I J A B E R, Vol. 13, No. 1, (2015): 439-459

OPTIMAL FUZZY COOPERATION IN DETERMINISTIC INVENTORY SITUATIONS

K. Kalaiarasi[1] and R. Sivasubramanian[2]

Abstract: *Game theory is the mathematical theory of interactive decision situations. Supply chain management is related to the coordination of materials, products and information flows among suppliers, manufactures, distributers, retailers and customers involved in producing and delivering a final product or service. In this paper we provide a review of the applications of cooperative game theory and an allocation problem in the management of centralized inventory systems. The agents involved in the inventory situation. This agree to cooperate and the characteristic function is given by an explicit formula. We deal with situations where the cooperation among the agent is not an assumption and the main issue is to analyse the coalition formation process. This paper develops an approach to determine the optimum economic order quantity and average inventory cost per unit time under the fuzzy arithmetic operations of function principle are proposed. A full fuzzy model is developed where the input parameters demand, shortage cost, holding cost, purchase cost are fuzzy trapezoidal numbers. The optimal policy for the fuzzy cooperation inventory model is determined using the algorithm of extension of the Lagrangean method for solving inequality constraint problem and Graded mean integration method is used for defuzzifying the fuzzy total average integrated cost. A numerical example is used to show the feasibility of the proposed integration models.*

Keywords: *Fuzzy inventory, Cooperation in deterministic inventory model, Function Principle, Graded Mean Integration Representation, The average inventory cost.*

1. INTRODUCTION

In this paper, we consider the situation of cooperation in deterministic inventory models. We consider a model to determine an optimal average inventory cost under conditions of coalition. The centralization of inventory management and coordination of actions, to further reduce costs and improve customer service level. Operations management focused on single-firm anlysis in the past. Its goal was to provide managers with suitable tools to improve the performance of their firms. Nowadays business decisions are dominated by the globalization of markets and should take into account the increasing competition among firms. Further more and more products reach the customer through supply chains that are compose of independent firms M. G. Fiestras-Janerio, I. Garcia-jurado, A. Meca, M. A. Mosquera

1 Department of Mathematics, Vel Tech University, Avadi, Chennai. Tamil Nadu
2 Saraswathi Velu College of Engineering, Sholighur, Vellore Dt, Tamil Nadu

(2011). Following these trends, research in supply chain has shifted its focus from single –firm analysis to multifirm analysis , in particular to improving the efficiency and performance of supply chains under decentralized control. The main characteristics of such chains are that the firms in the chain are independent actors who try to optimize their individual objectives and that the decisions taken by a firm do also affect the performance of the other parties in the supply chain. These interactions among firms decisions ask for alignment and coordination of actions and therefore game theory is very wellsuited to deal with these interactions. Cachon and Netessine (2004).

The authors discuss both non-cooperative and co-operative game theory in static and dynamic settings .Additionally, Cachon (1998) reviews competitive supply chain inventory management, and Cachon (2003) reviews and extends the supply chain literature on the management of incentive conflicts with contracts. A very recent survey on applications of cooperative game theory to supply chain management is called supply chain collaboration Meca and Timmer (2008).An important aspect of supply chain management is a good management of the inventories by the firms or retailers. The management of inventory started at the beginning of 20th century when manufacturing industries and engineering grew rapidly (Harris 1913), (Hadley and whitin 1963: Hax and Candea (1984), Tersine 1994, Zipkin 2000). The objective of inventory management is to minimize the average cost per time unit incurred by the inventory system, while guaranteeing a pre-specified minimal level of service. In cooperative games it is assumed that the grand coalition is formed whenever it leads to some profit. One of the goals of cooperative game theory is to find allocations of the total profit in such a way that no subset of players has incentives to leave the grand coalition and form its own coalition that is allocations that are stable with regard to one-step deviations of coalitions Harris (1913).

Basically the coalition formation process is analysed here using a two-stage approach. In stage 1 , suppliers form coalitions for sending a kit of components to the assembler. In stage 2, there is an interaction between the assembler and the coalitions of suppliers formed in stage 1. Granot and Yin (2008) and Nagarajan and Sosic (2007, 2008, 2009). In this paper we consider average cooperation in deterministic inventory models with fuzzy input parameters. Here demand and cost are represented as a trapezoidal fuzzy number. Chen's (1985) function principle is proposed for arithmetic operation of fuzzy number and Lagrangean method is used for optimization . Graded mean integration is used for defuzzifying the average cost.

1.1. The Fuzzy Arithmetical Operations Under Function Principle

Function principle is proposed to be as the fuzzy arithmetical operations by trapezoidal fuzzy numbers. We define some fuzzy arithmetical operations under Function Principle as follows :

Suppose $\widetilde{A} = (a_1, a_2, a_3, a_4)$ & $\widetilde{B} = (b_1, b_2, b_3, b_4)$ are 2 trapezoidal fuzzy numbers. Then

(1) The addition of $\widetilde{A}$ and $\widetilde{B}$ is

$\widetilde{A} \oplus \widetilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$
where $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are any real numbers.

(2) The multiplication of $\widetilde{A}$ and $\widetilde{B}$ is

$\widetilde{A} \otimes \widetilde{B} = (C_1, C_2, C_3, C_4)$
where $T = \{a_1b_1, a_2b_2, a_3b_3, a_4b_4\}$

$T_1 = \{a_2b_2, a_2b_3, a_3b_2, a_3b_3\}$

$C_1 = \min T_1, C_2 = \min T_1, C_3 = \max T_1, C_4 = \max T_1$

If $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are all zero positive real numbers then

$\widetilde{A} \otimes \widetilde{B} = (a_1b_1, a_2b_2, a_3b_3, a_4b_4)$

(3) $-\widetilde{B} = (-b_4, -b_3, -b_2, -b_1)$ then the subtraction of $\widetilde{A}$ and $\widetilde{B}$ is

$\widetilde{A}\ \widetilde{B} = \{a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1\}$ where $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are any real numbers.

(4) $\dfrac{1}{\widetilde{B}} = \widetilde{B}^{-1}\left(\dfrac{1}{b_4}, \dfrac{1}{b_3}, \dfrac{1}{b_2}, \dfrac{1}{b_1}\right)$ where b_1, b_2, b_3, b_4 are all positive real numbers. If $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are all nonzero positive real numbers then the division $\widetilde{A}$ and $\widetilde{B}$ is $\widetilde{A} \oslash \widetilde{B} = \left(\dfrac{a_1}{b_4}, \dfrac{a_2}{b_3}, \dfrac{a_3}{b_2}, \dfrac{a_4}{b_1}\right)$

(5) Let $\alpha \in R$, then

(i) $\alpha \geq 0, \alpha \otimes \widetilde{A} = (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4)$

(ii) $\alpha \geq 0, \alpha \otimes \widetilde{A} = (\alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1)$

1.2. Extension of the Lagrangean Method

Taha [19] discussed how to solve the optimum solution of nonlinear programming problem with equality constraints by using Lagrangean Method, and showed how the Lagrangean method may be extended to solve inequality constraints. The general idea of extending the Lagrangean procedure is that if the unconstrained optimum the problem does not satisfy all constraints, the constrained optimum must occur at a boundary point of the solution space. Suppose that the problem is given by

Minimize $y = f(x)$

Sub to $g_i(x) \geq 0$, i = 1, 2, . . . , m.

