

Now, we define

$$z_3 = 2x_1 + 2x_2 + x_3 \tag{69}$$

Using (69), we can simplify (68) as

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2 z_3 \tag{70}$$

Finally, we define a quadratic Lyapunov function

$$V(\mathbf{z}, e_a, e_b, e_c) = V_2(z_1, z_2) + \frac{1}{2} z_3^2 + \frac{1}{2}(e_a^2 + e_b^2 + e_c^2) \tag{71}$$

From (71), it is clear that V is a positive definite function on R^6.

Differentiating V along the dynamics (56) and (58), we obtain

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 + z_3 S - e_a \dot{\hat{a}} - e_b \dot{\hat{b}} - e_c \dot{\hat{c}} \tag{72}$$

where

$$S = z_3 + z_2 + \dot{z}_2 = z_3 + z_2 + 2\dot{e}_1 + 2\dot{e}_2 + \dot{e}_3 \tag{73}$$

Simplifying the equation (73), we obtain

$$S = 2e_1 + (5-b)e_2 + (3-a)e_3 - y_1^2 + x_1^2 + c(y_2^2 - x_2^2) + u \tag{74}$$

Substituting the control law (59) into (74), we get

$$S = -[b - \hat{b}(t)]e_2 - [a - \hat{a}(t)]e_3 + [c - \hat{c}(t)](y_2^2 - x_2^2) - kz_3 \tag{75}$$

Using the definitions in (57), we can simplify the equation (75) as

$$S = -e_b e_2 - e_a e_3 + e_c(y_2^2 - x_2^2) - kz_3 \tag{76}$$

Substituting (76) into (72), we obtain

$$\dot{V} = -z_1^2 - z_2^2 - (1+k)z_3^2 + e_a\left[-e_3 z_3 - \dot{\hat{a}}\right] + e_b\left[-e_2 z_3 - \dot{\hat{b}}\right] + e_c\left[\left(y_2^2 - x_2^2\right) z_3 - \dot{\hat{c}}\right] \tag{77}$$

Substituting the parameter update law (61) into (77), we obtain

$$\dot{V} = -z_1^2 - z_2^2 - (1+k)z_3^2 \tag{78}$$

Thus, it is clear that $\dot{V}$ is a negative semi-definite function on R^6.

From (78), it is clear that the vector $z(t) = (z_1(t), z_2(t), z_3(t))$ and the parameter estimation error $(e_a(t), e_b(t), e_c(t))$, are globally bounded, *i.e.*

$$\begin{bmatrix} z_1(t) & z_2(t) & z_3(t) & e_a(t) & e_b(t) & e_c(t) \end{bmatrix}^T \in \boldsymbol{L}_\infty \tag{79}$$

Also, it follows from (78) that

$$\dot{V} \le -z_1^2 - z_2^2 - z_3^2 = -\|\boldsymbol{z}\|^2 \tag{80}$$

or

$$\|\boldsymbol{z(t)}\|^2 \le -\dot{V}(t) \tag{81}$$

Integrating the inequality (81) from 0 to t, we get

$$\int_0^t \|z(\tau)\|^2 d\tau \le V(0) - V(t) \tag{82}$$

From (82), it follows that $z(t) \in L_2$.

From (56), it can be deduced that $z(t) \in L_\infty$.

Thus, using Barbalat's lemma [174], we can conclude that $z(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $z(0) \in R^3$.

Hence, it is immediate that $e(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $e(0) \in R^3$.

This completes the proof. ■

For numerical simulations, the classical fourth-order Runge-Kutta method with step-size $h = 10^{-8}$ is used to solve the systems (53), (54) and (61), when the adaptive control law (59) is applied.

We take the parameter values of the jerk systems (53) and (54) as in the chaotic case, *i.e.* $a = 0.44$, $b = 1.1$ and $c = 0.06$. We take the positive gain constant as $k = 12$.

As initial conditions of the master system (53), we take

$$x_1(0) = -1.2, \quad x_2(0) = 0.2, \quad x_3(0) = 0.6 \tag{83}$$

As initial conditions of the slave system (54), we take

$$y_1(0) = 1.1, \quad y_2(0) = -1.2, \quad y_3(0) = 1.3 \tag{84}$$

As initial conditions of the parameter estimates, we take

$$\hat{a}(0) = 2.5, \quad \hat{b}(0) = 4.2, \quad \hat{c}(0) = 3.4 \tag{85}$$

Figures 7-9 depict the synchronization of the novel jerk chaotic systems (53) and (54).

Figure 10 depicts the time-history of the complete synchronization errors e_1, e_2, e_3.

