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AN EOQ MODEL WITH RAMP TYPE DEMAND RATE, WEIBULL DETERIORATION RATE AND PARTIAL BACKLOGGING, UNDER INFLATION

Dinesh Prasad, S. R. Singh & M. P. Singh

Abstract: In this paper, an inventory model with ramp type demand rate with inflation and time value of money is developed. The deterioration rate follows three-parameter Weibull distribution with the concept of life time of an item. Shortages are also allowed and unsatisfied demand is partially backlogged. The model is fairly general in practice, as the demand of some items such as fashionable items increases up to the time point of its stabilization. The backlogging rate is any non-increasing function of the waiting time up to the next replenishment. Numerical example is also considered to illustrate the results. Sensitivity analysis is also carried out.

Keywords: Ramp type demand, Weibull deterioration, Inflation, Partial backlogging.

1. INTRODUCTION

In real life situation, the effect of deterioration plays an important role in many inventory systems. Deterioration may be defined as decay, damage, spoilage, evaporation, obsolescence, pilferage, loss of utility or loss of marginal value of a commodity that results in decreasing usefulness. Most of the physical goods undergo decay or deterioration over time, the examples being fruits, medicine, volatile liquids, blood banks, and others. Consequently, the production and inventory problem of deteriorating items has been extensively studied by researchers. Ghare and Schrader [1] first derived an EOQ model for exponential decaying inventory with constant deterioration rate. Covert and Philip [2] extended Ghare and Schrader's constant deterioration rate to a two-parameter Weibull distribution. Donaldson [3] was first studied inventory model with linearly time varying demand. Wee and Wang [4] analyzed a production-inventory model for deteriorating items with time varying demand, finite production rate and shortages, over a known planning horizon. More information regarding production-inventory models for deteriorating items can be found in the review articles by Raafat [5], Goyal and Giri [6]. Since then many researchers investigated deteriorating inventory models with time-dependent demand *e.g.* Goswami and Chaudhuri [7], Hariga [8], Sakaguchi [9]. Recently He *et al*., [10] developed a production inventory model for deteriorating items with multiple-market demand, where each market has a different selling season and a different constant demand rate.

Many business practices show that the presence of a larger quantity of goods displayed may attract more customers than that with a smaller quantity of goods. This phenomenon implies that the demand may have a positive correlation with stock level. Under such circumstances, a firm should seriously consider its pricing and ordering strategy since the demand for their goods may be affected by their selling prices and inventory level. Gupta and Vrat [11] first developed a model with stock-dependent consumption rate with the assumption that demand is a function of the initial stock level. Further stock-dependent demand rate was investigated by Balkhi and Benkherouf [12], Zhou and Yang [13] Dye *et al*., [14]. Inventory model with stock-dependent demand rate and variable holding cost was analyzed by Alfares [15]. You and Hsieh [16] studied an EOQ model with stock and price sensitive demand. Goyal and Chang [17] developed an inventory model with stock-dependent demand rate in which they assumed that the retailer gets the delivery of the item and some of the items are displayed in the shop while the rest of the items are kept in the backroom/warehouse.

The demand of some fashionable items increases up to a certain moment and then stabilizes to a constant demand, and if customers are satisfied with quality and price, it then stabilizes to a constant rate. For this we use ramp type demand rate. Hill [18] proposed an inventory model with increasing demand followed by a constant demand. Later inventory model with ramp type demand rate was studied by Mandal and Pal [19], Wu *et al*., [20], Wu and Ouyang [21], Wu [22]. Giri *et al*., [23] extended the ramp type demand inventory model with a more generalized Weibull distribution deterioration. Manna and Chaudhuri [24] analyzed an inventory model with ramp type demand rate allowing shortages which are completely backlogged. Deng *et al*., [25] investigated an inventory model to amend the incompleteness of the models given by Wu and Ouyang [21].

