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# SER and BER Performance Analysis of Digital Modulation Scheme over Multipath Fading Channel

Kommabatla Mahender<sup>1\*</sup>, Tipparti Anil Kumar<sup>2</sup> and K S Ramesh<sup>3</sup>

<sup>1</sup>Department of Electronics & Communication Engineering, Tallapadmavathi College of Engineering, Warangal Telangana, India, E-mail: kmsharma2@yahoo.co.in

<sup>2</sup>Department of Electronics & Communication Engineering, MVSR Engineering College, Hyderabad, Telangana, India E-mail: tvakumar2000@yahoo.co.in

<sup>3</sup>Department of Electronics & Communication Engineering, KL University, Vaddeswaram, Guntur, Andhra Pradesh, India E-mail: dr.ramesh@kluniversity.in

**Abstract:** Cellular and wireless communications link performance affected due to multipath fading. The analysis of link performance must be evaluated in the presence of fading channels for digital modulation schemes. In this paper, we analyze the proper uses of Rician and Rayleigh fading channels. Simulation results for link performance is obtained using these channel modeling in Binary phase shift keying (BPSK), Quadrature phase shift keying (QPSK) and M-ary phase shift keying (MPSK) systems and obtained results are approximate to the theoretical results. Simulation results are shown that to understand the effect of fading channels with different digital modulation schemes and use to verify the best suitable fading channels expressed in terms of bit error rate (BER) and symbol error rate (SER) performances.

**Keywords:** M-Ary Modulation Scheme; BER;SER;Fading Channels

## 1. INTRODUCTION

Fading and interference are the major performance degrading factors in wireless/mobile communications. In order to improve and testify the system's effectiveness to resist fading, modeling and simulation of communication system under fading channel is of great significance in the design of communication system. For different propagation environment, the characteristic of fading channel is diverse and complex. Therefore, design of proper fading model in particular communication circumstance is essential in this regard.

Rayleigh fading and Rician fading models are the most commonly used small scale fading models in wireless communication as shown in figure 1 [1]. However, as wireless sensor networks (WSN) migrate into vastly different applications, conventional Rayleigh and Rician channel model don't fit in every WSN environment. Recent research [2] [3] shows that some WSN applications where sensor nodes deployed within cavity environment suffer from more severe fading than predicted Rayleigh fading. Herein, development of a more applicable fading model which fits in some particular WSN circumstance has become an important issue.

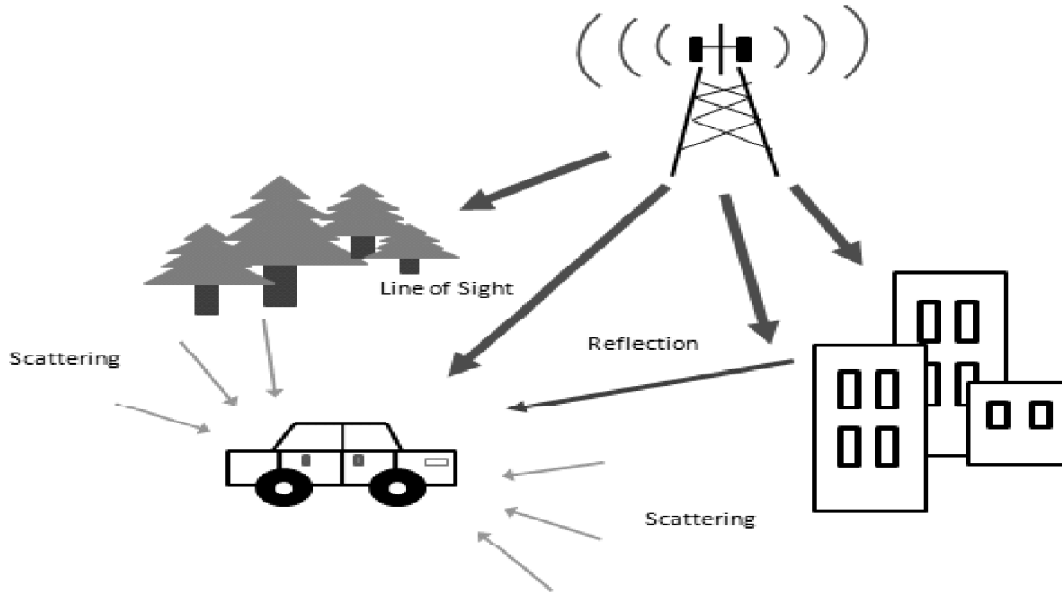


Figure 1: Wireless communication environment

Bit-error-rate (BER) is a key factor to measure the capacity and performance of communication system. Much effort has been made to explore the characteristic and BER performance of hyper-Rayleigh fading [5] [6].

