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A Novel 4-D Hyperchaotic System with Three Quadratic Nonlinearities, its Adaptive Control and Circuit Simulation

Sivaperumal Sampath[1], Sundarapandian Vaidyanathan and Viet-Thanh Pham†**

Abstract: Recently, Vaidyanathan *et al.* (2015) announced a seven-term novel 3-D chaotic system with three quadratic nonlinearities. The Lyapunov exponents of the novel Vaidyanathan chaotic system are found as L_1 = 2.71916, L_2 = 0 and L_3 = –13.72776. The Kaplan-Yorke dimension of the novel Vaidyanathan chaotic system is derived as D_{KY} = 2.19808. In this paper, we derive a novel 4-D hyperchaotic system by introducing a feedback control to the novel 3-D Vaidyanathan chaotic system. The phase portraits of the novel 4-D hyperchaotic system are displayed and the mathematical properties are discussed. The novel 4-D hyperchaotic system has a unique equilibrium at the origin, which is a saddle-point. The Lyapunov exponents of the novel 4-D hyperchaotic system are obtained as L_1 = 3.05638, L_2 = 0.08646, L_3 = 0 and L_4 = –20.12087. Also, the Kaplan-Yorke dimension of the novel 4-D hyperchaotic system is derived as D_{KY} = 3.15619. Next, an adaptive controller is designed to globally stabilize the novel 4-D hyperchaotic system with unknown parameters. Finally, an electronic circuit simulation of the novel hyperchaotic system is presented using SPICE to confirm the feasibility of the theoretical model.

Keywords: Chaos, chaotic systems, hyperchaos, hyperchaotic systems, circuit simulation.

1. INTRODUCTION

Chaos theory describes the qualitative study of unstable aperiodic behaviour in deterministic nonlinear dynamical systems. A dynamical system is called *chaotic* if it satisfies the three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1]. Chaos theory has applications in several areas in Science and Engineering.

A significant development in chaos theory occurred when Lorenz discovered a 3-D chaotic system of a weather model [2]. Subsequently, Rössler found a 3-D chaotic system [3], which is algebraically simpler than the Lorenz system. Indeed, Lorenz's system is a seven-term chaotic system with two quadratic nonlinearities, while Rössler's system is a seven-term chaotic system with just one quadratic nonlinearity.

Some well-known paradigms of 3-D chaotic systems are Arneodo system [4], Sprott systems [5], Chen system [6], Lü-Chen system [7], Liu system [8], Cai system [9], Tigan system [10], etc.

In the last two decades, many new chaotic systems have been also discovered like Li system [11], Sundarapandian systems [12-13], Vaidyanathan systems [14-33], Pehlivan systems [34-35], Pham systems [36-37], Jafari system [38], etc.

Chaos theory has applications in several fields of science and engineering such as lasers [39], oscillators [40], chemical reactions [41-50], biology [51-66], ecology [67], artificial neural networks [68-69], robotics [70-71], fuzzy logic [72], electrical circuits [73-76], cryptosystems [77-78], memristors [79-81], etc.

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A *hyperchaotic* attractor is typically defined as chaotic behavior with at least two positive Lyapunov exponents. The first hyperchaotic system was proposed by Rössler in 1979 [82]. The Lyapunov exponents of the hyperchaotic Rössler system were determined as $L_1 = 0.112$, $L_2 = 0.019$, $L_3 = 0$ and $L_4 = -25.188$. In the last few decades, many hyperchaotic systems have been reported in the literature such as hyperchaotic Lorenz system [83], hyperchaotic Lü system [84], hyperchaotic Chen system [85], hyperchaotic Li system [86], hyperchaotic Vaidyanathan systems [87-95], etc.

In this paper, we announce a novel 4-D hyperchaotic system with three quadratic nonlinearities. The novel 4-D hyperchaotic system has been derived by introducing a state feedback control to novel 3-D Vaidyanathan chaotic system [27]. We discuss the qualitative properties of the novel 4-D hyperchaotic system and display the phase portraits of the novel 4-D hyperchaotic system. The proposed novel 4-D hyperchaotic system has a unique equilibrium at the origin, which is a saddle-point.

The Lyapunov exponents of the novel 3-D Vaidyanathan chaotic system [27] are obtained as $L_1 = 2.71916$, $L_2 = 0$ and $L_3 = -13.72776$. The Kaplan-Yorke dimension of the novel Vaidyanathan chaotic system is derived as $D_{KY} = 2.19808$. The Lyapunov exponents of the novel 4-D hyperchaotic system are obtained as $L_1 = 3.05638$, $L_2 = 0.08646$, $L_3 = 0$ and $L_4 = -20.12087$. Also, the Kaplan-Yorke dimension of the novel conservative chaotic system is derived as $D_{KY} = 3.15619$.

Next, this paper derives an adaptive control law that stabilizes the novel hyperchaotic system with unknown system parameters. This paper also derives an adaptive control law that achieves global chaos synchronization of identical conservative chaotic systems with unknown parameters.

Synchronization of chaotic systems is a phenomenon that may occur when a chaotic oscillator drives another chaotic oscillator. In most of the synchronization approaches, the *master-slave* or *drive-response* formalism is used. If a particular chaotic system is called the master or drive system, and another chaotic system is called the slave or response system, then the idea of synchronization is to use the output of the master system to control the response of the slave system so that the slave system tracks the output of the master system asymptotically.

In the chaos literature, a variety of techniques have been proposed to solve the problem of chaos synchronization such as PC method [96], active control method [91-112], adaptive control method [113-125], backstepping control method [126-134], fuzzy control method [135-136], sliding mode control method [137-146], etc. Anti-chaos control and synchronization of chaotic and hyperchaotic systems are important research problems in the chaos literature.