The nonnegativity constraints $x \geq 0$ if any are included in the m constraints. Then the procedure of the Extension of the Lagrangean method involves the following steps.

Step 1: Solve the unconstrained problem

Min $y = f(x)$

If the resulting optimum satisfies all the constraints, stop because all constraints are redundant. Otherwise set $K = 1$ and go to step 2.

Step 2 : Activate any K constraints ((ie) convert them into equality) and optimize f(x) subject to the K active constraints by the Lagrangean method. If the resulting solution is feasible with respect to the remaining constraints and repeat the step. If all sets of active constraints taken K at a time are considered without encountering a feasible solution, go to step 3.

Step 3 : If $K = m$, stop; no feasible solution exists. Otherwise set $K = K + 1$ and go to step 2.

1.3. Methodology

1.3.1. Graded Mean Integration Reprsentation Method

Chen & Hsieh [1999] introduced Graded mean Integration Representation Method based on the integral value of graded mean h-level of generalized fuzzy number for defuzzifying generalized fuzzy number. Here, we fist define generalized fuzzy number as follows :

Suppose $\tilde{A}$ is a generalized fuzzy number as shown in Fig. 1. It is described as any fuzzy subset of the real line R, whose membership function $\mu_{\tilde{A}}$ satisfies the following conditions.

1. $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to [0, 1],
2. $\mu_{\tilde{A}}(x) = 0, -\infty < x \leq a_1$,
3. $\mu_{\tilde{A}}(x) = L(x)$ is strictly increasing on $[a_1, a_2]$,
4. $\mu_{\tilde{A}}(x) = W_A, a_2 \leq x \leq a_3$,

5. $\mu_{\tilde{A}}(x) = R(x)$ is strictly decreasing on $[a_3, a_4]$,

6. $\mu_{\tilde{A}}(x) = 0,\ a_4 \le x < \infty$,

where $0 < W_A \le 1$ and a_1, a_2, a_3 and a_4 are real numbers.

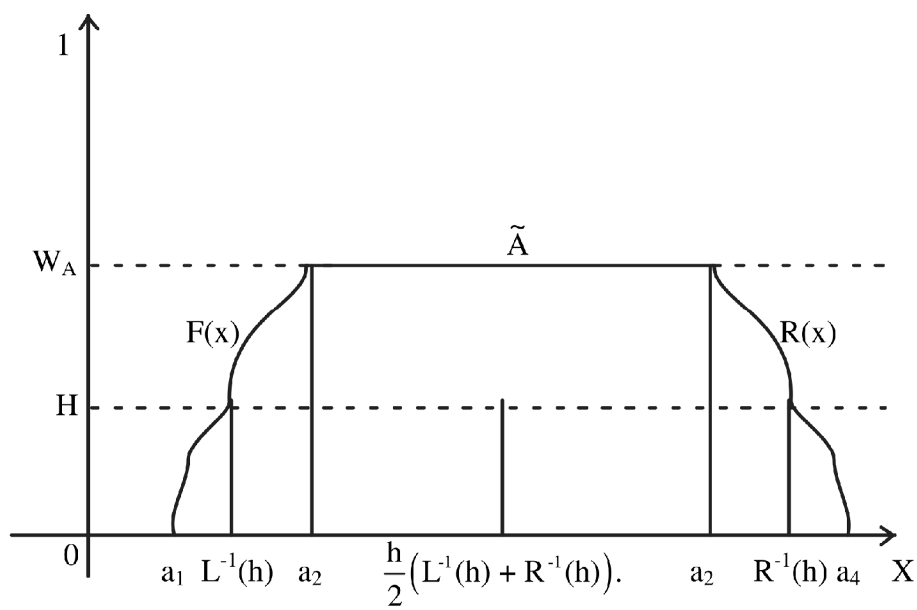
This type of generalized fuzzy numbers are denoted as $\tilde{A} = (a_1, a_2, a_3, a_4; \omega_A)_{LR}$ and $\tilde{A} = (a_1, a_2, a_3, a_4 : w_A)_{LR}$. When $\omega_A = 1$, it can be formed as $\tilde{A} = (a_1, a_2, a_3, a_4; \omega_A)_{LR}$. Second, by Graded Mean Integration Representation Method, L^{-1} and R^{-1} are the inverse functions of L and R respectively and the graded mean h-level value of generalized fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4; \omega_A)_{LR}$ is g

Then the graded Mean Integration Representation of $P(\tilde{A})$ with grade w_A, where

$$P(\tilde{A}) = \frac{\int_0^{\omega_A} \frac{h}{2}\left(L^{-1}(h) + R^{-1}(h)\right) dh}{\int_0^{\omega_A} h\ dh} \quad \ldots$$

where $0 < h \le w_A$ and $0 < w_A \le 1$.

Figure 1: The graded mean h-level value of generalized fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4 : w_A)_{LR.}$



Throughout this paper, we only use popular trapezoidal fuzzy number as the type of all fuzzy parameters in our proposed fuzzy production inventorymodels. Let $\tilde{B}$ be a trapezoidal fuzzy number and be denoted as $\tilde{B} = (b_1, b_2, b_3, b_4)$. Then we can get the Graded Mean Integration Representation of $\tilde{B}$ by the formula (1) as

$$P(\tilde{B}) = \frac{\int_0^1 \frac{h}{2}\left[\left(b_1+b_4\right)+h\left(b_2-b_1-b_4+b_3\right)\right] dh}{\int_0^1 h\ dh} = \frac{b_1+2b_2+2b_3+b_4}{6} \quad (2)$$

1.4. An Average Inventory Model with Cooperation in Deterministic Models

This was analysed by Meca *et al.* (2003). This section deals with allocation problem.

Notations

Q_i $\rightarrow$ ordersize

d_i $\rightarrow$ demand per time unit ($d_i \geq 0$)

a $\rightarrow$ Fixed cost of an order

r_i $\rightarrow$ Replacement rate of agent i

M_i $\rightarrow$ Maximum shortage

S_i $\rightarrow$ Cost of a shortage of one unit of the good for one time unit ($s_i > 0$)

Assumptions

Assume that there are n agents, N = {1,2,..............,n} each of them facing an economic prouction quantity (EPQ) problem with shortages . An EPQ model with shortages considers an agent i who places orders of a certain good that he sells. The demand that he must fulfill equals to d_i units per time unit ($d_i \geq 0$). The cost of keeping in stock one unit of this good per time unit is h_i ($h_i > 0$). The fixed cost of an order is 'a'. Agent i considers the possibility of not fulfilling all the demand in time ,but allowing a shortage of the good. The cost of a shortage of one unit of the good for one time unit is $s_i >$ o. When an order is placed, after a deterministic and constant lead time which can be assumed to be zero, without loss of generality, agent i receives the order gradually : r_i units of the good are received per time unit. It is assumed that $r_i > d_i$.