This paper is further organized as, BER and SER performance analysis over gaussian channels described in Section-II, Section-III explains about simulation results and discussions and finally conclusion in Section-IV.

## 2. BER AND SER PERFORMANCE ANALYSIS OVER GAUSSIAN CHANNEL

### (A) BER for BPSK Modulation

The received signal of the binary phase shift keying (BPSK) is given as

$$y = x + n \quad (1)$$

Where  $x \in \{-A, A\}$ ,  $n \sim (0, \sigma^2)$  and  $\sigma^2 = N_0$ . The real terms of the (1) equation can be written as  $y_{re} = x + n_{re}$

where  $n_{re} \sim N(0, \frac{\sigma^2}{2}) = (0; \frac{N_0}{2})$ .

In BPSK constellation  $d_{\min} = 2A$  and  $\gamma_b$  is defined as bit energy to noise ratio  $\frac{E_b}{N_0}$  and also known as SNR per bit. The BER is defined as

$$\gamma_b = \frac{E_b}{N_0} = \frac{A^2}{N_0} = \frac{d_{\min}^2}{4N_0} \quad (2)$$

Since the bit error probability is defined as

$$P_b = P\{n > A\} = \int_A^{\infty} \frac{1}{\sqrt{2\pi\sigma^2/2}} e^{-\frac{x^2}{2\sigma^2/2}} dx \quad (3)$$

Using Q-function Equation (3) can be rewritten as

$$P_b = Q\left(\sqrt{\frac{d_{\min}^2}{2N_0}}\right) = Q\left(\frac{d_{\min}}{\sqrt{2N_0}}\right) = Q(\sqrt{2\gamma_b}) \quad (4)$$

Here the Q function can be written as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{x^2}{2}} dx \quad (5)$$

## 2.2. BER for QPSK

The QPSK modulation defined with the help of two binary phase shift keying modulation of the signal. The BER of each channel namely in phase and quadrature channels is given as

$$P_b = Q(\sqrt{2\gamma_b}) \quad (6)$$

Each individual branch SER probability is given as

$$P_s = 1 - [1 - Q(\sqrt{2\gamma_b})]^2 \quad (7)$$

Therefore, the energy per bit of the symbol is divided equally into two channels with same bit rate  $\gamma_s = 2\gamma_b$  and hence further more is

$$P_s = 1 - [1 - Q(\sqrt{\gamma_s})]^2 \quad (8)$$

Here we calculated an upper limit for SER of QPSK. When QPSK is transmitted the zero symbols the corresponding probability of error ( $P_e$ ) of QPSK is expressed as the sum of probabilities of symbol state transitions between zero to one, zero to one and zero to 3. Therefore, the equation (8) can rewritten as

$$P_s \leq Q\left(\frac{d_{o1}}{\sqrt{2N_0}}\right) + Q\left(\frac{d_{o2}}{\sqrt{2N_0}}\right) + Q\left(\frac{d_{o3}}{\sqrt{2N_0}}\right) \quad (9)$$

$$= 2Q\left(\frac{A}{\sqrt{N_0}}\right) + Q\left(\frac{\sqrt{2}A}{\sqrt{2N_0}}\right) \quad (10)$$

Since  $\gamma_s = 2\gamma_b = \frac{A^2}{N_0}$ , the SER is expressed as

$$P_s \leq 2Q(\sqrt{\gamma_s}) + Q(\sqrt{2\gamma_s}) \leq 3Q(\sqrt{\gamma_s}) \quad (11)$$

Here Q function approximation for  $z \gg 0$

$$Q(z) \leq \frac{1}{z\sqrt{2\pi}} e^{-\frac{z^2}{2}} \quad (12)$$

Hence obtain SER is

$$P_s \leq \frac{3}{\sqrt{2\pi\gamma_s}} e^{-0.5\gamma_s} \quad (13)$$

With assuming that the high SNR values obtain when the BER occurs nearest neighbor BER ( $P_b$ ) and it is further defined as  $P_b \approx \frac{P_s}{2}$  using Gray coding.

