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Reduction of Interval Systems Based on Routh's Stability Table and Coefficients of Power Series Expansion

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Abstract: This paper presents a method for obtaining stable reduced-order model of high-order interval system using Routh's stability table and coefficients of power series expansion. The denominator of model is derived using Routh's stability table proposed by Dolgin. Once the denominator of reduced-order model is obtained, the numerator is calculated by matching coefficients of power series expansion of interval system and those of reduced-order interval model. The method proposed by Bandyopadhyay et. al., is utilized for obtaining the coefficients of power series expansion. Whole procedure is explained with one numerical example. The step responses of rational systems of high-order interval system and proposed reduced-order model are also provided for comparative study.

Keywords: Approximation; interval systems; Kharitonov polynomials; Routh table.

1. INTRODUCTION

A great number of practical problems exist in the literatures which are highly complex and large in dimension. Electrical power systems and robotic manipulators are two examples of such types of problems. The analysis and controller design for high-order systems are very complex and time consuming. To tackle these problems, the order reduction of such systems becomes mandatory.

A wide variety of methods exist for order reduction of systems having fixed coefficients (or nominal plant models) [1]. These methods include aggregation technique [2], Pade technique [3], Routh approximation method [4], time moment matching method [5], etc. All of these methods have been applied for continuous time as well as for discrete time systems having fixed coefficients. Some of these methods have recently been extended for order reduction of interval systems.

Pioneering work by Bandyopadhyay et. al., [6] in the field of model reduction of continuous interval systems attracted the attention of many researchers [7-10]. Bandyopadhyay et. al., [6] obtained the denominator of reduced-order interval model by truncating Routh table constructed using interval arithmetic [11]. The numerator of interval model was obtained by matching the power series expansion coefficients of high-order interval system to those of the reduced-order interval model. Dolgin and Zeheb [12] found that the technique proposed by Bandyopadhyay et. al., [6] may produce unstable interval model even though the high-order interval system is stable. In [12], a new method for construction of interval Routh table is also suggested. Yang [13], further, showed that the method proposed by Dolgin and Zeheb [12] may also result unstable interval model of stable high-order interval system. Two conditions were suggested in [14] to construct the interval Routh table which overcome the problem of unstable denominator polynomial.

In this paper, a method for order reduction of continuous interval systems is presented in which the denominator of model is determined using the modified method of formation of interval Routh

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table proposed by Dolgin [14] and the numerator of model is obtained by matching first r power series expansion coefficients of high-order interval system to those of the reduced-order interval model. The method proposed by Bandyopadhyay et. al., [6] is used for obtaining the coefficients of power series expansions of system and model. This method guarantees the stability of reduced-order interval model. Moreover, only first r power series expansion coefficients are required for deriving the reduced-order model.

The rest of the paper is organized as follows: section II describes the problem formulation, the proposed method is discussed in section III, section IV deals with numerical example and discussion and finally, whole work is concluded in section V.

2. PROBLEM FORMULATION

Consider a single-input-single-output nth-order continuous interval system

$$\mathrm{G}(s) = \frac{\mathrm{N}(s)}{\mathrm{D}(s)} = \frac{\left[\mathrm{N}_0^-, \mathrm{N}_0^+\right] + \left[\mathrm{N}_1^-, \mathrm{N}_1^+\right]s + \ldots + \left[\mathrm{N}_{n-1}^-, \mathrm{N}_{n-1}^+\right]s^{n-1}}{\left[\mathrm{D}_0^-, \mathrm{D}_0^+\right] + \left[\mathrm{D}_1^-, \mathrm{D}_1^+\right]s + \ldots + \left[\mathrm{D}_n^-, \mathrm{D}_n^+\right]s^n} \tag{1}$$

where, G(s) is the transfer function of nth-order interval system, $[\mathrm{N}_i^-, \mathrm{N}_i^+]$ for $i = 0, 1, \ldots, (n - 1)$ are the coefficients of numerator, and $[\mathrm{D}_i^-, \mathrm{D}_i^+]$ for $i = 0, 1, \ldots, n$ are the coefficients of denominator of interval system.

Suppose, it is desired to approximate the system, , by an rth-order interval model given by

$$\mathrm{H}(s) = \frac{n(s)}{d(s)} = \frac{\left[n_0^-, n_0^+\right] + \left[n_1^-, n_1^+\right]s + \ldots + \left[n_{r-1}^-, n_{r-1}^+\right]s^{r-1}}{\left[d_0^-, d_0^+\right] + \left[d_1^-, d_1^+\right]s + \ldots + \left[d_r^-, d_r^+\right]s^r} \tag{2}$$

such that $r < n$, where, H(s) denotes the transfer function of rth-order interval approximant (i.e. model), and $[n_i^-, n_i^+]$ for $i = 0, 1, \ldots, (r - 1)$ and $[d_i^-, d_i^+]$ for $i = 0, 1, \ldots, r$ are, respectively, the coefficients of numerator and denominator of interval model.

