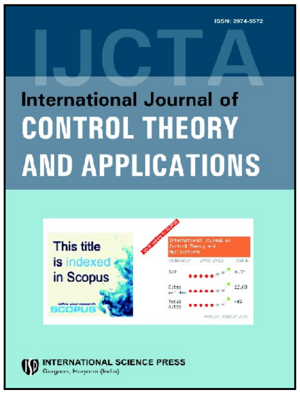

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An Analytical Method to Optimize Generation Schedule with a Fuel Restricted Plant Including Transmission Losses

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Abstract: Optimization of generation schedule by various methods of coordinating incremental generation costs with or without transmission losses has been reported in the literature. When one of the generating units of all thermal power system is having a restriction on the quantum of fuel reserve for any one plant, then the optimum generation schedule turns out to be a 'double iterative' process even when the transmission losses are not taken into consideration. This paper presents an analytical approach which reduces the double iterative process of Economic Dispatch calculation (EDC) into a simple single iterative process even when the transmission losses are included.

Keywords: Fuel-restricted plant, Economic Dispatch Calculation, Optimum Generation Schedule

1. INTRODUCTION

The minimization of total fuel consumption of all participating units while delivering power to all the consumers is the sole requirement of the economic operation of power system with all thermal units. Various methods of EDC are in use when there is no restriction on their fuel supply [1][2][4].

When the fuel resource (coal, oil or gas) available to any one of the thermal plant is restricted, in an all thermal power system, then the entire dispatch calculation is to be done in a different way, such that it should account for what had happened in the past and what will happen in the future. This warrants for the determination of the composite cost function or equivalent cost function [4] of all participating thermal units having no restriction on their fuel availability. A novel method of economic dispatch of an all thermal system, with a fuel restricted plant, eliminating the double iterative process, neglecting transmission losses was reported in the literature [5].However when the transmission losses are included, the EDC necessitates modification in the conventional EDC framework. This paper attempts to make some modification and successfully evolves an analytical approach of EDC for an all thermal power system, with a fuel restricted plant, including the transmission losses.

2. DEVELOPMENT OF THE TECHNIQUE

Consider an all thermal power system of N plants, with one of its plant having a restriction on its fuel reserve. The optimization of generation schedule of such a system turned out to be a time consuming double iterative process because of the addition of "Shadow cost factor" γ apart from the usual "equal incremental cost factor" λ, even when the transmission losses are neglected [4] [5]. That is, apart from the usual incremental cost criterion (λ), when one of the plants is having restriction on fuel, EDC has to satisfy another criterion on γ [4][5] as follows (transmission losses neglected case):

$$\gamma = \frac{dF_s / dP_s}{dq_T / dP_T} \tag{1}$$

where F_s is the equivalent cost function of all plants excluding fuel restricted plant; dq_T/dP_T is the incremental fuel rate of the fuel restricted plant.

When the transmission line losses are included, numerator and denominator of equation (1) are no longer linear and static; they become non-linear and dynamically changing with power demand P_D.

2.1. Development of the Technique

Consider an all thermal power system with N generating plants, with N^{th} plant as the fuel-restricted plant.

The input-output characteristics of (N-1) generating plants without any restriction on fuel assume the form:

$$H_i(P_i) = a_l' P_i^2 + b_i' P_i + c_i \tag{2}$$

where $i = 1, 2\ldots.. (N-1)$.

Taking, the heat rate of fuel used = $h_s MBtu/h$ and cost of fuel = R Rs/t, the cost functions can be written as [2][6]:

$$F_i = a_i P_i^2 + b_i P_i^2 + c_i \tag{3}$$

where $a_i = a'_l \, R/h_s$

$b_i = b'_l \, R/h_s$

$c_i = c'_l \, R/h_s$

The input-output characteristic of the fuel-restricted plant is taken as:

$$H_T = a_T {P_t}^2 + b_T P_t + c_T \text{MBtu/h} \tag{4}$$

Taking the heat rate of gas fired unit= h_g MBtu/ft^3, quantity of gas consumed by this plant during any interval of time n_j becomes [4][5]:

$$q_{Tj} = (a_T/h_g){P_{tj}}^2 + (b_T/h_g)P_{tj} + (c_T/h_g)ft^3/h$$
$$q_{Tj} = a_t {P_{tj}}^2 + b_t P_{tj} + c_t ft^3/h \tag{5}$$