This paper is organized as follows. In Section 2, we describe the novel 4-D hyperchaotic system with three quadratic nonlinearities. In Section 3, we describe the qualitative properties of the novel 4-D hyperchaotic system. In Section 4, we detail the adaptive control design for the global chaos stabilization of the novel hyperchaotic system with unknown system parameters. In Section 5, we detail the SPICE simulation of the proposed novel hyperchaotic system. In Section 6, we give a summary of the main results derived in this research work.

2. A NOVEL 4-D HYPERCHAOTIC SYSTEM

In [27], Vaidyanathan et al. (2015) announced a novel 3-D chaotic system described by

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + dx_2 x_3 \\ \dot{x}_2 = bx_1 - x_1 x_3 \\ \dot{x}_3 = x_1 x_2 - cx_3 \end{cases} \tag{1}$$

In (1), x_1, x_2, x_3 are the states and a, b, c, d are constant, positive, parameters.

In (1), it was shown that the system (1) is *chaotic* when the parameter values are taken as

$$a = 10, \quad b = 15, \quad c = 1, \quad d = 12 \tag{2}$$

For numerical simulations, we take the initial values of the novel chaotic system (1) as

$$x_1(0) = 0.6, \; x_2(0) = 1.8, \; x_3(0) = 1.2 \tag{3}$$

The Lyapunov exponents of the novel Vaidyanathan system (1) are obtained as

$$L_1 = 2.71916, \quad L_2 = 0, \quad L_3 = -13.72776 \tag{4}$$

Figure 1 describes the *two-scroll strange attractor* of the novel Vaidyanathan chaotic system (1).

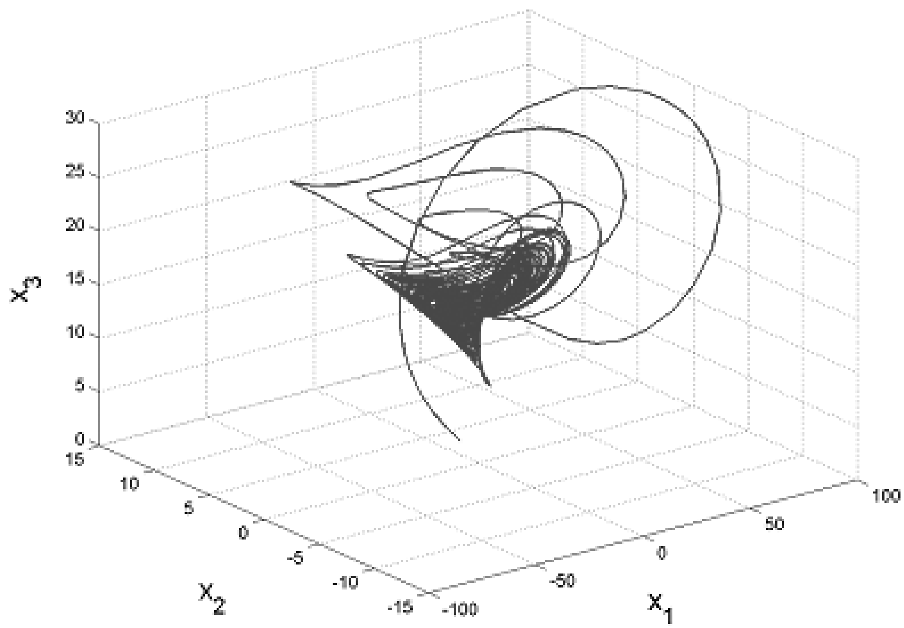

Figure 1: Phase portrait of the novel 3-D Vaidyanathan chaotic system

In this paper, we propose a novel 4-D hyperchaotic system by introducing a state feedback control to the novel Vaidyanathan chaotic system (1). Our system is described by the 4-D dynamics

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + dx_2x_3 \\ \dot{x}_2 = bx_1 - x_1x_3 + px_4 \\ \dot{x}_3 = x_1x_2 - cx_3 \\ \dot{x}_4 = -qx_1 \end{cases} \tag{5}$$

In (5), x_1, x_2, x_3, x_4 are the states and a, b, c, d, p, q are constant, positive, parameters.

The system (5) is *hyperchaotic* when we take the parameter values as

$$a = 15, \quad b = 18, \quad c = 2, \quad d = 13, \; p = 0.4, \quad q = 1.4 \tag{6}$$

For numerical simulations, we take the initial values of the novel 4-D system (5) as

$$x_1(0) = 0.6, \; x_2(0) = 1.8, \; x_3(0) = 1.2, \; x_4(0) = 0.5 \tag{7}$$

The Lyapunov exponents of the novel 4-D system (5) are numerically obtained as

$$L_1 = 3.05638, \; L_2 = 0.08646, \; L_3 = 0, \quad L_4 = -20.12087 \tag{8}$$

Since the novel 4-D system (5) has two positive Lyapunov exponents, it is hyperchaotic.

Figures 2-5 show the 3-D projections of the novel hyperchaotic system (5) in (x_1, x_2, x_3), (x_1, x_3, x_4) and (x_2, x_3, x_4) spaces, respectively. Figure 2 exhibits a two-scroll strange chaotic attractor of the novel hyperchaotic system (5) in the (x_1, x_2, x_3) space.