6. CONCLUSIONS

In this paper, we derived new results for a seven-term novel 3-D jerk chaotic system with two quadratic nonlinearities. The phase portraits of the novel jerk chaotic system are displayed and the mathematical properties are discussed. The proposed novel jerk chaotic system has two equilibrium points along the axis, which are both saddle-foci and unstable. The Lyapunov exponents of the novel jerk chaotic system are obtained as and Thus, the Maximal Lyapunov Exponent (MLE) of the novel jerk chaotic system is derived as Since the sum of the Lyapunov exponents of the novel jerk chaotic system is dissipative, the novel jerk chaotic system is dissipative. Also, the Kaplan-Yorke dimension of the novel jerk chaotic

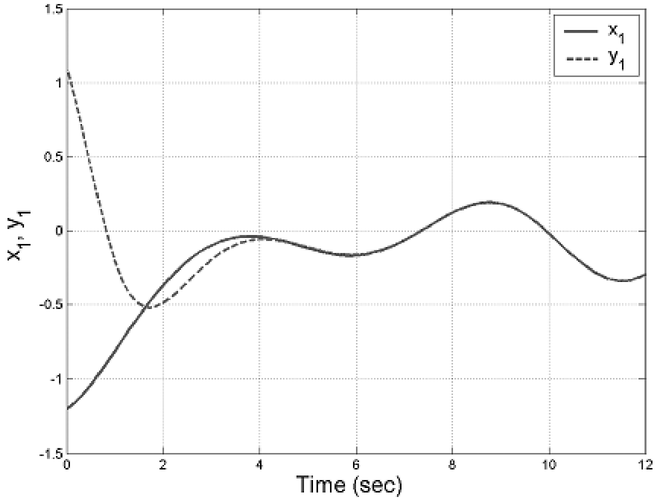

Figure 7: Synchronization of the states x_1 and y_1

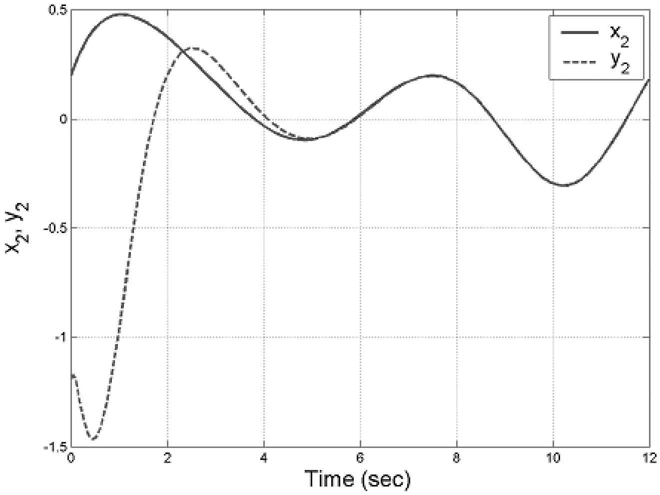

Figure 8:. Synchronization of the states x_2 and y_2

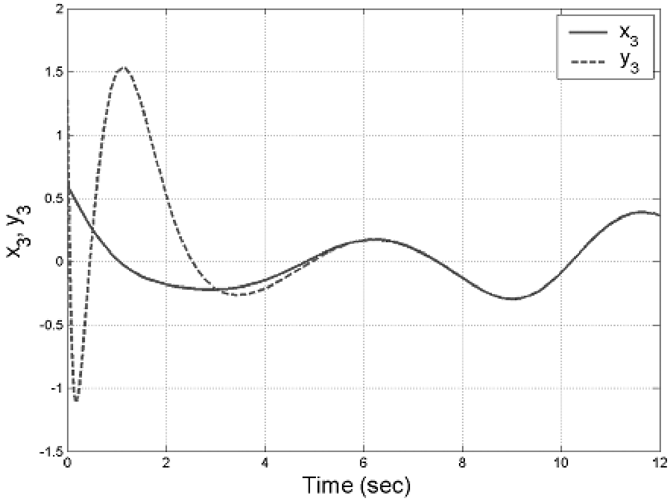

Figure 9: Synchronization of the states x_3 and y_3

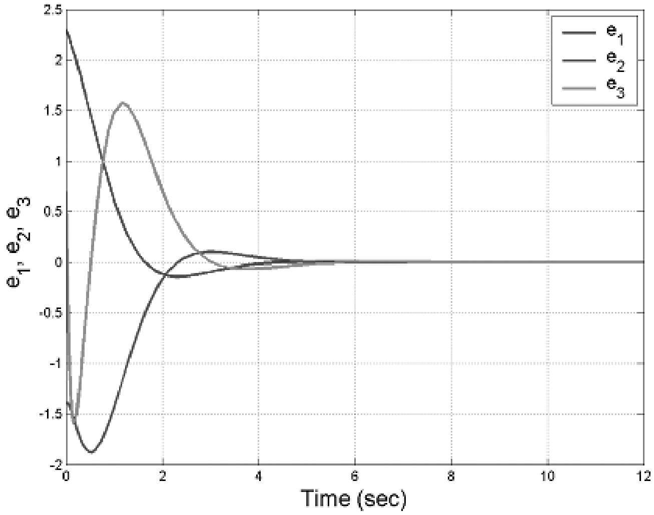

Figure 10: Time-history of the synchronization errors e_1, e_2, e_3

system is derived as Next, an adaptive controller is designed via backstepping control method to globally stabilize the novel jerk chaotic system with unknown parameters. Moreover, an adaptive controller is also designed via backstepping control method to achieve global and exponential synchronization of the identical novel jerk chaotic systems with unknown parameters. The main adaptive backstepping control results for stabilization and synchronization of the novel jerk chaotic system are established using Lyapunov stability theory.

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