LARGE DEFLECTION OF ANNULAR SANDWICH PLATES

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ABSTRACT

In this paper, a large deflection problem of an annular sandwich plate with a nondeformable rigid body at the center under uniform pressure q is solved by Yeh Kai-yuan and Liu Ren-huai's modified iteration method. Analytic solutions of the problem obtained may be applied directly to engineering design.

***Keywords*:** large deflection, annular sandwich plate, modified iteration method, analytical solution

1. INTRODUCTION

It is common knowledge that the sandwich plate is applied widely in aeronantical and astronautical engineering because of its essential feature of high rigidity and light weight. Therefore it is of great importance both theoretically and practically to study nonlinear bending problems for the plate. At first, Reissuer[1] established a nonlinear bending theory of a rectangular sandwich plate with a soft core and two very thin faces. Scholars[2] of the Chinese Research Institute of Mechanics solved a large deflection problem for the circular sandwich plate with a soft core and two very thin faces under the action of uniform lateral load by the perturbation method. The auther[3] solved yet a nonlinear bending problem of the plate under uniform edge moment and a more accurate third approximation solution was obtained using the modified iteration method. This method was suggested by Yeh Kai-yuan and Liu Ren-huai[4-6] in 1965. The method incorporates advantages of Chien Wei-zang's perturbation method[7] and usual successive approximations, and it is an effective, simple, accurate method for solving nonlinear differential equations.

After that, the author[8] presented a more accurate nonlinear bending theory of a circular sandwich plate with a soft core taking into account the bending rigidity of the faces, and also gave a simplified theory of the plate in the case of neglecting the bending rigidity of the faces. In the case of including the bending rigidity of the faces, the author[9] has first discussed the nonlinear bending problem for the plate by the author's modified power series method. Unfortunately, such studies are few yet because of quite complication of the problem.

So far as we know, nonlinear problems of bending and vibration for circular and annular sandwich plates with very thin faces behaving as membranes were studied by Liu Ren-huai[8,10-15], Du Guojun[16-20], Yang Jingning[21-22], Ho Chaosheng[23-24], Zhang Xiuli[25] and Kirichok[26], *et al.*

This paper is a further work of the author's previous papers[12,13]. A large deflection problem of an annular sandwich plate with a nondeformable rigid body at the center under uniform pressure is studied. We still use the modified iteration method to solve this problem. Analysic solutions presented here may be applied directly to the engineering design.

2. FUNDAMENTAL EQUATIONS

Now consider an annular sandwich plate with a nondeformable rigid body at the center under the action of uniform pressure q as shown in Fig.1. The outer edge of the plate is rigidly clamped and the inner edge is fixed on the non-deformable rigid body which can be moved up down. Here a is the outer radius, b is the inner radius, r is the radial coordinate.

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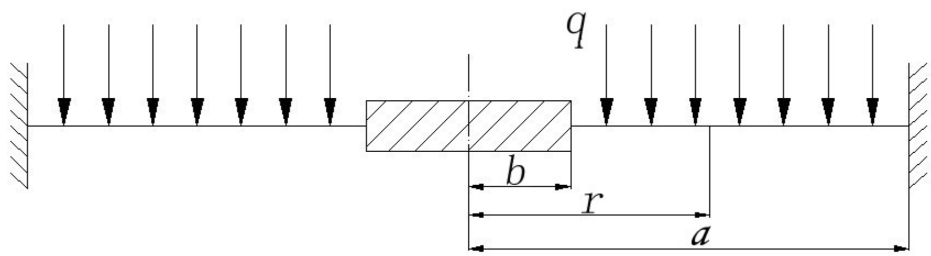

Figure 1

Using the same system of notation and the simplified equations of Ref.[8], we can easily obtain the fundamental equations of large deflection of the plate as follows.

$$Dr\frac{d}{dr}\frac{1}{r}\frac{d}{dr}r\frac{dw}{dr}-2t\left(1-\frac{D}{G_2h_0}r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\right)\left(r\sigma_{ro}\frac{dw}{dr}\right)-\frac{1}{2}q\left(r^2-b^2\right)=0\,,$$

$$r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\left(r^2\sigma_{r0}\right)+\frac{E}{2}\left(\frac{dw}{dr}\right)^2=0 \tag{1}$$

where w is the deflection of the middle plane of the plate, σ_{r0} is the radial stress of the middle plane of the plate, D is the flexural rigidity of the plate, E is Young's modulus of the face, ν is Poisson's ratio of the face, G_2 is the shear modulus of the core, t is the thickness of the face, h_0 is the distance from middle of thickness of the lower face to middle of thickness of the upper face,

$$D=\frac{Eth_0^{\ 2}}{2\left(1-v^2\right)} \tag{2}$$

Equations (1) will be solved under the following boundary conditions:

$$w = 0,\ \psi = 0,\ u = 0 \text{ at} \tag{3}$$
$$\psi = 0,\ u = 0 \text{ at } r = b$$

where ψ is the rotation of a normal to the middle plane of the plate in the diametral plane, u is the radial displacement of the middle plane of the plate,

$$\psi=-\frac{1}{G_2h_0r}\left[2tr\sigma_{r0}\frac{dw}{dr}+\frac{q}{2}\left(r^2-b^2\right)\right]-\frac{dw}{dr},$$

$$u=\frac{r}{E}\left[\frac{d}{dr}\left(r\sigma_{r0}\right)-v\sigma_{r0}\right] \tag{4}$$

In order to simplify the calculations, let us introduce the following nondimensional variables

$$\rho=\frac{r}{a},\ \alpha=\frac{b}{a},\ \ W=\sqrt{2\left(1-v^2\right)}\frac{w}{h_0},\ \phi=\frac{dw}{d\rho},\ S_r=\frac{2ta^2}{D}\sigma_{r0} \tag{5}$$

$$P=\frac{\sqrt{2\left(1-v^2\right)}}{2h_0D}a^4q,\ k=\frac{D}{G_2h_0a^2}$$

Using these nondimensional variables, the fundamental equations (1) and boundary conditions (3) become

$$L\left(\rho\phi\right)=\left(1-kL\right)\left(\rho S_r\phi\right)+P\left(\rho^2-\alpha^2\right),$$
$$L\left(\rho^2S_r\right)=-\phi^2 \tag{6a,b}$$

$$W=0,\ \phi=-k\left[S_r\phi-P\left(\alpha^2-1\right)\right],\frac{d}{d\rho}\left(\rho S_r\right)-vS_r=0 \quad at\ \rho=1; \tag{7a,b,c}$$

$$\phi=-kS_r\phi,,\ \frac{d}{d\rho}\left(\rho S_r\right)-\nu S_r=0 \quad \text{at}\ \ \rho=\alpha \tag{8a,b}$$

where L is a differential operator

$$L(...)=\rho\frac{d}{d\rho}\frac{1}{\rho}\frac{d}{d\rho}(...) \tag{9}$$

Thus Equations (6) and boundary conditions (7) and (8) constitute a nondimensional nonlinear boundary value problem for ϕ and S_r of the annular sandwich plate with a nondeformable rigid body at the center under uniform pressure.