Most of the countries have been suffering from large-scale inflation and sharp decline in the purchasing power of money for last several years. As a result, while determining the optimal inventory policy, the effects of inflation and time value of money cannot be ignored. Buzacott [26] first considered an EOQ model with inflation subject to different types of pricing policies, further Ray and Chaudhuri [27], Chen [28], Wee and Law [29], Yang *et al*., [30] investigated the effect of inflation, time value of money and deterioration on inventory models. Jaggi *et al*., [31] presented an optimal inventory replenishment policy for deteriorating items under inflationary conditions using discounted cash flow approach over a finite planning horizon. Recently Yang *et al*., [32] presented an inventory model under inflation for deteriorating items with stock-dependent consumption rate and partial backlogging shortages.

As deterioration may be affected by storage condition, weather condition etc, and also for some items deterioration does not start in the beginning but after certain period called life time of the item, which may vary from item to item. For this we have considered three-parameter Weibull distribution. An inventory model with ramp type demand rate under inflation and time value of money is developed. The model is fairly general in practice, as the demand of some items such as fashionable items increases up to the time point of its stabilization. The backlogging rate is a non-increasing function of the waiting time up to the next replenishment.

2. ASSUMPTIONS

We develop the inventory model under the following assumptions:

1. The life time of the item in inventory is random and follows a Weibull distribution. The deterioration rate is

$$\theta(t) = \alpha\beta(t-\gamma)^{\beta-1} \qquad \alpha, \beta > 0, \quad t > \gamma$$

2. There is no replacement of deteriorated items during the period T.

3. Shortages are allowed and are backlogged at a rate $g(x)$ which is a non-increasing function of x i.e. $g'(x) \le 0$ with the assumption $0 \le g(x) \le 1$, $g(0) = 1$ and x is the waiting time up to the next replenishment, i.e. $x = T - t_1$, moreover we assume that $g(x)$ satisfies the relation $Rg(x) + g'(x) \ge 0$, where $g'(x)$ is the derivative of $g(x)$ and R is the net discount rate of inflation. The cases $g(x) = 0$ (or 1) correspond to complete backlogging (or complete lost sales) models.

4. The demand rate $D(t)$ is a ramp type function of time given by

$$D(t) = \begin{cases} f(t) & t \le \mu \\ f(\mu) & t > \mu \end{cases} \qquad f(t) \text{ is a positive, continuous function of } t \in [0, T].$$

5. We assume $c_s - Rc_o \ge 0$, $\alpha << 1$, and c_o is sufficiently large than c_d so that $c_0 - c_d e^{\alpha(t_1-\gamma))^{\beta}} \ge 0$.

Notations:

T Scheduling period (cycle) which is constant.

t_1 The time when inventory level reaches zero.

A Ordering cost.

c_h Holding cost per unit per unit of time.

c_d Deterioration cost per unit per unit of time.

c_s Shortage cost per unit per unit of time.

c_o Opportunity cost per unit per unit of time due to lost sale.

μ Parameter of the ramp type demand function (time point).

3. THE MATHEMATICAL FORMULATION OF THE MODEL

3.1 Case I ($t_1 \leq \mu$)

The replenishment at the beginning of the cycle brings the inventory level up to I_m There is no deterioration during interval [0, γ], the depletion of inventory during this interval is only due to demand. During the interval [γ, t_1] the inventory level decreases with the combined effect of demand and deterioration, and falls to zero at $t = t_1$, thereafter shortages occur during the period [t_1, T], which are partially backlogged. The backlogged demand is satisfied at the next replenishment. The behavior of the inventory for this case is depicted in Fig. 1.