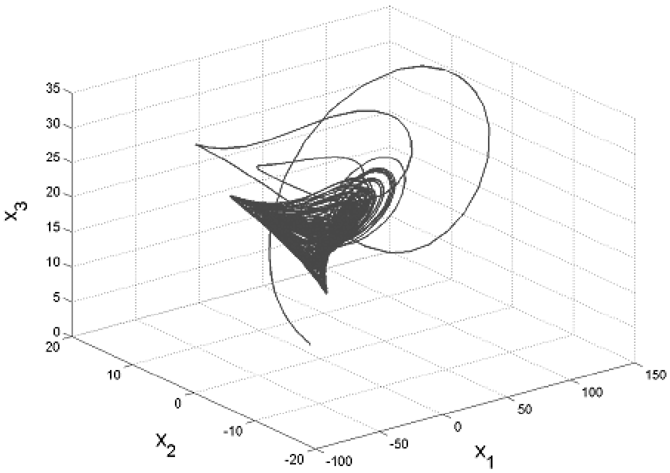

Figure 2: 3-D projection of the novel hyperchaotic system on the (x_1, x_2, x_3) space

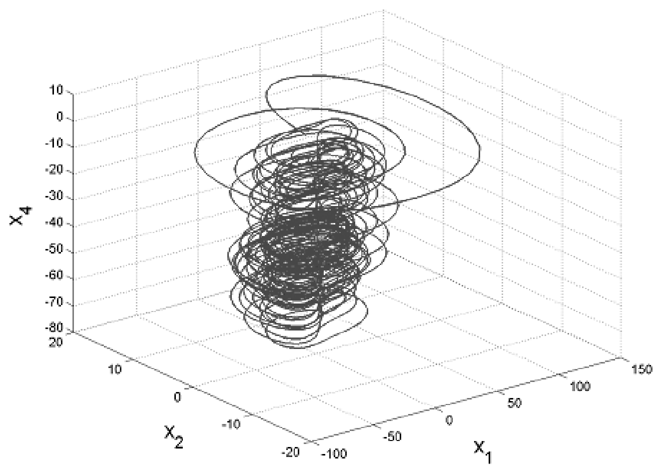

Figure 3: 3-D projection of the novel hyperchaotic system on the (x_1, x_2, x_4) space

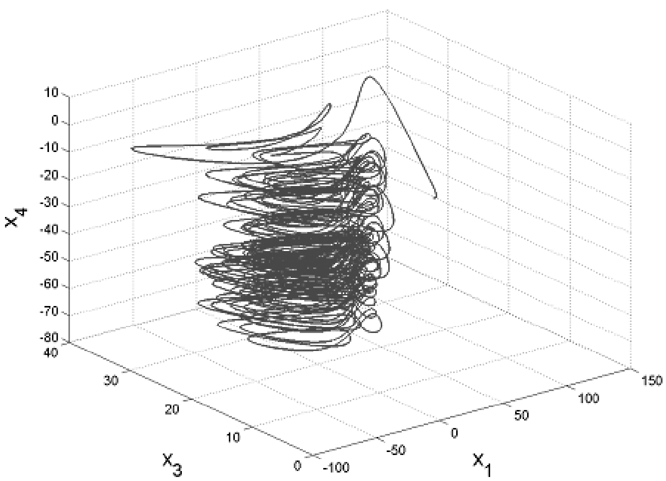

Figure 4. 3-D projection of the novel hyperchaotic system on the (x_1, x_3, x_4) space

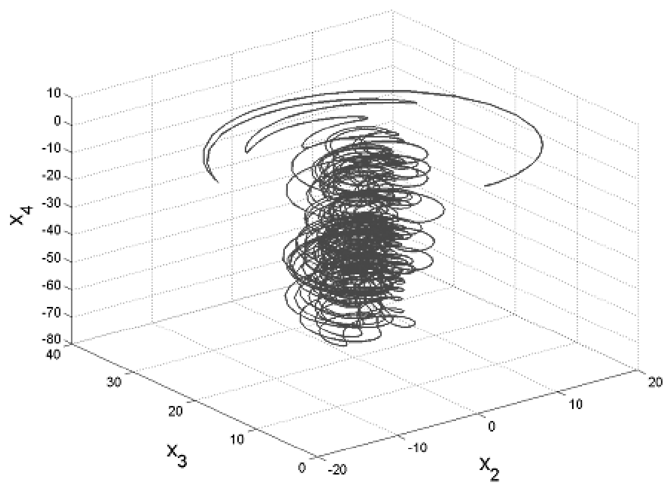

Figure 5. 3-D projection of the novel hyperchaotic system on the (x_2, x_3, x_4) space

3. PROPERTIES OF THE NOVEL HYPERCHAOTIC SYSTEM

In this section, we discuss the qualitative properties of the hyperchaotic system (5) introduced in Section 2. We suppose that the parameter values of the system (5) are as in the hyperchaotic case, i.e.

$$a = 15, \quad b = 18, \quad c = 2, \quad d = 13, \quad p = 0.4, \quad q = 1.4 \tag{9}$$

3.1. Dissipativity

In vector notation, we may express the 4-D hyperchaotic system (5) as

$$\dot{x} = f(x) = \begin{bmatrix} f_1(x_1, x_2, x_3, x_4) \\ f_2(x_1, x_2, x_3, x_4) \\ f_3(x_1, x_2, x_3, x_4) \\ f_4(x_1, x_2, x_3, x_4) \end{bmatrix} \tag{10}$$

where

$$\begin{cases} f_1(x_1,x_2,x_3,x_4) = a(x_2 - x_1) + dx_2x_3 \\ f_2(x_1,x_2,x_3,x_4) = bx_1 - x_1x_3 + px_4 \\ f_3(x_1,x_2,x_3,x_4) = x_1x_2 - cx_3 \\ f_4(x_1,x_2,x_3,x_4) = -qx_1 \end{cases} \tag{11}$$

Let Ω be any region in R^4 with a smooth boundary and also $\Omega(t) = \Phi_t(\Omega)$, where Φ_t is the flow of the vector field f. Furthermore, let $V(t)$ denote the hyper-volume of $\Omega(t)$.