### 2.3. SER for M-PSK

The SER can be easily calculated using nearest neighbor approximation for MPSK signaling, therefore SE is given by

$$P_s \approx 2Q\left(\frac{d_{\min}}{\sqrt{2N_0}}\right) = 2Q\left(\frac{2A \sin \frac{\pi}{M}}{\sqrt{2N_0}}\right) = 2Q\left(\sqrt{2\gamma_s} \sin\left(\frac{\pi}{M}\right)\right) \quad (14)$$

The equation (14) is good only for high SNR scenarios.

### 2.4. SER for QAM

The SER for M- QAM with  $M=2^N$  with use of two M-PAM on in phase and quadrature components is used. The QAM modulation probability of error is obtained individual branches/channels of M-PAM the probability errors and it is given as

$$P_s = 1 - \left(1 - \frac{2(\text{sqrt}M - 1)}{\text{sqrt}M} Q\left(\sqrt{\frac{3\bar{\gamma}_s}{M - 1}}\right)\right)^2 \quad (15)$$

If the nearest approximation uses for an M-QAM system<sup>9-10</sup>, the minimum distance between the four nearest neighbor  $d_{\min}$  is calculated shown in figure2. Therefore the SER for high SNR can be approximated by

$$P_s \approx 4Q\left(\sqrt{\frac{3\bar{\gamma}_s}{M - 1}}\right)$$

The mean energy per transmitted symbol is

$$\bar{E}_s = \frac{1}{M} \sum_{i=1}^M A_i^2 \tag{16}$$

Where  $A_i = (a_i + b_i)$ ,  $a_i$  and  $b_i \in \{2i - 1 - L\}$  for  $i=1, \dots, N$ .

Hence

$$\bar{E}_s = \frac{d_{\min}^2}{2L} \sum_{i=1}^L (2i - 1 - L)^2 \tag{17}$$

Let's consider an example for 16-QAM system the two nearest neighbor minimum distance is  $d_{\min} = 2$  and the corresponding mean energy per transmitted symbol is  $E_s = 10$ . Similarly, for 64-QAM  $d_{\min} = 2$ ,  $E_s = 21$ .

### 2.5. SER Approximation

The SER approximation values is given by

$$P_s(\gamma_s) \approx \alpha_M Q(\sqrt{\beta_M \gamma_s}) \tag{18}$$

Where type of approximation is  $\alpha_M$  and modulation type is  $\beta_M$  and summarized values for different modulation is shown in the table 1. And also bit error probability has the same value as SER. It is defined as

$$P_b(\gamma_b) \approx \hat{\alpha}_M Q(\sqrt{\hat{\beta}_M \gamma_b}) \tag{19}$$

Where

$$\hat{\alpha}_M = \frac{\alpha_M}{\log_2 M} \text{ and } \hat{\beta}_M = \frac{\beta_M}{\log_2 M}$$

$$\gamma_s = \frac{E_s}{N_0}, \gamma_b = \frac{E_b}{N_0}, \gamma_b = \frac{\gamma_s}{\log_2 M} \text{ and } P_b \approx \frac{P_s}{\log_2 M}$$

## 3. SIMULATION RESULTS AND DISCUSSIONS

### 3.1. Performance Analysis of Rayleigh Fading Channel

The transmitted signals arrive at the receiver over multipath due to many buildings, vehicles and other large objects in urban scenarios. These received multiple signals will cause fading. Rayleigh fading channel occur only in N-LOS. The PDF of Rayleigh channel expressed as

$$p(r_0) = \frac{r_0}{\sigma^2} \exp\left(-\frac{r_0^2}{2\sigma^2}\right) \text{ When } r_0 \geq 0 \tag{20}$$

According to chi-square channel distribution the PDF of the SNR is given as

$$p(\gamma_b) = \frac{1}{\gamma_b} e^{-\frac{\gamma_b}{\gamma_b}} \text{ For } \gamma_b \geq 0 \tag{21}$$

### 3.2. Performance of Rician Fading Channel

The Rician fading channel occurs when line of sight (LOS) communication exists out of multiple received signal paths between transmitter and receivers. The effect of Rician fading will be less than the Rayleigh fading on the transmitted signal because of LOS path. The Rician PDF of the received signal is given by

$$p(r_0) = \frac{r_0}{\sigma^2} \exp\left[-\frac{(r_0^2 + A^2)}{2\sigma^2}\right] I_0\left(\frac{r_0 A}{\sigma^2}\right)$$

For  $r_0 \geq 0, A \geq 0$  (22)

Where the modified Bessel function is  $I_0(\cdot)$  its order is zero and peak amplitude of signal component in LOS is A. In Rician fading channel the K-factor is one of the important parameter and defined as  $K = \frac{A^2}{2\sigma^2}$ .