Let $I(t)$ be the inventory level at any time t, $0 \leq t \leq T$. The differential equations governing

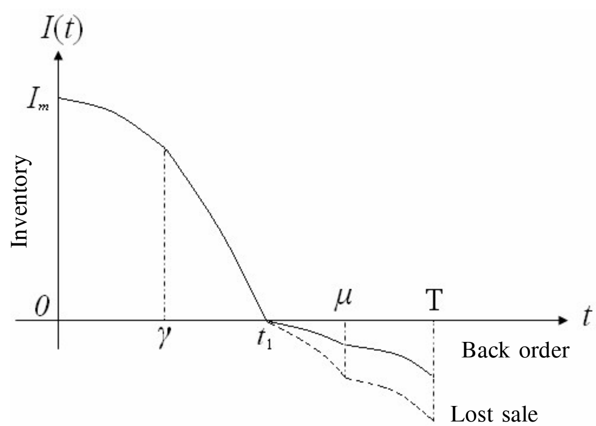

Figure 1: Inventory Level when $t_1 \leq \mu$

the instantaneous state of inventory level $I(t)$ in the interval [0, T] are given by:

$$I'(t) = -f(t) \qquad 0 \leq t \leq \gamma \tag{1}$$

$$I'(t) + \alpha\beta\,(t-\gamma)^{\beta-1} I(t) = -f(t) \text{ with } I(t_1) = 0 \qquad \gamma \leq t \leq t_1 \tag{2}$$

$$I'(t) = -f(t)\, g(T-t) \qquad t_1 \leq t \leq \mu \tag{3}$$

$$I'(t) = -f(\mu)\, g(T-t) \qquad \mu \leq t \leq T \tag{4}$$

The solutions of equations (1)-(4) are given by

$$I(t) = \int_t^{\gamma} f(x)\,dx + \int_{\gamma}^{t_1} f(x)\,e^{\alpha(x-\gamma)^{\beta}}\,dx \tag{5}$$

$$I(t) = e^{-\alpha(t-\gamma)^{\beta}} \int_t^{t_1} f(x)\,e^{-\alpha(x-\gamma)^{\beta}}\,dx \tag{6}$$

$$I(t) = -\int_{t_1}^{t} f(x)\,g(T-x)\,dx \tag{7}$$

$$I(t) = -f(u)\int_{\mu}^{t} g(T-x)\,dx - \int_{t_1}^{\mu} f(x)g(T-x)\,dx \tag{8}$$

Present value of holding cost of the inventory during $[0, t_1]$ is

$$H = c_h \int_0^{\gamma}\left[\int_t^{\gamma} f(x)dx + \int_{\gamma}^{t_1} f(x)\,e^{\alpha(x-\gamma)^{\beta}}dx\right]e^{-Rt}dt + c_h \int_{\gamma}^{t_1} e^{-\alpha(t-\gamma)^{\beta}}\left[\int_t^{t_1} f(x)\,e^{-\alpha(x-\gamma)^{\beta}}dx\right]e^{-Rt}dt \tag{9}$$

Present value of deterioration cost during $[0, t_1]$ is

$$D = c_d \int_{\gamma}^{t_1} f(t)e^{-\alpha(t-\gamma)^{\beta}} e^{-Rt}dt - c_d \int_{\gamma}^{t_1} f(t)e^{-Rt}dt \tag{10}$$

Present value of shortage cost during $[t_1, T]$ is

$$S = c_s \int_{t_1}^{\mu}\left[\int_{t_1}^{t} f(x)\,g(T-x)\,dx\right]e^{-Rt}dt + c_s \int_{\mu}^{T}\left[f(\mu)\int_{\mu}^{t} g(T-x)dx - \int_{t_1}^{\mu} f(x)\,g(T-x)\,dx\right]e^{-Rt}dt \tag{11}$$

Present value of opportunity cost due to lost sale during $[t_1, T]$ is

$$O = c_o \int_{t_1}^{\mu}[1-g(T-t)]\,f(t)e^{-Rt}dt + c_o f(\mu)\int_{\mu}^{T}[1-g(T-t)]e^{-Rt}dt \tag{12}$$