Taking cost of gas $\acute{R}$ Rs/ft^3,

$$F_{tj} = \acute{R} a_t {P_{tj}}^2 + \acute{R} b_t P_{tj} + \acute{R} c_t Rs/h \tag{6}$$

When transmission losses are not neglected, the power balance equation of the system in any given interval of time becomes [1][2][3][7]:

$$n_j \sum_{i=1}^{N-1} P_{ij} + P_{tj} - P_{Lj} - P_{Dj} = 0 \tag{7}$$

where P_{Lj} is transmission line losses during n_j and $j = 1, 2, \ldots. J$

One more condition to be satisfied for fuel restricted plant is [4][5]:

$$\sum_{j=1}^{J} n_j q_{tj} = QT \tag{8}$$

Any economic dispatch calculation should minimize the objective function given in equation (3) and equation (6) without violating the constraint equations given in equation (7) and equation (8) over the entire period of time T [4][5].

Mathematically the above problem becomes,

$$\text{Minimize } \sum_{i=1}^{N-1} \sum_{j=1}^{J} n_j F_{ij} + \sum_{j=1}^{J} n_j F_{tj} \tag{9}$$

Subject to:

$$\emptyset = \sum_{j=1}^{J} n_j q_{tj} - q_T = 0 \tag{10}$$

and

$$\varphi_j = \sum_{i=1}^{N-1} P_{ij} + P_{tj} - P_{Lj} - P_{Dj} = 0 \tag{11}$$

It should be noted that the term

$$\sum_{j=1}^{J} n_j F_{tj}$$

is constant as the total fuel consumed over the period of time T is fixed. Hence this term should be dropped from the objective function [4][5].

Now the lagrangian function becomes

$$L = \sum_{j=1}^{J} n_j \sum_{i=1}^{N-1} F_{ij} + \sum_{j=1}^{J} n_j \lambda_j \left(P_{Dj} + P_{Lj} - \sum_{i=1}^{N-1} P_{ij} - P_{tj}\right) + \gamma\left(\sum_{j=1}^{J} n_j q_{tj} - q_T\right) \tag{12}$$

For any given time period, j=k equation (12) becomes,

$$\frac{\partial L}{\partial P_{ik}} = n_k \frac{dF_{ik}}{dP_{ik}} - \lambda_k + \lambda_k \frac{\partial P_{Lk}}{\partial P_{ik}} = 0 \tag{13}$$

and

$$\frac{\partial L}{\partial P_{tk}} = -\lambda_k + \gamma n_k \frac{dq_{tk}}{dP_{tk}} + \lambda_k \frac{\partial P_L}{\partial P_{tk}} = 0 \tag{14}$$

And on arranging equation (13),

$$n_k \frac{dF_{ik}}{dP_{ik}} = \left(1 - \frac{\partial P_{Lk}}{\partial P_{ik}}\right) \lambda_k$$

$$n_k \frac{dF_{ik}}{dP_{ik}} \left[\frac{1}{1 - \frac{\partial P_{Lk}}{\partial P_{ik}}}\right] = \lambda_k$$

$$n_k \frac{dF_{ik}}{dP_{ik}} \left(1 + \frac{\partial P_{Lk}}{\partial P_{ik}}\right) = \lambda_k$$