1.5. Mathematical Model

The agent must choose n order size Q_i and a maximum shortage M_i minimizing his average inventory cost per time unit given by:

$$C(Q_i, M_i) = a\frac{d_i}{Q_i} + h_i \frac{\left(Q_i\left(1-\frac{d_i}{r_i}\right) - M_i\right)^2}{2Q_i\left(1-\frac{d_i}{r_i}\right)} + s_i \frac{M_i^{\ 2}}{2Q_i\left(1-\frac{d_i}{r_i}\right)}$$

$$C(Q_i, M_i) = a\frac{d_i}{r_i} + \frac{h_i}{2}\left[Q_i\left(1-\frac{d_i}{r_i}\right) - 2M_i + \frac{M_i^{\ 2}}{1-\frac{d_i}{r_i}}\right] + \frac{s_i M_i^{\ 2}}{2Q_i\left(1-\frac{d_i}{r_i}\right)}$$

The objective is to find the optimal order quantity which minimize the average inventory cost.

The necessary condition for minimum $\frac{\partial C}{\partial Q_i} = 0$

$$Q_i = \sqrt{\frac{2ad_i + \frac{(h_i + s_i)M_i^{\ 2}}{1-\frac{d_i}{r_i}}}{h_i\left(1-\frac{d_i}{r_i}\right)}}$$

2. AN INTEGRATED INVENTORY MODELS

2.1. Fuzzy Integrated Inventory Model for Crisp order Quantity

Throughout this paper, we use of the following variables in order to simplify the treatment of an integrated inventory models. Let $\tilde{d}_i, \tilde{h}_i, \tilde{r}_i, \tilde{M}_i, \tilde{s}_i$ be fuzzy parameters. We introduce an integrated inventory model with fuzzy parameters for crisp production quantity $C(Q_i, M_i)$ as follows.The annual integrated average inventory cost for the agent

$$C(Q_i, M_i) = \left\{ \begin{array}{l} \dfrac{a\,d_{i1}}{Q_i} + \dfrac{h_{i1}}{2}\left[Q_i\left(1-\dfrac{d_{i4}}{r_{i1}}\right) + \dfrac{{M_{i1}}^2}{\left(1-\dfrac{d_{i4}}{r_{i1}}\right)Q_i} - 2\,M_{i4} \right] + \dfrac{s_{i1}\,{M_{i1}}^2}{2\,Q_i\left(1-\dfrac{d_{i4}}{r_{i1}}\right)}, \\ \dfrac{ad_{i2}}{Q_i} + \dfrac{h_{i2}}{2}\left[Q_i\left(1-\dfrac{d_{i3}}{r_{i2}}\right) + \dfrac{{M_{i2}}^2}{\left(1-\dfrac{d_{i3}}{r_{i2}}\right)Q_i} - 2M_{i3} \right] + \dfrac{s_{i2}\,{M_{i2}}^2}{2Q_i\left(1-\dfrac{d_{i3}}{r_{i2}}\right)}, \\ \dfrac{ad_{i3}}{Q_i} + \dfrac{h_{i3}}{2}\left[Q_i\left(1-\dfrac{d_{i2}}{r_{i3}}\right) + \dfrac{{M_{i3}}^2}{\left(1-\dfrac{d_{i2}}{r_{i3}}\right)} - 2M_{i2} \right] + \dfrac{s_{i3}\,{M_{i3}}^2}{2Q_i\left(1-\dfrac{d_{i2}}{r_{i3}}\right)}, \\ \dfrac{ad_{i4}}{Q_i} + \dfrac{h_{i4}}{2}\left[Q_i\left(1-\dfrac{d_{i1}}{r_{i4}}\right) + \dfrac{{M_{i4}}^2}{\left(1-\dfrac{d_{i1}}{r_{i1}}\right)} - 2M_{i1} \right] + \dfrac{s_{i4}\,{M_{i4}}^2}{2Q_i\left(1-\dfrac{d_{i1}}{r_{i4}}\right)} \end{array} \right\}$$

$$\tilde{d}_i = (d_{i1}, d_{i2}, d_{i3}, d_{i4})$$
$$\tilde{Q}_i = (Q_{i1}, Q_{i2}, Q_{i3}, Q_{i4})$$
$$\tilde{r}_i = (r_{i1}, r_{i2}, r_{i3}, r_{i4})$$
$$\tilde{M}_i = (M_{i1}, M_{i2}, M_{i3}, M_{i4})$$

Suppose $\tilde{s}_i = (s_{i1}, s_{i2}, s_{i3}, s_{i4})$

are nonnegative trapezoidal fuzzy numbers. Then we solve the optimal production quantity of formula (5) as the following steps. Second, we defuzzify the fuzzy total production inventory for the vendor and buyer cost by formula (2).

$$P(C(Q_i, M_i)) = \frac{1}{6}\left\{\begin{array}{l} \dfrac{a\,d_{i1}}{Q_i} + \dfrac{h_{i1}}{2}\left[Q_i\left(1-\dfrac{d_{i4}}{r_{i1}}\right) + \dfrac{M_{i1}{}^2}{\left(1-\dfrac{d_{i4}}{r_{i1}}\right)Q_i} - 2M_{i4}\right] + \dfrac{s_{i1}\,M_{i1}{}^2}{2Q_i\left(1-\dfrac{d_{i4}}{r_{i1}}\right)}, \\ 2\left(\dfrac{ad_{i2}}{Q_i} + \dfrac{h_{i2}}{2}\left[Q_i\left(1-\dfrac{d_{i3}}{r_{i2}}\right) + \dfrac{M_{i2}{}^2}{\left(1-\dfrac{d_{i3}}{r_{i2}}\right)Q_i} - 2M_{i3}\right] + \dfrac{s_{i2}\,M_{i2}{}^2}{2Q_i\left(1-\dfrac{d_{i3}}{r_{i2}}\right)}\right), \\ 2\left(\dfrac{ad_{i3}}{Q_i} + \dfrac{h_{i3}}{2}\left[Q_i\left(1-\dfrac{d_{i2}}{r_{i3}}\right) + \dfrac{M_{i3}{}^2}{\left(1-\dfrac{d_{i2}}{r_{i3}}\right)} - 2M_{i2}\right] + \dfrac{s_{i3}\,M_{i3}{}^2}{2Q_i\left(1-\dfrac{d_{i2}}{r_{i3}}\right)}\right), \\ \dfrac{ad_{i4}}{Q_i} + \dfrac{h_{i4}}{2}\left[Q_i\left(1-\dfrac{d_{i1}}{r_{i4}}\right) + \dfrac{M_{i4}{}^2}{\left(1-\dfrac{d_{i1}}{r_{i1}}\right)} - 2M_{i1}\right] + \dfrac{s_{i4}\,M_{i4}{}^2}{2Q_i\left(1-\dfrac{d_{i1}}{r_{i4}}\right)} \end{array}\right\}$$

Third, we can get the optimal production quantity Q_i when $P(C(Q_i,M_i))$ is minimization.