By Liouville's theorem, we have

$$\dot{V} = \int_{\Omega(t)} (\nabla \cdot f)\, dx_1\, dx_2\, dx_3\, dx_4 \tag{12}$$

The divergence of the novel hyperchaotic system (5) is easily found as

$$\nabla \cdot f = \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} + \frac{\partial f_3}{\partial x_3} + \frac{\partial f_4}{\partial x_4} = -a + 0 - c + 0 = -(a + c) = -\mu \tag{13}$$

where $\mu = a + c = 17 > 0$.

Substituting (13) into (12), we obtain the first order ODE

$$\dot{V} = \int_{\Omega(t)} (-\mu)\, dx_1\, dx_2\, dx_3\, dx_4 = -\mu V \tag{14}$$

Integrating (14), we obtain the unique solution as

$$V(t) = \exp(-\mu t)\, V(0), \text{ for all } t \geq 0 \tag{15}$$

Since $\mu > 0$, it follows from Eq. (15) that $V(t) \to 0$ exponentially as $t \to \infty$.

Thus, the novel 4-D hyperchaotic system (5) is dissipative.

3.2. Symmetry and Invariance

It is easy to see that the hyperchaotic system (5) is invariant under the coordinates transformation

$$(x_1, x_2, x_3, x_4) = (-x_1, -x_2, x_3, -x_4) \tag{16}$$

Thus, the novel hyperchaotic system (5) has rotation symmetry about the x_3–axis. As a consequence, any non-trivial trajectory of the system (5) must have a twin trajectory.

It is also easy to check that the x_3–axis is invariant for the flow of the novel hyperchaotic system (5).

Also, the invariant motion along the x_3–axis is characterized by the scalar dynamics

$$\dot{x}_3 = -cx_3, \quad (c > 0) \tag{17}$$

which is globally exponentially stable.

3.3 Equilibrium Points

The equilibrium points of the system (5) are obtained by solving the system of equations

$$\begin{cases} a(x_2 - x_1) + dx_2 x_3 = 0 \\ bx_1 - x_1 x_3 + px_4 = 0 \\ x_1 x_2 - cx_3 = 0 \\ -qx_1 = 0 \end{cases} \tag{18}$$

Solving the system (18), we obtain the unique solution as

$$E_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{19}$$

The Jacobian matrix of the system (5) at the equilibrium point E_0 is given by

$$J_0 = J(E_0) = \begin{bmatrix} -a & a & 0 & 0 \\ b & 0 & 0 & p \\ 0 & 0 & -c & 0 \\ -q & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -15 & 15 & 0 & 0 \\ 18 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ -1.4 & 0 & 0 & 0 \end{bmatrix} \tag{20}$$

The matrix J_0 has the eigenvalues

$$\lambda_1 = 0, \quad \lambda_2 = -2, \ \lambda_3 = -25.5624, \quad \lambda_4 = 10.5624 \tag{21}$$

This shows that the equilibrium point E_0 is a saddle-focus point, which is unstable.

3.4. Lyapunov Exponents and Kaplan-yorke Dimension

We take the parameter values of the novel system (5) as in the hyperchaotic case, i.e.

$$a = 15, \quad b = 18, \quad c = 2, \quad d = 13, \ p = 0.4, \quad q = 1.4 \tag{22}$$

We choose the initial values of the state as

$$x_1(0) = 0.6, \ x_2(0) = 1.8, \ x_3(0) = 1.2, \ x_4(0) = 0.5 \tag{23}$$

The Lyapunov exponents of the novel 4-D system (5) are numerically obtained as

$$L_1 = 3.05638, \ L_2 = 0.08646, \ L_3 = 0, \ \ L_4 = -20.12087 \tag{24}$$

Since the novel 4-D system (5) has two positive Lyapunov exponents, it is hyperchaotic.

Figure 6 shows the Lyapunov exponents of the system (1) as determined by MATLAB.

Also, the Maximal Lyapunov Exponent of the system (1) is L_1 = 3.05638.

The Kaplan-Yorke dimension of the novel hyperchaotic system (5) is derived as

$$D_{KY} = 3 + \frac{L_1 + L_2 + L_3}{|L_4|} = 3.1562 \tag{25}$$

4. ADAPTIVE CONTROL DESIGN FOR THE STABILIZATION OF THE NOVEL HYPERCHAOTIC SYSTEM

In this section, we use adaptive control method to derive an adaptive feedback control law for globally and exponentially stabilizing the novel 4-D hyperchaotic system with unknown parameters.