3. ANALYTICAL SOLUTION

The nondimensional nonlinear boundary value problem (6), (7) and (8) will be solved by the modified iteration method. At first, we introduce a notation Wm of the nondimensional inner edge deflection as an iteration parameter

$$W_m=W\big|_{\rho=\alpha} \tag{10}$$

Using the fourth equation of expressions (5) and the boundary condition (7a), we obtain

$$W_m=-\int_{\alpha}^{1}\phi d\rho \tag{11}$$

For the first approximation, neglecting the nonlinear term $S_r\phi$ in Eq.(6a) and conditions (7b) and (8a) leads to the linear boundary value problem as follows

$$\begin{aligned}&L\left(\rho\phi_1\right)=P\left(\rho^2-\alpha^2\right),\\&L\left(\rho^2S_{r1}\right)=-\phi_1^{\ 2}\end{aligned} \tag{12a,b}$$

$$W_1=0,\ \phi_1=kP\left(\alpha^2-1\right),\ \frac{d}{d\rho}\left(\rho S_{r1}\right)-vS_{r1}=0 \quad \text{at}\ \ \rho=\alpha \tag{13a,b,c}$$

$$\phi_1=0,\ \frac{d}{d\rho}\left(\rho S_{r1}\right)-vS_{r1}=0 \quad \text{at}\ \rho=\alpha \tag{14a,b}$$

Eq. (12a) can be solved easily by direct integration in conjunction with the corresponding boundary conditions (13b) and (14a).Then we obtain

$$\phi_1=P\left(a_1\rho^3+a_2\rho\ln\rho+a_3\rho+a_4\rho^{-1}\right) \tag{15}$$

where

$$\begin{aligned}a_1&=\frac{1}{8},\\a_2&=-\frac{1}{2}\alpha^2,\\a_3&=\frac{1}{8\left(\alpha^2-1\right)}\left[4\alpha^4\ln\alpha-\alpha^4+1-8k\left(\alpha^2-1\right)\right],\\a_4&=-\frac{\alpha^2}{8\left(\alpha^2-1\right)}\left[4\alpha^2\ln\alpha-\alpha^2+1-8k\left(\alpha^2-1\right)\right]\end{aligned} \tag{16}$$

Substituting the solution (15) into (11), the linear characteristic relation is obtained as

$$P = \alpha_1 W_m \tag{17}$$

where

$$\alpha_1 = 4[\alpha_1(\alpha^4 - 1) + a_2 (2\alpha^2 \text{ In } \alpha - \alpha^2 + 1) + 2a_3 (\alpha^2 - 1) + 4a_4 \text{ In } \alpha]^{-1} \tag{18}$$

Using relation (17), the solution (15) may be written as

$$\phi_1 = \alpha_1 W_m (a_1 \rho^3 + a_2 \rho \text{ In } \rho + a_3 \rho + a_4 \rho^{-1}) \tag{19}$$

Using the solution (19) and integrating Eq.(12b) twice under conditions (13c) and (14b), the solution of Eq. (12b) is

$$\begin{aligned} S_{r1} = {} & -\alpha_1^{\,2} W_m^{\,2} (b_1 \rho^6 + b_2 \rho^4 \ln \rho + b_3 \rho^4 + b_4 \rho^2 \ln^2 \rho + b_5 \rho^2 \ln \rho \\ & + b_6 \rho^2 + b_7 \ln^2 \rho + b_8 \ln \rho + b_9 + b_{10} \rho^{-2} \ln \rho + b_{11} \rho^{-2}) \end{aligned} \tag{20}$$

where

$$b_1 = \frac{1}{48} a_1^{\,2},$$

$$b_2 = \frac{1}{12} a_1 a_2,$$

$$b_3 = \frac{a_1}{144} (12 a_3 - 5 a_2),$$

$$b_4 = \frac{1}{8} a_2^{\,2},$$

$$b_5 = \frac{a_2}{16} (4 a_3 - 3 a_2),$$

$$b_6 = \frac{1}{64} \left(7 a_2^{\,2} + 8 a_3^{\,2} + 16 a_1 a_4 - 12 a_2 a_3 \right)$$

$$b_7 = \frac{1}{2} a_2 a_4,$$

$$b_8 = \frac{a_4}{2} (2 a_3 - a_2),$$

$$\begin{aligned} b_9 = {} & -\frac{1}{\lambda_1 (\alpha^2 - 1)} \{ b_1 (7 - \nu)(\alpha^8 - 1) + b_2 \left[(5 - v) \alpha^6 \ln \alpha + \alpha^6 - 1 \right] + b_3 (5 - v)(\alpha^6 - 1) \\ & + b_4 \alpha^4 \ln \alpha \left[(3 - v) \ln \alpha + 2 \right] + b_5 \left[(3 - v) \alpha^4 \ln \alpha + \alpha^4 - 1 \right] + b_6 (3 - v)(\alpha^4 - 1) \\ & + b_7 \alpha^2 \ln \alpha (\lambda_1 \ln \alpha + 2) + b_8 (\lambda_1 \alpha^2 \ln \alpha + \alpha^2 - 1) - \lambda_2 b_{10} \ln \alpha \}, \end{aligned}$$

$$b_{10} = -\frac{1}{2} a_4^{\,2},$$

$$\begin{aligned} b_{11} = {} & -\frac{1}{\lambda_2 (\alpha^2 - 1)} \{ b_1 \alpha^2 (7 - v)(\alpha^6 - 1) + b_2 \alpha^2 \left[(5 - v) \alpha^4 \ln \alpha + \alpha^4 - 1 \right] \\ & + b_3 \alpha^2 (5 - v)(\alpha^4 - 1) + b_4 \alpha^4 \ln \alpha \left[(3 - v) \ln \alpha + 2 \right] + b_5 \alpha^2 \left[(3 - v) \alpha^2 \ln \alpha + \alpha^2 - 1 \right] \\ & + b_6 \alpha^2 (3 - v)(\alpha^2 - 1) + b_7 \alpha^2 \ln \alpha (\lambda_1 \ln \alpha + 2) + \lambda_1 b_8 \alpha^2 \ln \alpha - b_{10} (\alpha^2 + \lambda_2 \ln \alpha - 1) \}, \end{aligned}$$

$$\lambda_1 = 1 - v,$$

$$\lambda_2 = 1 + v \tag{21}$$

For the second approximation, from the problem (6)-(8) the following linear boundary value problem for ϕ is obtained

$$L(\rho\phi_2) = (1 - kL)(\rho S_{r1}\phi_1) + P(\rho^2 - \alpha^2) \tag{22}$$

$$W_2 = 0,\ \phi_2 = -k\left[S_{r1}\phi_1 - P(\alpha^2 - 1)\right] \text{ at } \rho = 1; \tag{23a,b}$$