In order to find the minimization of $P(C(Q_i,M_i))$

The derivative of $P(C(Qi,M_i))$ with Q_i is $\dfrac{\partial P(C(Q_i, M_i))}{\partial Q_i} = 0$

We find the optimal production quantity Q =

$$Qi^* = \sqrt{\frac{2(ad_{i1}, ad_{i2}, ad_{i3}, ad_{i4}) + \dfrac{(h_{i1}+s_{i1})M_{i1}{}^2}{2\left(1-\dfrac{d_{i4}}{r_{i1}}\right)} + \dfrac{(2h_{i2}+2s_{i2})M_{i2}{}^2}{2\left(1-\dfrac{d_{i3}}{r_{i2}}\right)} + \dfrac{(2h_{i3}+2s_{i3})M_{i3}{}^2}{2\left(1-\dfrac{d_{i2}}{r_{i3}}\right)} + \dfrac{(h_{i4}+s_{i4})M_{i4}{}^2}{2\left(1-\dfrac{d_{i1}}{r_{i4}}\right)}}{\dfrac{h_{i1}}{2}\left(1-\dfrac{d_{i4}}{r_{i1}}\right) + \dfrac{2h_{i2}}{2}\left(1-\dfrac{d_{i3}}{r_{i2}}\right) + \dfrac{2h_{i3}}{2}\left(1-\dfrac{d_{i2}}{r_{i3}}\right) + \dfrac{h_{i4}}{2}\left(1-\dfrac{d_{i1}}{r_{i4}}\right)}}$$

2.2. Fuzzy integrated inventory model for fuzzy order

In this section ,we introduce an integrated inventory model by changing the crisp order quantity into fuzzy order quantity.Suppose fuzzy order quantity $\tilde{Q}_i$ be a trapezoidal fuzzy number $\tilde{Q}_i$ =(Q_1,Q_2,Q_3,Q_4) with $0 < Q_{i1} \le Q_{i2} \le Q_{i3} \le Q_{i4}$. Thus we can get the fuzzy total order inventory cost

$$P(C(Q_i, M_i)) = \frac{1}{6}\left\{\begin{array}{l} \dfrac{a\,d_{i1}}{Q_{i4}} + \dfrac{h_{i1}}{2}\left[Q_{i1}\left(1-\dfrac{d_{i4}}{r_{i1}}\right) + \dfrac{M_{i1}^{\ 2}}{\left(1-\dfrac{d_{i4}}{r_{i1}}\right)Q_{i4}} - 2M_{i4}\right] + \dfrac{s_{i1}M_{i1}^{\ 2}}{2Q_{i4}\left(1-\dfrac{d_{i4}}{r_{i1}}\right)}, \\ 2\left(\dfrac{ad_{i2}}{Q_{i3}} + \dfrac{h_{i2}}{2}\left[Q_{i2}\left(1-\dfrac{d_{i3}}{r_{i2}}\right) + \dfrac{M_{i2}^{\ 2}}{\left(1-\dfrac{d_{i3}}{r_{i2}}\right)Q_{i3}} - 2M_{i3}\right] + \dfrac{s_{i2}M_{i2}^{\ 2}}{2Q_{i3}\left(1-\dfrac{d_{i3}}{r_{i2}}\right)}\right), \\ 2\left(\dfrac{ad_{i3}}{Q_{i2}} + \dfrac{h_{i3}}{2}\left[Q_{i3}\left(1-\dfrac{d_{i2}}{r_{i3}}\right) + \dfrac{M_{i3}^{\ 2}}{\left(1-\dfrac{d_{i2}}{r_{i3}}\right)} - 2M_{i2}\right] + \dfrac{s_{i3}M_{i3}^{\ 2}}{2Q_{i2}\left(1-\dfrac{d_{i2}}{r_{i3}}\right)}\right), \\ \dfrac{ad_{i4}}{Q_{i1}} + \dfrac{h_{i4}}{2}\left[Q_{i4}\left(1-\dfrac{d_{i1}}{r_{i4}}\right) + \dfrac{M_{i4}^{\ 2}}{\left(1-\dfrac{d_{i1}}{r_{i1}}\right)} - 2M_{i1}\right] + \dfrac{s_{i4}M_{i4}^{\ 2}}{2Q_{i1}\left(1-\dfrac{d_{i1}}{r_{i4}}\right)} \end{array}\right\} \quad (7)$$

with $0 < Q_{i1} \le Q_{i2} \le Q_{i3} \le Q_{i4}$.

It will not change the meaning of formula (7) if we replace inequality conditions $0 < Q_{i1} \le Q_{i2} \le Q_{i3} \le Q_{i4}$ into the following inequality$Q_{i2} - Q_{i1} \ge 0$, $Q_{i3} - Q_{i2} \ge 0$, $Q_{i4} - Q_{i3} \ge 0$, $Q_{i1} > 0$.

In the following steps, extension of the Lagrangean method is used to find the solutions of Q_{i1}, Q_{i2}, Q_{i3}, Q_{i4} to minimize $P(C(Q_i, M_i))$ in formula (7)

Step 1 : Solve the unconstraint problem. Consider min $P(C(Q_i,M_i))$

To find the min $P(C(Q_i,M_i))$, we have to find the derivative of $P(C(Q_i,M_i))$ with respect to Q_{i1}, Q_{i2}, Q_{i3}, Q_{i4}.

$$\frac{\partial P}{\partial Q_{i1}} = \frac{1}{6}\left\{\frac{h_{i1}}{2}\left(1-\frac{d_{i4}}{r_{i1}}\right)-\frac{a\,d_{i4}}{Q_{i1}^{\;2}}-\frac{M_{i4}^{\;2}\,h_{i4}}{\left(1-\frac{d_{i1}}{r_{i4}}\right)Q_{i1}^{\;2}}-\frac{s_{i4}\,M_{i4}^{\;2}}{2Q_{i1}^{\;2}\left(1-\frac{d_{i1}}{r_{i4}}\right)}\right\}$$

$$\frac{\partial P}{\partial Q_{i2}} = \frac{2}{6}\left\{\frac{h_{i2}}{2}\left(1-\frac{d_{i3}}{r_{i2}}\right)-\frac{a\,d_{i3}}{Q_{i2}^{\;2}}-\frac{M_{i3}^{\;2}\,h_{i3}}{\left(1-\frac{d_{i2}}{r_{i3}}\right)Q_{i2}^{\;2}}-\frac{s_{i3}\,M_{i3}^{\;2}}{2Q_{i2}^{\;2}\left(1-\frac{d_{i2}}{r_{i3}}\right)}\right\}$$

$$\frac{\partial P}{\partial Q_{i3}} = \frac{2}{6}\left\{\frac{h_{i3}}{2}\left(1-\frac{d_{i2}}{r_{i3}}\right)-\frac{a\,d_{i2}}{Q_{i3}^{\;2}}-\frac{M_{i2}^{\;2}\,h_{i2}}{\left(1-\frac{d_{i3}}{r_{i2}}\right)Q_{i1}^{\;2}}-\frac{s_{i4}\,M_{i2}^{\;2}}{2Q_{i3}^{\;2}\left(1-\frac{d_{i3}}{r_{i2}}\right)}\right\}$$

$$\frac{\partial P}{\partial Q_{i4}} = \frac{1}{6}\left\{\frac{h_{i4}}{2}\left(1-\frac{d_{i1}}{r_{i4}}\right)-\frac{a\,d_{i1}}{Q_{i4}^{\;2}}-\frac{M_{i1}^{\;2}\,h_{i1}}{\left(1-\frac{d_{i4}}{r_{i1}}\right)Q_{i4}^{\;2}}-\frac{s_{i1}\,M_{i1}^{\;2}}{2Q_{i4}^{\;2}\left(1-\frac{d_{i4}}{r_{i1}}\right)}\right\}$$

