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# DIDACTIC MEANING OF INTUITION

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***Abstract:*** The article reviews and analyzes the multi-aspect and versatile content of the concept of intuition; it reveals its native concepts and characteristic qualities. It analyzes the relation between this concept and logic in mathematics' didactics. It presents the details of the heuristic mechanism of intuition in the process of discovering mathematical truths. As a result of the conducted review, we define the quality of intuition related to a wider comprehension of the field of the subject of the study. The clear definition of this quality, which reveals, to some extent, the heuristic mechanism of intuition and which is used in didactics as a mean of heuristic learning, constitutes the novelty and theoretical significance of present article.

It shows further directions of this work, in particular, it points out further didactical reflection of the presented ideas.

***Keywords:*** Intuition, logics, qualities, definitions, heuristics, didactics, types of intuition, teaching mathematics.

## INTRODUCTION

The article presents a review of the definition of the intuition concept and analysis of its characteristic qualities with the aim of revealing its heuristic mechanism. Such aim appeared in the practice of teaching mathematics within the placement of hypotheses for studying the didactic role of this method. We hypothesized that it is possible to explicitly state an additional quality of intuition, which would clarify more its heuristic nature in the process of mathematics development and its more unambiguous and strict meaning, which would facilitate logically correct validation of didactic functioning of this method.

Therefore, the object of study in present article are theoretical statements in the definitions of the concepts, while the subject is the study of characteristic qualities of the concept of intuition as methodical means.

The concept of intuition has almost forty definitions in the research literature. They differ not only in some details in the definition of its characteristic qualities, but they also have differences even in the interpretation of the native concept (instinct, sensual form of cognition, unconscious primary principle of creation, ability to comprehend the truth, etc.). Due to this, it is not possible to speak about logically strict definitions, because, within such comprehension, the definitions have to be stated by one closest native concept and to have strictly defined, unambiguously

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comprehended characteristic qualities. On the other hand, the characteristic qualities can be stated in different ways and represent equivalent definitions.

We would like to present some of those definitions. We call them definitions, because they represent an attempt to reflect the content of this concept.

"Intuition (from Latin intuitio - looking closely). In the history of philosophy, the concept of intuition included highly multi-aspect and versatile content.

1. Intuition as common biological instinct, which directly points a living being to the things that are necessary for maintaining life…
2. Intuition as a sensual form of cognition - sensations and perceptions that are opposed to the mind and thinking…
3. Intuition as a rationalistic paradigm that finds primary knowledge on the operations of conscience and mind *per se* - so-called intellectual intuition…
4. Intuition as covert, hidden and unconscious principle of creation" (Philosophical encyclopedia, 1962, pp. 302-303).
5. "Intuition is a direct comprehension of truth without logical validation based on previous experience; feeling, insight" (Dictionary of foreign words, 2003).
6. "Intuition … is a thinking process that consists in finding a solution of a task based on search orienteers, which are not logically connected or insufficient for obtaining a logical conclusion" (Big psychological dictionary, 2003, p. 209).
7. "Intuition … is an ability to comprehend the truth by directly seeing it without a validation by verification" (Big Soviet encyclopedia, 1970).
8. "By intuition I mean not the faith in unstable confirmation of senses and not the deceiving statement of random imagination, but of the clear and attentive mind, which is so simple and vivid that it does not leave any doubt in the fact that we think, or equally, a robust understanding of a clear and attentive mind, which is produced only by the natural light of mind, and by its simplicity, it is more valid than the deduction itself…" - writes Descartes (1950, p. 80).
9. "Our 'intuitions' are simply opinions; our philosophical theories are the same. Some are commonsensical, some are sophisticated; some are particular, some general; some are more firmly held, some less. But they are all opinions…." (Lewis, 1983).
10. "In some cases, the tendencies that make certain beliefs attractive to us, that 'move' us in the direction of accepting certain propositions without taking us all the way to acceptance". (Van Inwagen, 1997, p. 306).