$$\phi_2 = -kS_{r1}\,\phi_1 \text{ at } \rho = \alpha \tag{24}$$

Using solutions (19) and (20),the solution of this problem is

$$\begin{aligned}\phi_2 = {} & P\left(a_1\rho^3 + a_2\rho\ln\rho + a_3\rho + a_4\rho^{-1}\right) + \alpha_1^{\,3}W_m^{\,3}(c_1\rho^{11} + c_2\rho^9\ln\rho + c_3\rho^9 + c_4\rho^7\ln^2\rho \\ & + c_5\rho^7\ln\rho + c_6\rho^7 + c_7\rho^5\ln^3\rho + c_8\rho^5\ln^2\rho + c_9\rho^5\ln\rho + c_{10}\rho^5 + c_{11}\rho^3\ln^3\rho \\ & + c_{12}\rho^3\ln^2\rho + c_{13}\rho^3\ln\rho + c_{14}\rho^3 + c_{15}\rho\ln^3\rho + c_{16}\rho\ln^2\rho + c_{17}\rho\ln\rho + c_{18}\rho \\ & + c_{19}\rho^{-1}\ln^2\rho + c_{20}\rho^{-1}\ln\rho + c_{21}\rho^{-1} + c_{22}\rho^{-1} + c_{22}\rho^{-3}\ln\rho + c_{23}\rho^{-3})\end{aligned} \tag{25}$$

where

$$c_1 = -\frac{1}{120}a_1b_1,$$

$$c_2 = -\frac{1}{80}(a_1b_2 + a_2b_1),$$

$$c_3 = \frac{9}{3200}(a_1b_2 + a_2b_1) - \frac{1}{80}(a_1b_3 + a_3b_1) + ka_1b_1,$$

$$c_4 = -\frac{1}{48}(a_1b_4 + a_2b_2),$$

$$c_5 = \frac{7}{576}(a_1b_4 + a_2b_2) - \frac{1}{48}(a_1b_5 + a_2b_3 + a_3b_2) + k(a_1b_2 + a_2b_1),$$

$$c_6 = -\frac{37}{13824}(a_1b_4 + a_2b_2) + \frac{7}{1152}(a_1b_5 + a_2b_3 + a_3b_2) - \frac{1}{48}(a_1b_6 + a_3b_3 + a_4b_1) + k(a_1b_3 + a_3b_1),$$

$$c_7 = -\frac{1}{24}a_2b_4,$$

$$c_8 = \frac{5}{96}a_2b_4 - \frac{1}{24}(a_1b_7 + a_2b_5 + a_3b_4) + k(a_1b_4 + a_2b_2),$$

$$c_9 = -\frac{19}{576}a_2b_4 + \frac{5}{144}(a_1b_7 + a_2b_5 + a_3b_4) - \frac{1}{24}(a_1b_8 + a_2b_6 + a_3b_5 + a_4b_2) + k(a_1b_5 + a_2b_3 + a_3b_2),$$

$$\begin{aligned}c_{10} = {} & \frac{65}{6912}a_2b_4 - \frac{19}{1728}(a_1b_7 + a_2b_5 + a_3b_4) + \frac{5}{288}(a_1b_8 + a_2b_6 + a_3b_5 + a_4b_2) - \frac{1}{24}(a_1b_9 + a_3b_6 + a_4b_3) \\ & + k(a_1b_6 + a_3b_3 + a_4b_1),\end{aligned}$$

$$c_{11} = -\frac{1}{8}a_2b_7 + ka_2b_4,$$

$$c_{12} = \frac{9}{32}a_2b_7 - \frac{1}{8}(a_2b_8 + a_3b_7 + a_4b_4) + k(a_1b_7 + a_2b_5 + a_3b_4),$$

$$\begin{aligned}c_{13} = {} & -\frac{21}{64}a_2b_7 + \frac{3}{16}(a_2b_8 + a_3b_7 + a_4b_4) - \frac{1}{8}(a_1b_{10} + a_2b_9 + a_3b_8 + a_4b_5) \\ & + k(a_1b_8 + a_2b_6 + a_3b_5 + a_4b_2),\end{aligned}$$

$$c_{14}=\frac{45}{256}a_2b_7-\frac{7}{64}\left(a_2b_8+a_3b_7+a_4b_4\right)+\frac{3}{32}\left(a_1b_{10}+a_2b_9+a_3b_8+a_4b_5\right)$$

$$-\frac{1}{8}\left(a_1b_{11}+a_3b_9+a_4b_6\right)+k\left(a_1b_9+a_3b_6+a_4b_3\right),$$

$$c_{15}=-\frac{1}{6}\left(a_2b_{10}+a_4b_7\right)+ka_2b_7,$$

$$c_{16}=\frac{1}{4}\left(a_2b_{10}-a_2b_{11}-a_3b_{10}+a_4b_7-a_4b_8\right)+k\left(a_2b_8+a_3b_7+a_4b_4\right),$$

$$c_{17}=-\frac{1}{4}\left(a_2b_{10}-a_2b_{11}-a_3b_{10}+2a_3b_{11}+a_4b_7-a_4b_8+2a_4b_9\right)+k(a_1b_{10}+a_2b_9+a_3b_8+a_4b_5),$$

$$c_{18}=d_1-\frac{d_3-\alpha d_4}{\alpha^2-1},$$

$$c_{19}=\frac{1}{4}a_4b_{10}+k\left(a_2b_{10}+a_4b_7\right),$$

$$c_{20}=\frac{1}{4}a_4\left(b_{10}+2b_{11}\right)+k\left(a_2b_{11}+a_3b_{10}+a_4b_8\right),$$

$$c_{21}=d_2+\frac{\alpha\left(\alpha d_3-d_4\right)}{\alpha^2-1},$$

$$c_{22}=ka_4b_{10},$$

$$c_{23}=ka_4b_{11},$$

$$d_1=\frac{1}{8}\left(a_2b_{10}-a_2b_{11}-a_3b_{10}+2a_3b_{11}+a_4b_7-a_4b_8+2a_4b_9\right)+k(a_1b_{11}+a_3b_9+a_4b_6),$$

$$d_2=k\left(a_3b_{11}+a_4b_9\right),$$

$$d_3=-\left(c_1+c_3+c_6+c_{10}+c_{14}+c_{23}+d_1+d_2\right)+k\left(a_1+a_3+a_4\right)\left(b_1+b_3+b_6+b_9+b_{11}\right),$$

$$d_4=-c_1\alpha^{11}-c_2\alpha^9\ln\alpha+\left(ka_1b_1-c_3\right)\alpha^9-c_4\alpha^7\ln^2\alpha-\left[c_5-k\left(a_1b_2+a_2b_1\right)\right]\alpha^7\ln\alpha$$