Let all the above results partial derivatives equal to zero and solve Q_{i1}, Q_{i2}, Q_{i3}, Q_{i4}. Let $\frac{\partial P}{\partial Q_{i1}} = 0$ then

$$Q_{i1} = \sqrt{\frac{2ad_{i4} + \dfrac{M_{i4}{}^2(h_{i4}+s_{i4})}{2\left(1-\dfrac{d_{i1}}{r_{i4}}\right)}}{\dfrac{h_{i1}}{2}\left(1-\dfrac{d_{i4}}{r_{i1}}\right)}}$$

Let $\dfrac{\partial P}{\partial Q_{i2}} = 0$ then $Q_{i2} = \sqrt{\dfrac{2(2ad_{i3}) + \dfrac{2M_{i3}{}^2(h_{i3}+s_{i3})}{2\left(1-\dfrac{d_{i2}}{r_{i3}}\right)}}{2\left(\dfrac{h_{i2}}{2}\left(1-\dfrac{d_{i3}}{r_{i2}}\right)\right)}}$

and $\dfrac{\partial P}{\partial Q_3} = 0$ then $Q_{i3} = \sqrt{\dfrac{2(2ad_{i2}) + \dfrac{2M_{i2}{}^2(h_{i2}+s_{i2})}{2\left(1-\dfrac{d_{i3}}{r_{i2}}\right)}}{2\left(\dfrac{h_{i3}}{2}\left(1-\dfrac{d_{i2}}{r_{i3}}\right)\right)}}$

and $\dfrac{\partial P}{\partial Q_4} = 0$ then $Q_{i4} = \sqrt{\dfrac{2ad_{i1} + \dfrac{M_{i1}{}^2(h_{i1}+s_{i1})}{2\left(1-\dfrac{d_{i4}}{r_{i1}}\right)}}{\dfrac{h_{i4}}{2}\left(1-\dfrac{d_{i1}}{r_{i4}}\right)}}$

Because the above show that $Q_{i1} > Q_{i2} > Q_{i3} > Q_{i4}$. It does not satisfy the constraint $0 < Q_{i1} > Q_{i2} > Q_{i3} > Q_{i4}$.

Therefore set K = 1 and go to Step 2.

Step 2 : Convert the inequality constraint $Q_{i2} - Q_{i1} \geq 0$ into equality constraint $Q_{i2} - Q_{i1} = 0$ and optimize $P(C(Q_i, M_i))$ subject to $Q_{i2} - Q_{i1} = 0$ by the Lagrangean Method. We have Lagrangean function as

$L(Q_{i1}, Q_{i2}, Q_{i3}, Q_{i4}, \lambda) = P(C(Q_i, M_i)) - \lambda(Q_{i2} - Q_{i1})$

Taking the partial derivatives of $L(Q_{i1}, Q_{i2}, Q_{i3}, Q_{i4}, \lambda)$ with respect to Q_{i1}, Q_{i2}, Q_{i3}, Q_{i4} and λ to find the minimization of $L(Q_{i1}, Q_{i2}, Q_{i3}, Q_{i4}, \lambda)$. Let all the partial derivatives equal to zero and solve $Q_{i1}, Q_{i2}, Q_{i3}, Q_{i4}$.

Then we get,

$$\frac{\partial L}{\partial Q_{i1}} = \frac{1}{6}\left\{\frac{h_{i1}}{2}\left(1-\frac{d_{i4}}{r_{i1}}\right)-\frac{a\,d_{i4}}{Q_{i1}^{2}}-\frac{M_{i4}^{2}\,h_{i4}}{\left(1-\frac{d_{i1}}{r_{i4}}\right)Q_{i1}^{2}}-\frac{s_{i4}\,M_{i4}^{2}}{2Q_{i1}^{2}\left(1-\frac{d_{i1}}{r_{i4}}\right)}\right\}+\lambda=0$$

$$\frac{\partial L}{\partial Q_{i2}} = \frac{2}{6}\left\{\frac{h_{i2}}{2}\left(1-\frac{d_{i3}}{r_{i2}}\right)-\frac{a\,d_{i3}}{Q_{i2}^{2}}-\frac{M_{i3}^{2}\,h_{i3}}{\left(1-\frac{d_{i2}}{r_{i3}}\right)Q_{i2}^{2}}-\frac{s_{i3}\,M_{i3}^{2}}{2Q_{i2}^{2}\left(1-\frac{d_{i2}}{r_{i3}}\right)}\right\}-\lambda=0$$

$$\frac{\partial L}{\partial Q_{i3}} = \frac{2}{6}\left\{\frac{h_{i3}}{2}\left(1-\frac{d_{i2}}{r_{i3}}\right)-\frac{a\,d_{i2}}{Q_{i3}^{2}}-\frac{M_{i2}^{2}\,h_{i2}}{\left(1-\frac{d_{i3}}{r_{i2}}\right)Q_{i1}^{2}}-\frac{s_{i4}\,M_{i2}^{2}}{2Q_{i3}^{2}\left(1-\frac{d_{i3}}{r_{i2}}\right)}\right\}=0$$

$$\frac{\partial L}{\partial Q_{i4}} = \frac{1}{6}\left\{\frac{h_{i4}}{2}\left(1-\frac{d_{i1}}{r_{i4}}\right)-\frac{a\,d_{i1}}{Q_{i4}^{2}}-\frac{M_{i1}^{2}\,h_{i1}}{\left(1-\frac{d_{i4}}{r_{i1}}\right)Q_{i4}^{2}}-\frac{s_{i1}\,M_{i1}^{2}}{2Q_{i4}^{2}\left(1-\frac{d_{i4}}{r_{i1}}\right)}\right\}=0$$

$$\frac{\partial L}{\partial \lambda} = -(Q_{i2} - Q_{i1}) = 0$$

$$Q_{i1} = Q_{i2} = \sqrt{\frac{2\left(\mathrm{ad}_{i4} + 2ad_{i3}\right) + \dfrac{M_{i4}{}^2\left(h_{i4} + s_{i4}\right)}{2\left(1 - \dfrac{d_{i1}}{r_{i4}}\right)} + \dfrac{2M_{i3}{}^2\left(h_{i3} + s_{i3}\right)}{2\left(1 - \dfrac{d_{i2}}{r_{i3}}\right)}}{\dfrac{\mathrm{h}_{i1}}{2}\left(1 - \dfrac{d_{i4}}{r_{i1}}\right) + \dfrac{2h_{i2}}{2}\left(1 - \dfrac{d_{i3}}{r_{i2}}\right)}}$$

$$Q_{i3} = \sqrt{\frac{2\left(2\mathrm{ad}_{i2}\right) + \dfrac{2M_{i2}{}^2\left(h_{i2} + s_{i2}\right)}{2\left(1 - \dfrac{d_{i3}}{r_{i2}}\right)}}{2\left(\dfrac{\mathrm{h}_{i3}}{2}\left(1 - \dfrac{d_{i2}}{r_{i3}}\right)\right)}}$$

$$Q_{i4} = \sqrt{\frac{2\mathrm{ad}_{i1} + \dfrac{M_{i1}{}^2\left(h_{i1} + s_{i1}\right)}{2\left(1 - \dfrac{d_{i4}}{r_{i1}}\right)}}{\dfrac{\mathrm{h}_{i4}}{2}\left(1 - \dfrac{d_{i1}}{r_{i4}}\right)}}$$

Because the above results show that $Q_{i3} > Q_{i4}$, it does not satisfy the constraint $0 < Q_{i1} > Q_{i2} > Q_{i3} > Q_{i4}$. Therefore it is not a local optimum. Similarly we can get the same result if we select any other one inequality constraint to be equality constraint, therefore set K = 2 and go to Step 3.

