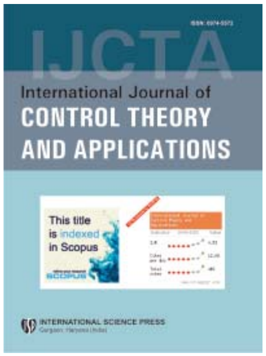

International Journal of Control Theory and Applications

ISSN : 0974–5572

Volume 9 • Number 42 • 2016

A New Insight Into the Schema Survival for Genetic Algorithms Consisting of Chromosomes Having Bits with Different Vulnerability for Mutation

Apoorva Mishra[a] and Anupam Shukla[a]

[a]ABV -Indian Institute of Information Technology and Management Gwalior, Madhya Pradesh, India – 474010

E-mail: apoorvamish1989@gmail.com, dranupamiiitm@gmail.com

Abstract: Genetic Algorithms mimic the functioning of natural evolution and try to use its power to solve several optimization problems. Schema theory forms a mathematical basis for the success of genetic algorithms. Traditional genetic algorithms assume that all the bits of a chromosome are equally vulnerable to mutation. In this paper, a novel case of genetic algorithms consisting of chromosomes having bits with different vulnerability for mutation is meticulously analyzed. The analysis is carried out for two different scenarios. The first scenario deals with the derivation of a mathematical expression, representing the probability of survival of a schema after traditional crossover and mutation for genetic algorithms involving chromosomes having bits with different vulnerability for mutation, and in the second scenario the derivation for the same is done after the application of ternary crossover and mutation. These two mathematical equations provide a new insight regarding schema percolation in genetic algorithms involving chromosomes having bits with different vulnerability for mutation.

Keywords: *Genetic Algorithms, Ternary Crossover, Mutation, Chromosomes, Schema.*

1. INTRODUCTION

Over the past four decades, genetic algorithms have been applied to solve innumerable problems across different domains [1–7]. Genetic Algorithms are inspired by the process of natural evolution [8]. The working principle of the genetic algorithms is based on Darwin's theory of "the survival of the fittest." The most common genetic operators are selection, crossover and mutation. The theoretical explanation for the success of genetic algorithms is given by schema theory [9,10], and the chance of survival of a group of schemata (plural of schema) has also been derived[11].

Let us consider an example of a schema to understand the various terms related to it.

$$\text{Schema (H)} = 1*0*1**$$

For this schema,

$$\text{Length of the schema } l(H) = 7 \text{ (Total number of bits in H)}$$

$$\text{Defining Length d(H)} = 5 - 1$$

	=	4 (Difference between the positions of first and last fixed bits)
Order o(H)	=	3 (Number of fixed bits in H)
Number of don't care bits dc(H)	=	l(H) – o(H)
	=	7 – 3
	=	4

The rest of the paper is organized in the following manner: section-2 describes the derivation of the probability of survival of a schema after crossover and mutation for traditional genetic algorithms.

In section-3, the likelihood of survival of a schema after the ternary crossover and mutation is derived.

Section-4(A) describes the derivation of the probability of survival of a schema after traditional crossover and mutation for genetic algorithms consisting of chromosomes having bits with different vulnerability for mutation.

In section-4(B), the probability of survival of a schema after the application of ternary crossover and mutation for genetic algorithms consisting of chromosomes having bits with different vulnerability for mutation is represented mathematically.

Section-5 discusses the implications of the results of the mathematical analysis.

Section-6 concludes the various findings of the mathematical analysis.

2. SCHEMA SURVIVAL IN TRADITIONAL GENETIC ALGORITHMS

A traditional one point crossover operator usually involves the recombination of two parents to form two new offspring. A schema H is said to survive a crossover operation involving two parents, if at least one of the resulting two offspring is an instance of H [10].

E.g. Consider the same schema as mentioned in section-1; H = 1*0*1**

Let the parent1 (P1) = 1 1 0 1 1 1 1

and Parent2 (P2) = 1 0 1 0 0 1 0

It can be observed that P1 is an instance of H, whereas P2 is not. Let us also assume that O1 and O2 are the two offspring created after the crossover operation.