$$-\left[c_6-k\left(a_1b_3+a_3b_1\right)\right]\alpha^7-c_7\alpha^5\ln^3\alpha-\left[c_8-k\left(a_1b_4+a_2b_2\right)\right]\alpha^5\ln^2\alpha$$

$$-\left[c_9-k\left(a_1b_5+a_2b_3+a_3b_2\right)\right]\alpha^5\ln\alpha-\left[c_{10}-k\left(a_1b_6+a_3b_3+a_4b_1\right)\right]\alpha^5$$

$$-\left(c_{11}-ka_2b_4\right)\alpha^3\ln^3\alpha-\left[c_{12}-k\left(a_1b_7+a_2b_5+a_3b_4\right)\right]\alpha^3\ln^2\alpha$$

$$-\left[c_{13}-k\left(a_1b_8+a_2b_6+a_3b_5+a_4b_2\right)\right]\alpha^3\ln\alpha-\left[c_{14}-k\left(a_1b_9+a_3b_6+a_4b_3\right)\right]\alpha^3$$

$$-\left(c_{15}-ka_2b_7\right)\alpha\ln^3\alpha-\left[c_{16}-k\left(a_2b_8+a_3b_7+a_4b_4\right)\right]\alpha\ln^2\alpha$$

$$-\left[c_{17}-k\left(a_1b_{10}+a_2b_9+a_3b_8+a_4b_5\right)\right]\alpha\ln\alpha-\left[d_1-k\left(a_1b_{11}+a_3b_9+a_4b_6\right)\right]\alpha$$

$$-\frac{1}{4}a_4b_{10}\alpha^{-1}\ln^2\alpha-\frac{1}{4}a_4\left(b_{10}+2b_{11}\right)\alpha^{-1}\ln\alpha \tag{26}$$

Substituting the solution (25) into (11), we obtain the nonlinear characteristic relation of the annular sandwich plate

$$P=\alpha_1W_m+\alpha_3{W_m}^3 \tag{27}$$

where

$$\alpha_3=-\alpha_1^{\ 4}\Big[\frac{c_1}{12}\left(\alpha^{12}-1\right)+\frac{c_2}{100}\left(10\alpha^{10}\ln\alpha-\alpha^{10}+1\right)+\frac{c_3}{10}\left(\alpha^{10}-1\right)+\frac{c_4}{256}\left(32\alpha^8\ln^2\alpha-8\alpha^8\ln\alpha+\alpha^8-1\right)$$

$$+\frac{c_5}{64}\left(8\alpha^8\ln\alpha-\alpha^8+1\right)+\frac{c_6}{8}\left(\alpha^8-1\right)+\frac{c_7}{216}\left(36\alpha^6\ln^3\alpha-18\alpha^6\ln^2\alpha+6\alpha^6\ln\alpha-\alpha^6+1\right)$$

$$+\frac{c_8}{108}\left(18\alpha^6\ln^2\alpha-6\alpha^6\ln\alpha+\alpha^6-1\right)+\frac{c_9}{36}\left(6\alpha^6\ln\alpha-\alpha^6+1\right)+\frac{c_{10}}{6}\left(\alpha^6-1\right)$$

$$+\frac{c_{11}}{128}(32\alpha^4\ln^3\alpha-24\alpha^4\ln^2\alpha+12\alpha^4\ln\alpha-3\alpha^4+3)+\frac{c_{12}}{32}\left(8\alpha^4\ln^2\alpha-4\alpha^4\ln\alpha+\alpha^4-1\right)$$

$$+\frac{c_{13}}{16}\left(4\alpha^4\ln\alpha-\alpha^4+1\right)+\frac{c_{14}}{4}\left(\alpha^4-1\right)+\frac{c_{15}}{8}\left(4\alpha^2\ln^3\alpha-6\alpha^2\ln^2\alpha+6\alpha^2\ln\alpha-3\alpha^2+3\right)$$

$$+\frac{c_{16}}{4}\left(2\alpha^2\ln^2\alpha-2\alpha^2\ln\alpha+\alpha^2-1\right)+\frac{c_{17}}{4}\left(2\alpha^2\ln\alpha-\alpha^2+1\right)+\frac{c_{18}}{2}\left(\alpha^2-1\right)+\frac{c_{19}}{3}\ln^3\alpha$$

$$+\frac{c_{20}}{2}\ln^2\alpha+c_{21}\ln\alpha-\frac{c_{22}}{4}\left(2\alpha^{-2}\ln\alpha+\alpha^{-2}-1\right)-\frac{c_{23}}{2}\left(\alpha^{-2}-1\right)\ \Big] \tag{28}$$

4. RESULTS AND DISCUSSION

Now let us introduce the nondimensional variable S_θ for the tangential stress $\sigma_{\theta 0}$ of the middle plane of the annular sandwich plate

$$S_\theta=\frac{2ta^2}{D}\sigma_{\theta 0} \tag{29}$$

where

$$\sigma_{\theta 0}=\frac{d}{dr}\left(r\sigma_{r0}\right) \tag{30}$$

Using Eq.(30) and (5), expression (29) becomes

$$S_\theta=\frac{d}{d\rho}\left(\rho S_r\right) \tag{31}$$

Substituting solution (20) into Eq.(31), we obtain the following formula of the nondimensional tangential stress for the first approximation

$$S_\theta=-\alpha_1^{\ 2}W_m^{\ 2}\Big[7b_1\rho^6+b_2\rho^4\left(5\ln\rho+1\right)+5b_3\rho^4+b_4\rho^2\ln\rho(3\ln\rho+2)$$
$$+b_5\rho^2\left(3\ln\rho+1\right)+3b_6\rho^2+b_7\ln\rho\left(\ln\rho+2\right)+b_8\left(\ln\rho+1\right)+b_9$$
$$-b_{10}\rho^{-2}\left(\ln\rho-1\right)-b_{11}\rho^{-2}\Big] \tag{32}$$

Finally, from formulas (20) and (32) we obtain the stresses at the inner and outer edges of the annular sandwich plate

$$S_r(1)=-\alpha_1^{\ 2}W_m^{\ 2}(b_1+b_3+b_6+b_9+b_{11}),$$

$$S_\theta(1)=-\alpha_1^{\ 2}W_m^{\ 2}(7b_1+b_2+5b_3+b_5+3b_6+b_8+b_9+b_{10}-b_{11}),$$

$$S_r(\alpha)=-\alpha_1^{\ 2}W_m^{\ 2}(b_1\alpha^6+b_2\alpha^4\ln\alpha+b_3\alpha^4+b_4\alpha^2\ln^2\alpha+b_5\alpha^2\ln\alpha+b_6\alpha^2+b_7\ln^2\alpha$$
$$+b_8\ln\alpha+b_9+b_{10}\alpha^{-2}\ln\alpha+b_{11}\alpha^{-2}),$$

$$S_\theta(\alpha) = -\alpha_1^{\;2} W_m^{\;2}[7b_1\alpha^6 + b_2\alpha^4(5\ln\alpha + 1) + 5b_3\alpha^4 + b_4\alpha^2\ln\alpha(3\ln\alpha + 2) + b_5\alpha^2(3\ln\alpha + 1)$$
$$+3b_6\alpha^2 + b_7\ln\alpha(\ln\alpha + 2) + b_8(\ln\alpha + 1) + b_9 - b_{10}\alpha^{-2}(\ln\alpha - 1) - b_{11}\alpha^{-2}] \qquad (33)$$

According to the above formulae (27), (20), (32) and (33), the numerical results of nonlinear bending of the annular sandwich plate for various nondimensional characteristic parameter k and nondimensional inner radius α by assuming Poisson's ratio $\nu = 0.3$ are represented graphically in Figs. 2-8.

