

# Tuning of Extended Kalman Filter for Power Systems using Two Lbest Particle Swarm Optimization

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**Abstract:** State estimation of power systems is essential for continuous monitoring and control of power systems. Since the Power system is nonlinear, Extended Kalman Filter (EKF) is useful for the estimation of dynamic states. One of the major challenges in the design of the EKF is tuning of covariance matrices. Tuning of EKF by trial and error method requires huge computation time and intense decision making. The objective of this paper is to design an Extended Kalman filter for a synchronous machine connected to an infinite bus and enhance the filter performance by tuning it using two lbest particle swarm optimization technique. The above designed filter is tested for various operating conditions and the simulation results show the superior performance of the filter in terms of measurement and process noise rejection and accuracy.

**Keywords:** Extended Kalman Filter, PSO, Filter Tuning, Power systems, Dynamic state estimation.

## 1. INTRODUCTION

For reliable operation of a power system, there is a need for continuous monitoring, analyzing and controlling the power system dynamics. After the occurrence of major blackouts in power systems throughout the world, the focus is shifted towards the development of wide-area measurements using Phasor Measurement Unit PMUs [1]. The measurements of PMUs can be utilized for state estimation of power systems because the data provided by PMUs are more accurate with high sampling rate when compared to SCADA. Improving the state estimation process leads to better utilization of existing power, automization of generation process, optimal power flow etc. [2].

Kalman filter is one of the greatest achievements in the history of estimation. It is very useful in the dynamic state estimation of linear systems. Schmidt extended the work of Kalman to non-linear systems and it is Extended Kalman filter (EKF) [3]. EKF is superior to many other filters in terms of less computation time, measurement and process noise rejection, high accuracy [4] etc. Because of these features, EKF has been used in several applications like electrical drives, freeway traffic state estimation, inertial and magnetic sensing, communications systems, etc [5], [6], [7], [8], [9]. It comes under the category of sub-optimal filter.

EKF is quite helpful in the dynamic state estimation of power systems [10]. States of multi machine are estimated using EKF in [11]. An Unscented Kalman Filter (UKF) for decentralized state estimation is employed for power systems [12]. A new model was proposed in [13] for estimation of electromechanical oscillations using an EKF. The major challenge in the implementation of EKF is tuning of covariance matrices (Q and R).

The performance of the filter is greatly influenced by the selection of Q and R matrices. Basically synchronous generator is considered to be a seventh order dynamic system. For simplicity fourth order dynamic model is considered in the design of EKF filter [10]. Along with unmodeled nonlinearities, further

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simplifying the model introduces huge errors in filter output during and after transient conditions. Tuning of covariance matrices is necessary to correct the errors related to model mismatch. Tuning can be done by trial and error method, but it requires huge computation time and requires intense decision making. This can be done only with expertise.

In this paper the matrices Q and R are tuned by two lbest PSO algorithm [15]. Two objective functions are formulated based on integral square error between actual values and their estimates. The performance of the filter is evaluated under various operating conditions and compared the estimates of the filter with the actual states.

The paper is organized as follows: section-2 discusses the dynamic model of power systems. Section-3 discusses about extended Kalman filter implementation. Section-4 discusses about Q and R tuning using trial & error method and PSO optimization technique. Section-5 analyses the simulation results.

## 2. DYNAMIC MODEL OF SINGLE MACHINE CONNECTED TO INFINITE BUS

Figure (1) shows a synchronous generator connected to infinite bus via a transformer and parallel transmission lines. It is assumed that the Phasor Measurement Unit (PMU) is placed at generating substation. The measured PMU data is given to the filter. Simplified fourth order dynamic model of single machine connected to infinite bus is given below [10].