Present value of the total cost

$$TC_1(t_1) = H + D + S + O$$

$$= c_h \int_0^{\gamma}\left[\int_t^{\gamma} f(x)dx + \int_{\gamma}^{t_1} f(x)e^{\alpha(x-\gamma)^{\beta}}dx\right]e^{-Rt}dt$$

$$+ c_h \int_{\gamma}^{t_1} e^{-\alpha(t-\gamma)^{\beta}}\left[\int_{\mu}^{t_1} f(x)e^{-\alpha(x-\gamma)^{\beta}}dx\right]e^{-Rt}dt$$

$$+ c_d \int_{\gamma}^{t_1} f(t)e^{-\alpha(t-\gamma)^{\beta}}e^{-Rt}dt - c_d \int_{\gamma}^{t_1} f(t)e^{-Rt}dt$$

$$+ c_s \int_{t_1}^{\mu}\left[\int_{t_1}^{t} f(x)g(T-x)dx\right]e^{-Rt}dt$$

$$+ c_s \int_{\mu}^{T}\left[f(\mu)\int_{\mu}^{t} g(T-x)dx + \int_{t_1}^{\mu} f(x)g(T-x)dx\right]e^{-Rt}dt$$

$$+ c_o \int_{t_1}^{\mu}[1-g(T-t)]f(t)e^{-Rt}dt + c_o f(\mu)\int_{\mu}^{T}[1-g(T-t)]e^{-Rt}dt \quad (13)$$

3.2 Case II ($t_1 > \mu$)

The behavior of the inventory for this case is depicted in Fig. 2. In this case the differential equations governing instantaneous state of $I(t)$ are given by:

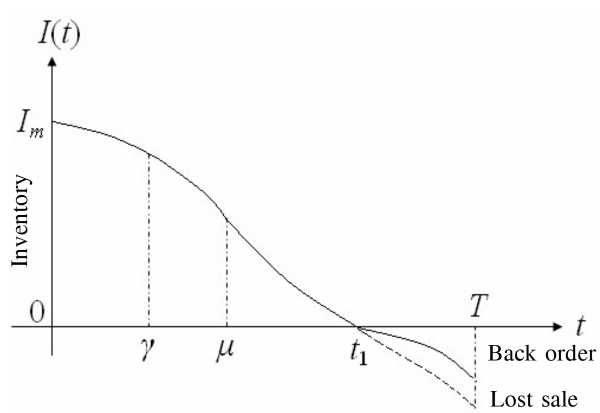

Figure 2: Inventory Level when $t_1 > \mu$

$$I'(t) = -f(t) \qquad \text{with } I(\gamma_-) = I(\gamma_+) \qquad 0 \le t \le \gamma \tag{14}$$

$$I'(t) + \alpha\beta(t-\gamma)^{\beta-1} I(t) = -f(t) \qquad \text{with } I(\mu_-) = I(\mu_+) \qquad \gamma \le t \le \mu \tag{15}$$

$$I'(t) + \alpha\beta(t-\gamma)^{\beta-1} I(t) = -f(\mu) \qquad \text{with } I(t_1) = 0 \qquad \mu \le t \le t_1 \tag{16}$$

$$I'(t) = -f(\mu)\, g\,(T-t) \qquad t_1 \le t \le T \tag{17}$$

Solutions of (14) – (17) are given by:

$$I(t) = \int_t^\gamma f(x)dx + \int_\gamma^\mu f(x)e^{\alpha(x-\gamma)^\beta}dx + f(\mu)\int_\mu^{t_1} e^{\alpha(x-\gamma)^\beta}dx \qquad 0 \le t \le \gamma \tag{18}$$

$$I(t) = e^{-\alpha(t-\gamma)^\beta}\left[\int_t^\mu f(x)e^{\alpha(x-\gamma)^\beta}dx + f(\mu)\int_\mu^{t_1} e^{\alpha(x-\gamma)^\beta}dx\right] \qquad \gamma \le t \le \mu \tag{19}$$