Figs. 2-3 indicate relations between the nondimensional uniform pressure P and the nondimensional inner edge deflection W_m for several values of k and α respectively. It is obvious that these curves rise monotonically. For the same value of P, the nondimensional inner edge deflection W_m of a annular sandwich plate with small α is larger, and W_m of the plate with small k is low.

The distributions of the nondimensional radial and tangential stresses Sr and S_θ along the nondimensional radius ρ in the case of $\alpha = 0.2$ and $k = 0.05$ are shown in Fig. 4a,b respectively. Obviously, the maximum stress of the annular sandwich plate is the radial stress, and is located at the inner edge of the plate.

Fig. 5 shows the curves for the nondimensional inner edge radial stress $S_r(\alpha)$. It can be seen from the figure that, the curves rise monotonically, and for the same value of the inner edge deflection, the radial stress $S_r(\alpha)$ at the inner edge induced in a plate with larger α is high.

Finally, the results of numerical calculation for the stresses $S_r(1)$, $S_\theta(\alpha)$ and $S_\theta(1)$ of the annular sandwich plate for the several values of α are given in Figs. 6-8 respectively.

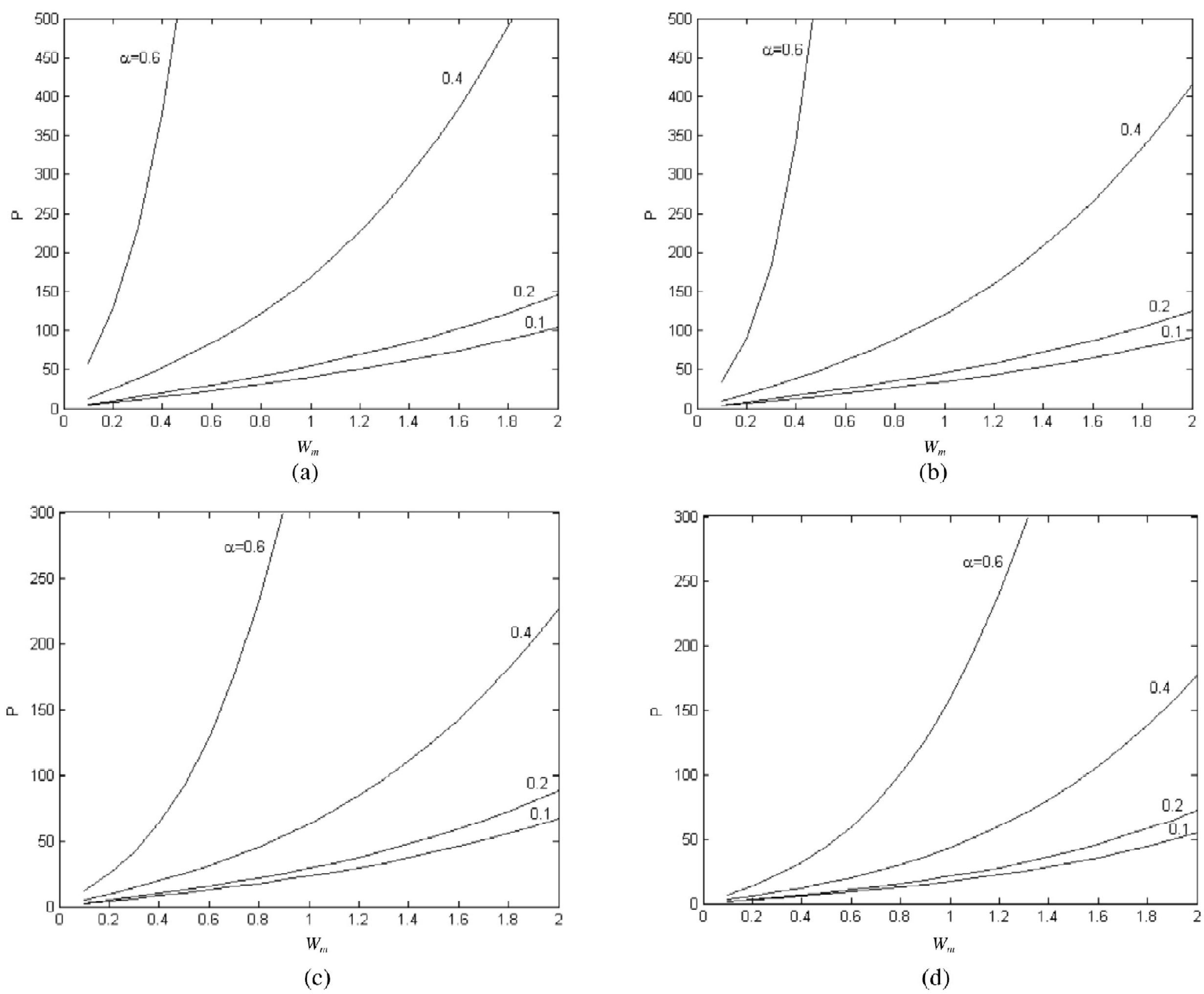

Figure 2: Variation of the Pressure P with the Inner edge Deflection W_m for Several Values of α ($\nu = 0.3$) . (a) $k = 0$, (b) $k = 0.01$, (c) $k = 0.05$, (d) $k = 0.1$

