

Analysis of Bifurcation in Voltage Mode Controlled Buck Converter Using Symbolic Sequence Method

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ABSTRACT

In this paper, the nonlinear dynamics of voltage mode controlled buck converter is examined in detail using symbolic sequence method with the input voltage taken as bifurcation parameter. In this method, the main and secondary symbolic sequences are established to classify different variety of standard and border-collision bifurcations. Extensive computer simulations and programming are performed using MATLAB software to demonstrate the concept revealed.

Keywords: Bifurcation, Border collision, Chaos, Switching block, Symbolic sequence.

1. INTRODUCTION

Design engineers come across various abnormal operations like bifurcation, quasi-periodicity, and chaos while testing power electronics circuit. Such a nonlinear behavior has been elaborated in those switching power converters [1 – 7] in past few decades. Thorough knowledge of the system behavior under all practical situations is essential for a power electronics engineers to design a stable system.

Power spectrum [8], Lyapunov exponents [9], entropy and fractal dimension are used to analyze the abnormal dynamics in dc-dc converters. However, these tools are tedious for the engineering applications, taking long time to investigate. Symbolic sequence method has gained its attention to differentiate various smooth and non-smooth bifurcations undergone by the converter with minimum time, when certain parameters are varied [10 – 13]. In this study, the main and secondary symbolic sequences are generated for the system to classify its bifurcation behavior over a range of system parameter values chosen. Moreover, only a few works has been performed using symbolic sequence method to scrutinize the bifurcation types.

This paper is ordered as follows: Section 2 presents the generation of the main switching block and its sequence. With reference to the threshold value, the secondary symbolic sequence is established. In Section 3, by using some norms, the standard bifurcations and the border collision bifurcations are categorized based on the periodicity of the main symbolic sequence and secondary symbolic sequence. In Section 4, computer simulations results for the voltage mode controlled buck converter is illustrated. Finally, the essence of this work is concluded in section 5.

2. SYMBOLIC SEQUENCE METHOD

2.1. Main symbolic sequence

On inspecting the switching modes in one cycle, the main switching block is developed, and its respective symbolic sequence is obtained by following the main switching block over infinite switching period.

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Normally in power converter consisting of two switching elements like switch and diode, the toggling states that exist includes (ON, OFF), (OFF, ON) and (OFF, OFF). The discontinuous conduction mode of operation (see Fig. 1(a)) have the sequence of switching states as (ON, OFF) \rightarrow (OFF, ON) \rightarrow (OFF, OFF) during one switching period, while in continuous conduction mode, the switching state (OFF, OFF) does not come into view as shown in the Fig. 1(b).

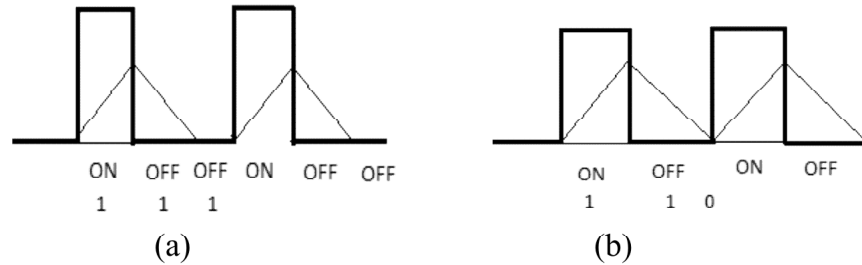


Figure 1: Main symbolic sequence when (a) Discontinuous mode (b) Continuous mode

The three transition states in one switching period are represented using binary digits b_1 , b_2 , and b_3 to establish the main switching block. Let $b_i = 0$ or 1 , signify whether the prevailing switching state exists or not during one cycle. Main switching block is given as

$$(b_1 b_2 b_3)_2 = (0)_{10} \quad (1)$$

Where, O is a decimal value (0-7).