Step 3 : Convert the inequality constraints $Q_{i2} - Q_{i1} \geq 0$, into equality constraints $Q_{i2} - Q_{i1} = 0$ and $Q_{i3} - Q_{i1} = 0$. We optimize $P(C(Q_i, M_i))$

Subject to $Q_{i2} - Q_{i1} = 0$ and $Q_{i3} - Q_{i2} = 0$ by the Lagrangean Method. Then the Lagrangean method is $L(Q_{i1}, Q_{i2}, Q_{i3}, Q_{i4}, \lambda) = P(C(Q_i, M_i)) - \lambda_1(Q_{i2} - Q_{i1}) - \lambda_2(Q_{i3} - Q_{i2})$

In order to find the minimization of $L(Q_{i1}, Q_{i2}, Q_{i3}, Q_{i4}, \lambda_1, \lambda_2)$, we take the partial derivatives of $L(Q_{i1}, Q_{i2}, Q_{i3}, Q_{i4}, \lambda_1, \lambda_2)$ with respect to $Q_{i1}, Q_{i2}, Q_{i3}, Q_{i4}, \lambda_1, \lambda_2$ and let all the partial derivatives equal to zero and solve $Q_{i1}, Q_{i2}, Q_{i3}, Q_{i4}$,

$$\frac{\partial L}{\partial Q_{i1}} = \frac{1}{6}\left\{\frac{h_{i1}}{2}\left(1-\frac{d_{i4}}{r_{i1}}\right) - \frac{a\,d_{i4}}{Q_{i1}^{2}} - \frac{M_{i4}^{2}\,h_{i4}}{\left(1-\frac{d_{i1}}{r_{i4}}\right)Q_{i1}^{2}} - \frac{s_{i4}\,M_{i4}^{2}}{2\,Q_{i1}^{2}\left(1-\frac{d_{i1}}{r_{i4}}\right)}\right\} + \lambda_1 = 0$$

$$\frac{\partial L}{\partial Q_{i2}} = \frac{2}{6}\left\{\frac{h_{i2}}{2}\left(1-\frac{d_{i3}}{r_{i2}}\right) - \frac{a\,d_{i3}}{Q_{i2}^{2}} - \frac{M_{i3}^{2}\,h_{i3}}{\left(1-\frac{d_{i2}}{r_{i3}}\right)Q_{i2}^{2}} - \frac{s_{i3}\,M_{i3}^{2}}{2\,Q_{i2}^{2}\left(1-\frac{d_{i2}}{r_{i3}}\right)}\right\} - \lambda_1 + \lambda_2 = 0$$

$$\frac{\partial L}{\partial Q_{i3}} = \frac{2}{6}\left\{\frac{h_{i3}}{2}\left(1-\frac{d_{i2}}{r_{i3}}\right) - \frac{a\,d_{i2}}{Q_{i3}^{2}} - \frac{M_{i2}^{2}\,h_{i2}}{\left(1-\frac{d_{i3}}{r_{i2}}\right)Q_{i1}^{2}} - \frac{s_{i4}\,M_{i2}^{2}}{2\,Q_{i3}^{2}\left(1-\frac{d_{i3}}{r_{i2}}\right)}\right\} - \lambda_2 = 0$$

$$\frac{\partial L}{\partial Q_{i4}} = \frac{1}{6}\left\{\frac{h_{i4}}{2}\left(1-\frac{d_{i1}}{r_{i4}}\right) - \frac{a\,d_{i1}}{Q_{i4}^{2}} - \frac{M_{i1}^{2}\,h_{i1}}{\left(1-\frac{d_{i4}}{r_{i1}}\right)Q_{i4}^{2}} - \frac{s_{i1}\,M_{i1}^{2}}{2\,Q_{i4}^{2}\left(1-\frac{d_{i4}}{r_{i1}}\right)}\right\} = 0$$

$$\frac{\partial L}{\partial \lambda_1} = -\,(Q_{i2} - Q_{i1})$$

$$\frac{\partial L}{\partial \lambda_2} = -\,(Q_{i3} - Q_{i2})$$

$$Q_{i1} = Q_{i2} = Q_{i3} = \sqrt{\frac{2\left(ad_{i4} + 2ad_{i3} + 2ad_{i2}\right) + \frac{M_{i4}^{2}\left(h_{i4}+s_{i4}\right)}{2\left(1-\frac{d_{i1}}{r_{i4}}\right)} + \frac{2M_{i3}^{2}\left(h_{i3}+s_{i3}\right)}{2\left(1-\frac{d_{i2}}{r_{i3}}\right)} + \frac{2M_{i2}^{2}\left(h_{i3}+s_{i2}\right)}{2\left(1-\frac{d_{i3}}{r_{i2}}\right)}}{\frac{h_{i1}}{2}\left(1-\frac{d_{i4}}{r_{i1}}\right) + \frac{2h_{i2}}{2}\left(1-\frac{d_{i3}}{r_{i2}}\right) + \frac{2h_{i3}}{2}\left(1-\frac{d_{i2}}{r_{i3}}\right)}}$$

$$Q_{i4} = \sqrt{\frac{2ad_{i1} + \dfrac{M_{i1}^{\ 2}\left(h_{i1} + s_{i1}\right)}{2\left(1 - \dfrac{d_{i4}}{r_{i1}}\right)}}{\dfrac{h_{i4}}{2}\left(1 - \dfrac{d_{i1}}{r_{i4}}\right)}}$$

The above results $Q_{i1} > Q_{i4}$, does not satisfy the constraint Q_{i1}, Q_{i2}, Q_{i3}, Q_{i4}. Therefore it is not a local optimum. Similarly we can get the same result if we select any other two inequality constraints to be equality constraint, therefore set K = 3 and go to Step 4.

Step 4 : Convert the inequality constraints $Q_{i2} - Q_{i1} \geq 0$, $Q_{i3} - Q_{i2} \geq 0$ and $Q_{i4} - Q_{i3} \geq 0$ into equality constraints $Q_{i2} - Q_{i1} = 0$, $Q_{i3} - Q_{i1} = 0$ and $Q_{i4} - Q_{i3} = 0$.