11. "Knowledge from, an ability to understand or know something immediately based on your feelings rather than facts" (Cambridge dictionary).
12. The ability to understand something instinctively, without the need for conscious reasoning. (Oxford dictionary, 1989).
13. Intuition (from Latin intuitio - observation, direct perception, looking closely), in wide (folk) sense, is an ability of direct, unmediated comprehension of truth. Philosophical understandings of intuition vary in meaning and content: from unconscious insight to specific, and even highest, form of knowledge. (New philosophical encyclopedia, 2010).

Modern studies (Karmin, 2007) separate intuition into six types of cognitive phenomena:

1. Intuition as an instinctive reaction;
2. Disposition intuition;
3. Perceptive (sensual) intuition;
4. Associative (imaginative) intuition;
5. Logical intuition;
6. Heuristic intuition.

However, the aims and subject of present study find the classification of V.Ya. Perminov (1987) more acceptable.

He separates several types of intuition: intellectual, subject or praxeological, empirical, logical, spatial or categorical, paradigmatic. Furthermore, possibilities of correct conclusion of the intuitive way of cognition are systematized in the form of apodictic (not revised by logic), which includes praxeological and logical, or assertoric (the rest of intuition kinds), which is a mean of heuristics and truth-like reasoning.

While the definitions 1) to 4) present an attempt to clarify the nature of this concept *per se*, in the definitions 5) to 8), the content of the concept is presented by the indication of qualities that characterize this method of thinking process. In turn, 5) makes an attempt to explain the origins of this phenomenon. In 9), intuition is presented as an opinion, in 10) – as a tendency, which contains the mechanism of moving toward a statement; in 11) – as the ability of sensual comprehension; in 12) – as an instinct. In some of the definitions, the native quality is replaced by other concepts, which themselves have an equal amount of unclarities in their interpretation. Descartes' explanation refers to intellectual intuition. For example, the most solid facts of arithmetic, like $2 \times 2 = 4$, were historically generated by such intuition.

The definition of the New philosophic dictionary (2010) shows that there is no especially novel change in comprehension of intuition in comparison with previous philosophic definition (1-4). Explanation of intuition proposed by A. Bergson (1912) and V.F. Asmus (1963) still remains relevant.

To summarize, the logical structure of definitions is not strict and is based on different native concepts; there is no unambiguous comprehension of characteristic qualities.

At the same time, there is another interesting fact: even in the routine life, people unambiguously understand this phenomenon and do not confuse it with other concepts. Therefore, in reality, they deal with non-strict definitions of this concept, i.e. they perceive the intuition itself intuitively, whereas going deeper into its structure does not help understanding it better. However, the routine use mostly refers to those interpretations that are presented in 1) and 12) (biological instinct), 5) (feeling, insight), 2) and 11) (sensual form of cognition).

Some studies mention wrong definitions. Such claims complicate the problem even more, because a definition is essentially a way of presenting a certain concept, which has to be perceived in a way that was stated in the definition. They are not statements, i.e. it is impossible to claim whether a definition is true or false; it is possible to speak about the absence of controversies in its presented qualities.

With all multitude of interpretation of the intuition concept, one of its characteristic qualities is always present in almost all of the definitions – it is the fact that intuitive way of comprehending the truth is conducted without the use of logical reasoning methods; but such interpretation creates such comprehension of the intuition concept that is too broad.

Naturally, all these points create difficulties for logical validation of many moments of this concept's functioning. Overcoming these difficulties is rather significant for the studies of this concept. Therefore, it is possible to talk about the relevance of present work. Scientific novelty of our suggestion is based on the fact that none of the existing definitions mention the quality that we state, and furthermore, its didactic meaning is not studied.

## METHODOLOGY

In the theoretical part of the study of the proposed new points, we used the methodology of methods, which is actively applied, for example, in such classical scientific works as "Mathematics and Plausible Reasoning" and "Mathematical Discovery" by G. Polya (1970, 1975).