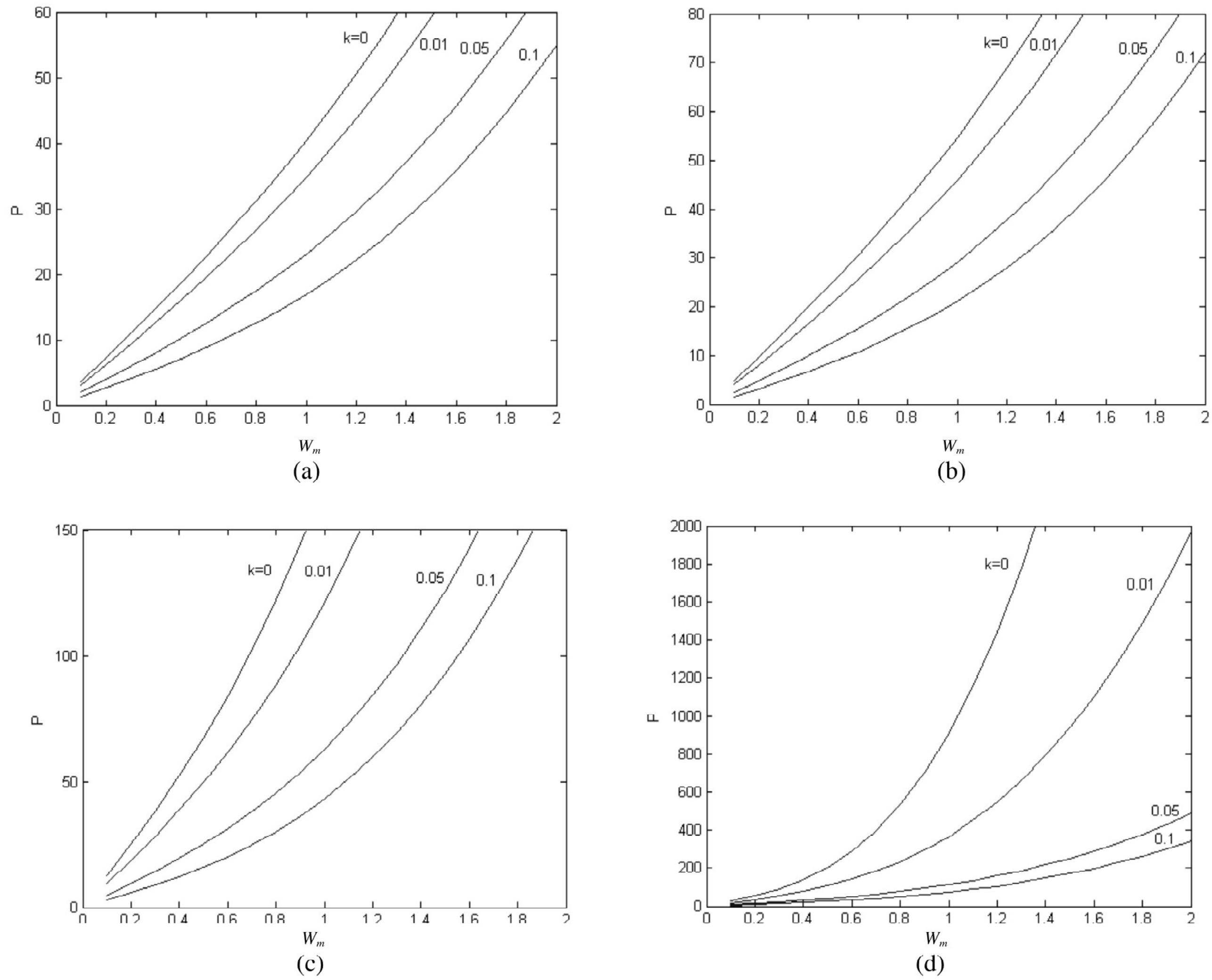

Figure 3: Variation of the Pressure *P* with the Inner Edge Deflection W_m for Several Values of *k* (ν = 0.3) . (a) α = 0.1, (b) α = 0.2, (c) α = 0.4, (d) α = 0.6

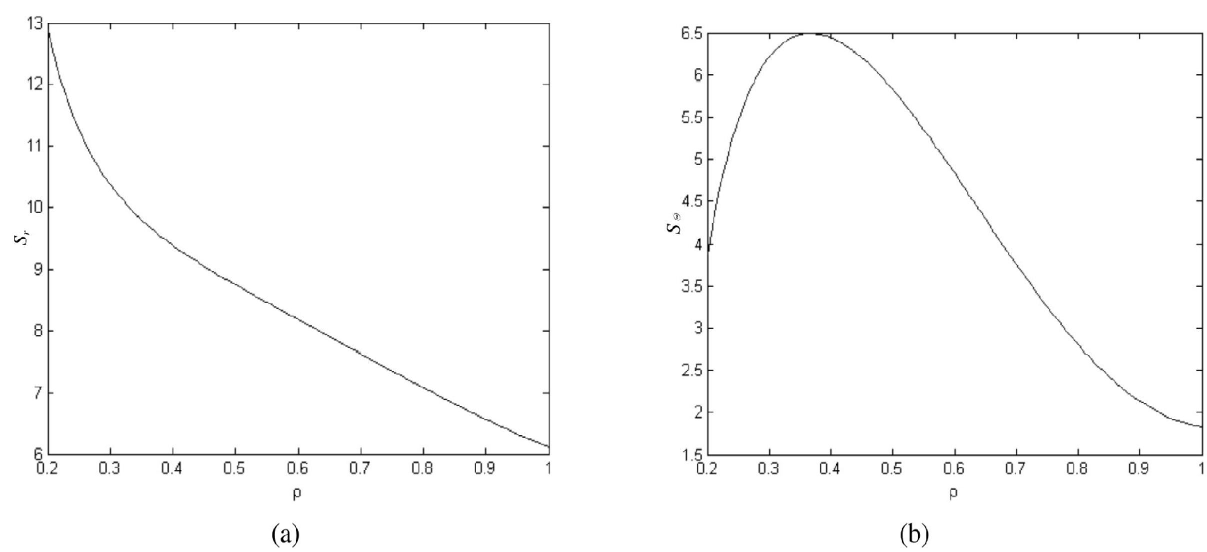

Figure 4: The Variation of the Radial and Tangential Stress S_r, S_θ alone the radius ρ (ν = 0.3, α = 0.2, *k* = 0.05) (a) S_r, (b) S_θ

