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A New Method To Find Octagonal Fuzzy Number (OFN)

P. Selvam* A.Rajkumar and J.Sudha Easwari *****

Abstract : In this paper we define Octagonal Fuzzy Number and also arithmetic operations like addition, subtraction, multiplication, division, inverse integral, maximum, minimum, scalar multiplication and symmetric image are defined with numerical examples. This is the alternate method to find octagonal fuzzy number with trapezoidal shape.

Keywords : Fuzzy numbers, Triangle Fuzzy Number, Octagonal Fuzzy Number, Fuzzy Arithmetic Operations, Alpha Cut.

1. INTRODUCTION

In 1965 fuzzy set theory was introduced by Lotfi.A.Zadeh and it is used to assess the graded membership function described in the interval [0,1]. But it is incomplete and imprecise. Interval arithmetic was developed by Dwyer in 1951. After that in 1978 D.Dubois and H.Prade defined any of the fuzzy numbers as a fuzzy subset of the line. A fuzzy number is a quantity whose value is precise rather than exact as in the case with ordinary single valued numbers. Based on the membership function, fuzzy numbers are classified into triangular fuzzy number, trapezoidal fuzzy number , interval value etc., fuzzy numbers are used to present real numbers in a fuzzy environment for analyzing fuzziness and fuzzy data that have various application as in linguistic, control, database system. The concept of octagonal fuzzy numbers was introduced in [1]. In this paper we introduced an another method to find octagonal fuzzy numbers for trapezoidal shape along with arithmetic operations like addition, subtraction, multiplication, division, inverse integral, maximum, minimum, scalar multiplication and symmetric image with numerical examples

2. PRELIMINARIES AND NOTATIONS

2.1. Definition: Fuzzy set

A fuzzy set is characterized by a membership function mapping the elements of a domain, space or universe of discourse X to the unit interval [0, 1].

A fuzzy set A in a universe of discourse X is defined as the following set of pairs :

$$A = \{(x, \mu_A(x))); x \in X\}.$$

Here $\mu_A : X \to [0, 1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \in X$ in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0, 1].

* Research scholar, Hindustan Institute of Technology and Science, Chennai -603 103, Tamil Nadu, India . & Assistant professor, Adithya Institute of Technology, Coimbatore-641 107, Tamil Nadu, India. thiruvibhaa@gmail.com.

** Assistant professor, Hindustan Institute of Technology and Science, Chennai -603 103, Tamil Nadu, India. arajkumar@hindustanuniv.ac.in

*** B.T Asistant. Govt. Hr. Sec. School, Thirupur- 641 606,Tamil Nadu, India. sarvapithama@gmail.com.

2.2 Definition : α-cut set [2][3]

The α -cut set A_α is made up of members whose membership is not less than α. $A_\alpha = \{x \in X \mid \mu_A(x) \geq \alpha\}$ note that α is arbitrary. This α -cut set is a crisp set.

For all x, $\alpha \in [0, 1]$ the upper α-cut of x is denoted as $x(\alpha)$ and is defined as

$$x^\alpha = \begin{cases} 1 & \text{if} \quad x \geq \alpha \\ 0 & \text{if} \quad x < \alpha \end{cases}$$ and the lower α-cut of x is denoted as $x(\alpha)$ and is defined as

$$x_\alpha = \begin{cases} x & \text{if} \quad x \geq \alpha \\ 0 & \text{if} \quad x < \alpha \end{cases}$$

2.3. Definition(Fuzzy number)

A fuzzy set A defined on the set of real numbers R is said to be a fuzzy number if its membership function $A : \Re \longrightarrow [0,1]$ has the following definitions

- **Convex fuzzy set :** A fuzzy set A of X is called convex if $^\alpha[A]$ is a convex subset of X, $\forall\, \alpha \in [0, 1]$
- **Normal fuzzy set:** A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that there exists at least one $x \in X$ such that $\mu_A(x) = 1$.
- A is membership function is piecewise continuous.
- A is defined in the real number

2.4. Definition (Octagonal Fuzzy Numbers) [1]

A fuzzy number $\tilde{A}$ is a normal octagonal fuzzy number denoted by $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ where $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8$ are real numbers and its membership function $\mu_{\tilde{A}}(x)$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} k\left(\dfrac{x - a_1}{a_2 - a_1}\right) & a_1 \leq x \leq a_2 \\ k & a_2 \leq x \leq a_3 \\ k + (1-k)\left(\dfrac{x - a_3}{a_4 - a_3}\right) & a_3 \leq x \leq a_4 \\ 1 & a_4 \leq x \leq a_5 \\ k + (1-k)\left(\dfrac{a_6 - x}{a_6 - a_5}\right) & a_5 \leq x \leq a_6 \\ k & a_6 \leq x \leq a_7 \\ k\left(\dfrac{a_8 - x}{a_8 - a_7}\right) & a_7 \leq x \leq a_8 \\ 0 & x < a_1, x > a_8 \end{cases}$$

Where $0 < k < 1$

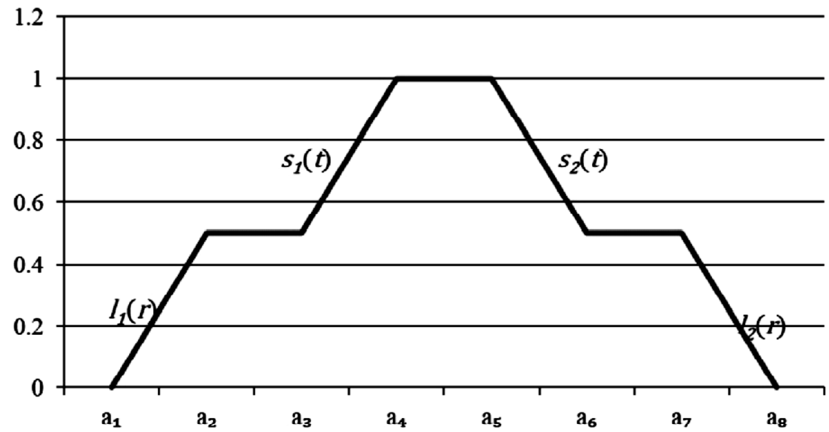

Fig. 1. Graphical representation of a normal octagonal fuzzy number for k = 0.5.

3. NEW OCTAGONAL FUZZY NUMBERS(OFN)

Definition 3.1

A fuzzy number $A_{\overline{OFN}}$ is a octagonal fuzzy number denoted by $A_{\overline{OFN}} = (a_1, a_2, a_{3,}\, a_4, a_5, a_6, a_7, a_8)$ where $a_1, a_2, a_{3,}\, a_4, a_5, a_6, a_7, a_8$ are real numbers and its membership function.

$$\mu_{\overline{OFN}}(x) = \begin{cases} \frac{1}{3}\left(\frac{x-a_1}{a_2-a_1}\right) & a_1 \le x \le a_2 \\ \frac{1}{3}+\frac{1}{3}\left(\frac{x-a_2}{a_3-a_2}\right) & a_2 \le x \le a_3 \\ \frac{2}{3}+\frac{1}{3}\left(\frac{x-a_3}{a_4-a_3}\right) & a_3 \le x \le a_4 \\ 1 & a_4 \le x \le a_5 \\ 1-\frac{1}{3}\left(\frac{x-a_5}{a_6-a_5}\right) & a_5 \le x \le a_6 \\ \frac{2}{3}-\frac{1}{3}\left(\frac{x-a_6}{a_7-a_6}\right) & a_6 \le x \le a_7 \\ \frac{1}{3}\left(\frac{a_8-x}{a_8-a_7}\right) & a_7 \le x \le a_8 \\ 0 & x < a_1, x > a_8 \end{cases}$$

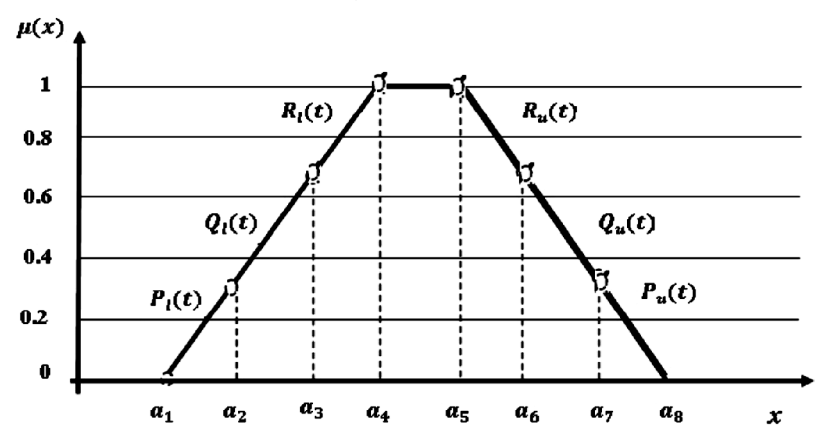

Fig. 2. Graphical representation of a normal octagonal fuzzy numbers for $x \in$ [0, 1].

