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CO–ISOLATED LOCATING DOMINATION MATRIX IN GRAPHS

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Abstract: Let G (V, E) be a connected graph. A dominating set $S \subseteq V$ is called a co–isolated locating dominating set, if for any two vertices $v, w \in V - S$, $N(v) \cap S \neq N(w) \cap S$ and there exists atleast one isolated vertex in $\langle V - S \rangle$. The minimum cardinality of a co–isolated locating dominating set is called the co–isolated locating domination number and is denoted by $\gamma_{cild}(G)$. This paper aims at the study of some new parameters namely Domination Matrix, Locating Domination Matrix and Co–isolated Locating Domination Matrix.

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1. INTRODUCTION

All the graphs considered here are finite, simple, connected and undirected. Let $G = (V, E)$ be a simple graph of order p. For any $v \in V(G)$, the **neighborhood** $N_G(v)$ (or simply $N(v)$) of v is the set of all vertices adjacent to v in G. A non – empty set $S \subseteq V(G)$ of a graph G is a dominating set, if every vertex in $V(G) - S$ is adjacent to atleast one vertex in S.

A special case of domination called a locating domination is defined by Rall and Slater [8]. A dominating set S in a graph G is called a **locating dominating set** in G, if for any two vertices $v, w \in V(G) - S$, $N_G(v) \cap S$ and $N_G(w) \cap S$ are distinct. The **locating domination number** $\gamma_L(G)$ of G is defined as the minimum number of vertices of a locating dominating set in G.

Muthammai and Meenal [4] introduced the concept of co–isolated locating dominating set. A locating dominating set $S \subseteq V(G)$ is called a **co–isolated locating dominating set** , if $\langle V - S \rangle$ contains atleast one isolated vertex. The minimum cardinality of a co–isolated locating dominating set is called the **co–isolated locating domination number** and is denoted by $\gamma_{cild}(G)$.

A dominating set with $\gamma(G)$ number of vertices is called a γ – set of G. Similarly, γ_L – set and γ_{cild} – set are defined.

In this paper, the concept of some new parameters namely Domination matrix, Locating domination matrix and Co–isolated locating domination matrix is introduced and studied with examples and illustrations.

2. PRIOR RESULTS

Theorem 2.1.[9]: For any graph G, $\gamma(G) \le p - \Delta(G)$.

Theorem 2.2.[9] : If G is a graph with no isolated vertices, then $\gamma(G) \le \frac{p}{2}$.

Theorem 2.3.[10]: (i) For a complete graph K_p, $\gamma_L(K_p) = p - 1$.

(ii) For paths and cycles,

$\gamma_L(P_{5k}) = \gamma_L(C_{5k}) = 2k$, and

$\gamma_L(P_{5k+1}) = \gamma_L(C_{5k+1}) = \gamma_L(P_{5k+2}) = \gamma_L(C_{5k+2}) = 2k + 1$ and

$\gamma_L(P_{5k+3}) = \gamma_L(C_{5k+3}) = \gamma_L(P_{5k+4}) = \gamma_L(C_{5k+4}) = 2k + 2$.

Theorem 2.4.[4]: For every non – trivial graph G, $1 \le \gamma_{cild}(G) \le p - 1$.

Theorem 2.5. [5]: If S is a co–isolated locating dominating set of G(V, E) with $|S|=k$, then $V(G) - S$ contains atmost $pC_1 + pC_2 + \dots + pC_k$ vertices.

Theorem 2.6.[4]: For the path P_p and cycle C_p $(p \ge 3)$, $\gamma_{cild}(P_p) = \gamma_{cild}(C_p) = \left\lceil \frac{2p}{5} \right\rceil$.

3. MAIN RESULTS

3.1. Preliminary Definitions

Definition 3.1.1. **Matrix** is a rectangular array of numbers, symbols or expression arranged in rows and columns and usually it is denoted by capital letters. The general form of m x n matrix is given as

$$\begin{array}{c} \\ R_1 \\ R_2 \\ R_. \\ R_i \\ R_. \\ R_m \end{array}
\begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & C_. & \cdot & \cdot & C_n \end{array} \\
\begin{pmatrix} a_{11} & a_{12} & a_{1.} & \cdot & \cdot & a_{1n} \\ a_{21} & a_{22} & a_{2.} & \cdot & \cdot & a_{2n} \\ & & & & & \\ a_{i1} & a_{i2} & \cdot & \cdot & \cdot & a_{in} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{m1} & a_{m2} & \cdot & \cdot & \cdot & a_{mn} \end{pmatrix} \end{array}$$

Definition 3.1.2. Given a matrix [A]. The **Submatrix** is a matrix obtained from the matrix [A] by deleting some of the row(s) and/or column(s) of [A].