3. PROPOSED METHOD

In the proposed method, the denominator of interval model, $d(s) = \sum_{i=0}^{r}\left[d_i^-, d_i^+\right]s^i$, is first determined by the modified method of formulation of Routh table [14]. The Routh table is formed using the denominator of interval system, D(s). Once the denominator of the model is determined, the numerator is obtained by matching the coefficients of power series expansion of high-order system to those of the reduced-order model. The detailed procedure to derive denominator and numerator of the model is explained as follows.

A. Procedure to Obtain the Denominator of Model

Dolgin [14] proposed the modified method of formation of Routh table. This method ensures the stability of reduced-order model. This method shrinks the uncertainty of the elements of last existing line of the Routh table for obtaining current line.

The modified Routh table for the denominator of system, D(s), is constructed as

Table 1
modified Routh table

$$
\begin{array}{llll}
D_{1,1} = \left[D_n^-, D_n^+\right] & D_{1,2} = \left[D_{n-2}^-, D_{n-2}^+\right] & D_{1,3} = \left[D_{n-4}^-, D_{n-4}^+\right] & \cdots \\
D_{2,1} = \left[D_{n-1}^-, D_{n-1}^+\right] & D_{2,2} = \left[D_{n-3}^-, D_{n-3}^+\right] & \cdots & \\
D_{3,1} & D_{3,2} & \cdots & \\
\vdots & \vdots & & \\
D_{n,1} & D_{n,2} & & \\
D_{n+1,1} & & &
\end{array}
$$

where, the elements of modified Routh table are calculated using

$$D_{i,j} = D_{i-2,j+1} - \frac{\tilde{D}_{i-2,1}}{\tilde{D}_{i-1,1}} D_{i-1,j+1} \tag{3}$$

for $i \geq 3$ and $1 \geq j \leq (n - i + 3)/2$. $\tilde{D}_{i,j}$ denotes a point in the interval $[D_{i,j}^-, D_{i,j}^+]$. The value of $\tilde{D}_{i,j}$ is usually taken as

$$\tilde{D}_{i,j} = \frac{1}{2}\left(D_{i,j}^- + D_{i,j}^+\right) \tag{4}$$

Two additional conditions are put to guarantee the consistency of all elements of modified Routh table.

Condition 1: This condition ensures the consistency of all elements of Routh table except the first element of first row of each pair of last two rows.

To ensure the existence of $D_{i,j}$, the element $D_{i-j,j+1}$ is shrunk as

$$D_{i-1,j+1} = \begin{bmatrix} \max\left(D_{i-1,j+1}^-, \hat{D}_{i-1,j+1} - 0.5\,K\,L_{i-2,j+1}\right), \\ \min\left(D_{i-1,j+1}^+, \hat{D}_{i-1,j+1} + 0.5\,K\,L_{i-2,j+1}\right) \end{bmatrix} \tag{5}$$

where, $L_{i-2,j+1} = D_{i-2,j+1}^+ - D_{i-2,j+1}^-$, $\hat{D}_{i-1,j+1}$ is mid-point of $D_{i-j,j+1}$ and $K = \left(1/k\right)\left|\tilde{D}_{i-1,1}/\tilde{D}_{i-2,1}\right|$ with $k > 1$. The value of k is determined as

$$k = \left(\left|\tilde{D}_{i-1,1}\right| + \left|\tilde{D}_{i-2,1}\right|\right) \Big/ \left|\tilde{D}_{i-2,1}\right| \tag{6}$$

Condition 2: This condition ensures the consistency of first element of each pair of rows.

The first coefficient of truncated table is taken as $\tilde{D}_{p+1,1}$ where $D_{p+1,1}$ is the first element of truncated table. $\tilde{D}_{p+1,1}$ is the value which is chosen to obtain the $(p + 2)$th row of Routh table.

The denominator, $d(s)$, of the rth-order model is calculated from $(n + 1 - r)$th and $(n + 2 - r)$th rows of Table I as

$$d(s) = \tilde{D}_{n+1-r,1} s^r + D_{n+2-r,1} s^{r-1} + D_{n+1-r,2} s^{r-2} + \cdots \tag{7}$$

where, $\tilde{D}_{n+1-r,1}$ is the point in interval $D_{n+1-r,1}$ which is used for obtaining the $(D_{n+2-r,1})$th element in Table I.