Taking $P_{Lk} = P_m B_{mn} P_n$ and neglecting off-diagonal elements of B_{mn} matrix, the above equation becomes [6][7],

$$n_k . 2(a_i + b_i B_{ii})P_{ik} + b_i = \lambda_k \tag{15}$$

On rearranging equation (14),

$$n_k \gamma \frac{dq_{tk}}{dP_{tk}} = n_k \lambda_k \left(1 - \frac{\partial P_{Lk}}{\partial P_{tk}}\right)$$

$$n_k \gamma \frac{dq_{tk}}{dP_{tk}} \left[\frac{1}{1 - \frac{\partial P_{Lk}}{\partial P_{tk}}}\right] = n_k \lambda_k$$

$$n_k \gamma \frac{dq_{tk}}{dP_{tk}} \left(1 + \frac{\partial P_{Lk}}{\partial P_{tk}}\right) = n_k \lambda_k$$

Taking $P_{Lk} = P_m B_{mn} P_n$, N^{th} plant is P_{tk} and neglecting off-diagonal elements of B_{mn} matrix, above equation becomes,

$$n_k \gamma (2a_t P_{tk} + b_t)(1 + 2B_{mn} P_{tk}) = n_k \lambda_k$$

Neglecting higher order terms, above equation becomes,

$$n_k \gamma [(2a_t + 2b_t B_{NN})P_{tk} + b_t] = n_k \lambda_k \tag{16}$$

Dividing equation (16) by equation (15),

$$\frac{\gamma [(2a_t + 2b_t B_{NN})P_{tk} + b_t]}{(2a_i + 2b_i B_{ii})P_{ik} + b_i} = 1$$

$$\text{Or } \gamma = \frac{(2a_i + 2b_i B_{ii})P_{ik} + b_i}{(2a_t + 2b_t B_{NN})P_{tk} + b_t} \tag{17}$$

$$\gamma = \frac{\propto_i P_{ik} + \beta_i}{\alpha_N P_{tk} + \beta_N} \tag{18}$$

where $\alpha_i = 2(a_i + b_i B_{ii})$; $\beta_i = b_i$; $\alpha_N = 2(a_t + b_t B_{NN})$ and $\beta_N = b_t$

$$\alpha_i P_{ik} + \beta_i = \gamma(\alpha_N P_{tk} + \beta_N)$$

$$P_{ik} = \frac{\gamma \alpha_N}{\alpha_i} P_{tk} + \frac{\gamma \beta_N - \beta_i}{\alpha_i}$$

$$P_{ik} = l_i P_{tk} + m_i \tag{19}$$

where $l_i = \frac{\gamma \alpha_N}{\alpha_i}$; $m_i = \frac{\gamma \beta_N - \beta_i}{\propto_i}$ and i=1, 2 ... (N-1)

Substituting the values of P_{ik} in terms of P_{tk} into the power balance equation (7) leads to

$$\sum_{i=1}^{N-1} (l_i P_{tk} + m_i) + P_{tk} - \sum_{i=1}^{N-1} B_{ii} (l_i P_{tk} + m_i)^2 - B_{NN} {P_{tk}}^2 - P_{Dk} = 0$$

Above equation becomes

$$-\left(\sum_{i=1}^{N-1} B_{ii}{l_i}^2\right){P_{tk}}^2\sum_{i=1}^{N-1}(l_i-2l_im_iB_{ii})P_{tk}-B_{NN}{P_{tk}}^2+P_{tk}+\sum_{i=1}^{N}(m_i-{m_i}^2B_{ii})-P_{Dk}=0 \tag{20}$$

Above equation can be written as

$$X_k{P_{tk}}^2+Y_kP_{tk}+Z_k-P_{Dk}=0 \tag{21}$$

where X_k, *Yk* and Z_k are functions of cost function coefficient, B_{mn} coefficients and shadow cost factor γ, during the time interval n_k. Since P_{tk} should be positive always, it is evaluated as the positive root of the above quadratic equation:

$$P_{tk}=\frac{-Y_k+\sqrt{{Y_k}^2-4X_k(Z_k-P_{Dk})}}{2X_k} \tag{22}$$

For the given load P_{Dk} and assumed value of γ, in equation (22) gives the optimum generation of P_{tk} but this value should satisfy the constraint in equation (10), otherwise γ value should be modified as described as follows.