$$I(t) = e^{-\alpha(t-\gamma)^\beta} f(\mu)\int_t^{t_1} e^{\alpha(x-\gamma)^\beta}dx \qquad \mu \le t \le t_1 \tag{20}$$

$$I(t) = -f(\mu)\int_{t_1}^{t} g(T-x)\,dx \qquad t_1 \le t \le T \tag{21}$$

Present value of the total cost in this case

$$\begin{aligned} TC_2(t_1) = \; & c_h\left[\int_0^\gamma\left[\int_t^\gamma f(x)dx + \int_\gamma^\mu f(x)e^{\alpha(x-\gamma)^\beta}dx + f(\mu)\int_\mu^{t_1} e^{\alpha(x-\gamma)^\beta}dx\right]e^{-Rt}dt\right. \\ & + \int_\gamma^\mu e^{-\alpha(t-\gamma)^\beta}\left[\int_t^\mu f(x)e^{\alpha(x-\gamma)^\beta}dx + f(\mu)\int_\mu^{t_1} e^{\alpha(x-\gamma)^\beta}dx\right]e^{-Rt}dt \\ & \left. + \int_\mu^{t_1} e^{-\alpha(t-\gamma)^\beta}\left[f(\mu)\int_t^{t_1} e^{\alpha(x-\gamma)^\beta}dx\right]e^{-Rt}dt\right] + c_d\int_\gamma^\mu f(t)(e^{\alpha(t-\gamma)^\beta}-1)\,e^{-Rt}dt \\ & + c_d f(\mu)\int_\mu^{t_1}(e^{\alpha(t-\gamma)^\beta}-1)\,e^{-Rt}dt + c_s f(\mu)\int_{t_1}^{T}\left[\int_{t_1}^{t} g(T-x)\,dx\right]e^{-Rt}dt \\ & + c_o f(\mu)\int_{t_1}^{T}[1-g(T-t)]\,e^{-Rt}dt \end{aligned} \tag{22}$$

The total present value of the system over [0, T] takes the form:

$$TC(t_1) = \begin{cases} TC_1(t_1) & \text{if } t_1 \le \mu \\ TC_2(t_1) & \text{if } t_1 > \mu \end{cases} \tag{23}$$

It can be easily verified that $TC(t_1)$ is a continuous function at $t = \mu$. Our problem is to minimize the present value of the total cost function $TC(t_1)$ having its two branches.

4. OPTIMAL REPLENISHMENT POLICY

We present the result which ensures the existence of a unique t_1 say t_1^* which minimizes the total cost function $TC(t_1)$.

We have from equation (13)

$$\frac{dTC_1(t_1)}{dt_1} = f(t_1)\, h(t_1) \tag{24}$$

where

$$h(t_1) = c_h e^{\alpha(t_1-\gamma)^\beta}\left[\int_\gamma^{t_1} e^{-\alpha(t-\gamma)^\beta} e^{-Rt}dt + \frac{1-e^{-R\gamma}}{R}\right] - \frac{c_s}{R} g(T-t_1)(e^{-Rt_1}-e^{-RT})$$
$$+c_d e^{-Rt_1}(e^{\alpha(t_1-\gamma)^\beta}-1) - c_o(1-g(T-t_1))e^{-Rt_1} \tag{25}$$

we have

$$h(0) = -\frac{c_s}{R} g(T)(1-e^{-RT}) - c_o(1-g(T)) < 0 \qquad (\text{as} \quad 0 \le g(x) \le 1) \tag{26}$$

and

$$h(T) = c_h e^{\alpha(T-\gamma)^\beta}\left[\int_\gamma^{T} e^{-\alpha(t-\gamma)^\beta} e^{-Rt}dt + \frac{1-e^{-R\gamma}}{R}\right] + c_d e^{-RT}(e^{\alpha(T-\gamma)^\beta}-1) > 0 \ \text{ as } g(0)=1 \tag{27}$$

i.e. there exists a root between 0 and T.