B. Procedure to Determine the Numerator of Model

The numerator of reduced-order interval model is obtained by matching first r power series expansion coefficients of system to those of the model. The power series expansion coefficients of the high-order system and reduced-order model are obtained as follow:

The power series expansion [6] of high-order interval system, (1), can be defined as

$$G(s) = \frac{N(s)}{D(s)} = N(s)M(s) \tag{8}$$

where,

$$M(s) = [m_0^-, m_0^+] + [m_1^-, m_1^+]s + [m_2^-, m_2^+]\, s^2 + \ldots \tag{9}$$

From (8), it is clear that

$$D(s)M(s) = 1 \tag{10}$$

By solving (10), a set of linear interval equations can be obtained as

$$\begin{aligned} [D_0^-, D_0^+]\,[m_0^-, m_0^+] &= [0, 1] \\ [D_1^-, D_1^+]\,[m_0^-, m_0^+] + [D_0^-, D_0^+]\,[m_1^-, m_1^+] &= [0, 0] \\ &\vdots \\ [D_n^-, D_n^+]\,[m_0^-, m_0^+] + \cdots + [D_0^-, D_0^+]\,[m_n^-, m_n^+] &= [0, 0] \end{aligned} \tag{11}$$

Solving (11), $[m_i^-, m_i^+]$ for $i = 0, 1, \ldots, n$ can be obtained. Thus, (8) becomes

$$\begin{aligned} G(s) &= ([N_0^-, N_0^+] + [N_1^-, N_1^+]s + \ldots + [N_{n-1}^-, N_{n-1}^+]\, s^{n-1})\,([m_0^-, m_0^+] + [m_1^-, m_1^+]s + \ldots) \\ &= [p_0^-, p_0^+] + [p_1^-, p_1^+]s + [p_2^-, p_2^+]\, s^2 + \ldots \end{aligned} \tag{12}$$

Similarly, the expansion of reduced-order model, (2), can be written as

$$H(s) = [q_0^-, q_0^+] + [q_1^-, q_1^+]s + [q_2^-, q_2^+]\, s^2 + \ldots \tag{13}$$

The parameters $[p_i^-, p_i^+]$ for $i = 0, 1, \ldots$ and $[q_i^-, q_i^+]$ for $i = 0, 1, \ldots$ are coefficients of power series expansion of high-order system and reduced-order model, respectively. The numerator of model is obtained by matching the first r power series expansion coefficients as given below.

$$[p_i^-, p_i^+] = [q_i^-, q_i^+] \text{ for } i = 0, 1, \ldots, (r-1) \tag{14}$$

4. NUMERICAL SECTION

Consider a seventh-order interval system [10] given as

$$G(s) = \frac{\begin{gathered}[1.9, 2.1]s^6 + [24.7, 27.3]s^5 + [157.7, 174.3]s^4 + [541.975, 599.025]s^3 + \\ [929.955, 1027.845]s^2 + [721.81, 797.79]s + [187.055, 206.745]\end{gathered}}{\begin{gathered}[0.95, 1.05]s^7 + [8.779, 9.703]s^6 + [52.231, 57.729]s^5 + [190, 194.98]s^4 + \\ [429.02, 474.18]s^3 + [582.23, 622.97]s^2 + [325.28, 359.52]s + [57.352, 63.389]\end{gathered}} \tag{15}$$

Suppose, a second-order model ($r = 2$) described by

$$H(s) = \frac{n(s)}{d(s)} = \frac{\left[n_0^-, n_0^+\right] + \left[n_1^-, n_1^+\right]s}{\left[d_0^-, d_0^+\right] + \left[d_1^-, d_1^+\right]s + \left[d_2^-, d_2^+\right]s^2} \tag{16}$$

is desired.

For (15), the Table I takes the form as given in Table 2 (see modified Routh Table 2).

Table 2
Modified Routh table

$[0.95, 1.05]$	$[52.231, 57.729]$	$[429.02, 474.18]$	$[325.28, 359.52]$
$[8.779, 9.703]$	$[190, 194.48]$	$[582.23, 622.97]$	$[57.352, 63.389]$
$[31.67, 36.63]$	$[384.43, 388.35]$	$[319.83, 351.90]$	
$[86.2, 90.126]$	$[510.5, 513.32]$	$[57.325, 63.389]$	
$[186.73, 189.7]$	$[311.5, 313.46]$		
$[364.72, 366.62]$	$[59.74, 61]$		
$[281.08, 282.35]$			
$[59.74, 61]$			