The string of consecutive main switching blocks considered over an infinite switching cycles generates the main symbolic sequence

$$O = (O_0 O_1 O_2 \dots O_n \dots) \quad (2)$$

2.2. Secondary symbolic sequence

Since the main symbolic sequence provides only the partial information about the onset of border collision bifurcation, the idea of the secondary symbolic block and the respective sequence is presented to further identify the various types of standard and border collision bifurcation.

The output voltage is sampled at integral multiples of switching period, and the maximum of sampled output voltage is considered as a threshold value V_b . With reference to V_b , the secondary switching block is obtained. At n^{th} switching instant, the secondary switching block is given as

$$l_n = \begin{cases} 1, & V_0 < V_b \\ 0, & V_0 \geq V_b \end{cases} \quad (3)$$

This block carried over an infinite switching period produces the secondary symbolic sequence given by

$$L = (l_0 l_1 l_2 \dots l_n \dots) \quad (4)$$

The periodicity of the generated sequences is utilized for bifurcation analysis.

3. BIFURCATION ANALYSIS BASED ON SYMBOLIC SEQUENCES

As in literature [12], switching converter is given by the following iterative map $X_{n+1} = f(X_n, a)$, where a is a parameter, and X_n is the state variable sampled at integer multiples of switching time. With the variation in parameter a , the converter experiences a standard bifurcation where the form of f remains unaffected before and after the incidence of bifurcation. While, when the converter demonstrates a border collision

bifurcation, the form of f change as a is varied [12]. Hence, in the symbolic sequence method, as the main symbolic sequence relates the form of f , the change in the form of f , governs the change in the main symbolic sequence. Moreover, the occurrence of border collision bifurcation is easily identified by inspecting the main symbolic sequence.

The secondary symbolic sequence which reveals the periodicity of the waveform plays a major role in distinguishing some standard bifurcation types such as period doubling bifurcation, saddle node bifurcation, and border collision bifurcation from period- n to period- n , period- m or chaos. Furthermore, the series in which the sequences are analyzed to classify the bifurcation types is given as $O \rightarrow L$.

Let $a_1 < a_c < a_2$ in which a_c is the critical parameter value. P_{mi} and P_{si} symbolizes the periodicity of the main symbolic sequence and secondary symbolic sequence respectively for $a = a_i$ with $i = 1$ or 2 . Subsequently, by following the norms as tabulated in Table 1, all kinds of bifurcation are explored [13].

Table 1
Characterizing the bifurcation types based on symbolic sequences

Standard Bifurcation	Period Doubling	$O_1 = O_2$ $P_{m1} = P_{m2}$	$L_1 \neq L_2$ $2P_{s1} = P_{s2}$
	Saddle Node	$O_1 = O_2$ $P_{m1} = P_{m2}$	$L_1 \neq L_2$ $P_{s1} \gg P_{s2}$
	Hopf	$O_1 = O_2$ $P_{m1} = P_{m2}$	$L_1 \neq L_2$ $P_{s1} \ll P_{s2}$
Border collision Bifurcation	Period- n to Period- n	$O_1 \neq O_2$ $P_{m1} \neq P_{m2}$	$L_1 = L_2$ $P_{s1} = P_{s2}$
	Period- n to Period- m	$O_1 \neq O_2$ $P_{m1} \neq P_{m2}$	$L_1 \neq L_2$ $P_{s1} \neq P_{s2}$
	Period- n to chaos	$O_1 \neq O_2$ $P_{m1} \ll P_{m2}$	$L_1 \neq L_2$ $P_{s1} \ll P_{s2}$

4. APPLIED EXAMPLE

With the specification as in Table 2, the voltage mode controlled buck converter (see Fig. 2) is considered to analyze the bifurcation scenario by keeping the input voltage (E) as the bifurcation parameter.