Depending upon the value of K Rician and Rayleigh fading channels distributed, the Rician and Rayleigh channels is equal if K=0, where as K increases the Rician fading approximately equal to gaussian distribution with mean.

The PDF of Rician fading is defined as<sup>4</sup>

$$pdf_{\gamma_b}(\gamma_b) = \frac{1+K}{\bar{\gamma}_b} \exp\left(-\frac{\gamma_b(1+K) + K\bar{\gamma}_b}{\bar{\gamma}_b}\right) I_0\left(\sqrt{\frac{4(1+K)K\gamma_b}{\bar{\gamma}_b}}\right)$$
 (23)

The non coherent BER for BPSK in Rician fading is given as

$$P_b = \frac{1}{2} \left[ 1 - Q(\sqrt{b}, \sqrt{a}) + Q(\sqrt{a}, \sqrt{b}) - \frac{A}{2} \exp\left(-\frac{a+b}{2}\right) I_0(\sqrt{ab}) \right]$$
 (24)

Where  $\begin{Bmatrix} a \\ b \end{Bmatrix} = \frac{1}{2} \left( \sqrt{\frac{\frac{K}{1+K} G_{\bar{\gamma}}}{1 + \frac{1}{1+K} G_{\bar{\gamma}}}} + \sqrt{\frac{\frac{K}{1+K} \bar{\gamma}}{1 + \frac{1}{1+K} \bar{\gamma}}} \right)^2$  and

$$A = \frac{\frac{\sqrt{G}}{1+K} \bar{\gamma}}{\sqrt{\left(\frac{G}{1+K} \bar{\gamma} + 1\right) \left(\frac{1}{1+K} \bar{\gamma} + 1\right)}}$$

Where the coherent non ideal parameter is  $G$ , the Rician fading channel is very complex<sup>5</sup>.

Figure 3 shows that BER performance of theoretical and simulations of BPSK over AWGN and Rayleigh fading channels, both the channels BER probability. Therefore the link performance of BSPK over Rayleigh fading channel is worse than AWGN channel with no fading.