The crossover operation can be represented as follows:

P1 =	1 1 0	1 1 1 1	
P2 =	1 0 1	0 0 1 0	Crossover point is 3 (Single point crossover)
O1 =	1 1 0	0 0 1 0	(Does not belong to H)
O2 =	1 0 1	1 1 1 1	(Does not belong to H)

Here, the place of crossover is 3, which is within the defining length of H; hence the schema has not been able to survive. If we change the position of crossover to 5, then, we have:

P1 =	1 1 0 1 1	1 1	
P2 =	1 0 1 0 0	1 0	Crossover point is 5 (Single point crossover)
O1 =	1 1 0 1 1	1 0	(Belongs to H)
O2 =	1 0 1 0 0	1 1	(Does not belong to H)

Now, since O1 is an instance of H, the schema has survived the crossover operation.

Let the probability of the occurrence of crossover be denoted by P_c and the number of cases in which it occurs within the defining length be denoted by d(H). For a schema of length l, the total possible points of crossover are (l – 1). Hence, the probability of crossover taking place within the defining length P_{dl} is given by:

$$P_{dl} = \frac{d(H)}{(l-1)} \tag{1}$$

From the above discussion, it is clear that a schema is destroyed, if crossover occurs and it occurs within the defining length. We can represent this fact mathematically by the following equation

$$P_d = P_c \times P_{dl} \tag{2}$$

Here, P_d denotes the probability of a schema being destroyed.

Substituting the value of P_{dl} from Eq. (1) we get

$$P_d = P_c \times \frac{d(H)}{(l-1)} \tag{3}$$

Let P_s be the probability of survival of a schema. Then,

$$P_s = 1 - P_d \tag{4}$$

Substituting the value of P_d from Eq. (3)

$$P_s = 1 - P_c \times \frac{d(H)}{(l-1)} \tag{5}$$

This equation represents the probability of survival of a schema after traditional crossover operation. Mutation is another common operation that occurs after crossover. In traditional genetic algorithms, all the bits of a chromosome are considered to be equally vulnerable to mutation. Hence, for all the bits, the probability of getting mutated is same.

Let the probability of any bit getting mutated be given by Pm, the order of a schema be represented by o(H) and the probability of survival of a schema after mutation be denoted by P_{surv}.

As, for each particular bit, the probability of getting mutated is Pm, the probability of it not getting mutated is $(1 - P_m)$.

Since the number of fixed bits is o(H), and each one of them should not be altered for a schema to survive, the probability of survival of a schema after mutation in case of traditional genetic algorithms could be represented as follows[10]:

$$P_{surv} = (1 - P_m) \times (1 - P_m) \times \ldots \ldots \ldots \ldots \times (1 - P_m),\ o(\mathrm{H})\ \text{time.}$$

Hence, we get,

$$P_{surv} = (1 - P_m)^{o(H)} \tag{6}$$

From Eq. (5) & Eq. (6), the probability of the schema survival after crossover as well as mutation (P_{sur}) can be mathematically represented as follows:

$$P_{sur} = \left[1 - P_c \times \frac{d(H)}{(l-1)}\right] \times [(1 - P_m)^{o(H)}] \tag{7}$$

3. SCHEMA SURVIVAL AFTER TERNARY CROSSOVER OPERATOR AND MUTATION

Ternary crossover involves the mating of the three parents to generate three new offspring [12]. Each parent is assumed to have contributed equally to the formation of each offspring. From the mathematical analysis, the probability of survival of a schema after the application of the ternary crossover operator is obtained as follows [12]

$$P_{S_c} = P_c \times \left[1 - \frac{d(H)}{(l-1)}\right] \times \left[1 - \frac{d(H)}{(l-2)}\right] \times 0.8$$

Here, P_{S_c} = Probability of survival of a schema when ternary crossover occurs

P_c = Probability that the crossover takes place

$d(H)$ = Defining Length of the given schema

l = Length of the given schema

Let the probability that the crossover will not take place be represented by P_c', then,

$$P_c' = 1 - P_c \tag{9}$$

Now, the schema will survive if either of the following cases occurs

1. Crossover does not take place.
2. If crossover takes place and both the points of crossover lie outside the defining length, then, the probability of survival of the schema is 0.8.