4. OPERATIONS OF OCTAGONAL FUZZY NUMBERS

Definition 4.1.

Let $\mathrm{A}_{\overline{\mathrm{OFN}}} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ and $\mathrm{B}_{\overline{\mathrm{OFN}}} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$ be their corresponding octagonal fuzzy numbers then

1. **Addition :** $\mathrm{A}_{\overline{\mathrm{OFN}}} + \mathrm{B}_{\overline{\mathrm{OFN}}} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6, a_7 + b_7, a_8 + b_8)$
2. **Subtraction :** $\mathrm{A}_{\overline{\mathrm{OFN}}} - \mathrm{B}_{\overline{\mathrm{OFN}}} = (a_1 - b_8, a_2 - b_7, a_3 - b_6, a_4 - b_5, a_5 - b_4, a_6 - b_3, a_7 - b_2, a_8 - b_1)$
3. **Multiplication :** $$\mathrm{A}_{\overline{\mathrm{OFN}}} * \mathrm{B}_{\overline{\mathrm{OFN}}} = \begin{pmatrix} (a_1*b_1 \wedge a_1*b_8 \wedge a_8*b_1 \wedge a_8*b_8),(a_2*b_2 \wedge a_2*b_7 \wedge a_7*b_2 \wedge a_7*b_7), \\ (a_3*b_3 \wedge a_3*b_6 \wedge a_6*b_3 \wedge a_6*b_6),(a_4*b_4 \wedge a_4*b_5 \wedge a_5*b_4 \wedge a_5*b_5), \\ (a_4*b_4 \vee a_4*b_5 \vee a_5*b_4 \vee a_5*b_5),(a_3*b_3 \vee a_3*b_6 \vee a_6*b_3 \vee a_6*b_6), \\ (a_2*b_2 \vee a_2*b_7 \vee a_7*b_2 \vee a_7*b_7),(a_1*b_1 \vee a_1*b_8 \vee a_8*b_1 \vee a_8*b_8) \end{pmatrix}$$
4. **Division :** $$\mathrm{A}_{\overline{\mathrm{OFN}}}\,(/)\,\mathrm{B}_{\overline{\mathrm{OFN}}} = \begin{pmatrix} \left(\frac{a_1}{b_1} \wedge \frac{a_1}{b_8} \wedge \frac{a_8}{b_1} \wedge \frac{a_8}{b_8}\right), \left(\frac{a_2}{b_2} \wedge \frac{a_2}{b_7} \wedge \frac{a_7}{b_2} \wedge \frac{a_7}{b_7}\right) \cdots \left(\frac{a_4}{b_4} \wedge \frac{a_4}{b_5} \wedge \frac{a_5}{b_4} \wedge \frac{a_5}{b_5}\right), \\ \left(\frac{a_4}{b_4} \vee \frac{a_4}{b_5} \vee \frac{a_5}{b_4} \vee \frac{a_5}{b_5}\right) \cdots \left(\frac{a_2}{b_2} \vee \frac{a_2}{b_7} \vee \frac{a_7}{b_2} \vee \frac{a_7}{b_7}\right), \left(\frac{a_1}{b_1} \vee \frac{a_1}{b_8} \vee \frac{a_8}{b_1} \vee \frac{a_8}{b_8}\right) \end{pmatrix}$$
   Excluding the case $b_1 = 0$ or $b_2 = 0$ or $b_3 = 0$ or $b_4 = 0$ or $b_5 = 0$ or $b_6 = 0$ or $b_7 = 0$ or $b_8 = 0$
5. **Inverse interval :** $$[\mathrm{A}_{\overline{\mathrm{OFN}}}]^{-1} = \left(\left(\frac{1}{a_1} \wedge \frac{1}{a_8}\right), \left(\frac{1}{a_2} \wedge \frac{1}{a_7}\right), \ldots \left(\frac{1}{a_4} \wedge \frac{1}{a_5}\right), \left(\frac{1}{a_4} \vee \frac{1}{a_5}\right), \ldots \left(\frac{1}{a_2} \vee \frac{1}{a_7}\right), \left(\frac{1}{a_1} \vee \frac{1}{a_8}\right)\right)$$
   Excluding the case $a_1 = 0$ or $a_2 = 0$ or $a_3 = 0$ or $a_4 = 0$ or $a_5 = 0$ or $a_6 = 0$ or $a_7 = 0$ or $a_8 = 0$
6. **Minimum :** $\mathrm{A}_{\overline{\mathrm{OFN}}} \wedge \mathrm{B}_{\overline{\mathrm{OFN}}} = [(a_1 \wedge b_1), (a_2 \wedge b_2), (a_3 \wedge b_3), \ldots, (a_7 \wedge b_7), (a_8 \wedge b_8)]$
7. **Maximum :** $\mathrm{A}_{\overline{\mathrm{OFN}}} \vee \mathrm{B}_{\overline{\mathrm{OFN}}} = [(a_1 \vee b_1), (a_2 \vee b_2), (a_3 \vee b_3), \ldots, (a_7 \vee b_7), (a_8 \vee b_8)]$
8. **Scalar Multiplication :** $$c * \mathrm{A}_{\overline{\mathrm{OFN}}} = \begin{cases} c*a_1, c*a_2, \ldots c*a_7, c*a_8 & if \quad c \geq 0 \\ c*a_8, c*a_7, \ldots c*a_2, c*a_1 & if \quad c < 0 \end{cases}$$
9. **Symmetric image :** $(-)\mathrm{A}_{\overline{\mathrm{OFN}}} = (-a_8, -a_7, -a_6, -a_5, -a_4, -a_3, -a_2, -a_1)$

   When previous sets $\mathrm{A}_{\overline{\mathrm{OFN}}}$ and $\mathrm{B}_{\overline{\mathrm{OFN}}}$ is defined in the positive real number $\Re^+$, the operations of multiplication, division, and inverse interval are written as,

   (3') **Multiplication :** $\mathrm{A}_{\overline{\mathrm{OFN}}} * \mathrm{B}_{\overline{\mathrm{OFN}}} = ((a_1 * b_1), (a_2 * b_2), (a_3 * b_3), \ldots, (a_7 * b_7), (a_8 * b_8))$

   (4') **Division:** $$\mathrm{A}_{\overline{\mathrm{OFN}}}(/)\mathrm{B}_{\overline{\mathrm{OFN}}} = \left(\left(\frac{a_1}{b_8}\right), \left(\frac{a_2}{b_7}\right), \left(\frac{a_3}{b_6}\right), \left(\frac{a_4}{b_5}\right), \left(\frac{a_5}{b_4}\right), \left(\frac{a_6}{b_3}\right), \left(\frac{a_7}{b_2}\right), \left(\frac{a_8}{b_1}\right)\right)$$

   (5') **Inverse Interval :** $$[\mathrm{A}_{\overline{\mathrm{OFN}}}]^{-1} = \left(\frac{1}{a_8}, \frac{1}{a_7}, \frac{1}{a_6}, \frac{1}{a_5}, \frac{1}{a_4}, \frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1}\right)$$

Example : 4.1

Let $\mathrm{A}_{\overline{\mathrm{OFN}}} = (3,6,9,12,15,18,21,24)$ and $\mathrm{B}_{\overline{\mathrm{OFN}}} = (2,7,12,17,22,27,32,37)$ be two octagonal fuzzy numbers then

- $\mathrm{A}_{\overline{\mathrm{OFN}}} + \mathrm{B}_{\overline{\mathrm{OFN}}} = (5,13,21,29,37,45,53,61)$
- $\mathrm{A}_{\overline{\mathrm{OFN}}} - \mathrm{B}_{\overline{\mathrm{OFN}}} = (-34,-26,-18,-10,-2,6,14,22)$
- $\mathrm{A}_{\overline{\mathrm{OFN}}} * \mathrm{B}_{\overline{\mathrm{OFN}}} = (6,42,108,204,330,486,672,888)$

- $A_{\overline{OFN}}(/)B_{\overline{OFN}} = (0.0811, 0.1875, 0.3333, 0.5455, 0.8824, 1.5, 3.0, 12.0)$
- $[A_{\overline{OFN}}]^{-1} = (0.0417, 0.0476, 0.0556, 0.0667, 0.0833, 0.1111, 0.1667, 0.3333)$
- $A_{\overline{OFN}}(\wedge)B_{\overline{OFN}} = (2, 6, 9, 12, 15, 18, 21, 24)$
- $A_{\overline{OFN}}(\vee)B_{\overline{OFN}} = (3, 7, 12, 17, 22, 27, 32, 37)$
- $3 * A_{\overline{OFN}} = (9, 18, 27, 36, 45, 54, 63, 72)$
- $-A_{\overline{OFN}} = (-24, -21, -18, -15, -12, -9, -6, -3)$