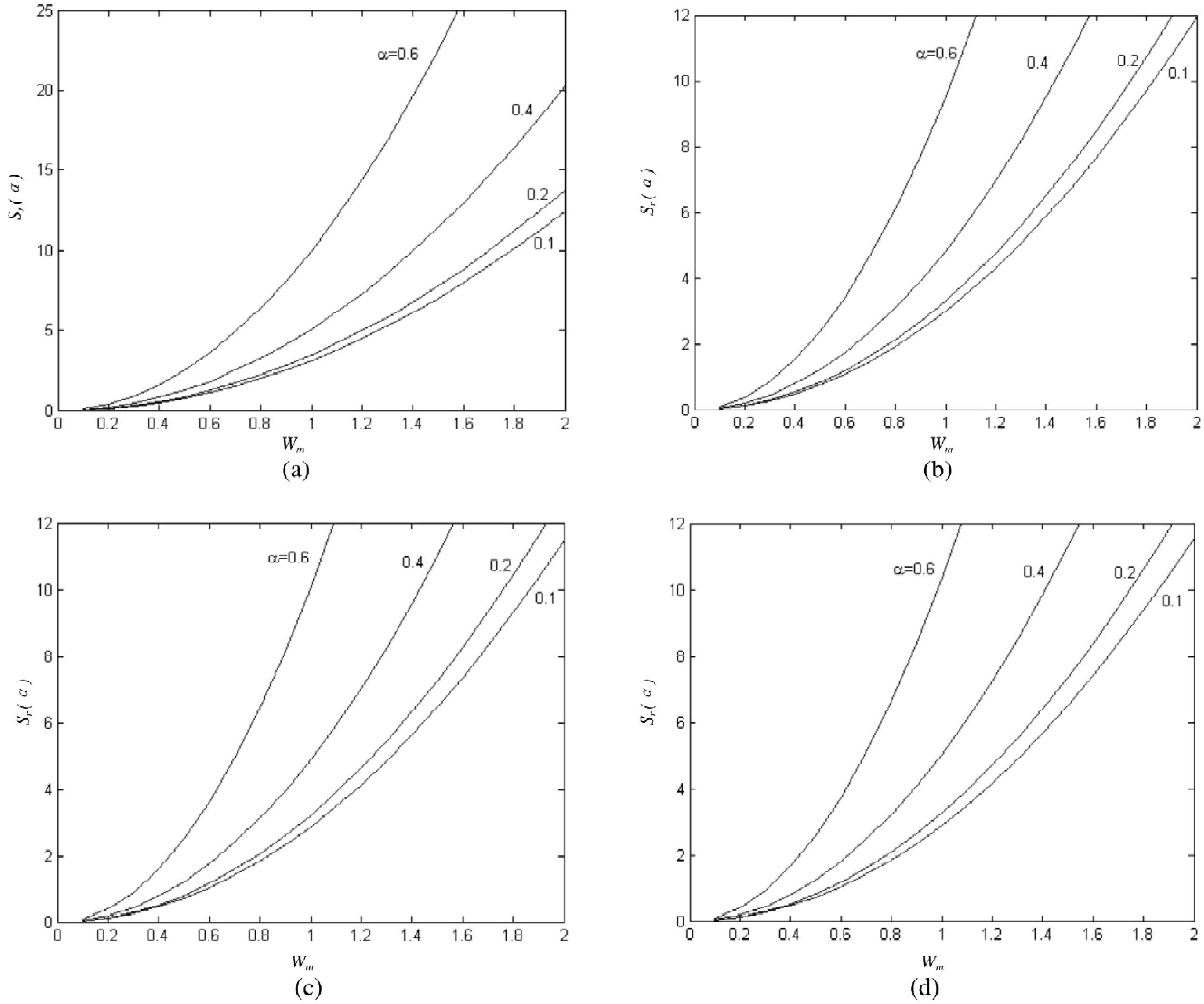

Figure 5: The Radial Stress $S_r(\alpha)$ at the Inner Edge of the Annular Sandwich Plate for Several Values of α (ν = 0.3) (a) k = 0, (b) k = 0.01, (c) k = 0.05, (d) k = 0.1

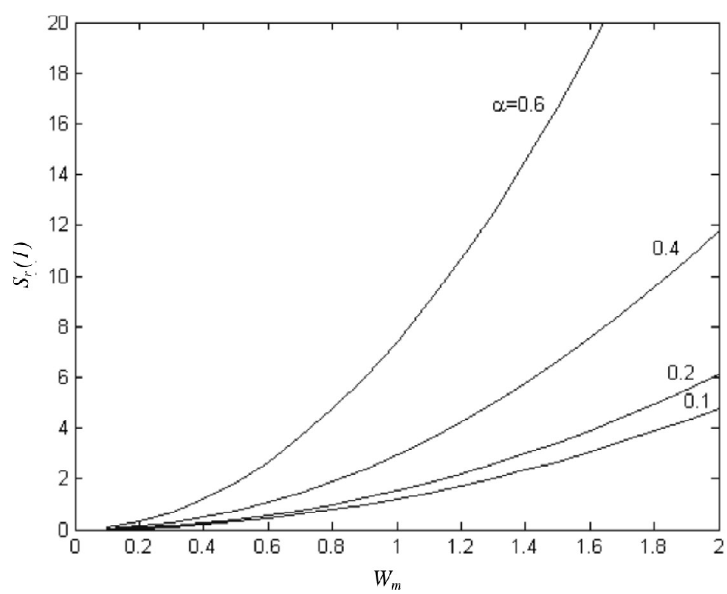

Figure 6: The Radial Stress $S_r(1)$ at the Outer Edge of the Annular Sandwich Plate for Several Values of α (ν = 0.3, k = 0.05)

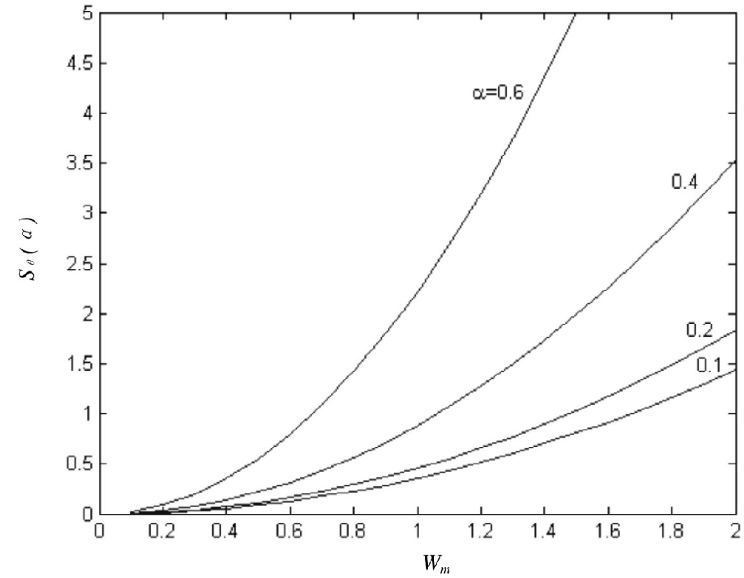

Figure 7: The Tangential Stress $S_\theta(\alpha)$ at the Inner Edge of the Annular Sandwich Plate for Several Values of $\alpha(\nu = 0.3, k = 0.05)$

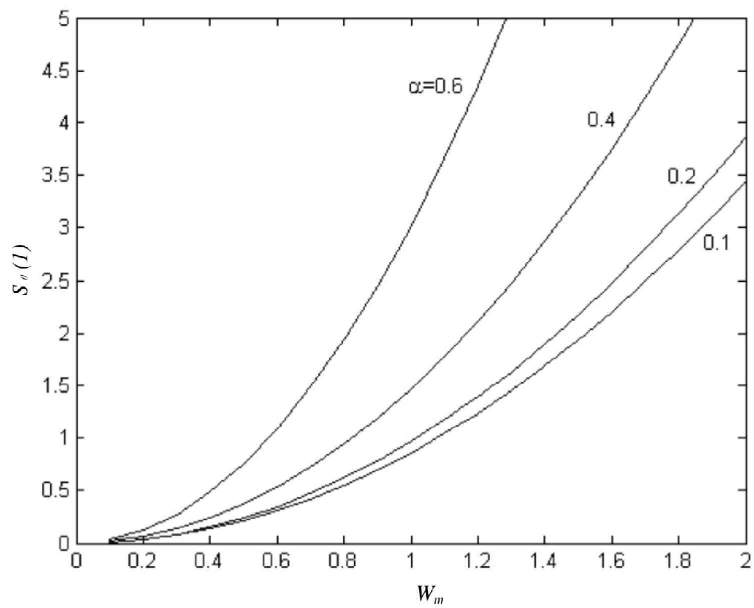

Figure 8: The Tangential Stress $S_\theta(1)$ at the Outer Edge of the Annular Sandwich Plate for Several Values of α $(\nu = 0.3, k = 0.05)$

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References

[1] Resissner, E.. Finite Deflection of Sandwich Plates[J], *Journal of the Aeronautical Sciences*, 1948, **15**(7): 435-440; 1950, **17**(2): 125-130.