Many mathematicians point out the important role of heuristic. For example, M. Kline analyzed large amount of material on the history of mathematics development and made the following significant conclusion about the role of intuition: "In general, this meant that the foundation of mathematics is not logic, but common sense and intuition. In the words of J. Hadamard, strictness only reveals what is conquered by intuition" (Kline, 1984, p. 225; Hadamard, 1970). He also points out important pedagogical meaning of intuition and convinces the readers that "knowledge is achieved intuitively, and logical statement is, at best, a secondary and additional help in teaching, and at worst, a definitive obstacle. … With the help of intuition, a student has to "fly to" the conclusion, "land", and only then can he use logic in order to see the general way leading to the goal. If this thought is correct, then the intuitive approach has to be the primary one during the introduction of new materials on all levels" (Kline, 1984, p. 47).

Similar position is shared by many recognized mathematicians – H. Poincare (1990), R. Thom (1978), H. Freudenthal (1983), G.V. Dorofeev (2000), and others.

These problems are addressed in more detail in the monograph "Intuition in teaching mathematics" of one of the authors of present article, T.S. Malikov. Present study continues the ideas that were developed in the monograph (Malikov, 2014).

In the field of mathematical cognition and education, H. Poincare defines the meaning of intuition in a wide sense and expresses it in the following statement: "In order to create geometry, or any other science, it is necessary to have something else rather than pure logic. We do not have any other word, apart from "intuition", for referring to this something…" (Poincare, 1990, p. 210). With such interpretation, all statements that are not based on the rules of logical conclusion have to be considered intuitive. At least, in the field of didactics of mathematics such interpretation of intuition seems more appropriate for solving the tasks of this field of cognition. Being more precise, we mean the functioning of intuition in the assertoric sense. For example, conclusions by induction, when the general conclusion is drawn from several specific statements, are not conducted based on the rules of logical conclusion; therefore, this reasoning can also be attributed to intuition. Obviously, because of the same reason, analogy should also be considered an intuitive conclusion. However, this reasoning by induction and analogy, apart from the intuitive component, also has a significant analytical component, a well-comprehended hint for the conclusion. On the other hand, such form as intuition, an insight, is not only conducted without a logical validation, but is seemingly not controlled by the conscience at all.

In our opinion, ambiguous comprehension of intuition creates one more factor: in the research literature, intuition appears in a rather strange manner, without clearly defined opposite way of reasoning. In fact, if analysis is opposed to synthesis in the dialectical unity, induction is opposed to deduction, specification to generalization, etc., intuition does not have an unambiguously defined "pair". Depending from the choice of a certain interpretation, different methods of reasoning are opposed to the intuition concept. As we stated above, H. Poincare opposes logic, and therefore, he considers induction and analogy as the manifestations of intuition. In turn, J. Bruner interprets intuition more in accordance with definition 4), and therefore, he opposes analytical methods of thinking, which include induction and analogy, to intuition (Bruner, 1962), i.e., in this case, induction and analogy are already the methods that are opposed to intuition. Such opposition occurs because his interpretation of intuition points out the factor of its unconscious nature.

Heuristic role of the sensual form of cognition in the functioning of intuition also remains unclear. In some cases, sensations and perceptions obviously help conducting the reasoning without a logical conclusion. For example, it is possible to propose a hypothesis that all diagonals of a parallelepiped intercept in one point because of the same conclusion about the quality of diagonals of a parallelogram. In other cases, the stereotype of thinking, which develops due to experience, sensual perception and the same analogy, leads to a false conclusion. Heuristic value of true intuitive conclusion often lies in overcoming a stereotype developed by the experience of sensual perception. We will demonstrate this influence on the following example. While studying the topic of "Parallel alignment of lines and planes" in the stereometrics course, the students learn the quality of parallel alignment of a line and a plane. "If a line that does not belong to a plane is parallel to a certain line in this plane, it is parallel to the plane itself". Then, during the studying of the quality of parallel alignment of planes, if the students are asked to propose a hypothesis about this quality, almost 95% of 420 respondents made the same mistake (surprisingly!). They say that, if a certain line on one plane is parallel to a line on the other plane, these planes are parallel. Obviously, this mistake is provoked by previous experience – the analogy with the quality of parallel alignment of a line and a plane.