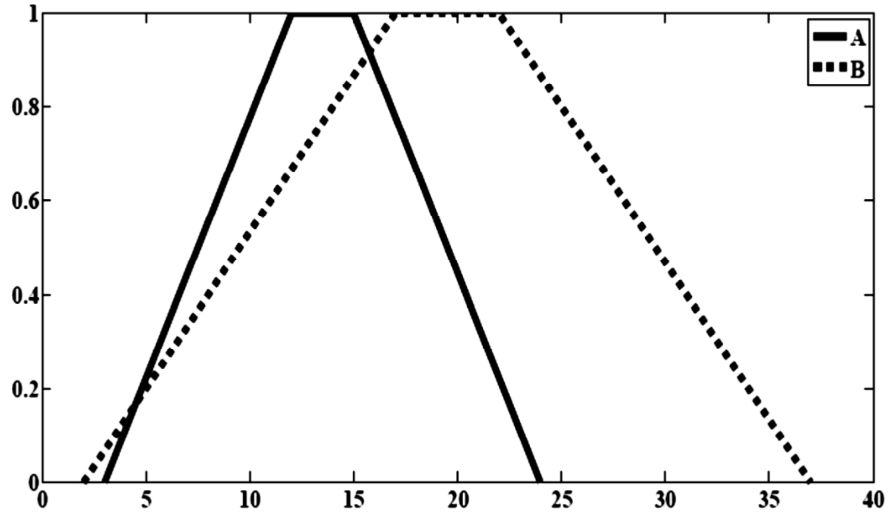

Fig. 3. Two OFN A and B.

Definition 4.2

A Octagonal fuzzy number $\overline{OFN}$ can also be defined as $\overline{OFN} = P_l(t), Q_l(u), R_l(v), P_u(t), Q_u(u), R_u(v)$, here $t \in [0, 0.3333], u \in [0.3333, 0.6666], v \in [0.6666, 1.0000]$ where

$P_l(t) = \frac{1}{3}\left(\frac{x - a_1}{a_2 - a_1}\right)$	$P_u(t) = \frac{1}{3}\left(\frac{a_8 - x}{a_8 - a_7}\right)$
$Q_l(u) = \frac{1}{3} + \frac{1}{3}\left(\frac{x - a_2}{a_3 - a_2}\right)$	$Q_u(u) = \frac{2}{3} - \frac{1}{3}\left(\frac{x - a_6}{a_7 - a_6}\right)$
$R_l(v) = \frac{2}{3} + \frac{1}{3}\left(\frac{x - a_3}{a_4 - a_3}\right)$	$R_u(v) = 1 - \frac{1}{3}\left(\frac{x - a_5}{a_6 - a_5}\right)$

$P_l(t), Q_l(u), R_l(v)$ is bounded and continuous increasing function over [0,0.3333], [0.3333,0.6666] and [0.6666,1.0] respectively.

$P_u(t), Q_u(u), R_u(v)$ is bounded and continuous decreasing function over [0,0.3333], [0.3333,0.6666] and [0.6666,1.0] respectively.

Definition 4.3

The α – cut of the fuzzy set of the universe of discourse X is defined as $\overline{OFN} = \{x \in X / \mu_{\bar{A}}(x) \geq \alpha\}$ where $\alpha \in [0,1]$

$$\overline{\text{OFN}}_\alpha = \begin{cases} [P_l(\alpha), P_u(\alpha)] & \text{for} \quad \alpha \in [0.0000, 0.3333) \\ [Q_l(\alpha), Q_u(\alpha)] & \text{for} \quad \alpha \in [0.3333, 0.6666) \\ [R_l(\alpha), R_u(\alpha)] & \text{for} \quad \alpha \in [0.6666, 1.0000] \end{cases}$$

Definition 4.4

If $P_l(\alpha) = \alpha$ and $P_u(\alpha) = \alpha$, then α – cut operations interval $\overline{\text{OFN}}_\alpha$ is obtained as

- $[P_l(\alpha), P_u(\alpha)] = [3\alpha(a_2 - a_1) + a_1, 3\alpha(a_7 - a_8) + a_8]$

  Similarly we can obtain α – cut operation interval $\overline{\text{OFN}}_\alpha$ for $[Q_l(\alpha), Q_u(\alpha)]$ and $[R_l(\alpha), R_u(\alpha)]$ as follows:

- $[Q_l(\alpha), Q_u(\alpha)] = [3\alpha(a_3 - a_2) + 2a_2 - a_3, 3\alpha(a_6 - a_7) + 2a_7 - a_6]$
- $[R_l(\alpha), R_u(\alpha)] = [3\alpha(a_4 - a_3) + 3a_3 - 2a_4, 3\alpha(a_5 - a_6) + 3a_6 - 2a_5]$

Hence α – cut of Octagonal fuzzy number

$$\overline{\text{OFN}}_\alpha = \begin{cases} [3\alpha(a_2 - a_1) + a_1, 3\alpha(a_7 - a_8) + a_8] & \text{for} \quad \alpha \in [0.0000, 0.3333) \\ [3\alpha(a_3 - a_2) + 2a_2 - a_3, 3\alpha(a_6 - a_7) + 2a_7 - a_6] & \text{for} \quad \alpha \in [0.3333, 0.6666) \\ [3\alpha(a_4 - a_3) + 3a_3 - 2a_4, 3\alpha(a_5 - a_6) + 3a_6 - 2a_5] & \text{for} \quad \alpha \in [0.6666, 1.0000] \end{cases}$$

5. A NEW OPERATION FOR ADDITION, SUBTRACTION, MULTIPLICATION, DIVISION, INVERSE, MAXIMUM, MINIMUM, SCALAR MULTIPLICATION AND SYMMETRIC IMAGE ON OCTAGONAL FUZZY NUMBER

5.1. Addition Of Two Octagonal Fuzzy Numbers

Definition 5.1.

Let $A_{\overline{\text{OFN}}} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ and $B_{\overline{\text{OFN}}} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$ be their corresponding octagonal fuzzy numbers for all $\alpha \in [0,1]$. Let us add α – cut ${}^\alpha A$ and ${}^\alpha B$ of $A_{\overline{\text{OFN}}}$ and $B_{\overline{\text{OFN}}}$ using interval arithmetic.

$$A_{\overline{\text{OFN}}} + B_{\overline{\text{OFN}}} = \begin{cases} \begin{pmatrix} [3\alpha((a_2 + b_2) - (a_1 + b_1)) + (a_1 + b_1), \\ 3\alpha((a_7 + b_7) - (a_8 + b_8)) + (a_8 + b_8)] \end{pmatrix} & \text{for} \quad \alpha \in [0.0000, 0.3333) \\ \begin{pmatrix} [3\alpha((a_3 + b_3) - (a_2 + b_2)) + 2(a_2 + b_2) - (a_3 + b_3,), \\ 3\alpha((a_6 + b_6) - (a_7 + b_7)) + 2(a_7 + b_7) - (a_6 + b_6)] \end{pmatrix} & \text{for} \quad \alpha \in [0.3333, 0.6666) \\ \begin{pmatrix} [3\alpha((a_4 + b_4) - (a_3 + b_3)) + 3(a_3 + b_3) - 2(a_4 + b_4,), \\ 3\alpha((a_5 + b_5) - (a_6 + b_6)) + 3(a_6 + b_6) - 2(a_5 + b_5)] \end{pmatrix} & \text{for} \quad \alpha \in [0.6666, 1.0000] \end{cases}$$

To verify this new operation with addition operation, we take the arithmetic example 4.1 $A_{\overline{\text{OFN}}} = (3,6,9,12,15,18,21,24)$ and $B_{\overline{\text{OFN}}} = (2,7,12,17,22,27,32,37)$

For $\alpha \in [0.0000, 0.3333)$	${}^\alpha A_{\overline{\text{OFN}}} = (9\alpha + 3, -9\alpha + 24)$	${}^\alpha(A+B)_{\overline{\text{OFN}}} = (24\alpha + 5, -24\alpha + 61)$
For $\alpha \in [0.3333, 0.6666)$		
For $\alpha \in [0.6666, 1.0000]$	${}^\alpha B_{\overline{\text{OFN}}} = (15\alpha + 2, -15\alpha + 37)$	

Since for $\alpha \in [0.0000, 0.3333)$, $\alpha \in [0.3333, 0.6666)$ and $\alpha \in [0.6666, 1.0000]$ arithmetic intervals are same ${}^\alpha(A+B)_{\overline{\text{OFN}}} = (24\alpha + 5, -24\alpha + 61)$ for all $\alpha \in [0,1]$

when
$$\alpha = 0.0000 \Rightarrow {}^{\alpha}(A+B) = [5,61]$$
$$\alpha = 0.3333 \Rightarrow {}^{\alpha}(A+B) = [13,53]$$
$$\alpha = 0.6666 \Rightarrow {}^{\alpha}(A+B) = [21,45]$$
$$\alpha = 1.0000 \Rightarrow {}^{\alpha}(A+B) = [29,37]$$

Hence ${}^{\alpha}(A+B) = (5,13,21,29,37,45,53,61)$ hence all the points coincide with the addition of the two octagonal fuzzy number.