$$\frac{dh(t_1)}{dt_1} = c_h \alpha\beta(t_1-\gamma)^{\beta-1} e^{\alpha(t_1-\gamma)^\beta}\left[\int_\gamma^{t_1} e^{-\alpha(t-\gamma)^\beta} e^{-Rt}dt + \frac{1-e^{-R\gamma}}{R}\right] + c_h e^{-Rt_1} - \frac{c_s}{R} g'(T-t_1)e^{-RT}$$
$$+\left(\frac{c_s}{R}-c_o\right)[Rg(T-t_1)+g'(T-t_1)]e^{-Rt} + c_d[\alpha\beta(t_1-\gamma)^{\beta-1}e^{\alpha(t_1-\gamma)^\beta}+R]e^{-Rt_1}$$
$$+R(c_o - c_d e^{\alpha(t_1-\gamma)^\beta})e^{-Rt_1} > 0 \qquad (\text{Using assumption 3 and 5}) \tag{28}$$

This implies that $h(t_1)$ is strictly increasing function of t_1, and we have $h(0) < 0$ and $h(T) > 0$, and by our assumption $f(t_1) > 0$, so the derivative $\frac{dTC_1(t_1)}{dt_1}$ vanishes at t_1^*, with $0 < t_1^* < T$, which is unique root of $h(t_1) = 0$, for this t_1^* we have

$$\frac{d^2TC_1(t_1^*)}{dt_1^{*2}} = \frac{df(t_1^*)}{dt_1^*} h(t_1^*) + f(t_1^*)\frac{dh(t_1^*)}{dt_1^*} = f(t_1^*)\frac{dh(t_1^*)}{dt_1^*} > 0 \qquad \text{by equation} \quad (28)$$

t_1^* corresponds to the unconstraint global minimum of $TC_1(t_1)$, if $t_1^* \le \mu$, i.e., if t_1^* is feasible, the minimum cost $TC(t_1)$ is given by equation (13) and optimal order quantity Q^* is given by

$$Q^* = \int_0^{\gamma} f(t)\, dt + \int_{\gamma}^{t_1^*} f(t)\, e^{\alpha(t-\gamma)^{\beta}} dt + \int_{t_1^*}^{\mu} f(t)\, g(T-t)\, dt + f(\mu) \int_{\mu}^{T} g(T-t)\, dt \quad (29)$$

when

$$g(x) = 0, \text{ we have } h(0) < 0 \text{ and } \frac{dh(t_1)}{dt_1} > 0,$$

and

$$h(T) = c_h e^{\alpha(T-\gamma)^{\beta}} \left[\int_{\gamma}^{T} e^{-\alpha(t-\gamma)^{\beta}} e^{-Rt} dt + \frac{1-e^{-R\gamma}}{R} \right] + c_d e^{-Rt} (e^{\alpha(T-\gamma)^{\beta}} - 1) - c_o,$$

which may be positive or negative as $h(T) \le 0$ is a consequence of high lost sale cost c_o.

If $h(T) > 0$, t_1^* corresponds to the unconstraint global minimum of $TC_1(t_1)$, moreover if $t_1^* \le \mu$, the optimal value of the order quantity is given by equation (29). If $h(T) < 0$, $TC_1(t_1)$ is strictly decreasing and attaining its minimum at T.