Theoretical studies attempt to explain heuristic capacities of intuition. For example, dialectical materialism "sees the rational grain of the concept of Intuition in the characteristic of the unmediated moment in the cognition, which is a unity of sensual and rational" (Big Soviet encyclopedia, 1970). H. Poincare thinks that, during the selection of a correct solution by the intuition, aesthetic perception of the problem and similar things have a significant effect (Poincare, 1990). However, these thoughts do not have an experimental foundation, which is probably highly difficult; they are stated on the level of hypotheses.

We would like to focus on one of qualities of intuition, which is noted by some researchers, but the significance of which is somewhat unrecognized. We think that this quality allows clarifying to some extent the mechanism of heuristic capacities of intuition; it can be studied and validated rationally. This quality consists in the fact that intuitive thinking process is always conducted by integral reflection of not only the object of the study, but also of the whole system of connections or, at least, of an extended system that includes these objects. During the reasoning by the rules of logical conclusion, these connections are cut out of the integral picture of the reflection of the reality; they remain outside an abstractly separated field of study. For example, our survey (420 students), which was conducted with the aim of studying intuitive representation of parallelogram that the students operate while they solve the problems, showed that this figure occurs in students' conscience integrally with all its qualities. Moreover, this representation does not have any logical order in the multitude of these qualities. Hence, they do not distinguish the characteristic qualities (parallel alignment of the opposite sides), which lie at the basis of the definition of this concept, and the qualities, which are its consequences. Furthermore, the field of vision of this representation also includes wider connections of parallelogram with other figures on the plane.

And on the logical stage of a study, after defining the object and constructing the model, it is already impossible to include the studied object in other connections of general nature, because the logic is originated from one area of the reality, relations, qualities and statements, and draws the consequences that concern the statements from this field. Being more precise, this can be demonstrated on the basis of the Modus ponens logical conclusion ($P, P \rightarrow Q \models Q$), which is fundamental for other rules of conclusion in the axiomatic construction of mathematical logics. The statements that are the consequence of the general statement $P \rightarrow Q$ are concluded logically, whereas the solution can, in fact, lie outside the defined field of phenomena, and therefore it is already unreachable by logic. Integral general overview of all phenomena is available only to intuition.

Then, through intuition, the range of the $P \rightarrow Q$ statement is expanded until the searched solution falls into the area of these statements.

We will firstly attempt to illustrate this mechanism of heuristic capacities of intuition on a simple example. A popular entertaining puzzle about creating four equilateral triangles out of six matches with every side having the length of one match is difficult for many people who try to solve it. This happens primarily because they are initially predisposed to solving the required construction in one plane. But the puzzle does not have a solution on a single plane, regardless of the applied logical transformations. A person is at a dead end, because the logic does not expand the object of the study: a plane is not expanded to a three-dimensional space. But as soon as a guess that the problem has to be addressed in a more general situation (in a three-dimensional space) comes, the construction in the form of a tetrahedron becomes correct. This guess occurs not because of the logic but because of another thinking process, which, in the words of H. Poincare, cannot not be called anything other than "intuition". Moreover, this happens as a consequence of its capacity to integrally overview more complete situation, due to the transition to three-dimensional space, development of thought from specific to general and expansion of $P \rightarrow Q$ unity.

This mechanism of heuristic action of intuition practically exists in the process of discovering new truths of any scale. In fact, in the physics of the beginning of the

XXth century, the experiments of Michelson–Morley contradicted the postulates of classical physics, which lead to seemingly unsolvable controversies. Further development of this science showed that the difficulties were related to the attempts of explaining the results of the experiment within the classical interpretation of the structure of time and space (similarly to the attempts of creating four triangles with matches on a plane). As soon as A. Einstein was able to transition to wider connections between the objects of the study and to review the structure of the concept of time and space *per se*, it became possible to logically combine new facts with traditional physics, which was successful in his relativity theory. A. Einstein wrote the following about the role of intuition in this process: "… highest obligation of physicists is to search for those elementary laws, from which it is possible to obtain the picture of the world through deduction. It is not possible to reach these laws through a logical way, but only through intuition that is based on reaching the essence of experiments" (Einstein, 1965).