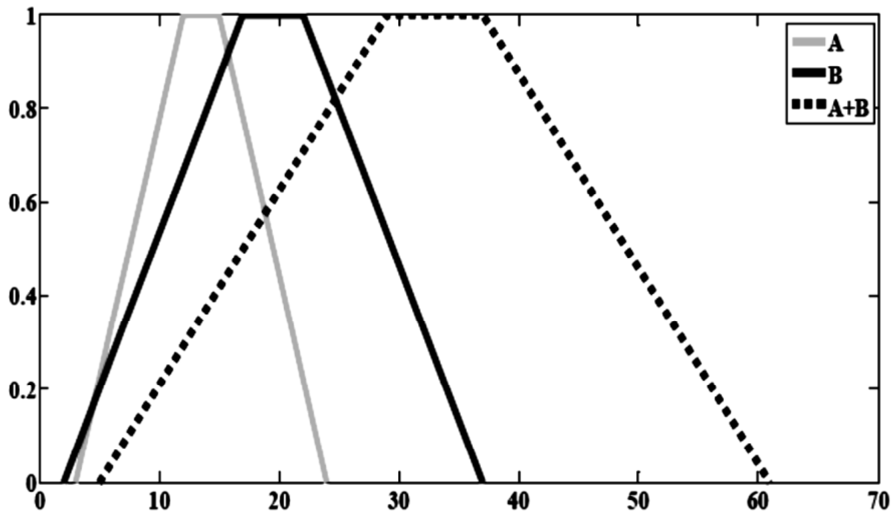

Fig. 4. Addition of two OFN A and B.

Definition 5.2

Let $A_{\overline{OFN}} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ and $B_{\overline{OFN}} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$ be their corresponding octagonal fuzzy numbers for all $\alpha \in [0,1]$. Let us subtract $\alpha -$ cut ${}^{\alpha}A$ and ${}^{\alpha}B$ of $A_{\overline{OFN}}$ and $B_{\overline{OFN}}$ using interval arithmetic.

$$A_{\overline{OFN}}(-)B_{\overline{OFN}} = \begin{cases} \begin{pmatrix} [3\alpha((a_2+b_8)-(a_1+b_7))+(a_1-b_8), \\ 3\alpha((a_7+b_1)-(a_8+b_2))+(a_8-b_1)] \end{pmatrix} & \text{for} \quad \alpha \in [0.0000, 0.3333) \\ \begin{pmatrix} [3\alpha((a_3+b_7)-(a_2+b_6))+2(a_2-b_7)-(a_3-b_6), \\ 3\alpha((a_6+b_2)-(a_7+b_3))+2(a_7-b_2)-(a_6-b_3)] \end{pmatrix} & \text{for} \quad \alpha \in [0.3333, 0.6666) \\ \begin{pmatrix} [3\alpha((a_4+b_6)-(a_3+b_5))+3(a_3-b_6)-2(a_4-b_5), \\ 3\alpha((a_5+b_3)-(a_6+b_4))+3(a_6-b_3)-2(a_5-b_4)] \end{pmatrix} & \text{for} \quad \alpha \in [0.6666, 1.0000] \end{cases}$$

To verify this new operation with subtract operation, we take the arithmetic example 4.1 $A_{\overline{OFN}} = (3,6,9,12,15,18,21,24)$ and $B_{\overline{OFN}} = (2,7,12,17,22,27,32,37)$

For $\alpha \in [0.0000, 0.3333)$	${}^{\alpha}A_{\overline{OFN}} = (9\alpha+3, -9\alpha+24)$	
For $\alpha \in [0.3333, 0.6666)$	${}^{\alpha}B_{\overline{OFN}} = (15\alpha+2, -15\alpha+37)$	${}^{\alpha}(A-B)_{\overline{OFN}} = (24\alpha-34, -24\alpha+22)$
For $\alpha \in [0.6666, 1.0000]$		

Since for $\alpha \in [0.0000, 0.3333)$, $\alpha \in [0.3333, 0.6666)$ and $\alpha \in [0.6666, 1.0000]$ arithmetic intervals are same ${}^{\alpha}(A-B)_{\overline{OFN}} = (24\alpha-34, -24\alpha+22)$ for all $\alpha \in [0,1]$

when

$$\alpha = 0.0000 \quad \Rightarrow \quad {}^{\alpha}(\mathrm{A}-\mathrm{B}) = [-34, 22]$$
$$\alpha = 0.3333 \quad \Rightarrow \quad {}^{\alpha}(\mathrm{A}-\mathrm{B}) = [-26, 14]$$
$$\alpha = 0.6666 \quad \Rightarrow \quad {}^{\alpha}(\mathrm{A}-\mathrm{B}) = [-18, 6]$$
$$\alpha = 1.0000 \quad \Rightarrow \quad {}^{\alpha}(\mathrm{A}-\mathrm{B}) = [-10, -2]$$

Hence ${}^{\alpha}(\mathrm{A}-\mathrm{B}) = (-34, -26, -18, -10, -2, 6, 14, 22)$. Hence all the points coincide with the subtraction of the two octagonal fuzzy number.

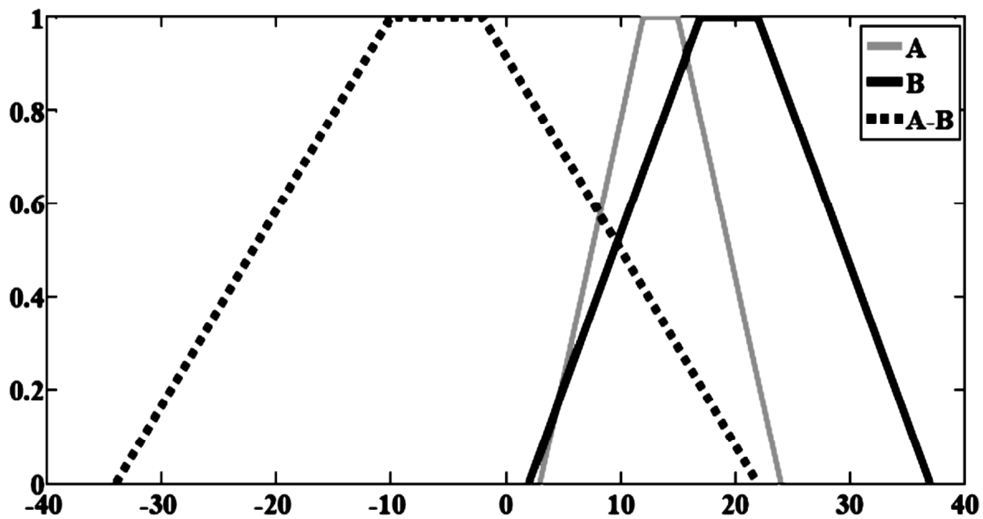

Fig. 5. Subtraction of two OFN A and B.

Definition 5.3

Let $\mathrm{A}_{\overline{\mathrm{OFN}}} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ and $\mathrm{B}_{\overline{\mathrm{OFN}}} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$ be their corresponding octagonal fuzzy numbers for all $\alpha \in [0,1]$. Let us multiply $\alpha-$ cut ${}^{\alpha}$A and ${}^{\alpha}$B of $\mathrm{A}_{\overline{\mathrm{OFN}}}$ and $\mathrm{B}_{\overline{\mathrm{OFN}}}$ using interval arithmetic.

$$\mathrm{A}_{\overline{\mathrm{OFN}}}(-)\mathrm{B}_{\overline{\mathrm{OFN}}} = \begin{cases} \begin{pmatrix} [3\alpha(a_2-a_1)+a_1, 3\alpha(a_7-a_8)+a_8]* \\ [3\alpha(b_2-b_1)+b_1, 3\alpha(b_7-b_8)+b_8] \end{pmatrix} & \textit{for} \quad \alpha \in [0.0000, 0.3333) \\ \begin{pmatrix} [3\alpha(a_3-a_2)+2a_2-a_3, 3\alpha(a_6-a_7)+2a_7-a_6]* \\ [3\alpha(b_3-b_2)+2b_2-b_3, 3\alpha(b_6-b_7)+2b_7-b_6)] \end{pmatrix} & \textit{for} \quad \alpha \in [0.3333, 0.6666) \\ \begin{pmatrix} [3\alpha(a_4-a_3)+3a_3-2a_4, 3\alpha(a_5-a_6)+3a_6-2a_5]* \\ [3\alpha(b_4-b_3)+3b_3-2b_4, 3\alpha(b_5-b_6)+3b_6-2b_5] \end{pmatrix} & \textit{for} \quad \alpha \in [0.6666, 1.0000] \end{cases}$$

To verify this new operation with multiplication operation , we take the arithmetic example 4.1 $\mathrm{A}_{\overline{\mathrm{OFN}}} = (3, 6, 9, 12, 15, 18, 21, 24)$ and $\mathrm{B}_{\overline{\mathrm{OFN}}} = (2, 7, 12, 17, 22, 27, 32, 37)$