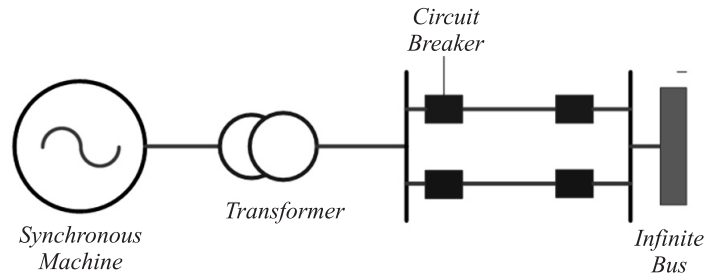


Figure 1: Synchronous generator connected to infinite bus

$$\dot{\delta} = \omega \quad (1)$$

$$\dot{\omega} = -\frac{D}{J} \omega - \frac{\omega_0}{J} (P_e - P_m) \quad (2)$$

$$\dot{E}'_q = \frac{1}{T'_{do}} (E_f - E'_q - (x_d - x'_d) i_d) \quad (3)$$

$$\dot{E}'_d = \frac{1}{T'_{qo}} (-E'_d - (x_q - x'_q) i_q) \quad (4)$$

where,  $\delta$  is power angle in rad;  $\omega$  is relative speed in rad/s;  $E'_q$  is quadratic axis transient voltage in pu;  $E'_d$  is direct axis transient voltage in pu;  $P_e$  is power generated by synchronous machine in pu;  $P_m$  is mechanical power input to synchronous machine in pu;  $i_d$  is direct axis current in pu;  $i_q$  is quadratic axis current in pu;  $x_d$  is direct axis reactance in pu;  $x'_d$  is direct axis transient reactance in pu;  $x_q$  is quadratic axis reactance in pu;  $x'_q$  is quadratic axis transient reactance in pu;  $D$  is damping constant in pu;  $J$  is inertia constant in pu;  $\omega_0$  is synchronous speed in rad/s;  $E_f$  is field excitation voltage in pu;  $T'_{do}$  is  $d$ -axis open circuit time constant in pu;  $T'_{qo}$  is  $q$ -axis open circuit time constant in pu;

The direct axis and quadratic axis currents are given by

$$i_d = \frac{E'_q - V_t \cos \delta}{x'_d} \quad (5)$$

$$i_q = \frac{V_t \sin \delta}{x_q} \quad (6)$$

Where,  $V_t$  is the generator terminal voltage in pu;

Let the state vector be  $x = [x_1 \quad x_2 \quad x_3 \quad x_4]^T = [\delta \quad \omega \quad E'_q \quad E'_d]^T$

Mechanical power input of the generator, Field excitation and terminal voltage are taken as filter inputs.

Input vector is  $u = [u_1 \quad u_2 \quad u_3]^T = [P_m \quad E_f \quad V_t]^T$

The power delivered by the generator is

$$P_e = \frac{V_t}{x'_d} E'_q \sin \delta + \frac{V_t^2}{2} \left( \frac{1}{x_q} - \frac{1}{x'_d} \right) \sin 2\delta \quad (7)$$

Generated Power is considered to be the measured output of the system.

Let us consider Output vector is  $y = P_e$

Replacing the above model in generic form

$$\dot{x} = \mathbb{F}(x, u) + w \quad (8)$$

$$y = \mathbb{S}(x, u) + v \quad (9)$$

Where,  $w$  is the process noise and  $v$  is the measurement noise.

Discretize the system by considering the  $\Delta t$  as sample interval. Now the system can be written as

$$x_k = \mathbb{F}(x_{k-1}, u_{k-1}) + v_{k-1} \quad (10)$$

$$y_k = \mathbb{S}(x_k, u_k) + v_k \quad (11)$$

Here  $x_k$ ,  $u_k$  and  $y_k$  are state, input and output respectively at time  $k \Delta t$

### 3. EXTENDED KALMAN FILTER

An extended Kalman filter is an extension of Kalman filter to nonlinear systems and it is based on linearization of nonlinear system using Taylor's expansion. EKF comes under the category of sub-optimal filter. Refer [3] for more details on EKF.