We will further analyze other examples. Attempts to prove the axiom of parallel alignment due to the limitation by the aim of proving this statement as a theorem, based on other axioms of Euclidian geometry. However, rejecting such limitation, analysis of this axiom as independent and construction of new geometrically equal geometry has led to success. It is possible to see that in this case there is also a transition from narrow field (multitude of axioms and their logical consequences) to wider field of cognitive activity.

Such analyses of the mechanism of discovery in mathematics led us to the statement of this new quality as a factor that clarifies heuristic qualities of intuition, which can be used in mathematics' didactics.

According to the stated above, we conducted an experimental work in order to verify the following hypothesis: in order to develop students' heuristic thinking, it is necessary to specifically develop their ability to conduct generalization, ability to go over the limits of the field, in which a task or a problem is set. Specifically, these qualities were evaluated by increasing the level of the problem-solving ability.

The experiment was conducted during many years with the help of interns in Kokshetau schools and teachers of MS #12. The number of students in the control classes is 265 people, in the experimental – 283 people.

In the beginning of the experiment, we checked the equality of the characteristics of learning mathematics in experimental and control classes on the basis of grades for typical control examinations. The conclusions were also made according to the results of the typical control examination at the end of the term.

We randomly selected the samples from experimental and control classes that contained 60 people each. The grades were given according to the criteria developed in the didactics: "unsatisfactory", "satisfactory", "good" and "excellent", i.e. the measurement was conducted on the order scale with four categories (C = 4).

Due to the fact that the samples were random and independent, and the studied quality has continuous distribution, as well as the small number of categories, it was appropriate to use the $\chi^2$ criterion in its two-tailed variation.

We would like to present the 2x4 table on the basis of the data of the samples obtained at the end of the experiment (table 1).

$n_1$ and $n_2$ – size of samples from experimental and control classes respectively, $O_{1i}$ – number of students in the experimental sample in each category $i = 1, 2, 3, 4$, $O_{2i}$ = number of students in the control group in each category $i = 1, 2, 3, 4$.

**TABLE 1: THE DATA OF THE SAMPLES OBTAINED AT THE END OF THE EXPERIMENT**

| *Categories Samples* | *Category 1 (unsatisfactory)* | *Category 2 (satisfactory)* | *Category 3 (good)* | *Category 4 (excellent)* |
|---|---|---|---|---|
| Sample #1 $n_1 = 60$ (experimental) | $O_{11} = 6$ | $O_{12} = 13$ | $O_{13} = 26$ | $O_{14} = 15$ |
| Sample #2 $n_2 = 60$ (control) | $O_{21} = 12$ | $O_{22} = 24$ | $O_{23} = 18$ | $O_{24} = 6$ |

None of the values of $O_{1i}$ and $O_{2i}$ are less than 5, therefore, approximation of the statistic distribution would not be approximate (Grabar & Krasnyanskaya, 1977, p. 102).

The formula of the value of statistic of the $\chi^2$ criterion with $n_1 = n_2$ would become more simple and would take the form of:

$$T = \sum_{i=1}^{4} \frac{(O_{2i} - O_{1i})^2}{O_{1i} + O_{2i}}$$

We will calculate the value of the statistic of the criterion by this formula after the conduction of the experiment ($T^2_{observation}$)

$$T^2_{observation} \approx 10{,}582$$

Level of significance $\alpha = 0{,}05$. According to the G table (Grabar & Krasnyanskaya, 1977, p. 130), for the degree of freedom $\nu = 4 - 1 = 3$ and level of significance $\alpha = 0{,}05$, according to the conditions of using the two-tailed $\chi^2$ criterion, critical value of the statistic of the criterion would be: $T_{critical} = 7{,}815$

Then: 7,815 < 10,582, т.е.

$$T_{critical} < T^2_{observation}$$

Based on this inequation, according to the rule of decision-making (Grabar & Krasnyanskaya, 1977, p. 102), null hypothesis $H_0$ is rejected and the alternative hypothesis $H_1$ is accepted with the validity of 95%. This means that there is a significant difference in the characteristics of the efficiency of learning in experimental and control classes, and the comparison of the results by each grade demonstrates that this difference manifests in the fact that the proposed technique serves as a mean of increasing the efficiency of learning.