For $\alpha \in [0.0000, 0.3333)$ For $\alpha \in [0.3333, 0.6666)$ For $\alpha \in [0.6666, 1.0000]$	${}^{\alpha}\mathrm{A}_{\overline{\mathrm{OFN}}} = (9\alpha+3, -9\alpha+24)$ ${}^{\alpha}\mathrm{B}_{\overline{\mathrm{OFN}}} = (15\alpha+2, -15\alpha+37)$	${}^{\alpha}(\mathrm{A}*\mathrm{B})_{\overline{\mathrm{OFN}}} = \begin{pmatrix} 135\alpha^2+63\alpha+6, \\ 135\alpha^2-693\alpha+888 \end{pmatrix}$

Since for $\alpha \in [0.0000, 0.3333)$, $\alpha \in [0.3333, 0.6666)$ and $\alpha \in [0.6666, 1.0000]$ arithmetic intervals are same ${}^{\alpha}(\mathrm{A}*\mathrm{B})_{\overline{\mathrm{OFN}}} = (135\alpha^2+63\alpha+6, 135\alpha^2-693\alpha+888)$ for all $\alpha \in [0,1]$

when

$$\alpha = 0.0000 \;\Rightarrow\; {}^{\alpha}(A*B) = [6, 888]$$
$$\alpha = 0.3333 \;\Rightarrow\; {}^{\alpha}(A*B) = [42, 672]$$
$$\alpha = 0.6666 \;\Rightarrow\; {}^{\alpha}(A*B) = [108, 486]$$
$$\alpha = 1.0000 \;\Rightarrow\; {}^{\alpha}(A*B) = [204, 330]$$

Hence ${}^{\alpha}(A*B) = (6, 42, 108, 204, 330, 486, 672, 888)$. Hence all the points coincide with the multiplication of the two octagonal fuzzy number.

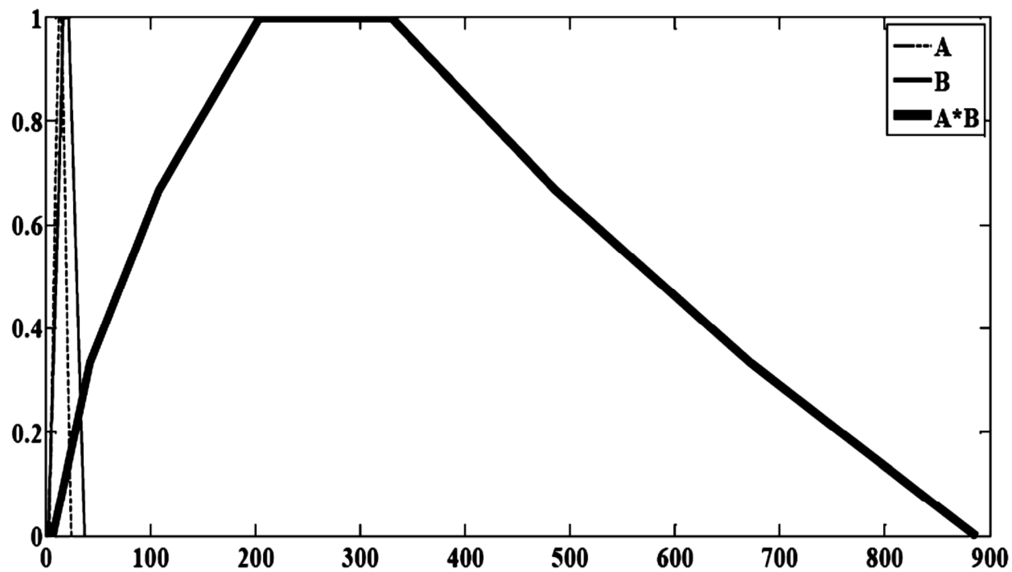

Fig. 6. Multiplication of two OFN A and B.

Definition 5.4

Let $A_{\overline{OFN}} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ and $B_{\overline{OFN}} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$ be their corresponding octagonal fuzzy numbers for all $\alpha \in [0,1]$. Let us division $\alpha-$ cut ${}^{\alpha}A$ and ${}^{\alpha}B$ of $A_{\overline{OFN}}$ and $B_{\overline{OFN}}$ using interval arithmetic.

$$A_{\overline{OFN}} (/) B_{\overline{OFN}} = \begin{cases} \left[\dfrac{3\alpha(a_2-a_1)+a_1}{3\alpha(b_7-b_8)+b_8}, \dfrac{3\alpha(a_7-a_8)+a_8}{3\alpha(b_2-b_1)+b_1}\right] & \textit{for} \quad \alpha \in [0.0000, 0.3333) \\ \left[\dfrac{3\alpha(a_3-a_2)+2a_2-a_3}{3\alpha(b_6-b_7)+2b_7-b_6}, \dfrac{3\alpha(a_6-a_7)+2a_7-a_6}{3\alpha(b_3-b_2)+2b_2-b_3}\right] & \textit{for} \quad \alpha \in [0.3333, 0.6666) \\ \left[\dfrac{3\alpha(a_4-a_3)+3a_3-2a_4}{3\alpha(b_5-b_6)+3b_6-2b_5}, \dfrac{3\alpha(a_5-a_6)+3a_6-2a_5}{3\alpha(b_4-b_3)+3b_3-2b_4}\right] & \textit{for} \quad \alpha \in [0.6666, 1.0000] \end{cases}$$

To verify this new operation with division operation , we take the arithmetic example 4.1 $A_{\overline{OFN}} = (3, 6, 9, 12, 15, 18, 21, 24)$ and $B_{\overline{OFN}} = (2, 7, 12, 17, 22, 27, 32, 37)$

For $\alpha \in [0.0000, 0.3333)$ For $\alpha \in [0.3333, 0.6666)$ For $\alpha \in [0.6666, 1.0000]$	${}^{\alpha}A_{\overline{OFN}} = (9\alpha+3, -9\alpha+24)$ ${}^{\alpha}B_{\overline{OFN}} = (15\alpha+2, -15\alpha+37)$	$\dfrac{{}^{\alpha}A_{\overline{OFN}}}{{}^{\alpha}B_{\overline{OFN}}} = \left(\dfrac{9\alpha+3}{-15\alpha+37}, \dfrac{-9\alpha+24}{15\alpha+2}\right)$

Since for $\alpha \in [0.0000, 0.3333)$, $\alpha \in [0.3333, 0.6666)$ and $\alpha \in [0.6666, 1.0000]$ arithmetic intervals are same $\dfrac{{}^{\alpha}A_{\overline{OFN}}}{{}^{\alpha}B_{\overline{OFN}}} = \left(\dfrac{9\alpha+3}{-15\alpha+37}, \dfrac{-9\alpha+24}{15\alpha+2}\right)$ for all $\alpha \in [0,1]$

when

$$\alpha = 0.0000 \;\Rightarrow\; {}^{\alpha}(A/B) = [0.08116, 12.0]$$
$$\alpha = 0.3333 \;\Rightarrow\; {}^{\alpha}(A/B) = [0.1875, 3.000]$$
$$\alpha = 0.6666 \;\Rightarrow\; {}^{\alpha}(A/B) = [0.3333, 1.500]$$
$$\alpha = 1.0000 \;\Rightarrow\; {}^{\alpha}(A/B) = [0.5455, 0.8824]$$

Hence ${}^{\alpha}(A/B) = (0.0811, 0.1875, 0.3333, 0.5455, 0.8824, 1.5, 3.0, 12.0)$. Hence all the points coincide with the division of the two octagonal fuzzy number.

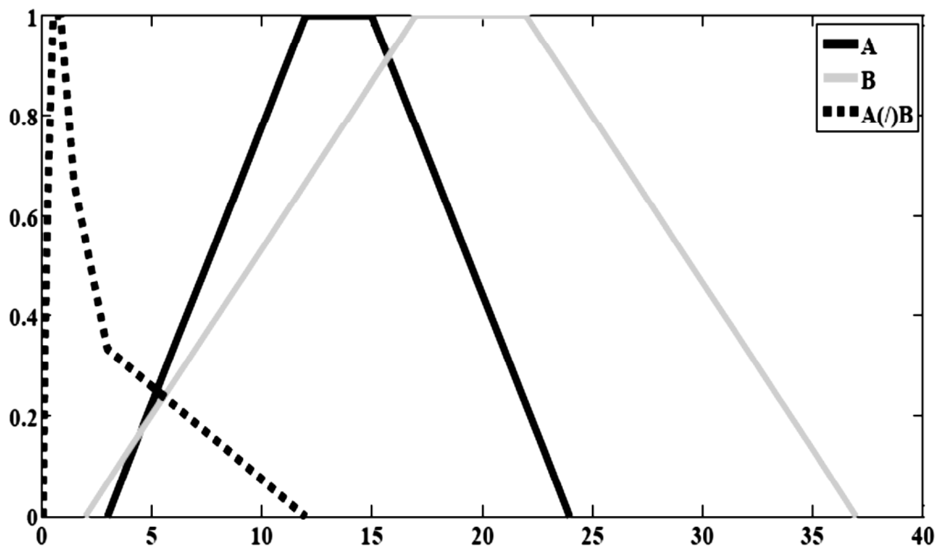

Fig. 7. Division of two OFN A and B.