For second-order model, (7) becomes

$$d(s) = 365.67s^2 + [281.08, 282.35]s + [59.70, 61] \tag{17}$$

The power series expansions of system and model, given by (12) and (13), turn out to be

$$\mathrm{G}(s) = [2.94, 3.60] + [-11.21, -1.23]s + [-26.88, 68.05]s^2 + \cdots \tag{18}$$

$$\mathrm{H}(s) = [0.0164, 0.0167][n_0^-, n_0^+] + \{[-0.079, -0.075][n_0^-, n_0^+] + [0.0164, 0.0167][n_1^-, n_1^+]\}s + \cdots \tag{19}$$

From (14), we obtain

$$\left.\begin{array}{l}[p_0^-, p_0^+] = [q_0^-, q_0^+] \\ [p_1^-, p_1^+] = [q_1^-, q_1^+]\end{array}\right\} \Rightarrow \left\{\begin{array}{l}[2.94, 3.60] = [0.0164, 0.0167][n_0^-, n_0^+] \\ [-11.21, -1.23] = [-0.079, -0.075][n_0^-, n_0^+] + [0.0164, 0.0167][n_1^-, n_1^+]\end{array}\right. \tag{20}$$

Solving (20), the numerator coefficients of model obtained are

$$[n_0^-, n_0^+] = [176.10, 219.94],\ [n_1^-, n_1^+] = [124.46, 987.24] \tag{21}$$

From (17) and (21), the second-order model obtained is

$$\mathrm{H}(s) = \frac{[124.46, 987.24]s + [176.10, 219.94]}{365.67s^2 + [281.08, 282.35]s + [59.70, 61]} \tag{22}$$

while the second-order models calculated using the techniques proposed by Bandyopadhyay et. al., [6], Selvaganesan [10], Sastry and Rao [9], Dolgin and Zeheb [12], and Sharma et. al., [15] turn out to be, respectively, (23)-(27).

$$\mathrm{H_B}(s) = \frac{[-743472.49, 447092.40]s + [-755.42, 377.06]}{[-2614.87, 1360.25]s^2 + [-61581.55, 102981]s + [-209.56, 104.60]} \tag{23}$$

$$\mathrm{H_S}(s) = \frac{[260.95, 861.33]s + [175.23, 218.58]}{[364.72, 366.62]s^2 + [281.08, 282.35]s + [59.70, 61]} \tag{24}$$

$$\mathrm{H_{SR}}(s) = \frac{[0.961, 2.194]s + [0.249, 0.568]}{[1, 1]s^2 + [0.433, 0.988]s + [0.0763, 0.174]} \tag{25}$$

$$\mathrm{H_{DZ}}(s) = \frac{[59.97, 1379.80]s + [169.17, 228.58]}{[331.48, 399.36]s^2 + [260.51, 301.77]s + [57.352, 63.389]} \tag{26}$$

$$H_{SS}(s) = \frac{[0.305, 0.467]s + [1.038, 1.221]}{[0.093, 0.144]s^2 + [0.586, 0.735]s + [1, 1]} \tag{27}$$

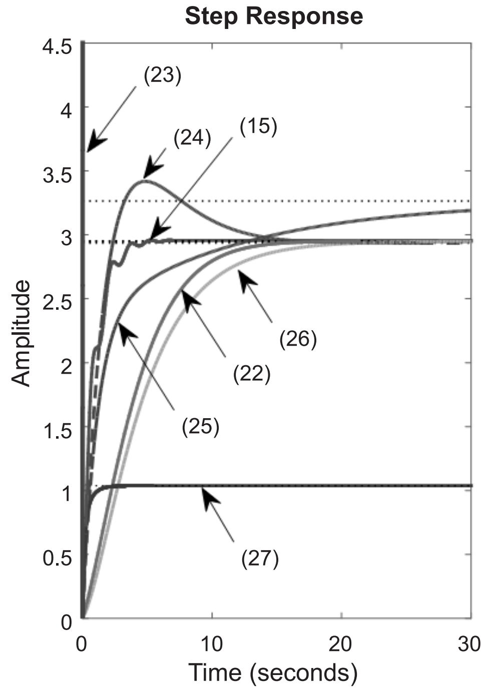

Figure 1: Unit step response of rational systems

Figure 1 shows the unit step responses of rational systems of (15), (22)-(27). It is clear from Figure 1 that the time response of proposed model, (22), is better matched to high-order interval system, (15), compared to other models, (23)-(27).

5. CONCLUSION

This paper proposed reduction of continuous interval systems using Routh's stability table and coefficients of power series expansion. The denominator of model is derived using Routh's stability table and the numerator is obtained by matching first *r* coefficients of power series expansion. The proposed method produced better time response approximation, for all cases considered, when compared to other techniques.

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