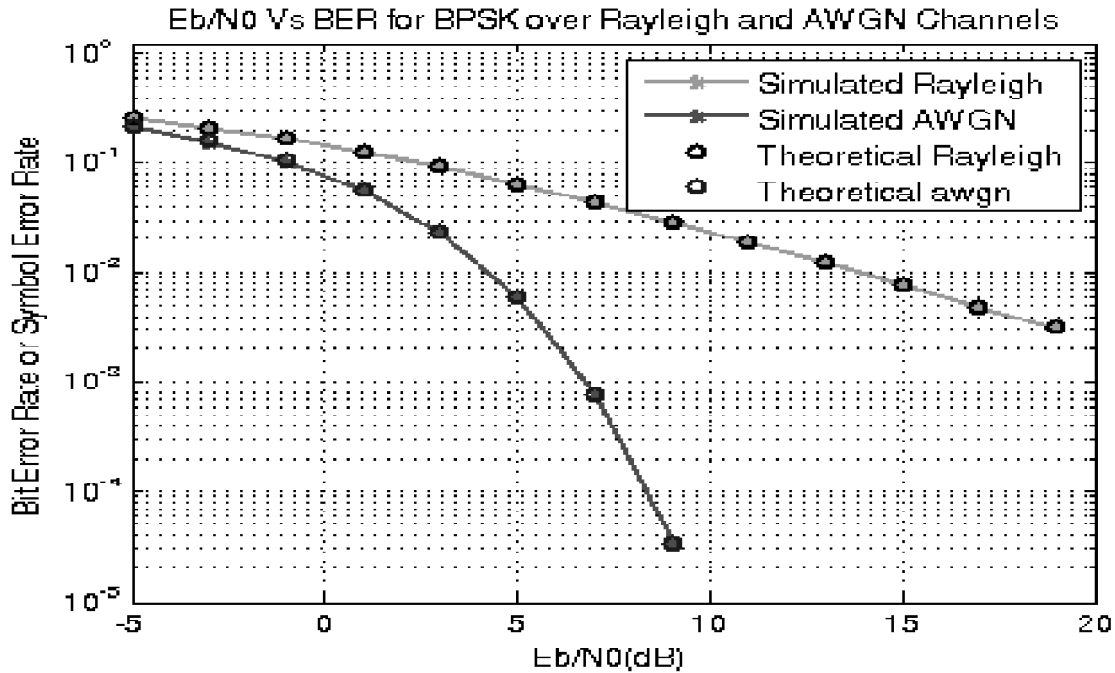


Figure 3: BER for BPSK over Rayleigh fading and AWGN Channels