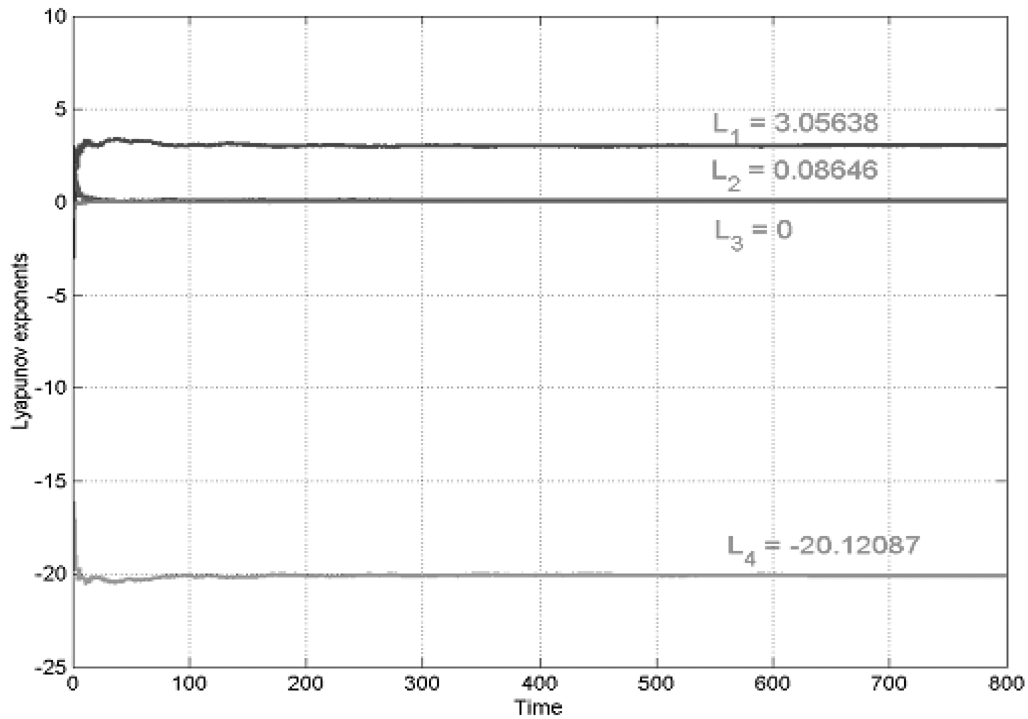

Figure 6: Lyapunov exponents of the novel hyperchaotic system

Thus, we consider the novel 4-D hyperchaotic system given by

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + dx_2x_3 + u_1 \\ \dot{x}_2 = bx_1 - x_1x_3 + px_4 + u_2 \\ \dot{x}_3 = x_1x_2 - cx_3 + u_3 \\ \dot{x}_4 = -qx_1 + u_4 \end{cases} \tag{26}$$

In (26), x_1, x_2, x_3, x_4 are the states and u_1, u_2, u_3, u_4 are adaptive controls to be determined using estimates of the unknown system parameters.

We consider the adaptive control law defined by

$$\begin{cases} u_1 = -\hat{a}(t)(x_2 - x_1) - \hat{d}(t)x_2x_3 - k_1x_1 \\ u_2 = -\hat{b}(t)x_1 + x_1x_3 - \hat{p}x_4 - k_2x_2 \\ u_3 = -x_1x_2 + \hat{c}(t)x_3 - k_3x_3 \\ u_4 = \hat{q}(t)x_1 - k_4x_4 \end{cases} \tag{27}$$

where k_1, k_2, k_3, k_4 are positive gain constants.

Substituting (27) into (26), we get the closed-loop plant dynamics as

$$\begin{cases} \dot{x}_1 = [a - \hat{a}(t)](x_2 - x_1) + [d - \hat{d}(t)]x_2x_3 - k_1x_1 \\ \dot{x}_2 = [b - \hat{b}(t)]x_1 + [p - \hat{p}(t)]x_4 - k_2x_2 \\ \dot{x}_3 = -[c - \hat{c}(t)]x_3 - k_3x_3 \\ \dot{x}_4 = -[q - \hat{q}(t)]x_1 - k_4x_4 \end{cases} \tag{28}$$

The parameter estimation errors are defined as

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_b(t) = b - \hat{b}(t) \\ e_c(t) = c - \hat{c}(t) \\ e_d(t) = d - \hat{d}(t) \\ e_p(t) = p - \hat{p}(t) \\ e_q(t) = q - \hat{q}(t) \end{cases} \tag{29}$$

Using (29), we can simplify the closed-loop plant dynamics (28) as

$$\begin{cases} \dot{x}_1 = e_a(x_2 - x_1) + e_d x_2 x_3 - k_1 x_1 \\ \dot{x}_2 = e_b x_1 + e_p x_4 - k_2 x_2 \\ \dot{x}_3 = -e_c x_3 - k_3 x_3 \\ \dot{x}_4 = -e_q x_1 - k_4 x_4 \end{cases} \tag{30}$$

Differentiating (25) with respect to t, we obtain

$$\begin{cases} \dot{e}_a(t) = -\dot{\hat{a}}(t) \\ \dot{e}_b(t) = -\dot{\hat{b}}(t) \\ \dot{e}_c(t) = -\dot{\hat{c}}(t) \\ \dot{e}_d(t) = -\dot{\hat{d}}(t) \\ \dot{e}_p(t) = -\dot{\hat{p}}(t) \\ \dot{e}_q(t) = -\dot{\hat{q}}(t) \end{cases} \tag{31}$$

We use adaptive control theory to find an update law for the parameter estimates.

We consider the quadratic candidate Lyapunov function defined by

$$V(x, e_a, e_b, e_c, e_d, e_p, e_q) = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2 + x_4^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2 + e_p^2 + e_q^2\right) \tag{32}$$

Clearly, V is a positive definite function on R^{10}.