Table 2
System specification

Parameters	Values
Supply Voltage, V_{in}	22 – 33 V
Switching Frequency, f	2500 Hz
Load Resistance, R	22 Ω
Inductance, L	20 mH
Capacitance, C	47 μ F
Reference voltage, V_{ref}	11 V
Gain, A	8.4
Ramp signal lower voltage, V_L	3.8 V
Ramp signal upper voltage, V_U	8.2 V

Computer simulations are performed to disclose the complete dynamics of the system for variation in input voltage. It is inferred that the converter bifurcates via period doubling cascade to chaos as the value

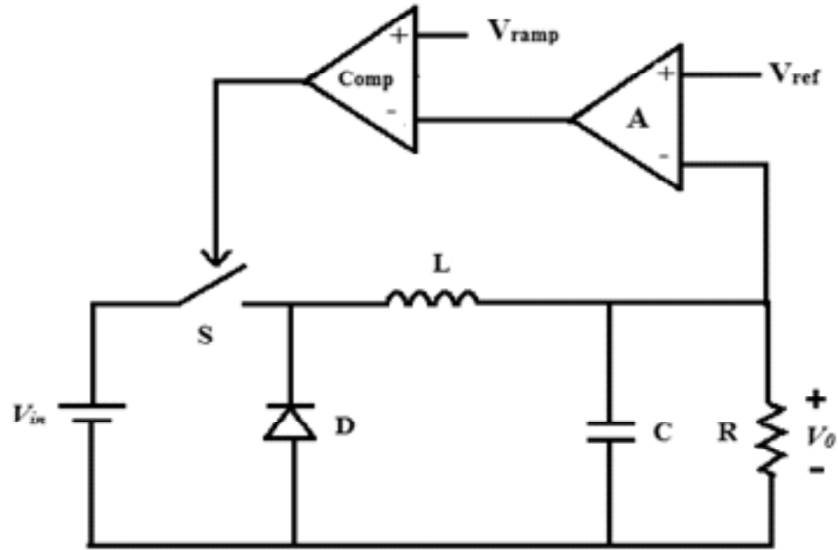


Figure 2: Voltage mode controlled buck converter

of V_{in} is varied from 22 V to 33 V. When input voltage drops below 32.4 V, the converter exhibits a chaotic operation.

The simulated result showing the generation of main symbolic sequence and secondary symbolic sequence for variation in the input voltage is shown in the Fig. 3 and Fig. 4, respectively.

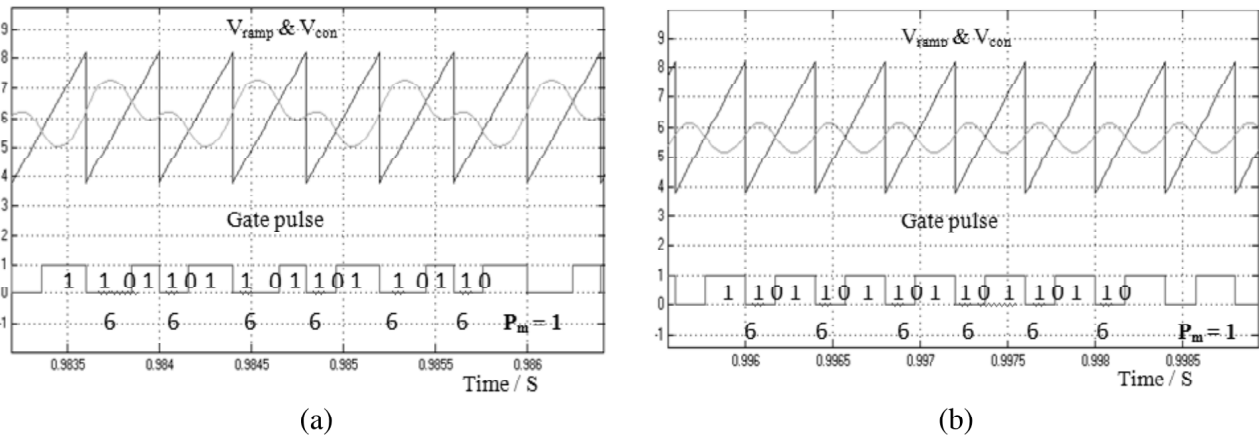


Figure 3: Simulation results illustrating the main symbolic sequence generation (a) Period 1 operation (b) Period 2 operation