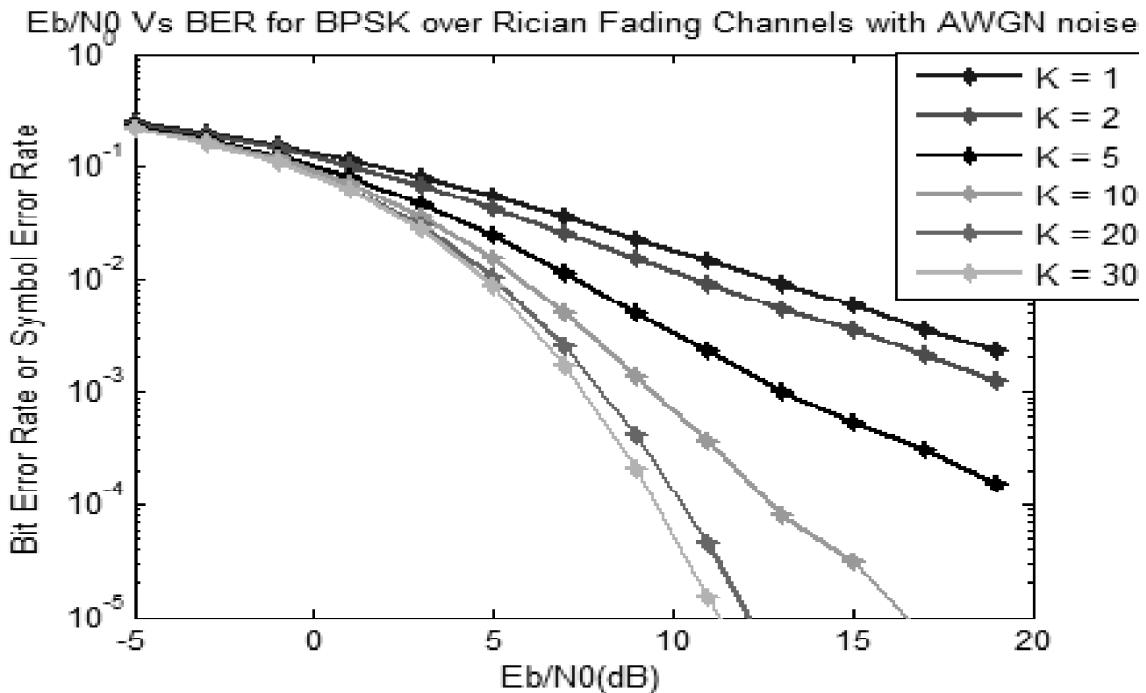
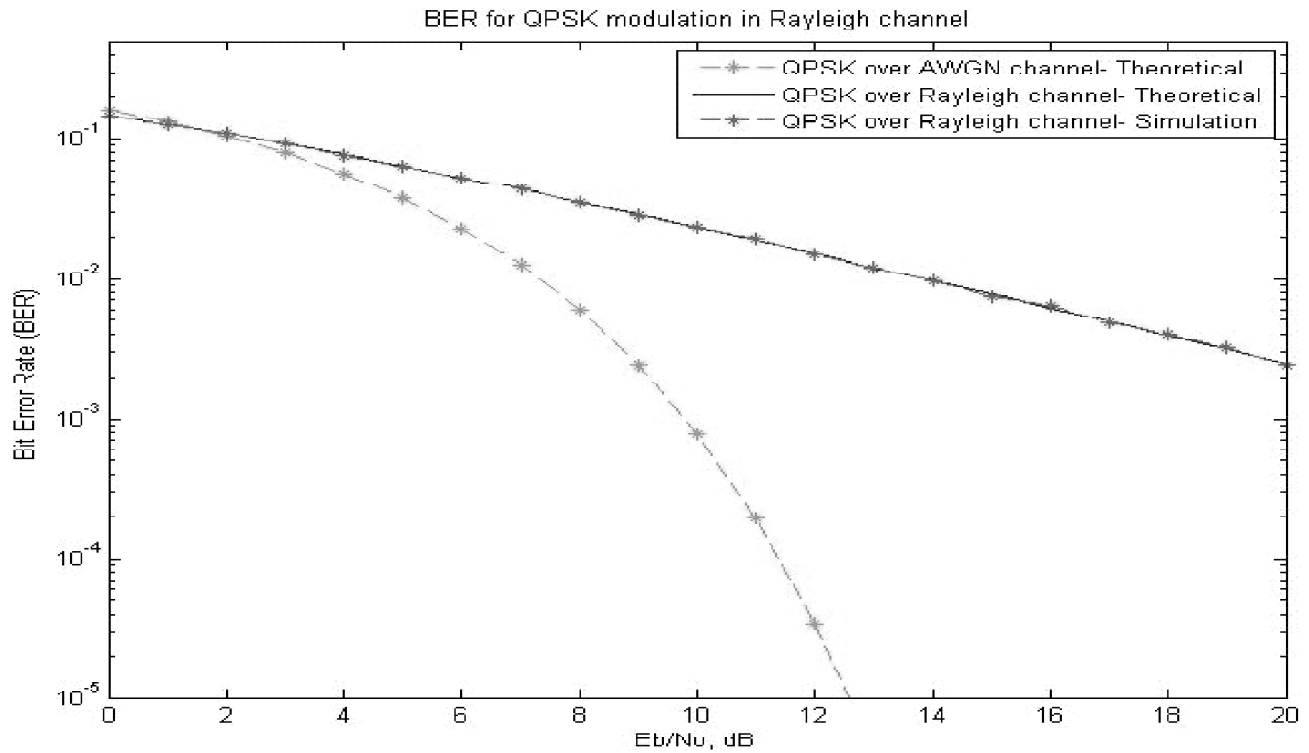
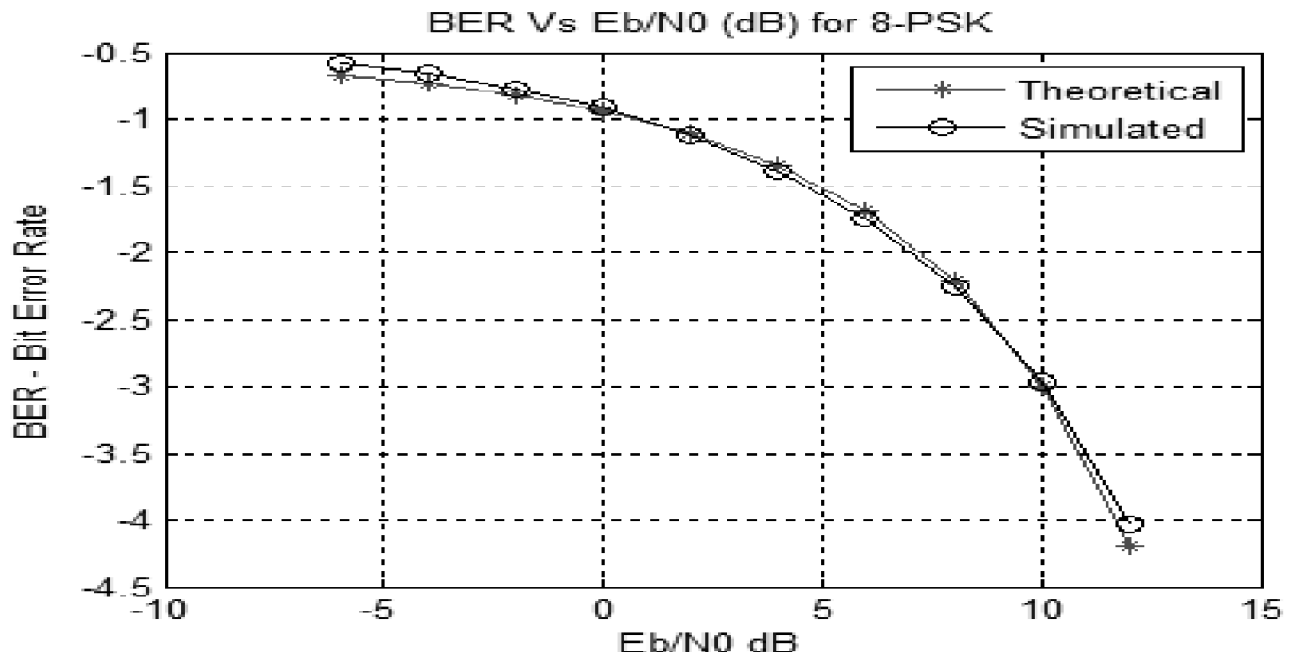