*Extended Kalman Filter Algorithm:*

Initialize state vector and state covariance matrix  $z_0$  &  $P_0$

Predict state vector and state covariance

$$z_k^- = \mathbb{F}_k(z_{k-1}) \quad P_k^+ = F_{k-1} P_{k-1}^+ F_{k-1}^T + Q \quad (12)$$

Update covariance matrix

$$P_k^+ = [I - K_k H_k] P_k^- \quad (13)$$

Kalman gain calculation & state vector updation:

$$K_k = P_k^- H_k^T [H_k P_k^- + R]^{-1} \quad (14)$$

$$z_k^+ = z_k^- + K_k [y_k - \mathbb{S}_k(z_k^-)] \quad (15)$$

Where,

$$F_{k-1} = \frac{\partial \mathbb{F}_{k-1}}{\partial x} \quad \text{and} \quad H_k = \frac{\partial \mathbb{S}_k}{\partial x}$$

### 4. TUNING OF EKF

In this section, we present trail & error and PSO methods to tune Q and R matrices for EKF.

#### 4.1. Trail & Error Tuning of EKF

The performance of the filter is greatly affected by the covariance matrices. Therefore a proper selection of covariance matrix is required for better performance of the filter. The covariance matrices are normally tuned by trial and error method. After exhaustive iteration process, Q is chosen as  $\text{diag}([0.042 \ 0.042 \ 0.042 \ 0.042])$  and R value as 10. Figure (2) and Figure (3) shows the power angle estimate and relative speed