Similarly, we validated that $T^{1}_{observation} < 7{,}815$, where $T^{1}_{observation}$ is the value of the statistic of the criterion before the conduction of the study. This inequation shows that the results of the samples obtained before the experiment do not present sufficient ground for rejecting the null hypothesis, which points to equal level of academic performance of students from control and experimental classes before the experiment.

Hence, the results of the study confirmed the proposed hypothesis.

Therefore, theoretical validation and the experiment show effective realization of the heuristic quality of intuition in the didactics of mathematics by a more generalized analysis of the object of the study.

## RESULTS, DISCUSSION

Presented reasoning shows that one of the essential points in the intuitive thinking, apart from the previously known, is the fact that intuitive thinking is a mean of integral and general overview of components and connections of a studied phenomenon. Consequently, there is a possibility to engage a certain studied situation in new connections, a possibility to choose from many situations and a possibility to transition from the range of the addressed phenomena to a more general unknown level. It is also possible to state that these conditions are necessary for heuristic process; they explain, to some extent, the mechanism of its action, which can be applied in didactics.

Therefore, generalizations of the studied field can be considered as one of the essential qualities of at least intuition in its assertoric interpretation in the field of mathematics' didactics.

Within the present article, we do not aim for a very large-scale significance of the new statements that we proposed, but they have significant consequences for the science because we approached one of the fundamental concepts in the theory of cognition, namely, intuition. It is interesting that such classical concept was left without due attention; moreover, a psychologist V.P. Zinchenko claims that it was deliberately avoided by researchers, thus producing a knot of unsolved problems. He writes that intuition "seems to turn into a divine vessel or a garbage pit, where everything unclear, enigmatic and mysterious has been put" (Zinchenko, 2002). Therefore, any minor clarification of the mechanism of this was of thinking, or even statement of problems in its functioning, is relevant and has scientific significance.

Also, due to the fundamental nature of the studied concept, it is possible to say that even minor conclusions about its use can lead to significant consequences of both theoretical and practical nature, as well as provoke a discussion around those ideas.

However, controversial interpretations of intuition can cause critical thoughts about our conclusions. We can reply that we addressed only that specific interpretation of intuition, which is used in the field of didactics of mathematics, in particular, according to H. Poincare.

Other critical points might refer to the fact that the proposed statement, although not being defined explicitly, was always implied intuitively, etc. We can reply that conscious study would still lead to positive consequences.

It is also possible that a problem can be solved without any generalization, but the student who is solving it cannot see this simple solution. Yes, this is possible. But also in this case, the ability to transition to other fields would not be an obstacle for searching even a simple solution. The ability to switch to other situations and to see the studied field in a generalized way is ultimately always useful.

Therefore, it is possible to say that our hypothesis has certain scientific significance, and it was confirmed by a theoretical review validation and an experiment.

## CONCLUSION

Due to the fact that the process of learning modulates the process of cognition, it is highly significant that the students learn scientific research method. Hence, it is necessary to focus on using intuition as heuristic method of cognition.

Additionally, the studied subject can be continued by expanding the field of application of intuition not only in the mathematics didactics, but also in other fields of cognition.

It is possible to continue the work in order to find common points in the controversial manifestations of intuition that were expressed in multiple definitions (1) - (13) described above.

In the process of the study we had the following additional hypotheses: certain manifestations of intuition can be studied for the *a priori* nature, for example, intellectual intuition.

Many questions rise in the evaluation of the correctness of intuitive statements; some manifestations are only plausible (word used by G. Polya), while intellectual intuition, according to the above mentioned statement by Descartes, can compete with logic in terms of correctness.

Many problems also appear in comprehending intuition from the perspective of consciousness, for example, insight is not controlled by the conscience, whereas induction in the interpretation of H. Poincare, as a type of intuitive reasoning, is directed by specific examples rather consciously.

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