Definition 5.5

Let $A_{\overline{OFN}} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ the corresponding octagonal fuzzy numbers for all $\alpha \in [0,1]$. Let us inverse $\alpha-$ cut ${}^{\alpha}A$ of $A_{\overline{OFN}}$ interval arithmetic.

$$[A_{\overline{OFN}}]^{-1} = \begin{cases} \dfrac{1}{3\alpha(a_7 - a_8) + a_8}, \dfrac{1}{3\alpha(a_2 - a_1) + a_1} & for \quad \alpha \in [0.0000, 0.3333) \\ \dfrac{1}{3\alpha(a_6 - a_7) + 2a_7 - a_6}, \dfrac{1}{3\alpha(a_3 - a_2) + 2a_2 - a_3} & for \quad \alpha \in [0.3333, 0.6666) \\ \dfrac{1}{3\alpha(a_5 - a_6) + 3a_6 - 2a_5}, \dfrac{1}{3\alpha(a_4 - a_3) + 3a_3 - 2a_4} & for \quad \alpha \in [0.6666, 1.0000] \end{cases}$$

To verify this new operation with inverse operation , we take the arithmetic example 4.1 $A_{\overline{OFN}} = (3, 6, 9, 12, 15, 18, 21, 24)$

For $\alpha \in [0.0000, 0.3333)$ For $\alpha \in [0.3333, 0.6666)$ For $\alpha \in [0.6666, 1.0000]$	${}^{\alpha}\left(\dfrac{1}{a_m}\right) = \dfrac{1}{9\alpha + 3}$ ${}^{\alpha}\left(\dfrac{1}{a_n}\right) = \dfrac{1}{-9\alpha + 24}$	${}^{\alpha}[A_{\overline{OFN}}]^{-1} = {}^{\alpha}\left(\dfrac{1}{a_n}, \dfrac{1}{a_m}\right) = \left(\dfrac{1}{-9\alpha + 24}, \dfrac{1}{9\alpha + 3}\right)$ Where n = 1, 2, 3, 4 and m = 8, 7, 6, 5

Since for $\alpha \in [0.0000, 0.3333)$, $\alpha \in [0.3333, 0.6666)$ and $\alpha \in [0.6666, 1.0000]$ arithmetic intervals are same ${}^{\alpha}[A_{\overline{OFN}}]^{-1} = {}^{\alpha}\left(\dfrac{1}{a_n}, \dfrac{1}{a_m}\right) = \left(\dfrac{1}{-9\alpha + 24}, \dfrac{1}{9\alpha + 3}\right)$ for all $\alpha \in [0,1]$

when

$$\alpha = 0.0000 \quad \Rightarrow \quad {}^{\alpha}\left(\frac{1}{a_8}, \frac{1}{a_1}\right) = [0.0417, 0.3333]$$

$$\alpha = 0.3333 \quad \Rightarrow \quad {}^{\alpha}\left(\frac{1}{a_7}, \frac{1}{a_2}\right) = [0.0476, 0.1667]$$

$$\alpha = 0.6666 \quad \Rightarrow \quad {}^{\alpha}\left(\frac{1}{a_6}, \frac{1}{a_3}\right) = [0.0556, 0.1111]$$

$$\alpha = 1.0000 \quad \Rightarrow \quad {}^{\alpha}\left(\frac{1}{a_5}, \frac{1}{a_4}\right) = [0.0667, 0.0833]$$

Hence ${}^{\alpha}[\mathrm{A}_{\overline{\mathrm{OFN}}}]^{-1} = (0.0417, 0.0476, 0.0556, 0.0667, 0.0833, 0.1111, 0.1667, 0.3333)$. Hence all the points coincide with the inverse of a octagonal fuzzy number.

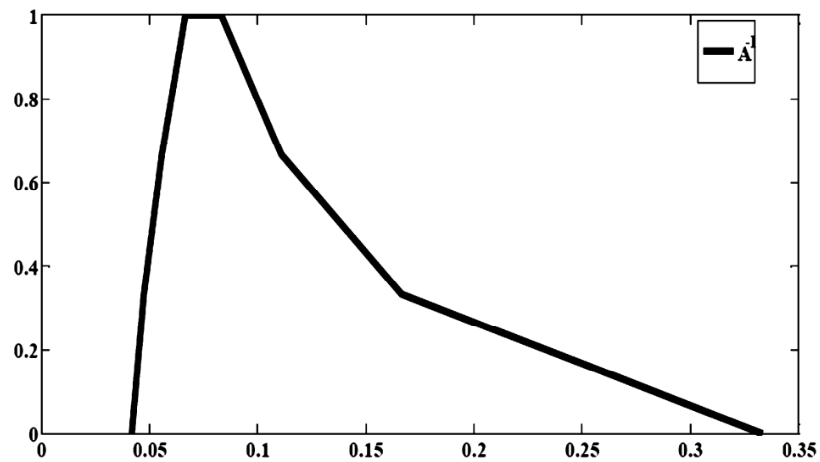

Fig. 8. Inverse interval of OFN A.

Definition 5.6

Let $\mathrm{A}_{\overline{\mathrm{OFN}}} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ and $\mathrm{B}_{\overline{\mathrm{OFN}}} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$ be their corresponding octagonal fuzzy numbers for all $\alpha \in [0,1]$. Let us minimum $\alpha -$ cut ${}^{\alpha}$A and ${}^{\alpha}$B of $\mathrm{A}_{\overline{\mathrm{OFN}}}$ and $\mathrm{B}_{\overline{\mathrm{OFN}}}$ using interval arithmetic.

$$\min{}^{\alpha}(\mathrm{A}_{\overline{\mathrm{OFN}}}, \mathrm{B}_{\overline{\mathrm{OFN}}}) = \begin{cases} \begin{pmatrix} (3\alpha(a_2 - a_1) + a_1) \wedge (3\alpha(b_2 - b_1) + b_1), \\ (3\alpha(a_7 - a_8) + a_8) \wedge (3\alpha(b_7 - b_8) + b_8) \end{pmatrix} & \textit{for} \quad \alpha \in [0.0000, 0.3333) \\ \begin{pmatrix} (3\alpha(a_3 - a_2) + 2a_2 - a_3) \wedge (3\alpha(b_3 - b_2) + 2b_2 - b_3), \\ (3\alpha(a_6 - a_7) + 2a_7 - a_6) \wedge (3\alpha(b_6 - b_7) + 2b_7 - b_6), \end{pmatrix} & \textit{for} \quad \alpha \in [0.3333, 0.6666) \\ \begin{pmatrix} (3\alpha(a_4 - a_3) + 3a_3 - 2a_4) \wedge (3\alpha(b_4 - b_3) + 3b_3 - 2b_4) \\ (3\alpha(a_5 - a_6) + 3a_6 - 2a_5) \wedge (3\alpha(b_5 - b_6) + 3b_6 - 2b_5) \end{pmatrix} & \textit{for} \quad \alpha \in [0.6666, 1.0000] \end{cases}$$

To verify this new operation with minimum operation , we take the arithmetic example 4.1 $\mathrm{A}_{\overline{\mathrm{OFN}}} = (3, 6, 9, 12, 15, 18, 21, 24)$ and $\mathrm{B}_{\overline{\mathrm{OFN}}} = (2, 7, 12, 17, 22, 27, 32, 37)$

For $\alpha \in [0.0000, 0.3333)$	${}^{\alpha}\mathrm{A}_{\overline{\mathrm{OFN}}} = (9\alpha + 3, -9\alpha + 24)$	
For $\alpha \in [0.3333, 0.6666)$	${}^{\alpha}\mathrm{B}_{\overline{\mathrm{OFN}}} = (15\alpha + 2, -15\alpha + 37)$	$\min{}^{\alpha}(\mathrm{A}, \mathrm{B}) = \begin{pmatrix} (9\alpha + 3) \wedge (15\alpha + 2), \\ (-9\alpha + 24) \vee (-15\alpha + 37) \end{pmatrix}$
For $\alpha \in [0.6666, 1.0000]$		

Since for $\alpha\in[0.0000,0.3333)$, $\alpha\in[0.3333,0.6666)$ and $\alpha\in[0.6666,1.0000]$ arithmetic intervals are same $\min^{\alpha}(A,B)=((9\alpha+3)\wedge(15\alpha+2),(-9\alpha+24)\vee(-15\alpha+37))$ for all $\alpha\in[0,1]$

when

$$\alpha=0.0000 \Rightarrow \min^{\alpha}(A,B)=[2,24]$$
$$\alpha=0.3333 \Rightarrow \min^{\alpha}(A,B)=[6,21]$$
$$\alpha=0.6666 \Rightarrow \min^{\alpha}(A,B)=[9,18]$$
$$\alpha=1.0000 \Rightarrow \min^{\alpha}(A,B)=[12,15]$$

Hence $\min^{\alpha}(A,B)=(2,6,9,12,15,18,21,24)$ hence all the points coincide with the minimum of the two octagonal fuzzy number.