Differentiating V along the trajectories of (30) and (31), we obtain

$$\begin{aligned} \dot{V} = &-k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 - k_4 x_4^2 + e_a\left[x_1(x_2 - x_1) - \dot{\hat{a}}\right] + e_b\left[x_1 x_2 - \dot{\hat{b}}\right] \\ &+ e_c\left[-x_3^2 - \dot{\hat{c}}\right] + e_d\left[x_1 x_2 x_3 - \dot{\hat{d}}\right] + e_p\left[x_2 x_4 - \dot{\hat{p}}\right] + e_q\left[-x_1 x_4 - \dot{\hat{q}}\right] \end{aligned} \tag{33}$$

In view of (33), we take the parameter update law as follows:

$$\begin{cases} \dot{\hat{a}} = x_1(x_2 - x_1) \\ \dot{\hat{b}} = x_1 x_2 \\ \dot{\hat{c}} = -x_3^2 \\ \dot{\hat{d}} = x_1 x_2 x_3 \\ \dot{\hat{p}} = x_2 x_4 \\ \dot{\hat{q}} = -x_1 x_4 \end{cases} \tag{34}$$

Theorem 1. The novel 4-D hyperchaotic system (26) with unknown system parameters is globally and exponentially stabilized for all initial conditions $x(0)\in R^4$ by the adaptive control law (27) and the parameter update law (34), where k_1, k_2, k_3, k_4 are positive gain constants.

Proof. We prove this result by using Lyapunov stability theory [147]. We consider the quadratic Lyapunov function defined by (32), which is positive definite on R^{10}.

By substituting the parameter update law (34) into (33), we obtain the time derivative of V as

$$\dot{V} = -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 - k_4 x_4^2 \tag{35}$$

From (35), it is clear that $\dot{V}$ is a negative semi-definite function on R^{10}.

Thus, we can conclude that the state vector $x(t)$ and the parameter estimation error are globally bounded, *i.e.*

$$\left[\boldsymbol{x}(t) \quad e_a(t) \quad e_b(t) \quad e_c(t) \quad e_d(t) \quad e_p(t) \quad e_q(t) \right]^T \in L_\infty \tag{36}$$

We define $k = \min \{k_1, k_2, k_3, k_4\}$.

Thus, it follows from (35) that

$$\dot{V} \le -k \left\| \boldsymbol{x}(t) \right\|^2 \tag{37}$$

Thus, we have

$$k \left\| \boldsymbol{x}(t) \right\|^2 \le -\dot{V} \tag{38}$$

Integrating the inequality (38) from 0 to t, we get

$$k \int_0^t \left\| \boldsymbol{x}(\tau) \right\|^2 d\tau \le V(0) - V(t) \tag{39}$$

From (39), it follows that $x\in L_2$. Using (30), we can conclude that $\dot{x} \in L_\infty$.

Using Barbalat's lemma [147], we can conclude that $x(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $x(0)\in R^4$.

This completes the proof. ■

For numerical simulations, the classical fourth-order Runge-Kutta method with step size $h = 10^{-8}$ is used to solve the systems (26) and (34), when the adaptive control law (27) is applied.

The parameter values of the novel hyperchaotic system are taken as in the hyperchaotic case, i.e.

$$a = 15, \quad b = 18, \quad c = 2, \quad d = 13, \quad p = 0.4, \quad q = 1.4 \tag{40}$$

We take the positive gain constants as $k_i = 5$ for $i = 1, 2, 3, 4$.

Furthermore, as initial conditions of the novel hyperchaotic system (22), we take

$$x_1(0) = 8.5, \quad x_2(0) = 4.7, \quad x_3(0) = -5.9, \quad x_4(0) = 12.8 \tag{41}$$

Also, as initial conditions of the parameter estimates, we take

$$\hat{a}(0) = 15, \ \hat{b}(0) = 9, \ \hat{c}(0) = 4, \ \hat{d}(0) = 3, \ \hat{p}(0) = 11, \ \hat{q}(0) = 5 \tag{42}$$

Figure 7 shows the exponential convergence of the controlled state trajectories of the 4-D novel hyperchaotic system (26).

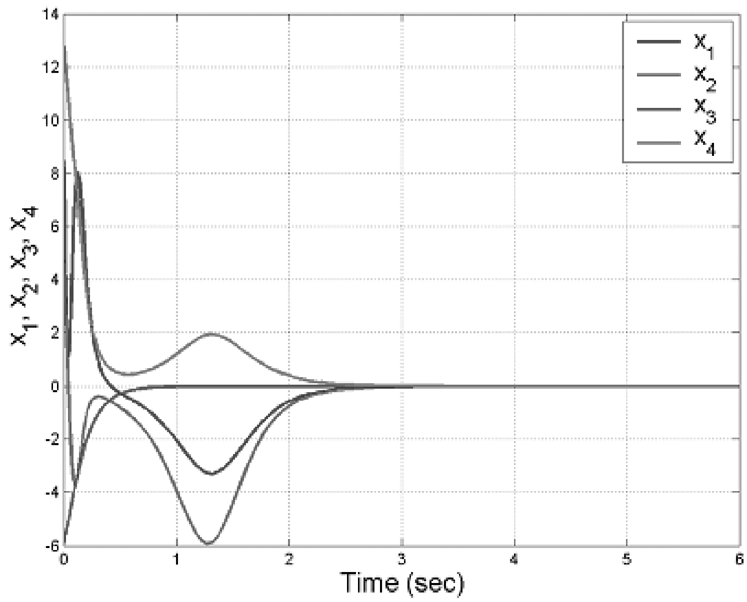

Figure 7: Time-history of the controlled state trajectories of the hyperchaotic system

5. CIRCUIT SIMULATION OF THE NOVEL HYPERCHAOTIC SYSTEM

In this section, an electronic circuit is designed to show the feasibility and complex dynamics of the theoretical model (5). The circuit employs common electronic components such as resistors, operational amplifiers, analog multipliers, and capacitors.