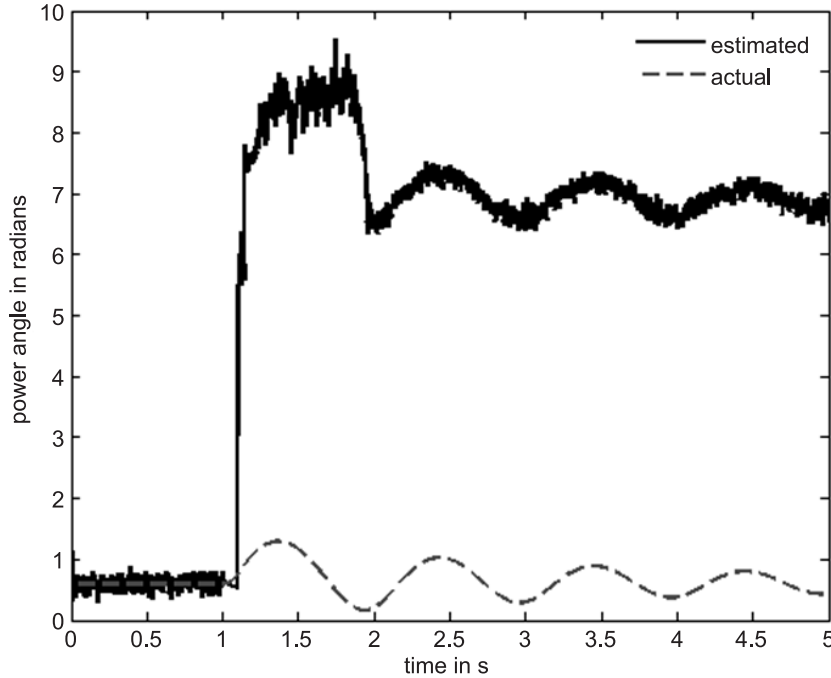


Figure 2: Actual and estimated power angle

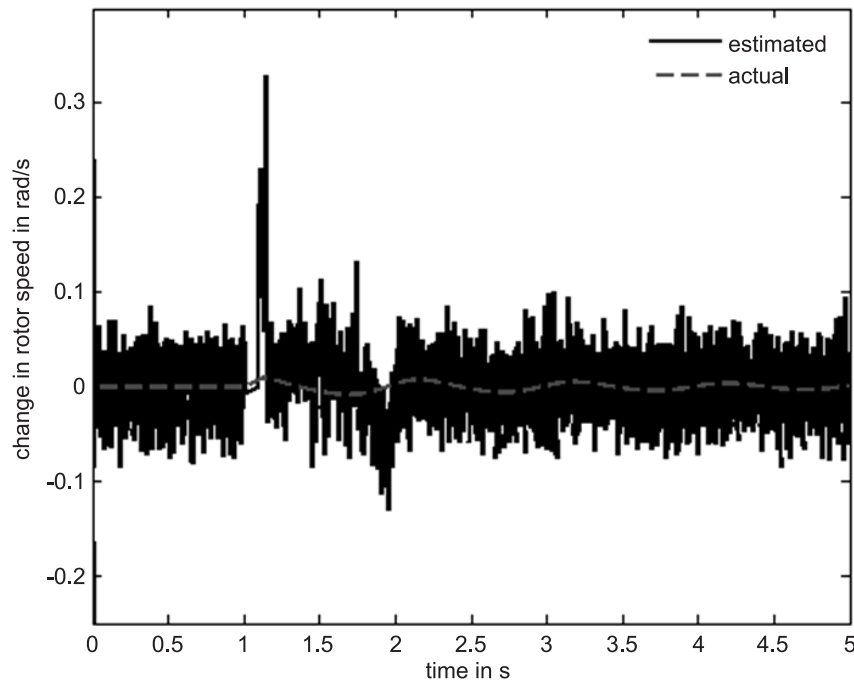


Figure 3: Actual and estimated change in rotor speed

estimate of the filter with measurement noise of standard deviation  $[0, 0.0001]$  during the fault condition. The states without fault condition are observed to be noisy and under fault conditions the performance of the system further deteriorates. Thus we can conclude that the performance of the filter is not optimum as heuristic approach is adopted in selecting the covariance matrices.

#### 4.2. Tuning of EKF using Advanced PSO Algorithm

Consider the tuning matrices advanced PSO as

Let  $Q = \text{diag}[q_1 \quad q_2 \quad q_3 \quad q_4]$  and  $R = r$

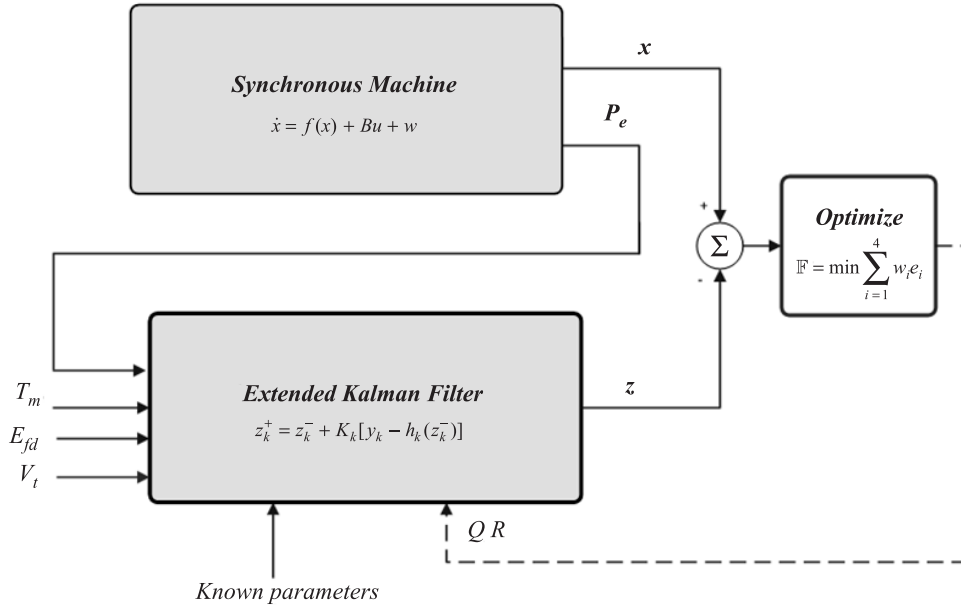


Figure 4: Proposed Method

The filter performance can be improved substantially by tuning the diagonal elements of covariance matrices  $q_1$ ,  $q_2$ ,  $q_3$ ,  $q_4$  and  $r$  using optimization techniques. Only diagonal elements are selected to minimize the number of tuning parameters and the computation time. The main objective is to select the covariance matrix elements that minimize the error between the state variables and its estimates. Integral Square Error (ISE) is used as it is very effective in reducing the steady state error, settling time and peak overshoot.