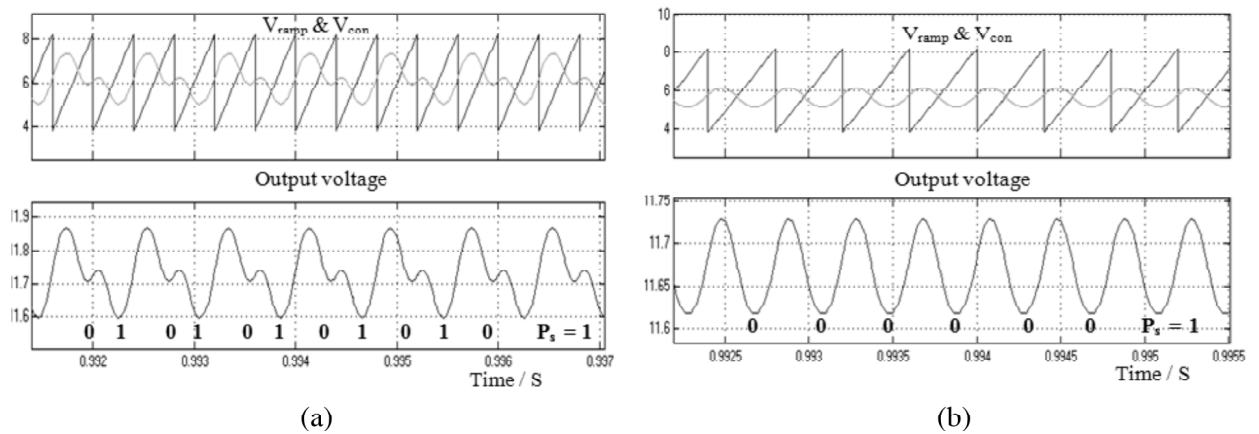


Figure 4: Simulation results illustrating the secondary symbolic sequence generation (a) Period 1 operation (b) Period 2 operation

4.1. Analysis of bifurcation behavior based on symbolic sequences

For variation in input voltage, the symbolic sequences and their periodicities are identified as tabulated in Table 3.

Table 3
Main and secondary symbolic sequences

Input voltage (V)	Main Sequence	P_m	Secondary Sequence	P_s
(22 – 24.7)	(6) ∞	1	(0) ∞	1
(24.8 – 31.7)	(6) ∞	1	(01) ∞	2
(31.8 – 32.3)	(6) ∞	1	(1011) ∞	4
< 32.4	∞	$+\infty$	∞	$+\infty$

In the range of input voltage (22 – 24.7) V, the converter operates in stable period-1 operation. It is inferred that when the input voltage (V_{in}) is increased from 24.7 V to 24.8 V, there is no difference in the periodic number of main sequence (P_m) i.e., $P_{m1} = P_{m2}$, but periodicity of secondary sequence (P_s) is doubled i.e., $2P_{s1} = P_{s2}$, showing the occurrence of period doubling bifurcation. Further when the input voltage is increased to 31.8 V, $P_{m1} = P_{m2}$ and $2P_{s1} = P_{s2}$. It implies that standard bifurcation has occurred, and jumps from period-2 orbit to period-4 orbit. Moreover, when the input voltage is increased beyond 32.4 V, both the sequences become more random exemplifying that the system enters into the chaotic regime via border collision bifurcation since $P_{m1} \neq P_{m2}$.

5. CONCLUSION

In this paper, the nonlinear behavior of a voltage mode controlled buck converter in continuous conduction mode is investigated using symbolic sequence method. It is observed that as the input voltage is varied, the system enters chaotic regime via period doubling route. With the help of symbolic sequences, the events of bifurcation phenomena are identified. Such an analysis is beneficial for design engineers to avoid the unexpected complex behavior of the system.

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