Equation (22) can be written as,

$$P_{tk}=(-Y_k/2X_k)+(1/2X_k)\sqrt{{Y_k}^2-4X_k(Z_k-P_{Dk})}$$

$$=(-Y_k/2\,X_k)\;+(Y_k/2X_k)\{1+\left(4X_k(P_{Dk}-Z_k)\right)/{Y_k}^2\}^{1/2} \tag{23}$$

But

$$(1+x)^{1/2}=1+(1/2)x-(1/8)x^2+(1/16)x^3 \tag{24}$$

Using equation (24), and neglecting higher order terms factors inside the braces of equation (23) becomes [3][7]:

$$\begin{aligned}\{1+\left(4X_k(P_{Dk}-Z_k)/{Y_k}^2\right)\}^{1/2}\\ &=1+(1/2)\left(4X_k(P_{Dk}-Z_k)/{Y_k}^2\right)-(1/8)\left(16{X_k}^2\left({P_{Dk}}^2+{Z_k}^2-2Z_kP_{Dk}\right)/{Y_k}^4\right)\\ &=\{-\;2{X_k}^2/{Y_k}^4\}{P_{Dk}}^2+\{(2X_k/{Y_k}^2)+\;\left(4{X_k}^2Z_k/{Y_k}^4\right)\}P_{Dk}\\ &\quad+\{1-\left(2X_kZ_k/{Y_k}^2\right)\left(2{X_k}^2{Z_k}^2/{Y_k}^4\right)\}\end{aligned} \tag{25}$$

Substituting the above into equation (23) and regrouping leads to:

$$P_{tk}=\{-X_k/{Y_k}^3\}{P_{Dk}}^2+\{(1/Y_k)+\;\left(2X_kZ_k/{Y_k}^3\right)\}P_{Dk}-\{(Z_k/Y_k)+\left(X_k{Z_k}^2/{Y_k}^3\right)\} \tag{26}$$

Note that (tracing back from equation (26) to equation (17)) X_k, Y_k and Z_k are the functions of cost equation coefficients, γ and elements of $[B_{mn}]$ matrix. For a given load demand P_{Dk} and assumed value of γ, the generation of P_{tk} can be calculated. Substituting this value of P_{tk} into equation (8) the quantity of fuel Q_T consumed by the fuel-restricted plant can be found. However this should be equal to the total availability of the fuel for the scheduled time. An iterative procedure has been developed in accordance with the algorithm developed in the next section.

3. PROGRAM DEVELOPMENT

A program has been developed based on the following algorithm to obtain the economic dispatch calculations for the known load pattern and the stipulated quantum of fuel with the fuel restricted plant as presented below:

1. Read the cost function coefficients of (N-1) non-fuel restricted plants, input-output coefficients and heat rate (h_g) and total quantity of fuel Q_T for the fuel restricted plant, load pattern P_{Dj} and elements of loss coefficient matrix [B_{mn}].
2. Set $\gamma = 0.85$
3. Compute α_i, β_i as given in equation (18) by varying i from 1 to N-1.
4. Compute l_i and m_i as given in equation (19) by varying i from 1 to N-1.
5. Set XX=YY=ZZ=0.
6. Compute $XX = XX - B_{ii} * {l_i}^2$;

   $YY = YY + l_i - 2 * l_i * m_i * B_{ii}$;

   $ZZ = ZZ + m_i - m_i * m_i * B_{ii}$;

   by varying i from 1 to N
7. Set $X_j = XX$;$Y_j = YY$; Z_j=ZZ
8. Compute P_{tj} using equation (26) for P_{Dj}. Compute $QT = QT + a_t * {P_{tj}}^2 + b_t * P_{tj} + c_t$ by varying j from 1 to J.
9. If $|Q_T - QT| \geq -0.0001$, and $|Q_T - QT| \leq 0.0001$ go to step 10. Else continue.
10. If $|Q_T - QT| \geq 0$ then $\gamma = \gamma - 0.001$, go to step 4 else if $|Q_T - QT| \leq 0$ then $\gamma = \gamma + 0.001$ and go to step 4
11. Print P_{ij}, F_{ij},P_{tj} and F_{tj}.
12. End.