$$\text{Let } e_i = \int |x_i - z_i|^2 dt \quad (16)$$

The optimization function is given by

$$\mathfrak{M} = \min (\mathfrak{M}_1, \mathfrak{M}_2)$$

$$\mathfrak{M}_1 = \sum_{i=1}^4 w_i e_i = e_1 + 50e_2 + 0.01e_3 + 0.001e_4 \quad (17)$$

$$\mathfrak{M}_2 = 3e_1 + 200e_2 + 0.01e_3 + 0.001e_4 \quad (18)$$

where  $e_i$  is integral square error of  $i^{\text{th}}$  state variable and  $w_i$  is the corresponding weight.

$F_1$  and  $F_2$  are the two objective functions selected with different weights. The second objective function is chosen with larger weights for power angle error and speed error. Two objective functions are selected because the optimization algorithm may end up with local minimum value when single optimization

function is used. EKF is implemented using embedded MATLAB function in Simulink using Ranga-Kutta solver. An error of standard deviation [0, 0.0001] is added in the measurements and an error of standard deviation [0, 0.0000001] is added to the actual states as process-noise. The elements are tuned using two lbests Multi-Objective Particle swarm optimization technique [15]. In other PSO algorithms, searching of optimum value is directed by the positions of pbest (local best) and gbest (global best). If the positions of pbest and gbest are far away from each other, then search direction of optimum value is not very effective. To address this issue, bins are developed and two lbests are selected in the nearby bins. The detailed explanation is given in [14] and [15]. For optimization the following are initialized in two lbest PSO algorithm.

- i) Dimension  $d$  of the problem is 5 and Population size NP is 50.
- ii) Lower limit  $lu = [1 \times 10^{-8} \quad 1 \times 10^{-11} \quad 1 \times 10^{-8} \quad 1 \times 10^{-8} \quad 1 \times 10^{-4}]$
- iii) Upper limit  $up = [1 \times 10^{-3} \quad 1 \times 10^{-4} \quad 1 \times 10^{-4} \quad 1 \times 10^{-4} \quad 1 \times 10^2]$
- iv) Number of bins is 10. Maximum limit of generation is  $Gen-max$  is 40.  $c_1 = 2.05$  and  $c_2 = 2.05$

### **Two lbest PSO Steps**

*Initialization:* Initialize a random array of particles (solutions) and uniformly distributed in the search space. Evaluate the fitness function  $\mathfrak{M}$  for each particle.

*Selection of two lbests:* Selection of lbest is random.  $lbest1$  is selected by randomly selecting one of the two objectives & randomly selecting a bin and  $lbest2$  is selected from a neighboring bin of the  $lbest1$  in the parameter space.

*Velocity Updation:* Velocity and position of the particle are updated.

$$V_p = w_j(V_p + c_1 r \text{ and } ()(lbest(j) - \text{particle}(j)) + c_2 r \text{ and } ()(lbest(j + NP) - \text{particle}(j))) \quad (19)$$

*Limit the velocity for each dimension  $d$ :*

$$V_i(d) = (\min(V \max(d), V_i(d)) \& \max(-V \max(d), V_i(d))) \quad (20)$$

*Update the position of each particle:*

$$X_i(d) = X_i(d) + V_i(d) \quad (21)$$

*Stopping Criteria:* The evaluation criteria will be stopped if total number of evaluations exceeds the  $Gen-max$ .

Print the best solution.

## **5. RESULT ANALYSIS**

The tuned Q and R matrices obtained after applying Advanced PSO are

$$Q = \text{diag} [10^{-3} \quad 10^{-10} \quad 10^{-3} \quad 3.56 \times 10^{-4}] \quad R = 10$$

The performance of the tuned optimal filter is tested on a synchronous generator under various operating conditions. The data of the synchronous machine is given in appendix.

### **Case 1: Sudden Change in Mechanical Power Input**

Mechanical input power to the generator is suddenly raised from 0.8 pu to 0.9 pu at  $t = 1$  s. Figure (5) shows the filter state estimates. Even though states deviate from the actual values initially, estimates tracks

the actual states with in a very short time. Figure (6) shows the measured active power and active power derived from the filter. The filtered active power has reduced measurement noise.