We optimize $P(C(Q_i,M_i))$ Subject to $Q_{i2} - Q_{i1} = 0$, $Q_{i3} - Q_{i2} = 0$ by the Lagrangean Method. The Lagrangean function is given by

$$L(Q_{i1}, Q_{i2}, Q_{i3}, Q_4, \lambda_1, \lambda_2, \lambda_3) = P(C(Q_i,M_i)) - \lambda_1(Q_{i2} - Q_{i1}) - \lambda_2(Q_{i3} - Q_{i2}) - \lambda_3(Q_{i4} - Q_{i3})$$

In order to find the minimization of $L(Q_{i1}, Q_{i2}, Q_{i3}, Q_{i4}, \lambda_1, \lambda_2, \lambda_3)$, we take the partial derivatives of $L(Q_{i1}, Q_{i2}, Q_{i3}, Q_{i4}, \lambda_1, \lambda_2, \lambda_3)$ with respect to Q_{i1}, Q_{i2}, Q_{i3}, Q_{i4}, λ_1, λ_2, λ_3 and let all the partial derivatives equal to zero and solve Q_{i1}, Q_{i2}, Q_{i3} and Q_{i4}. Then we get

$$\frac{\partial L}{\partial Q_{i1}} = \frac{1}{6}\left\{\frac{h_{i1}}{2}\left(1 - \frac{d_{i4}}{r_{i1}}\right) - \frac{a\,d_{i4}}{Q_{i1}^{\ 2}} - \frac{M_{i4}^{\ 2}\,h_{i4}}{\left(1 - \dfrac{d_{i1}}{r_{i4}}\right)Q_{i1}^{\ 2}} - \frac{s_{i4}\,M_{i4}^{\ 2}}{2\,Q_{i1}^{\ 2}\left(1 - \dfrac{d_{i1}}{r_{i4}}\right)}\right\} + \lambda_1 = 0$$

$$\frac{\partial L}{\partial Q_{i2}} = \frac{2}{6}\left\{\frac{h_{i2}}{2}\left(1 - \frac{d_{i3}}{r_{i2}}\right) - \frac{a\,d_{i3}}{Q_{i2}^{\ 2}} - \frac{M_{i3}^{\ 2}\,h_{i3}}{\left(1 - \dfrac{d_{i2}}{r_{i3}}\right)Q_{i2}^{\ 2}} - \frac{s_{i3}\,M_{i3}^{\ 2}}{2\,Q_{i2}^{\ 2}\left(1 - \dfrac{d_{i2}}{r_{i3}}\right)}\right\} - \lambda_1 + \lambda_2 = 0$$

$$\frac{\partial L}{\partial Q_{i3}} = \frac{2}{6}\left\{\frac{h_{i3}}{2}\left(1 - \frac{d_{i2}}{r_{i3}}\right) - \frac{a\,d_{i2}}{Q_{i3}^{\ 2}} - \frac{M_{i2}^{\ 2}\,h_{i2}}{\left(1 - \dfrac{d_{i3}}{r_{i2}}\right)Q_{i1}^{\ 2}} - \frac{s_{i4}\,M_{i2}^{\ 2}}{2\,Q_{i3}^{\ 2}\left(1 - \dfrac{d_{i3}}{r_{i2}}\right)}\right\} - \lambda_1 + \lambda_2 = 0$$

$$\frac{\partial \mathrm{L}}{\partial \mathrm{Q}_{i4}} = \frac{1}{6}\left\{\frac{\mathrm{h}_{i4}}{2}\left(1-\frac{d_{i1}}{r_{i4}}\right)-\frac{a\,d_{i1}}{Q_{i4}^{\ 2}}-\frac{M_{i1}^{\ 2}\,h_{i1}}{\left(1-\frac{d_{i4}}{r_{i1}}\right)Q_{i4}^{\ 2}}-\frac{s_{i1}\,M_{i1}^{\ 2}}{2\,Q_{i4}^{\ 2}\left(1-\frac{d_{i4}}{r_{i1}}\right)}\right\}-\lambda_3 = 0$$

$$\frac{\partial \mathrm{L}}{\partial \lambda_1} = -\,(\mathrm{Q}_{i2} - \mathrm{Q}_{i1})$$

$$\frac{\partial \mathrm{L}}{\partial \lambda_2} = -\,(\mathrm{Q}_{i3} - \mathrm{Q}_{i2})$$

$$\frac{\partial \mathrm{L}}{\partial \lambda_3} = -\,(\mathrm{Q}_{i4} - \mathrm{Q}_{i3})$$

$$\mathrm{Q}_{i1} = \mathrm{Q}_{i2} = \mathrm{Q}_{i3} = \mathrm{Q}_{i4} = \sqrt{\frac{2\left(\mathrm{ad}_{i4}+2ad_{i3}+2ad_{i2}\right)+\frac{M_{i4}^{\ 2}\left(h_{i4}+s_{i4}\right)}{2\left(1-\frac{d_{i1}}{r_{i4}}\right)}+\frac{2M_{i3}^{\ 2}\left(h_{i3}+s_{i3}\right)}{2\left(1-\frac{d_{i2}}{r_{i3}}\right)}+\frac{2M_{i2}^{\ 2}\left(h_{i3}+s_{i3}\right)}{2\left(1-\frac{d_{i3}}{r_{i2}}\right)}+\frac{M_{i1}^{\ 2}\left(h_{i1}+s_{i1}\right)}{2\left(1-\frac{d_{i4}}{r_{i1}}\right)}}{\frac{\mathrm{h}_{i1}}{2}\left(1-\frac{d_{i4}}{r_{i1}}\right)+\frac{2h_{i2}}{2}\left(1-\frac{d_{i3}}{r_{i2}}\right)+\frac{2h_{i3}}{2}\left(1-\frac{d_{i2}}{r_{i3}}\right)+\frac{h_{i4}}{2}\left(1-\frac{d_{i1}}{r_{i4}}\right)}}$$

Because the above solution $\widetilde{\mathrm{Q}}_i = (\mathrm{Q}_{i1}, \mathrm{Q}_{i2}, \mathrm{Q}_{i3}, \mathrm{Q}_{i4})$ satisfies all inequality constraints, the procedure terminates with $\widetilde{\mathrm{Q}}_i$ as a local optimum solution to the problem. Since the above local optimum solution is the only one feasible solution of formula (), So it is an optimum solution of the inventory model with fuzzy order quantity according to extension of the Lagrangean Method.

Let $\mathrm{Qi}_1 = \mathrm{Qi}_2 = \mathrm{Qi}_3 = \mathrm{Qi}_4 = \widetilde{\mathrm{Q}}_i^{\ *}$. Then the optimal fuzzy production quantity is

$$\tilde{Q}^{*} = (Qi^{*}, Qi^{*}, Qi^{*}, Qi^{*}) = \sqrt{\frac{2\left(\mathrm{ad}_{i4} + 2ad_{i3} + 2ad_{i2}\right) + \frac{M_{i4}^{2}\left(h_{i4}+s_{i4}\right)}{2\left(1-\frac{d_{i1}}{r_{i4}}\right)} + \frac{2M_{i3}^{2}\left(h_{i3}+s_{i3}\right)}{2\left(1-\frac{d_{i2}}{r_{i3}}\right)} + \frac{2M_{i2}^{2}\left(h_{i3}+s_{i3}\right)}{2\left(1-\frac{d_{i3}}{r_{i2}}\right)} + \frac{M_{i1}^{2}\left(h_{i1}+s_{i1}\right)}{2\left(1-\frac{d_{i4}}{r_{i1}}\right)}}{\frac{h_{i1}}{2}\left(1-\frac{d_{i4}}{r_{i1}}\right) + \frac{2h_{i2}}{2}\left(1-\frac{d_{i3}}{r_{i2}}\right) + \frac{2h_{i3}}{2}\left(1-\frac{d_{i2}}{r_{i3}}\right) + \frac{h_{i4}}{2}\left(1-\frac{d_{i1}}{r_{i4}}\right)}}$$

2.3. Numerical Examples

To illustrate the results obtained in this paper, the proposed analytic solution method is applied to efficiency solve the following numerical example. Consider an inventory system with the following characteristics.

d_i=2600 r_i =8000 s_i =150 M_i =300 a= 500 h_i=100 Qi*= 200.41 $C(Q_i,M_i)$ = 66411.5 In this example can be transferred into the fuzzy parameters as follows. Consider any problem in which an annual demand is more or less than 2600 units, replacement rate is more or less than 8000, unit stock-holding cost is more or less than 100.00 per item per year , cost of a shortage of one unit of the good for one time unit is more or less than 150, maximum shortage is more or less than 300 per units. Determine the optimum integrated averagel cost?