Let the total chance of survival of the schema after the application of ternary crossover operator be represented by $P_{Surv'}$ then,

$$P_{Surv'} = P_c' + P_{S_c} \tag{10}$$

Substituting the values of P_{S_c} and P_c' from Eq. (8) and Eq. (9) in Eq. (10) we get:

$$P_{Surv'} = (1 - P_c) + P_c \times \left[1 - \frac{d(H)}{(l-1)}\right] \times \left[1 - \frac{d(H)}{(l-2)}\right] \times 0.8 \tag{11}$$

Let the probability of the schema survival after ternary crossover and mutation be represented by $P_{Surv''}$ then,

$$P_{Surv''} = P_{Surv'} \times P_{Surv} \tag{12}$$

Substituting the values of $P_{Surv'}$ and P_{surv} from Eq. (11) and Eq. (6) in Eq. (12) we get:

$$P_{Surv''} = \left[(1 - P_c) + P_c \times \left\{1 - \frac{d(H)}{(l-1)}\right\} \times \left\{1 - \frac{d(H)}{(l-2)}\right\} \times 0.8\right] \times \left[(1 - P_m)^{o(H)}\right] \tag{13}$$

4. PROBABILITY OF SURVIVAL OF A SCHEMA FOR GENETIC ALGORITHMS CONSISTING OF CHROMOSOMES HAVING BITS WITH DIFFERENT VULNERABILITY FOR MUTATION

Let us consider the example of the same schema (H) = 1*0*1** as discussed above; suppose, each bit in H has a different vulnerability for mutation and $P_{m_1}, P_{m_2} \ldots\ldots P_{m_7}$ be the probabilities of mutation of the bits ‘1’, ‘2’…… ‘7’respectively. Now, for this particular schema, bits 1, 3 and 5 are fixed, and none of them should change, for the schema to survive the mutation operation.

The probabilities of bits 1, 3 and 5 not getting mutated are given by $(1 - P_{m_1})$, $(1 - P_{m_3})$ and $(1 - P_{m_5})$ respectively. Hence, the overall expression for the survival of this schema is given by:

$$P_{s'} = (1 - P_{m_1}) \times (1 - P_{m_3}) \times (1 - P_{m_5}) \tag{14}$$

Now, for a general schema having ‘*n*’ fixed bits *i.e*. ‘bit-1’, ‘bit-2’,……‘bit-*n*’, the expression representing the probability of survival of the schema is as follows:

$$P_{s'} = (1 - P_{m_1}) \times (1 - P_{m_3}) \times \ldots\ \ldots\ \ldots.\ (1 - P_{m_n}) \tag{15}$$

4.1. Probability of Schema Survival after Traditional Crossover and Mutation for Genetic Algorithms Consisting of Chromosomes having Bits with Different Vulnerability for Mutation

Let the combined probability of the schema survival after traditional crossover and mutation (with each bit having a different vulnerability for mutation) be denoted by $P_{s''}$. Then,

$$P_{s''} = P_s \times P_{s'} \tag{16}$$

Substituting the values of P_s and $P_{s'}$ f'rom Eq. (5) and Eq. (15) in Eq. (16), the value of $P_{s''}$ could be derived as follows:

$$P_{s''} = \left[1 - P_c \times \frac{d(H)}{(l-1)}\right] \times [(1-P_{m_1}) \times [(1-P_{m_2}) \times \ldots\ldots\ldots (1-P_{m_n})] \tag{17}$$

Probability of Schema Survival after Ternary Crossover and Mutation for Genetic Algorithms Consisting of Chromosomes having Bits with Different Vulnerability for Mutation

Let the combined probability of the schema survival after ternary crossover and mutation (with each bit having a different vulnerability for mutation) be denoted by $P_{s'''}$. Then,

$$P_{s'''} = P_{surv'} \times P_{s'} \tag{18}$$

Substituting the values of $P_{surv'}$ and $P_{s'}$ from Eq. (11) and Eq. (15) in Eq. (18), we get:

$$P_{s'''} = \left[(1-P_c) + P_c \times \left\{1 - \frac{d(H)}{(l-1)}\right\} \times \left\{1 - \frac{d(H)}{(l-2)}\right\} \times 0.8\right] \times [(1-P_{m_1}) \times [(1-P_{m_2}) \times \ldots\ldots [(1-P_{m_n})] \tag{19}$$

5. DISCUSSION

Eq. (17) represents the probability of schema survival after traditional crossover and mutation for genetic algorithms consisting of chromosomes having bits with different vulnerability for mutation; and Eq. (19) represents the same after ternary crossover and mutation. The results obtained through these equations can be used to determine whether a particular schema will survive in the next generation in genetic algorithms consisting of chromosomes having bits with different vulnerability for mutation, which provides a better insight regarding the penetration of schema for such cases.