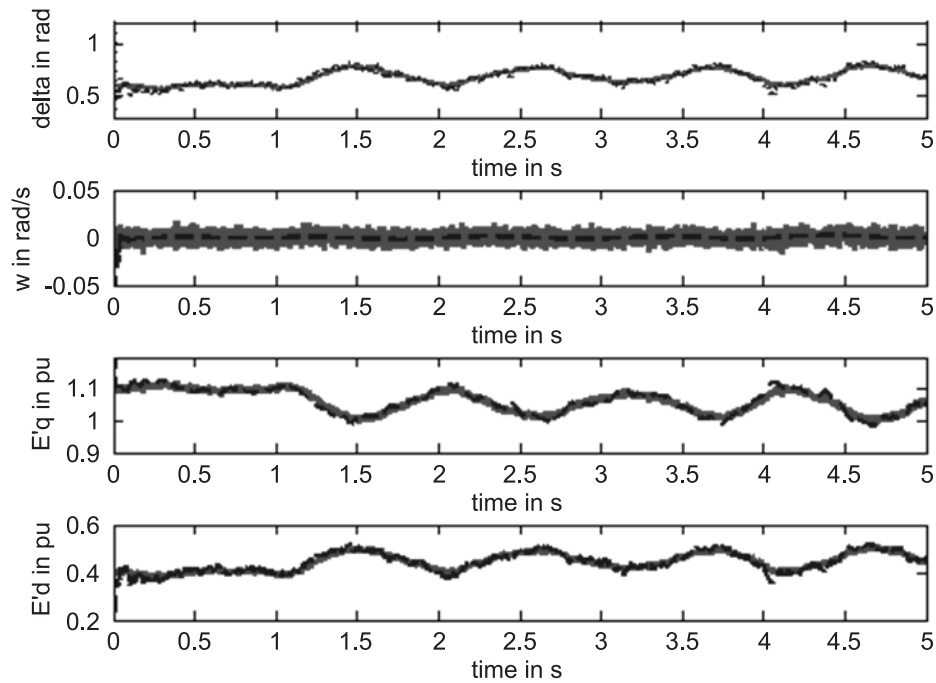


Figure 5: Actual and Estimated States. Red represents actual and blue represents estimated