Here we represent the case of value, "more or less than Y" as the type of trapezoidal fuzzy number.

Suppose Fuzzy annual demand is "more or less than 2600"

$\tilde{d}_i = (d_1, d_2, d_3, d_4) = (2400, 2500, 2700, 2800)$

Fuzzy replacement rate is "more or less than 8000"

$\tilde{r}_i = (r_1, r_2, r_3, r_4) = (7000, 7500, 8500, 9000)$

Fuzzy cost of keeping of one unit is "more or less than 100"

$\tilde{h}i = (h_{i1}, h_{i2}, h_{i3}, h_{i4}) = (80, 90, 110, 120)$

Fuzzy cost of shortage of one unit of the good for one time unit "more or less than 150"

$\tilde{s}_i = (s_{i1}, s_{i2}, s_{i3}, s_{i4}) = (130, 140, 160, 170)$

Fuzzy maximum shortage is "more or less than 300"

$$\widetilde{M}_i = (M_{i1}, M_{i2}, M_{i3}, M_{i4}) = (280,290,310,320)$$

Fuzzy order quantity

$$\widetilde{Q}_i = (Q_{i1}, Q_{i2}, Q_{i3}, Q_{i4}) = (757.24, 757.24,757.24,757.24)$$

with $0 < Q_{i1} \leq Q_{i2} \leq Q_{i3} \leq Q_{i4}$

The minimization fuzzy average order inventory cost for the agent i is

$P(C(Q_i,M_i)) =(12935.35, 15976.05, 23701.25,28468.16)$

3. CONCLUSION

This paper presents two fuzzy models for an optimal integrated inventory model and minimizing the average expected cost of the agent i. In the first model demand, replacement rate, maximum shortage,cost of a shortage of one unit represented demand cost represented by fuzzy number while Q_i is treated as a fixed constant. In the second model Q_i is also represented as a fuzzy number. For each fuzzy model a method of defuzzification, graded mean integration representation is applied to find the estimate of average expected cost of agent in the fuzzy type and then corresponding optimal order quantity is derived to maximize the total profit.

References

Cachon, G., (1998), Competitive supply chain inventory management. Tayur. S., Ganeshan, R., Magazine, M., Quantitative models for supply chain management, Kluwer Academic Publisher.

Cachon, G., (2003), Supply chain coordination with constants in: Graves,S.,Dekok. T., Handbooks in operations research and management science: supply chain management. North Holland.

Cachon, G., Netessine, S., (2004), Game theory in supply analysis, in: Simchilevi, D;Wu,S.D., Shen,. Z. J. Handbook of quantitative supply chain analysis. Modeling in the eBusiness Era Kluwer academic publishers.

Chand S., Ward J., (1987), A note on Economic order quantity under conditions of permissible delay in payments. *Journal of Operational Research Society* 38, 83-84.

Chang H.J., Hung C.H., Dye C.Y., (2001), An inventory model for deteriorating item with linear trend demand under the condition of permissible delay in payments. Production planning and control 12, 274-282.

Chung K.J., (1998), A theorem on the determination of economic order quantity under conditions of permissible delay in payments. *Computers and Operations Research* 25, 49-52.

Chung K.J., Huang C. K., (2009), An ordering policy with allowable shortage and permissible delay in payments. *Applied Mathematical Modeling* 33, 2518-2525.

Granot, D., Yin, S., (2008), Competition and cooperation in decentralized push and pull assembly systems. *Management Science* 54, 733-747.

Harris, F., (1913, How many parts to make at once.The magazine of management 10,135-136,152

Meca, A., Timmer. J. (2008), Supply chain collabration.

Mosquera, M. A., Fiestras-Janerio. G., Garcia-Jurado, I., (2008), A note on coalitional manipulation and centralized inventory management. *Annals of Operations Research* 158, 183-188.

Nagarajan, M., Bassok, Y., (2008), A bargainingframework in supply chains: The assembly problem Management science 54. 1482-1496.

Nagrajan, M., Sosic, G., (2007), Stable farsighted coalitions in competitive markets. *Management Science* 53, 29-45.

Gupta U. K., (1988), A Comment on Economic order quantity under conditions of permissible delay in payments. *Journal of Operational Research Society* 39, 322-323.

Huang H., Shin S. W., (1997), Retailer's Pricing and lot sizing for exponentially deteriorating products under the condition of permissible delay in payments. *Computers and Operations Research* 24, 539-547.

H. A. Taha, (1997), in: Operations Research, Prentice-Hall, Englewood Cliffs, NJ, USA, PP. 753-777.

K. S. Park, (1987), Fuzzy set thuretic interpretation of economic order quantity, IEEE Transactions on systems, Man, Cybernatics SMC 17, 1082-1084.

S.H. Chen, (1985), Operations on fuzzy numbers with function principle, *Tamkong Journal of Mangement Sciences* 6 (1) 13-26.

S.H. Chen, C.H. Heish, (1999), Graded mean integration representations of generalized fuzzy number. *Journal of Chinese Fuzzy Systems* 5 1-7.

S.H. Chen., (1998), C.H. Heish, Graded mean integration representations of generalized fuzzy numbers in: proceedings of the sixth conference on fuzzy theory and its applications Chinese fuzzy systems Association Taiwan,. pp. 1-6.

S.C. Chang, (1999), Fuzzy production inventory for fuzzy product with triangular fuzzy number, *Fuzzy Sets and Systems* 107, 37-57.

S.H. Chen, (1985), Fuzzy linear combination of fuzzy linear function principle, *Tamkang Journal of Management Science* 6 (1), 13-26.

S.H. Chen, C.H. Hsieh, (1999), Optmization of fuzzy inventory models in IEEE SMC99 Conference proceeding Tokyo, Japan Vol.1, 1999, 240-244.

S.H. Chen, C.H. Hsieh, Graded mean integration representation of generalized fuzzy number, *Journal of Chinese Fuzzy Systems* 5 (2) 1999, 1-7.

S.K. Goyal. (1985), Economic order quantity under conditions of permissible delay in payments. J. Oper. Res. So 36, 335-338.

S.C. Chang, (1999), Fuzzy production inventory for fuzzy product quantity with triangular fuzzy number. *Fuzzy Sets and Systems* 107 37-57.

Teng. J. T., (2002), On the economic order quantity under conditions of permissible delay in payments. *Journal of Operation Research Society* 53, 915-918.

Teng J. T. Chang C. T., (2009), Optimal manufacturels replenishment policies in the EPQ model under two levels of trade credit policy. *European Journal of Operational Research* 195, 358-363.

Zipkin, P. H. (2000), Foundations of inventory Management .Ms.Graw-Hill companies.