Figure 4: BER for BPSK over Rician Fading Channels



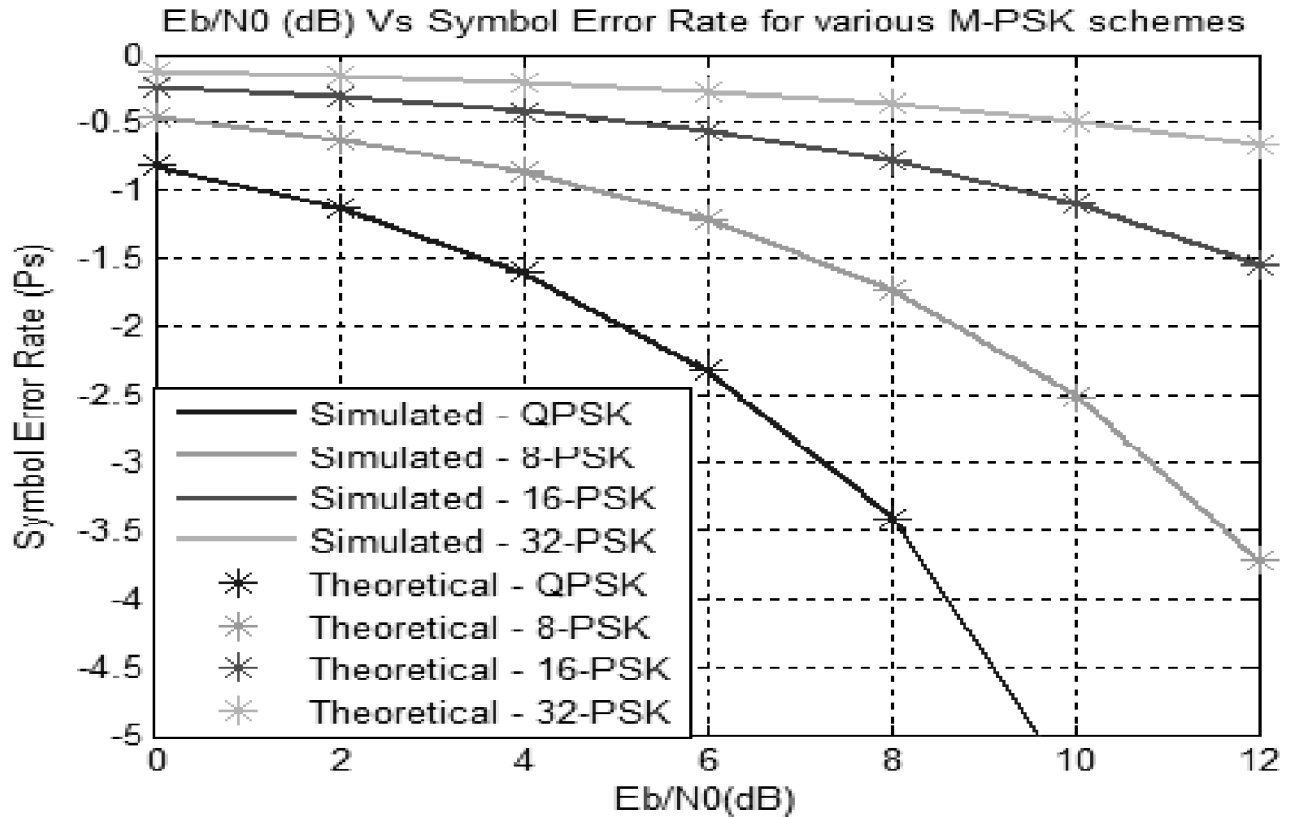
(a) QPSK



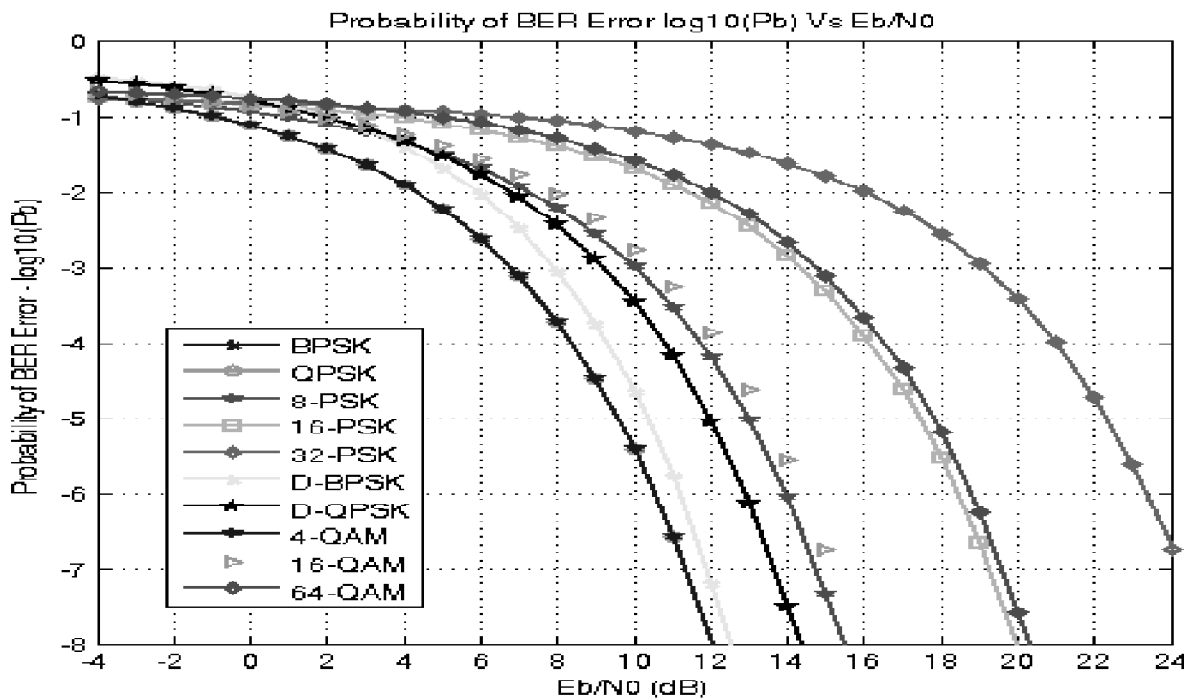
(b) 8PSK

Figure 5: SNR versus BER curve for QPSK and 8PSK over an AWGN





(a) M-PSK schemes



(b) M-ary Modulation

Figure 6: SER versus SNR for various M-PSK and M-ary Schemes over AWGN

In figure 4 illustrated that the theoretical and simulations of BPSK modulation over Rician fading channel BER performance with various K-factor. The probability of BPSK decreases when K-factor increases. For high K-factor values the BER theoretical and simulation for BSPK over Rician fading channel matches the AWGN channel.

Figure5 show the BER comparison between the theory and simulation of AWGN in QPSK and 8-PSK. The SER and probability of BER performances for various M-PSK and M-ary Schemes over multipath fading channels is shown in figure 6.

**Table 1**  
**Approximate coherent modulation SER and BER error probabilities**

Modulation	$P_s(\gamma_s)$	$P_b(\gamma_b)$
BPSK	—	$P_b = Q(\sqrt{2\gamma_b})$
QPSK	$P_s \approx 2Q(\sqrt{\gamma_s})$	$P_b \approx Q(\sqrt{2\gamma_b})$
M-PSK	$P_s \approx 2Q\left(\sqrt{2\gamma_s} \sin\left(\frac{\pi}{M}\right)\right)$	$P_b \approx \frac{2}{\log_2 M} Q\left(\sqrt{2\gamma_b \log_2 M} \sin\left(\frac{\pi}{M}\right)\right)$
M-QAM	$P_s \approx 4Q\left(\sqrt{\frac{3\gamma_s}{M-1}}\right)$	$P_b \approx \frac{4}{\log_2 M} Q\left(\sqrt{\frac{3\gamma_b \log_2 M}{M-1}}\right)$

#### 4. CONCLUSION

In this paper, the effect of multipath fading channels over various digital modulation techniques has been studied. With the proper utilization of these channels and has been analyzed, easy to implement complex channel model for next generation wireless communication systems. The link performance obtained by using these channel models in BPSK, QPSK and M-ary systems correspond closely to the theoretical results. And also the BER and SER theoretical and simulated results for digital modulation schemes have been studied and compared.

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