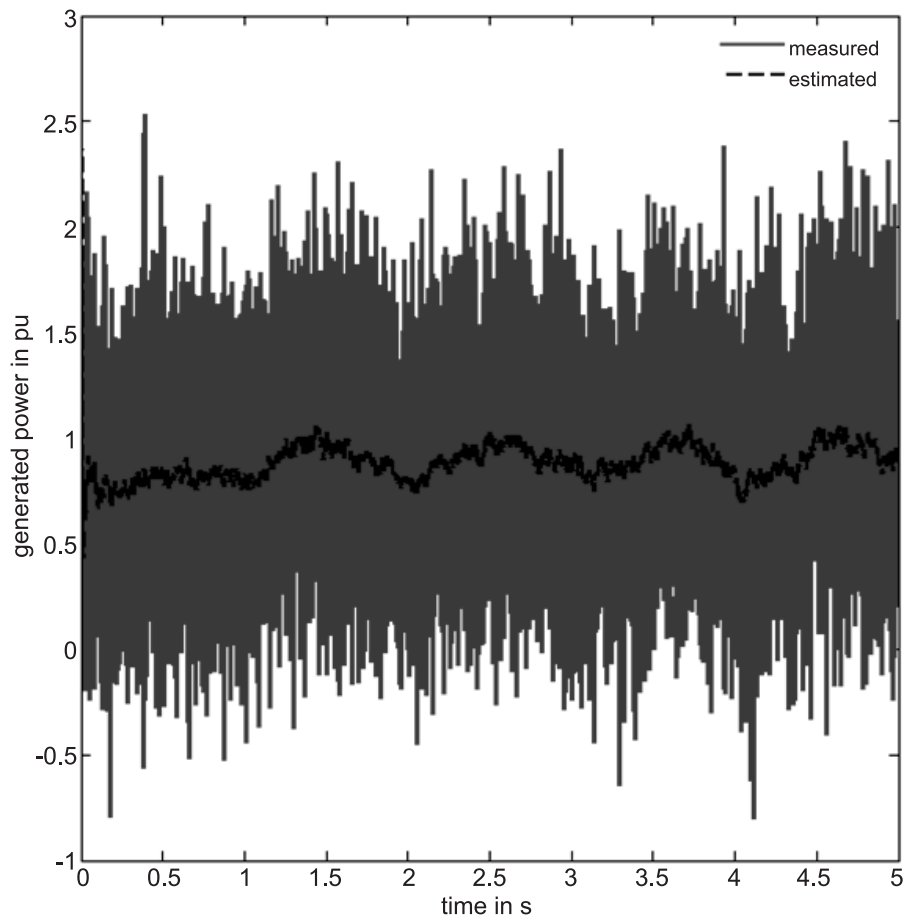


Figure 6: Measured and estimated active power

### Case 2: A Three Fault on the Transmission Line

The filter behaviour is analyzed with two different types of faults. A three phase symmetrical fault occurs at starting end of the transmission line and the fault is cleared by circuit breaker at 1.1s and system is stable even after the fault. In the second type the fault is cleared by circuit breaker at 1.2s, system is unstable after the fault. Estimated states without measurement noise and process noise are similar to estimates with measurement noise and process noise. Estimate states tracks the true states. Figure (7) shows the filter estimates with measurement noise and process noise. The EKF filters out noise in the states and the estimated states tracks the true value. But with Q and R obtained by trial and error, the filter estimates deviates from the true value. Figure (8) shows the measured active power and active power derived from the filter.