The schematic of the proposed circuit is shown in Fig. 8 in which the output voltages on the capacitors $(v_{c_1}, v_{c_2}, v_{c_3}, v_{c_4})$ correspond to the state variables (x_1, x_2, x_3, x_4) of the model (5). The circuital equations of the circuit are given by

$$\begin{cases} \dfrac{dv_{C_1}}{dt} = \dfrac{1}{R_1C_1}v_{C_2} - \dfrac{1}{R_2C_1}v_{C_1} + \dfrac{1}{R_3C_1}v_{C_2}v_{C_3} \\ \dfrac{dv_{C_2}}{dt} = \dfrac{1}{R_4C_2}v_{C_1} - \dfrac{1}{10R_5C_2}v_{C_1}v_{C_3} + \dfrac{1}{R_6C_2}v_{C_4} \\ \dfrac{dv_{C_3}}{dt} = \dfrac{1}{R_7C_3}v_{C_1}v_{C_2} - \dfrac{1}{R_8C_3}v_{C_3} \\ \dfrac{dv_{C_4}}{dt} = -\dfrac{1}{R_9C_4}v_{C_1} \end{cases} \tag{43}$$

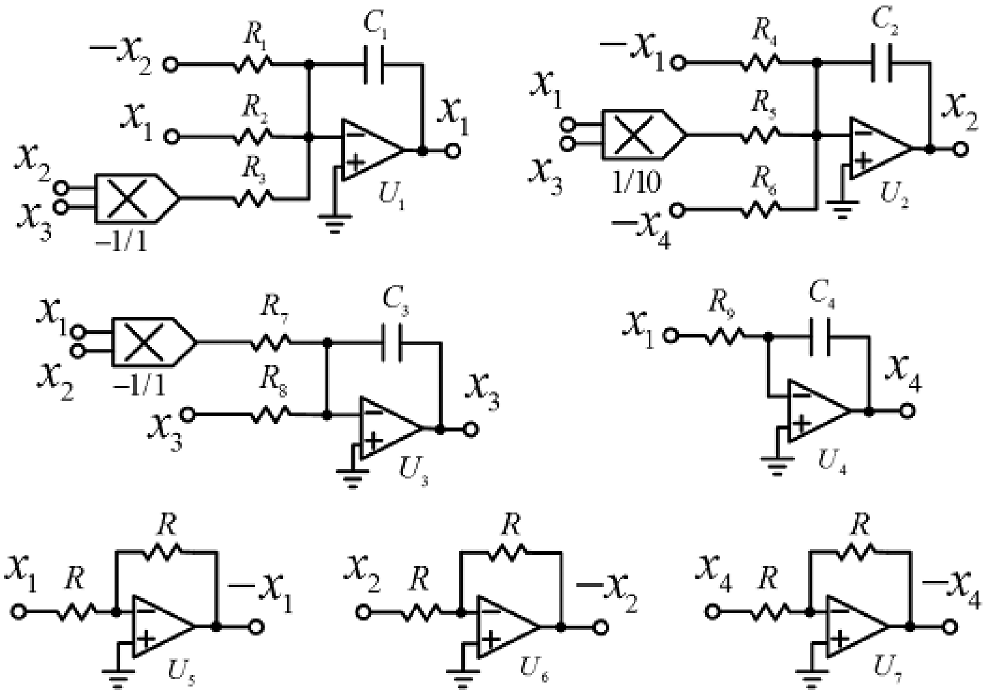

Figure 8: The electronic circuit of the novel 4-D hyperchaotic system

The values of electronic components in Fig. 8 are chosen as:

$R_1 = R_2 = 26.667k\Omega$, $R_3 = 3.077k\Omega$, $R_4 = 22.222k\Omega$, $R_5 = 4k\Omega$ $R_6 = 1M\Omega$, $R_7 = 40k\Omega$, $R_8 = 200k\Omega$, $R_9 = 285.714k\Omega$, $R = 400k\Omega$, and $C_1 = C_2 = C_3 = C_4 = 1nF$.

The outputs of the circuit in Fig. 8 are displayed in Figs. 9-12 by using OrCAD-PSpice. Obtained results indicate the feasibility of the theoretical hyperchaotic system (5). Moreover, this circuit is useful in potential chaos-based applications.

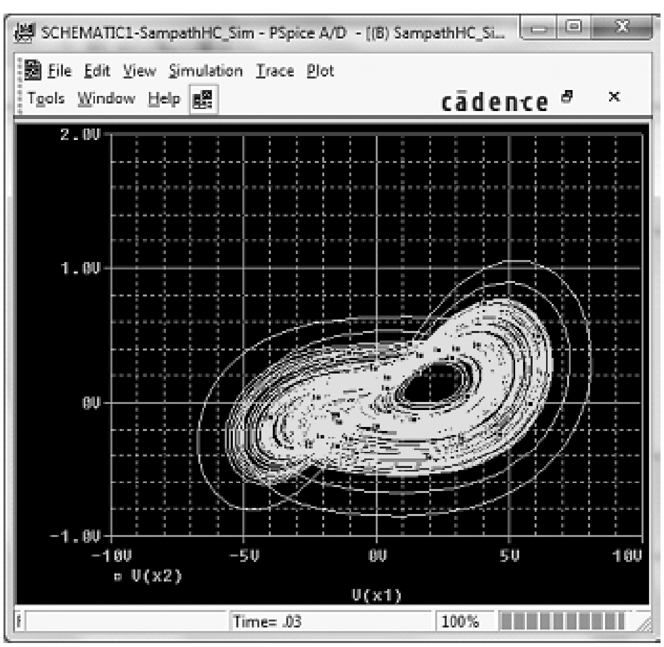

Figure 9: Phase portrait of the electronic circuit in the $v_{c_1} - v_{c_2}$ plane