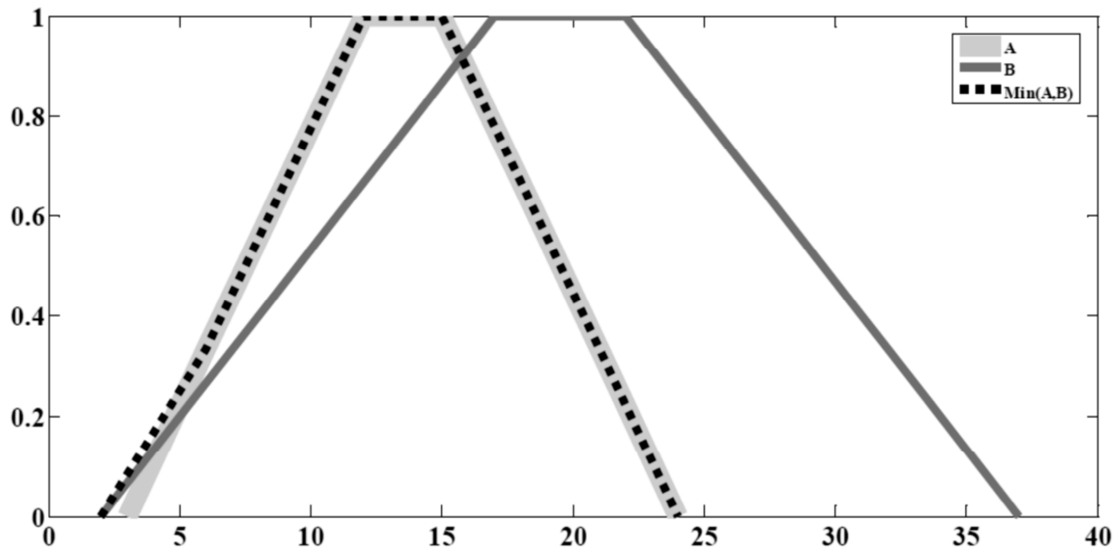

Fig. 9. Minimum of two OFN A and B.

Definition 5.7

Let $A_{\overline{OFN}}=(a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8)$ and $B_{\overline{OFN}}=(b_1,b_2,b_3,b_4,b_5,b_6,b_7,b_8)$ be their corresponding octagonal fuzzy numbers for all $\alpha\in[0,1]$. Let us maximum $\alpha-$ cut ${}^{\alpha}A$ and ${}^{\alpha}B$ of $A_{\overline{OFN}}$ and $B_{\overline{OFN}}$ using interval arithmetic.

$$\max{}^{\alpha}(A,B)=\begin{cases}\begin{pmatrix}(3\alpha(a_2-a_1)+a_1)\vee(3\alpha(b_2-b_1)+b_1),\\(3\alpha(a_7-a_8)+a_8)\vee(3\alpha(b_7-b_8)+b_8)\end{pmatrix} & \text{for} \quad \alpha\in[0.0000,0.3333)\\ \begin{pmatrix}(3\alpha(a_3-a_2)+2a_2-a_3)\vee(3\alpha(b_3-b_2)+2b_2-b_3),\\(3\alpha(a_6-a_7)+2a_7-a_6)\vee(3\alpha(b_6-b_7)+2b_7-b_6),\end{pmatrix} & \text{for} \quad \alpha\in[0.3333,0.6666)\\ \begin{pmatrix}(3\alpha(a_4-a_3)+3a_3-2a_4)\vee(3\alpha(b_4-b_3)+3b_3-2b_4)\\(3\alpha(a_5-a_6)+3a_6-2a_5)\vee(3\alpha(b_5-b_6)+3b_6-2b_5)\end{pmatrix} & \text{for} \quad \alpha\in[0.6666,1.0000]\end{cases}$$

To verify this new operation with maximum operation , we take the arithmetic example 4.1 $A_{\overline{OFN}}=(3,6,9,12,15,18,21,24)$ and $B_{\overline{OFN}}=(2,7,12,17,22,27,32,37)$

For $\alpha\in[0.0000,0.3333)$	${}^{\alpha}A_{\overline{OFN}}=(9\alpha+3,-9\alpha+24)$	$\max^{\alpha}(A,B)=\begin{pmatrix}(9\alpha+3)\vee(15\alpha+2),\\(-9\alpha+24)\vee(-15\alpha+37)\end{pmatrix}$
For $\alpha\in[0.3333,0.6666)$	${}^{\alpha}B_{\overline{OFN}}=(15\alpha+2,-15\alpha+37)$	
For $\alpha\in[0.6666,1.0000]$		

Since for $\alpha \in [0.0000, 0.3333)$, $\alpha \in [0.3333, 0.6666)$ and $\alpha \in [0.6666, 1.0000]$ arithmetic intervals are same $\max^{\alpha}(A,B) = ((9\alpha+3) \vee (15\alpha+2), (-9\alpha+24) \vee (-15\alpha+37))$ for all $\alpha \in [0,1]$

when

$$\alpha = 0.0000 \Rightarrow \max^{\alpha}(A, B) = [3, 37]$$
$$\alpha = 0.3333 \Rightarrow \max^{\alpha}(A, B) = [7, 32]$$
$$\alpha = 0.6666 \Rightarrow \max^{\alpha}(A, B) = [12, 27]$$
$$\alpha = 1.0000 \Rightarrow \max^{\alpha}(A, B) = [17, 22]$$

Hence $\max^{\alpha}(A, B) = (3, 7, 12, 17, 22, 27, 32, 37)$ hence all the points coincide with the maximum of the two octagonal fuzzy number.

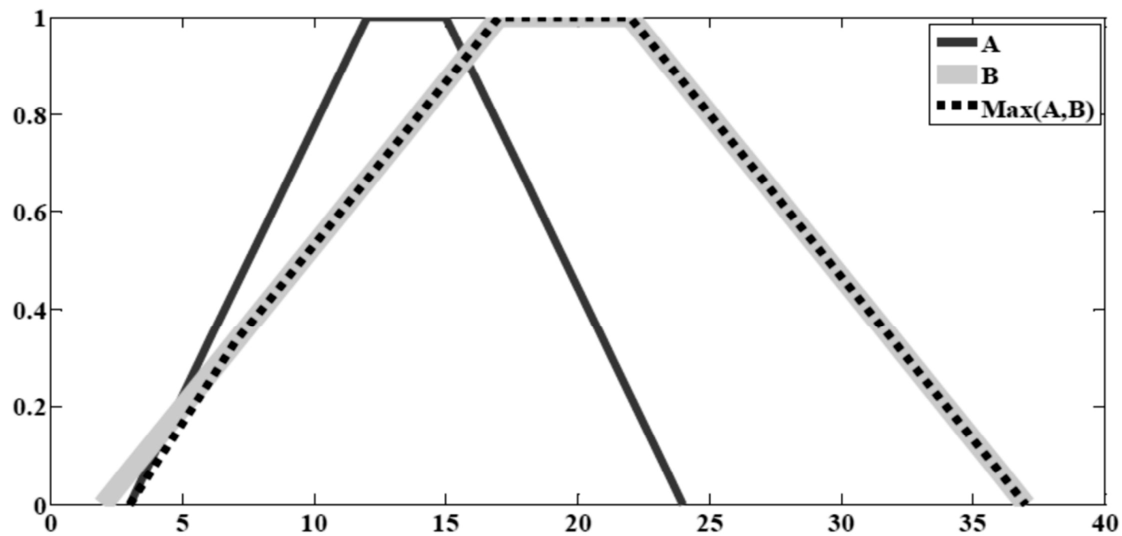

Fig. 10. Maximum of two OFN A and B.