[2] Chinese Research Institute of Mechanics, Bending, Stability and Vibration of Sandwich Plates and Shells[M], Science Press, Beijing, 1972, 42-47. (in Chinese).

[3] Liu Ren-huai. Nonlinear Axisymmetrical Bending of Circular Sandwich Plates under the Action of Uniform Edge Moment[J], *Journal of the China University of Science and Technology*, 1980, **10**(2): 56-67. (in Chinese).

[4] Yeh Kai-yuan, Liu Ren-huai, Ping Qing-yuan and Li si-lai. Nonlinear Stability of thin Circular Shallow Spherical Shell under Actions of Axisymmetric Uniform Distributed Line Loads[J], *Bulletin of Science*, 1965, **2**: 142-145. (in Chinese).

[5] Yeh Kai-yuan, Liu Ren-huai, Zhang Chuan-zhe and Xu Yi-fan. Nonlinear Stability of thin Circular Shallow Spherical Shell under the Action of Uniform Edge Moment[J], *Bulletin of Science*, 1965, **2**: 145-147. (in Chinese).

[6] Liu Ren-huai. Nonlinear Stability of Circular Shallow Spherical Shell with a Hole in the Center under the Action of Uniform Moment at the Inner Edge[J], *Bulletin of Science*, 1965, **3**: 253-255. (in Chinese).

[7] Chien Wei-zang. Large Deflection of a Circular Clamped Plate under Uniform Pressure[J], *Chinese Journal of Physics*, 1947, **7**(2): 102-113.

[8] Liu Ren-huai. Nonlinear Bending of Circular Sandwich Plates[J], *Applied Mathmatics and Mechanics*, 1981, **2**(2): 189-208.

[9] Liu Ren-huai and Zhu Gao-qiu. Further Study on Large Deflection of Circular Sandwich Plates[J], *Applied Mathematics and Mechanics,* 1989, **10**(12): 1099-1106.

[10] Liu Ren-huai and Shi Yun-fan. Exact Solution for Circular Sandwich Plate with Large Deflection[J], *Applied Mathematics and Mechanics*, 1982, **3**(1): 11-24.

[11] Liu Ren-huai. Nonlinear Bending of Circular Sandwich Plates under the Action of Axisymmetric Uniformly Distributed Line Loads[J], *Progress in Applied Mechanics(Edited by Yeh Kai-yuan)*, Martinus Nijhoff Publishers, Dordrecht, 1987, 293-321.

[12] Liu Ren-huai, Zhu Jin-fu and Zhang Xiao-guo. Nonlinear Bending of Annular Sandwich Plates[J], *Journal of Jinan University*, 1997, **18**(1): 1-11. (in Chinese).

[13] Xu Jia-chu, Wang Cheng and Liu Ren-huai. Nonlinear Bending of Annular Sandwich Plates with Variable Thickness[J], *Engineering Mechanics*, 2001, **18**(4): 28-37.(in Chinese).

[14] Liu Ren-huai. Study on Nonlinear Mechanics of Plates and Shells[M], Science *Press and Jinan University Pres*s, New York, Beijing and Guangzhou, 1998.

[15] Liu Ren-huai. Nonlinear Theroy and Analysis of Sandwich Plates and Shells[M], *Jinan University Press,* Guangzhou, 2007. (in Chinese).

[16] Du Guojun and Li Huijian. Nonlinear Vibration of Circular Sandwich Plate under the Uniformed Load[J], *Applied Mathmatics and Mechanics*, 2000, **21**(2): 217-226.

[17] Du Guojun and Ma Jianqing. Nonlinear Vibration of Circular Sandwich Plates Under Circumjacent Load[J], *Applied Mathmatics and Mechanics*, 2006, **27**(10): 1417-1424.

[18] Du Guojun and Ma Jianqing. Nonlinear Vibration and Buckling of Circular Sandwich Plates under Complex Load[J], *Applied Mathmatics and Mechanics*, 2007, **28**(8): 1081-1091.

[19] Du Guojun, Zhang Xiuli and Hu Yuda. Nonlinear Vibration and Solution Stability[J], *Journal of Vibration and Shock*, 2007, **26**(11): 156-159. (in Chinese).

[20] Du Guojun, Hu Yuda and Zhang Xiuli. Time Domain Characteristic Analysis of Large Amplitude Vibration on Circular Sandwich Plate under Static Load[J], *Engineering Mechanics*, 2008, **25**(4): 39-44. (in Chinese).

[21] Yang Jingning, Zhao Yonggang and Ma Liansheng. Numerical Solution for the Shooting Method of the Large Deflection of Sandwich Plates[J], *Journal of Lanzhou Railway University*, 2001, **20**(6): 111-114. (in Chinese).

[22] Yang Jingning, Zhao Yonggang, Qin Ping and Liu Caixue. Geometrically Nonlinear Deformation of Circular Sandwich Plates under Transversely Non-uniform Temperature Rise[J], *Key Engineering Materials*, 2007, 353-358(2): 1161-1164.

[23] Hou Chaosheng, Zhang Shoukai and Lin Feng. Cublic Spline Solutions of Axisymmetrical Nonlinear Bending and Buckling of Circular Sandwich Plates[J], *Applied Methmatics and Mechanics*, 2005, **26**(1): 131-138.

[24] Hou Chaosheng, Zhou Chengxiang and Yang Lijun. Axisymmetrical Nonlinear Bending and Buckling of Annular Sandwich Plates[J], *Journal of Tianjin University*, 2006, **39**(11): 1293-1298. (in Chinese).

[25] Zhang Xiuli. The Effect of Initial Deflection in Nonlinear Vibration Property of Circular Sandwich Plate[J], *Science and Technology Information*, 2007, 29: 64,83-84. (in Chinese).

[26] Kirichok, I. F., Karnaukhov, M. V. Single-frequency Vibrations and Vibrational Heating of a Piezoelectric Circular Sandwich Plate under Monoharmonic Electromechanical Loading [J], *International Applied Mechanics*, 2008, **44**(1): 65-72.

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