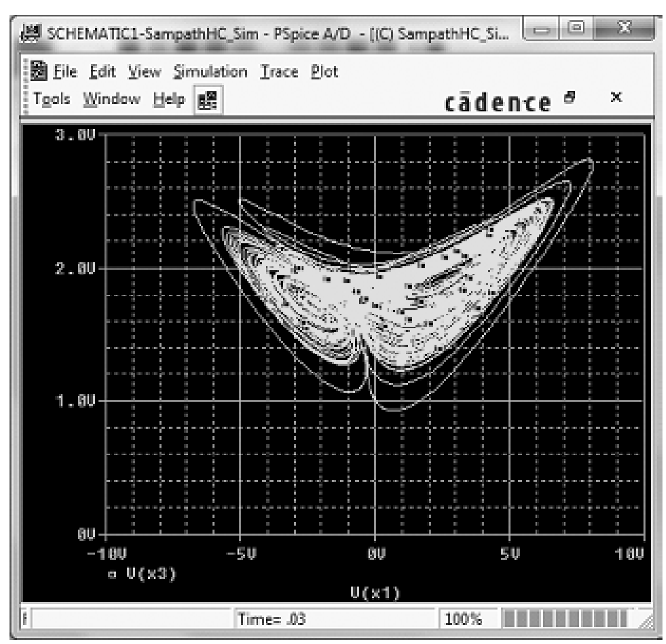

Figure 10: Phase portrait of the electronic circuit in the $v_{c_1} - v_{c_3}$ plane

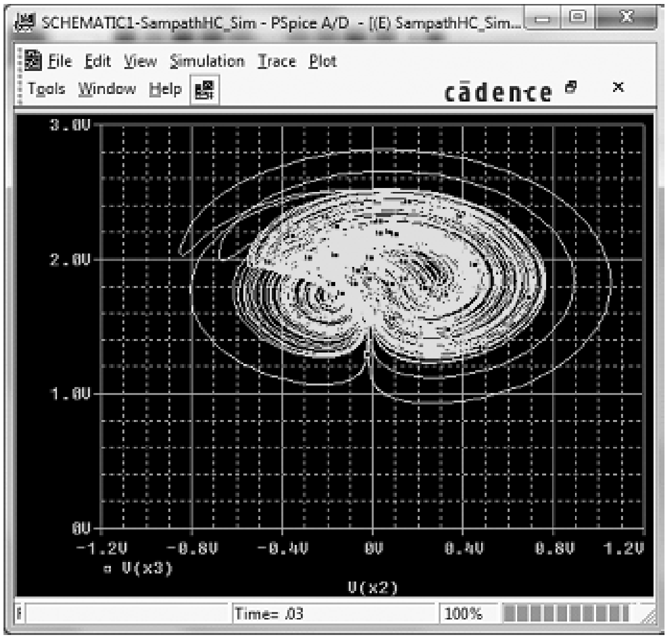

Figure 11: Phase portrait of the electronic circuit in the $v_{c_2} - v_{c_3}$ plane

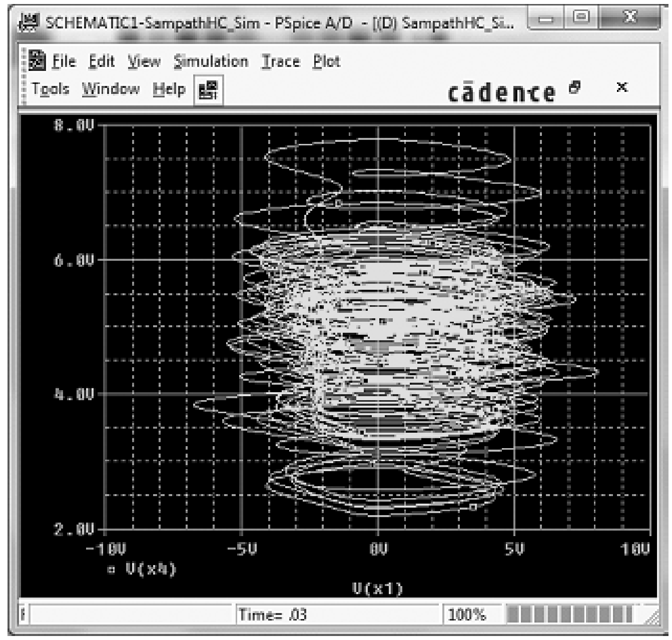

Figure 12: Phase portrait of the electronic circuit in the $v_{c_1} - v_{c_4}$ plane

6. CONCLUSIONS

In this paper, we derived a novel 4-D hyperchaotic system by introducing a feedback control to the novel 3-D Vaidyanathan chaotic system. The phase portraits of the novel 4-D hyperchaotic system are displayed and the mathematical properties are discussed. The novel 4-D hyperchaotic system has a unique equilibrium at the origin, which is a saddle-point. The Lyapunov exponents of the novel 4-D hyperchaotic system are obtained as $L_1 = 3.05638$, $L_2 = 0.08646$, $L_3 = 0$ and $L_4 = -20.12087$. Also, the Kaplan-Yorke dimension of the novel 4-D hyperchaotic system is derived as $D_{KY} = 3.15619$. Next, we designed an adaptive controller to globally stabilize the novel 4-D hyperchaotic system with unknown parameters. Finally, an electronic circuit simulation of the novel hyperchaotic system was presented using SPICE to confirm the feasibility of the theoretical model.

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