4. NUMERICAL EXAMPLE

A sample system [5][7] is considered with the following parameters to verify the simplicity and validity of the proposed method.

The input-output functions of the participating units are given as follows.

Fuel-Restricted Plant:

$H_T(P_T) = 0.0045{P_T}^2 + 4.75P_T + 950.0$ MBtu/h

Cost of gas=2.0 Rs/ccf

The gas is rated at 1100 BTU/ft^3

Non fuel restricted plants:

The input-output characteristics of all the remaining thermal units are given as:

$$H_1(P_1) = 0.0100{P_1}^2 + 6.00P_T + 100$$

$$H_2(P_2) = 0.008502^2 + 6.50P_T + 120$$

$$H_3(P_3) = 0.0150{P_3}^2 + 5.70P_T + 180$$

$$H_4(P_4) = 0.0170{P_4}^2 + 5.00P_T + 130$$

$$H_5(P_5) = 0.01255^2 + 7.30P_T + 110$$

The daily load pattern of the above system is as given in the following Table 1.

A program has been developed in MATLAB in accordance with the algorithm given in section 3. The value of γ for the given system (section 4) is found to be 0.862 as per the iterative procedure given in the algorithm. The plant generations for the given daily load pattern is given in Table 2.The operating cost of plants corresponding to the given load pattern is given in Table 3.

Diagonal elements of Bmn matrix are given below

B11=0.000200

B22=0.000300

B33=0.000100

B44=0.000150

B55=0.000250

B66=0.000210

Table 1
Daily Load pattern

S.No	*Time Period (n_j)*	*Load (P_{Dj})*
1	12AM-4AM	1000MW
2	4AM-8AM	1200MW
3	8AM-12PM	1800MW
4	12PM-4PM	950MW
5	4PM-8PM	800MW
6	8PM-12AM	750MW

Table 2
Plant generation and operating cost for the given load pattern

S.No	*P_{Dj} (MW)*	*P_{1j}(MW)*	*P_{2j}(MW)*	*P_{3j}(MW)*	*P_{4j}(MW)*	*P_{5j}(MW)*	*P_{tj}(MW)*	*F_{tj}(Rs.)*
1	1000	198.324	188.635	152.295	153.309	109.685	234.479	18613.2
2	1200	236.789	229.860	179.964	177.579	139.758	289.476	23629.4
3	1800	355.558	357.153	265.398	252.521	232.618	459.295	44874.4
4	950	188.796	178.423	145.441	147.297	102.235	220.855	18355.4
5	800	160.422	148.013	125.031	129.393	80.051	180.286	9278.8
6	750	151.035	137.951	118.278	123.470	72.711	166.863	8777.4

Table 3
Operating cost of plants for the daily load pattern

F_{ij} (Rs) \ P_{Dj} (MW)	1000	1200	1800	950	800	750
F_{1j}	1783.3	2181.4	3597.6	1689.2	1419.9	1334.30
F_{2j}	1748.6	2163.2	3625.7	1650.3	1368.3	1278.4
F_{3j}	1496	1791.6	2849.3	1426.3	1227.2	1164.0
F_{4j}	1396.1	1654.0	2576.6	1335.3	1161.6	1106.5
F_{5j}	1161.1	1474.4	2584.5	1087.0	874.5	806.9
F_{6j}	3819.9	4466.0	6744.8	3666.8	3227.3	3087.3
F_{totj}	11405	13730.6	21978.5	10854.9	9278.8	7976.5

5. CONCLUSION

A simple single iterative procedure has been developed to obtain the Economic Dispatch Calculation of an all thermal power system with a fuel-restricted plant. Even though conventional methods are available to obtain EDC with a fuel-restricted plant, there is a need to evolve a curve fitting technique even for the losses neglected case. However this paper has eliminated the requirement of evolving equivalent cost function and turned out to be a single iterative method, even when transmission losses are not neglected. Results cannot be compared, as there is no method for the fuel-restricted plant, with transmission losses included case.

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