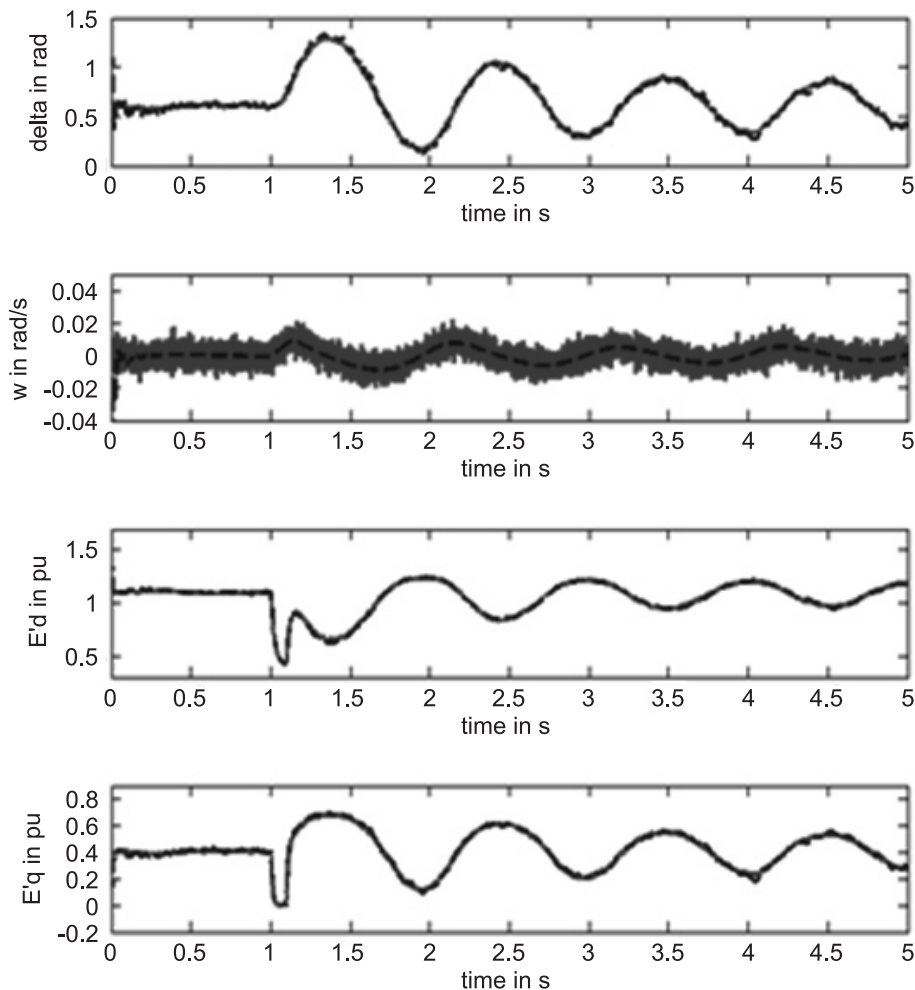
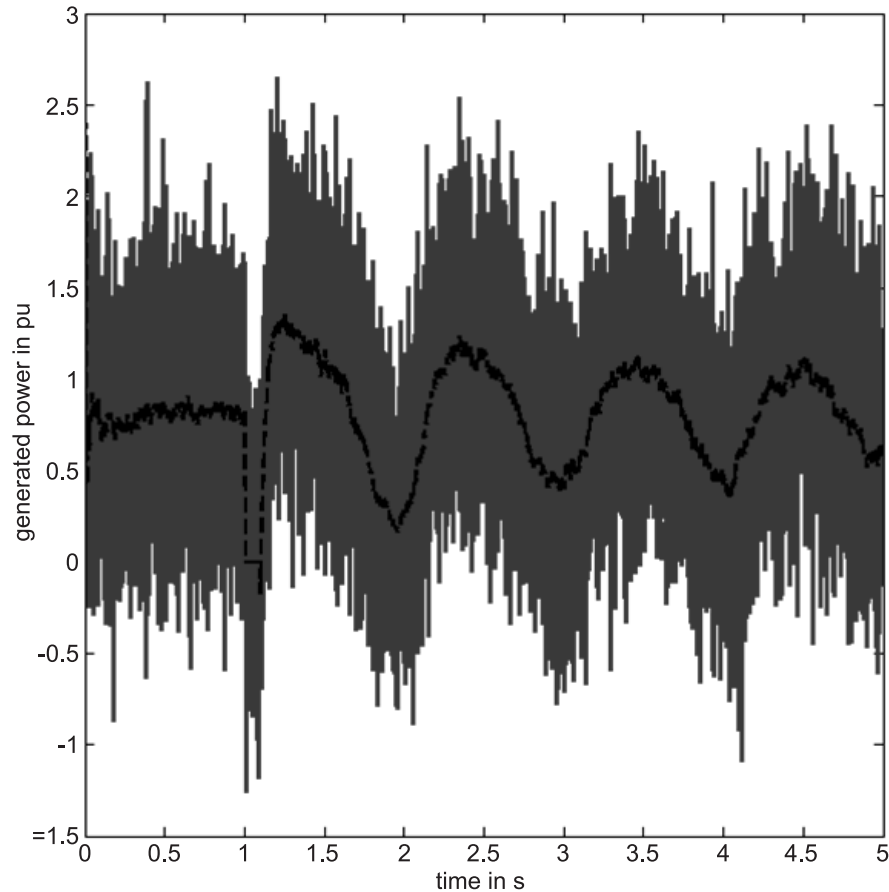


Figure 7: Actual and Estimated States. Red represents actual and blue represents estimated

## 6. CONCLUSION

An Extended kalman filter is designed for single machine connected to infinite bus. With the Covariance matrices obtained by trial & error method, filter performance is satisfactory under noise free environment and the estimates deviates from actual states under noisy conditions. In this paper, we propose the use of two lbest PSO algorithm to tune the matrices Q and R of EKF. The performance of tuned EKF is evaluated under different operating conditions. The Covariance matrices tuned with two lbests PSO algorithm gives accurate estimates under steady state and transient conditions. The performance of advanced PSO based tuned EKF is better than heuristic method. This work can be extended by online-tuning of covariance matrices so that the elements of the matrices change with change in operating conditions.





**Figure 8. Measured and estimated active power. Red represents measured and blue represents estimated**

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