Now consider the branch $TC_2(t_1)$. Its first and second order derivatives are

$$\frac{dTC_2(t_1)}{dt_1} = f(\mu)\, h(t_1), \qquad \frac{d^2TC_2(t_1)}{dt_1^2} = f(\mu) \frac{dh(t_1)}{dt_1} \quad (30)$$

where $h(t_1)$ is given by equation (25). $TC_2(t_1)$ is strictly increasing when $0 < g(x) \le 1$, and as above $\frac{dh(t_1)}{dt_1}$ has its unique root t_1^* with $0 < t_1^* < T$. This t_1^* corresponds to the unconstraint global minimum of $TC_2(t_1)$, if t_1^* is feasible *i.e.* if $t_1^* > \mu$, the minimum total cost $TC(t_1)$ is given by equation (22) and optimal order quantity Q^* is given by

$$Q^* = \int_0^{\gamma} f(t)\, dt + \int_{\gamma}^{\mu} f(t)\, e^{\alpha(t-\gamma)^{\beta}} dt + f(\mu) \int_{\mu}^{t_1^*} e^{\alpha(t-\gamma)^{\beta}} dt + f(\mu) \int_{t_1^*}^{T} g(T-t)\, dt \quad (31)$$

For the case $g(x) = 0$, $\frac{dh(t_1)}{dt_1} > 0$ and $h(0) > 0$, again $h(T)$ may be less than or greater than zero. If $h(T) > 0$, t_1^* corresponds to the unconstraint global minimum of $TC_2(t_1)$, moreover if t_1^* is feasible *i.e.* if $t_1^* > \mu$, the optimal value of order quantity is given by equations (31). If $h(T) < 0$, $TC_2(t_1)$ is strictly decreasing and attaining its minimum at T.

The analysis done so far shows that the two functions $TC_1(t_1)$ and $TC_2(t_1)$ have the same (unique) unconstraint minimum point $t_1^* \in (0, T)$, which is determined by equation (25). From equation (23) we can further show that the function $TC(t_1)$ is differentiable at the point μ, so μ cannot be a corner point for $TC(t_1)$.

5. NUMERICAL EXAMPLES

Example 1: Let us take $\alpha = 0.01$, $\beta = 2$, $\gamma = 0.3$ year, $\mu = 0.9$ year, $c_h = 3$ unit per unit per year, $c_d = 5$ unit per unit per year, $c_s = 15$ unit per unit per year, $c_o = 20$ unit per unit per year, and $R = 0.2$, $T = 1$ year, $f(t) = 3\, e^{4.5t}$, and $g(x) = e^{-0.2x}$, solving equation (25) by Newton Raphson method we get the optimal value of t_1 as $t_1^* = 0.8472 < \mu$. The optimal ordering quantity is $Q^* = 54.4905$ from equation (29) and the minimum cost is $TC(t_1^*) = 82.51$ from equation (13).

Example 2. This example is identical to example 1, except $\mu = 0.6$ year. The optimal value of t_1 is $t_1^* = 0.8472 > \mu$. The optimal ordering quantity is $Q^* = 27.029$ from equation (31) and the minimum cost is $TC(t_1^*) = 41.64$ from equation (22).

6. SENSITIVITY ANALYSIS

Table 1 shows the sensitivity analysis of total cost which is performed by changing (increasing or decreasing) the parameters by 25% and 50%, taking one parameter at a time and keeping the remaining parameters unchanged.