Definition 5.8

Let $A_{\overline{OFN}} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ the corresponding octagonal fuzzy numbers for all $\alpha \in [0,1]$. Let us scalar multiplication α– cut $^{\alpha}A$ of $A_{\overline{OFN}}$ interval arithmetic.

$$C * A_{\overline{OFN}} = \begin{cases} \begin{pmatrix} c*(3\alpha(a_2 - a_1) + a_1) \wedge c*(3\alpha(a_7 - a_8) + a_8), \\ c*(3\alpha(a_2 - a_1) + a_1) \vee c*(3\alpha(a_7 - a_8) + a_8) \end{pmatrix} & \text{for} \quad \alpha \in [0.0000, 0.3333) \\ \begin{pmatrix} c*(3\alpha(a_3 - a_2) + 2a_2 - a_3) \wedge c*(3\alpha(a_6 - a_7) + 2a_7 - a_6), \\ c*(3\alpha(a_3 - a_2) + 2a_2 - a_3) \vee c*(3\alpha(a_6 - a_7) + 2a_7 - a_6) \end{pmatrix} & \text{for} \quad \alpha \in [0.3333, 0.6666) \\ \begin{pmatrix} c*(3\alpha(a_4 - a_3) + 3a_3 - 2a_4) \wedge c*(3\alpha(a_5 - a_6) + 3a_6 - 2a_5), \\ c*(3\alpha(a_4 - a_3) + 3a_3 - 2a_4) \vee c*(3\alpha(a_5 - a_6) + 3a_6 - 2a_5) \end{pmatrix} & \text{for} \quad \alpha \in [0.6666, 1.0000] \end{cases}$$

To verify this new operation with scalar multiplication operation, we take the arithmetic example 4.1 $A_{\overline{OFN}} = (3, 6, 9, 12, 15, 18, 21, 24)$

For $\alpha \in [0.0000, 0.3333)$ For $\alpha \in [0.3333, 0.6666)$ For $\alpha \in [0.6666, 1.0000]$	$^{\alpha}A_{\overline{OFN}} = (9\alpha + 3, -9\alpha + 24)$	$c*^{\alpha}(A) = \begin{pmatrix} c*(9\alpha+3) \wedge c*(-9\alpha+24), \\ c*(9\alpha+3) \vee c*(-9\alpha+24) \end{pmatrix}$

Since for $\alpha \in [0.0000, 0.3333)$, $\alpha \in [0.3333, 0.6666)$ and $\alpha \in [0.6666, 1.0000]$ arithmetic intervals are same $\mathrm{sm}^{\alpha}(\mathrm{A}) = (c*(9\alpha+3) \wedge c*(-9\alpha+24), c*(9\alpha+3) \vee (-9\alpha+24))$ for all $\alpha \in [0,1]$ and $c = 3$

when

$$\alpha = 0.0000 \quad \Rightarrow \quad 3*^{\alpha}(\mathrm{A}) = [9, 72]$$
$$\alpha = 0.3333 \quad \Rightarrow \quad 3*^{\alpha}(\mathrm{A}) = [18, 63]$$
$$\alpha = 0.6666 \quad \Rightarrow \quad 3*^{\alpha}(\mathrm{A}) = [27, 54]$$
$$\alpha = 1.0000 \quad \Rightarrow \quad 3*^{\alpha}(\mathrm{A}) = [36, 45]$$

Hence $\mathrm{sm}^{\alpha}(\mathrm{A}) = (9, 18, 27, 36, 45, 54, 63, 72)$. Hence all the points coincide with the scalar multiplication of the a octagonal fuzzy number.

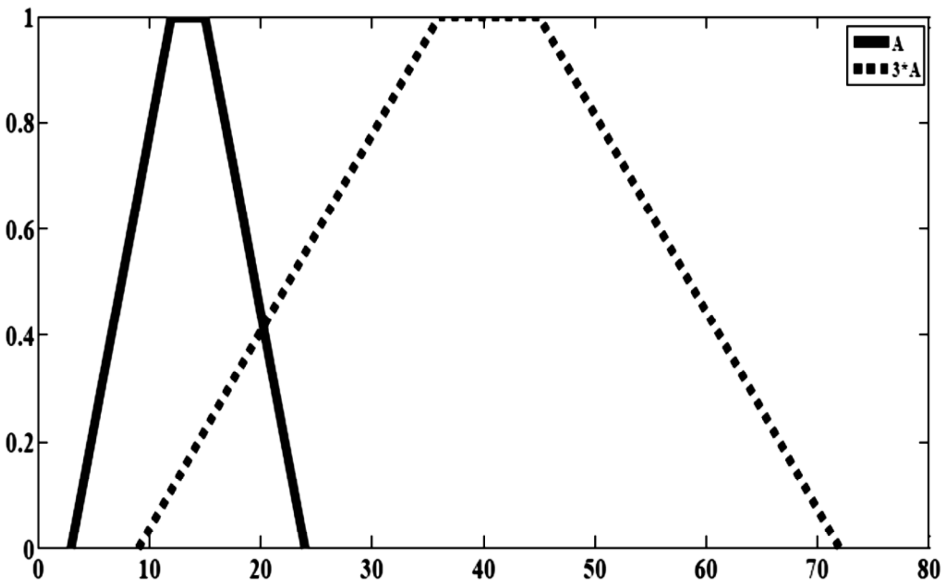

Fig. 11. Scalar Multiplication of OFN A.

Definition 5.9

Let $\mathrm{A}_{\overline{\mathrm{OFN}}} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ the corresponding octagonal fuzzy numbers for all $\alpha \in [0,1]$. Let us symmetric image $\alpha -$ cut ${}^{\alpha}\mathrm{A}$ of $\mathrm{A}_{\overline{\mathrm{OFN}}}$ interval arithmetic.

$$(-)\mathrm{A}_{\overline{\mathrm{OFN}}} = \begin{cases} [-(3\alpha(a_7 - a_8) + a_8), -(3\alpha(a_2 - a_1) + a_1)] & \textit{for} \quad \alpha \in [0.0000, 0.3333) \\ [-(3\alpha(a_6 - a_7) + 2a_7 - a_6), -(3\alpha(a_3 - a_2) + 2a_2 - a_3)] & \textit{for} \quad \alpha \in [0.3333, 0.6666) \\ [-(3\alpha(a_5 - a_6) + 3a_6 - 2a_5), -(3\alpha(a_4 - a_3) + 3a_3 - 2a_4)] & \textit{for} \quad \alpha \in [0.6666, 1.0000] \end{cases}$$

To verify this new operation with symmetric image operation, we take the arithmetic example 4.1 $\mathrm{A}_{\overline{\mathrm{OFN}}} = (3, 6, 9, 12, 15, 18, 21, 24)$

For $\alpha \in [0.0000, 0.3333)$ For $\alpha \in [0.3333, 0.6666)$ For $\alpha \in [0.6666, 1.0000]$	${}^{\alpha}\mathrm{A}_{\overline{\mathrm{OFN}}} = (9\alpha + 3, -9\alpha + 24)$	$(-)^{\alpha}(\mathrm{A})_{\overline{\mathrm{OFN}}} = (9\alpha - 24, -9\alpha - 3)$

Since for $\alpha \in [0.0000, 0.3333)$, $\alpha \in [0.3333, 0.6666)$ and $\alpha \in [0.6666, 1.0000]$ arithmetic intervals are same $(-)^{\alpha}(\mathrm{A}) = (9\alpha - 24, -9\alpha - 3)$ for all $\alpha \in [0,1]$

when

$$\alpha = 0.0000 \quad \Rightarrow \quad (-)^{\alpha}(A) = [-24, -3]$$
$$\alpha = 0.3333 \quad \Rightarrow \quad (-)^{\alpha}(A) = [-21, -6]$$
$$\alpha = 0.6666 \quad \Rightarrow \quad (-)^{\alpha}(A) = [-18, -9]$$
$$\alpha = 1.0000 \quad \Rightarrow \quad (-)^{\alpha}(A) = [-15, -12]$$

Hence $(-)^{\alpha}(A) = [-24, -21, -18, -15, -12, -9, -6, -3]$. Hence all the points coincide with the symmetric image the a octagonal fuzzy number.

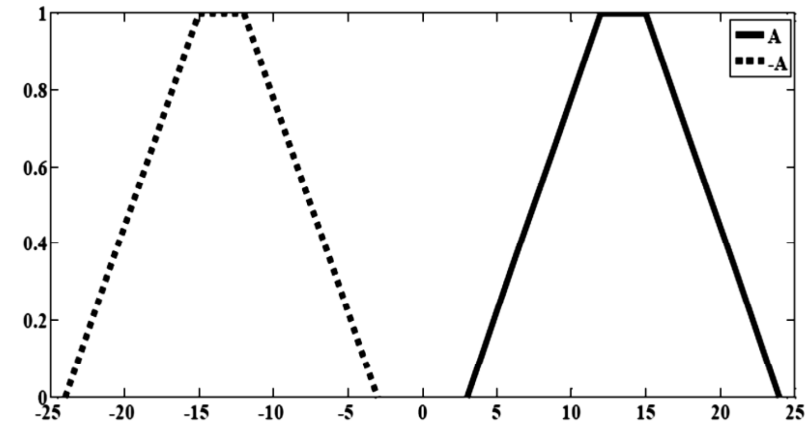

Fig. 12. Symmetri Image of OFN A.

6. CONCLUSION

In this article, the another method has been introduced by the Octagonal Fuzzy Number with arithmetic operations. Using these operations, we solved few examples. Octagonal Fuzzy Number can be applied to that problem which has eight points in representation. In future, it may be applied in many operations/ research problems, which has eight arithmetic and linguistic variables.

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