6. CONCLUDING REMARKS

In this paper, a new scenario of genetic algorithms consisting of chromosomes having bits with different vulnerability for mutation has been explored, and a mathematical expression representing the probability of survival of a schema after traditional crossover and mutation for this scenario is derived, as represented by Eq. (17) which is as follows:

$$P_{s''} = \left[1 - P_c \times \frac{d(H)}{(l-1)}\right] \times [(1-P_{m_1}) \times [(1-P_{m_2}) \times \ldots\ldots\ldots\ldots (1-P_{m_n})]$$

Here,
P_c = Probability that the crossover takes place
$d(H)$ = Defining Length of the given schema
l = Length of the given schema
$P_{m_1}, P_{m_2} \ldots P_{m_n}$ = Probabilities of mutation of the bits 1, 2…n respectively.

Another mathematical equation representing the probability of survival of a schema after the application of ternary crossover and mutation for the same scenario is also derived, as represented by Eq. (19), which is as follows:

$$P_{s'''} = \left[(1-P_c)+P_c \times \left\{1-\frac{d(H)}{(l-1)}\right\} \times \left\{1-\frac{d(H)}{(l-2)}\right\} \times 0.8\right] \times [(1-P_{m_1}) \times [(1-P_{m_2}) \times \ldots\ldots [(1-P_{m_n})]$$

The meaning of all the symbols used in this equation is same as defined above. These two mathematical expressions will prove to be very useful while applying these modified forms of genetic algorithms for solving various optimization problems.

7. ACKNOWLEDGMENT

The authors are grateful to Prof. S.G. Deshmukh, Director, ABV-Indian Institute of Information Technology & Management (An autonomous institute of government of India), Gwalior (M.P), for providing a cordial atmosphere of research in the institute.

REFERENCES

[1] Comput, N., Liang, M., Gao, C., & Zhang, Z. (2016). A new genetic algorithm based on modified Physarum network model for bandwidth-delay constrained least-cost multicast routing. Natural Computing, http://doi.org/10.1007/s11047-016-9545-6

[2] Kurdi, M. (2016). An effective new island model genetic algorithm for job shop scheduling problem. Computers & Operations Research, 132–142. http://doi.org/10.1016/j.cor.2015.10.005

[3] Hsu, C., & Cho, H. (2015). A genetic algorithm for the maximum edge-disjoint paths problem. Neurocomputing, 17–22. http://doi.org/10.1016/j.neucom.2012.10.046

[4] Mencía, R., Sierra, M. R., Menciá, C., & Varela, R. (2014). A genetic algorithm for job-shop scheduling with operators enhanced by weak Lamarckian evolution and search space narrowing. Natural Computing, 179–192. http://doi.org/10.1007/s11047-013-9373-x

[5] Stražar, M., Mraz, M., Zimic, N., & Moškon, M. (2014). An adaptive genetic algorithm for parameter estimation of biological oscillator models to achieve target quantitative system response. Natural Computing, 119–127. http://doi.org/10.1007/s11047-013-9383-8

[6] Amos, M., & Coldridge, J. (2012). A genetic algorithm for the Zen Puzzle Garden game. Natural Computing, 353–359. http://doi.org/10.1007/s11047-011-9284-7

[7] Pan, X., Jiao, L., & Liu, F. (2011). An improved multi-agent genetic algorithm for numerical optimization. Natural Computing, 487–506. http://doi.org/10.1007/s11047-010-9192-2

[8] Shukla, A., Tiwari, R., & Kala, R. (2010). Towards Hybrid and Adaptive Computing Studies in Computational Intelligence. Springer Verlag. ISBN: 9783642143434, http://doi.org/ 10.1007/978-3-642-14344-1.

[9] White, D. (2014). An overview of schema theory, Neural and Evolutionary Computing (cs.NE) 1–27. arXiv:1401.2651

[10] Goldberg, D. (1989). Genetic Algorithms in Search, Optimization, and Machine Learning, Reading, MA, Addison-Wesley.

[11] Mishra, A., & Shukla, A. (2016). Analysis of the Effect of Defining Length and Order of Schemata on Probability of Survival of a Group of Schemata, ICCSIS, 12–15. doi: 10.15242/IAE.IAE0416004

[12] Mishra, A., & Shukla, A. (2016). Mathematical analysis of the cumulative effect of novel ternary crossover operator and mutation on probability of survival of a schema. Theoretical Computer Science, 1–11. http://doi.org/10.1016/j.tcs.2016.07.035