Table 1

			Q^*		TC^*		% change in TC^*	
Parameters	*% change*	t_1^*	$t_1 \le \mu$	$t_1 > \mu$	$t_1 \le \mu$	$t_1 > \mu$	$t_1 \le \mu$	$t_1 > \mu$
α	−50	0.8477	54.47	27.02	82.38	41.57	−0.16	−0.17
	−25	0.8474	54.48	27.02	82.44	41.60	−0.08	−0.08
	+25	0.8469	54.50	27.03	82.58	41.67	+0.08	+0.08
	+50	0.8466	54.51	27.04	82.64	41.70	+0.16	+0.17
γ	−50	0.8464	54.53	27.06	82.78	41.80	+0.33	+0.39
	−25	0.8469	54.51	27.04	82.61	41.69	+0.12	+0.14
	+25	0.8474	54.48	27.02	82.43	41.59	−0.10	−0.11
	+50	0.8476	54.47	27.02	82.36	41.55	−0.18	−0.20
c_h	−50	0.9354	54.79	27.13	39.69	18.14	−51.89	−56.43
	−25	0.8811	54.64	27.07	67.76	39.00	−17.88	−6.32
	+25	0.8155	54.35	26.98	94.64	49.59	+14.70	+19.11
	+50	0.7859	54.21	26.92	104.72	56.77	+26.91	+36.36
c_d	−50	0.8475	54.49	27.03	82.41	41.58	−0.12	−0.12
	−25	0.8473	54.49	27.03	82.46	41.61	−0.06	−0.06
	+25	0.8470	54.49	27.03	82.56	41.66	+0.06	+0.06
	+50	0.8468	54.49	27.03	82.61	41.68	+0.12	+0.12
c_s	−50	0.7697	54.14	26.89	66.77	36.87	−19.08	−11.44
	−25	0.8161	54.35	26.98	75.88	39.73	−8.03	−4.57
	+25	0.8693	54.59	27.06	87.49	42.99	+6.04	+3.26
	+50	0.8858	54.65	27.08	91.34	44.00	+10.70	+5.69
c_o	−50	0.8315	54.42	27.00	79.14	40.68	−4.08	−2.28
	−25	0.8397	54.45	27.02	80.89	41.18	−1.96	−1.09
	+25	0.8539	54.55	27.04	84.00	42.05	+1.81	+0.99
	+50	0.8601	54.55	27.05	85.38	42.42	+3.48	+1.89
R	−50	0.8535	54.52	27.04	87.17	43.49	+5.65	+4.47
	−25	0.8504	54.50	27.04	84.81	42.55	+2.79	+2.21
	+25	0.8439	54.47	27.02	80.26	40.74	−2.72	−2.16
	+50	0.8405	54.46	27.02	78.08	39.86	−5.36	−4.26

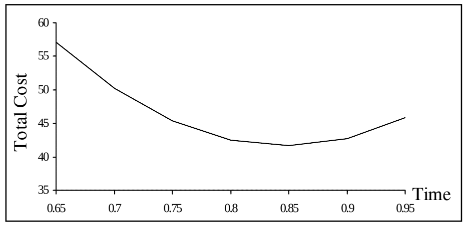

Figure 3: The Graphical Representation of the Total Cost Function when $t_1^* \leq \mu$

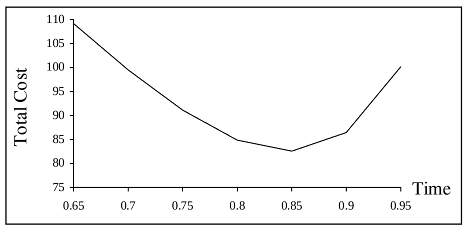

Figure 4: The Graphical Representation of the Total Cost Function when $t_1^* > \mu$

7. CONCLUSION

In this paper, we developed an order level inventory model for deteriorating items. We have taken three-parameter Weibull distribution deterioration. It can be applied to items with any initial value of the deterioration rate and items which start deteriorating only after a certain period of time. The model is fairly general as the demand rate is a ramp type function of time, and the backlogging rate is any non-increasing function of the waiting time, up to the next replenishment.

The proposed model can be extended in several ways, for instance we may consider finite planning horizon. Also we can extend the deterministic demand function to stochastic demand patterns. Further more, we can generalize the model to allow permissible delay in payments.

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Dinesh Prasad
Department of mathematics,
Vira College of Engineering, Bijnor, U. P., India.
E-mail: bhardwaj_dp82@yahoo.co.in

S. R. Singh
Department of mathematics,
D. N. (P. G.) College, Meerut, U. P., India.
E-mail: shivrajpundir@yahoo.com

M. P. Singh
Department of mathematics and Statistics,
Gurukul Kangri Vishwavidyalaya, Haridwar, U. K., India.