Definition 3.1.3. A **(0, 1) – matrix** is an integer matrix in which each entry in the matrix is either '0' or '1'. It is also called Relation matrix.

Definition 3.1.4. A dominating set $S \subseteq V$ is called a **Co–isolated locating dominating set** in G, if for any two vertices $v, w \in V(G) - S$, $N_G(v) \cap S$ and $N_G(w) \cap S$ are distinct and $<V - S>$ contains atleast one isolated vertex. The minimum cardinality of a co–isolated locating dominating set is called the **Co–isolated locating domination number** and is denoted by $\gamma_{cild}(G)$. The corresponding co–isolated locating dominating set is a γ_{cild}– set.

Illustration 3.1.5. Consider the graph G as shown in Figure 3.1.6.

The set $S_1 = \{v_2, v_5\}$ is a γ – set of G and $\gamma(G) = 2$.

The set $S_2 = \{v_1, v_3, v_4, v_6\}$ is a γ_L–set of G and $\gamma_L(G) = 4$.

$V - S_2 = \{v_2, v_5\}$, $N(v_2) \cap S_2 = \{v_1, v_3\}$ and $N(v_5) \cap S_2 = \{v_4, v_6\}$.

Both v_2 and v_5 are isolated vertices in $<V - S_2>$.

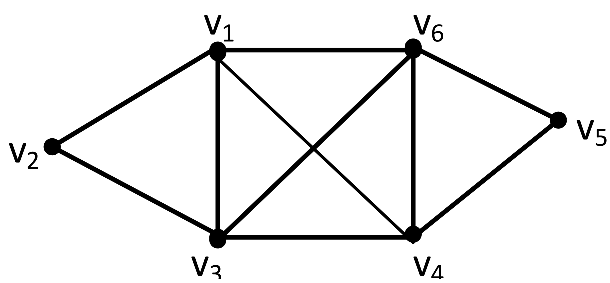

Figure 3.1.6

Hence the set S_2 is also a minimum co – isolated locating dominating set of G and $\gamma_{cild}(G) = 4$.

Definition 3.1.7. Let G be a graph of order p then the Adjacency matrix A(G) is a p x p matrix with the row labeling $V_1, V_2, \ldots, V_p$ and the column labeling $V_1, V_2, \ldots, V_p$ corresponding to the vertices $v_1, v_2, \ldots, v_p$ is given by

$$a_{ij} = \begin{cases} 1; \text{if there is an edge between the vertices } v_i \text{ and } v_j \\ -; \text{if } i = j \\ 0; \text{otherwise} \end{cases}.$$

a_{ij} denote the entries of the adjacency matrix in the i^{th} row and j^{th} column for all $v_i, v_j \in V(G)$.

Example 3.1.8. The Adjacency matrix of the graph G in Figure 3.1.6. is given below

$$A(G) = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{array} \begin{array}{c} \begin{array}{cccccc} V_1 & V_2 & V_3 & V_4 & V_5 & V_6 \end{array} \\ \begin{pmatrix} - & 1 & 1 & 1 & 0 & 1 \\ 1 & - & 1 & 0 & 0 & 0 \\ 1 & 1 & - & 1 & 0 & 1 \\ 1 & 0 & 1 & - & 1 & 1 \\ 0 & 0 & 0 & 1 & - & 1 \\ 1 & 0 & 1 & 1 & 1 & - \end{pmatrix} \end{array}$$

Definition 3.1.9. A matrix A is said to be **symmetric** if $A = A^t$ where A^t is the transpose of the matrix A.

Remark 3.1.10

i. Adjacency matrix of a graph G is a symmetric matrix since the graph is an undirected graph.

ii. Sum of the entries in a row (or a column) of the Adjacency matrix is equal to the degree of the corresponding vertex.

3.2. Domination matrix

Definition 3.2.1. Let G be a connected graph and $V(G) = \{v_1, v_2, \ldots, v_p\}$ be the vertex set of the graph G. Let A(G) be the adjacency matrix of the graph G. There exist a row with label V_i in A(G) for all i = 1, 2, …, p. Then the **Row operation** on the Adjacency matrix is defined as follows

Let V_i and V_k be the two rows in the Adjacency matrix A(G) corresponding to the vertices v_i and v_k of the graph G. Then

$$V_i \oplus V_k = a_{ij} \oplus a_{kj} = \begin{cases} 0; & \text{if } a_{ij} = a_{kj} = 0 \\ -; & \text{if } i = j \text{ (or) } k = j \\ 1; & \text{otherwise} \end{cases} \text{ for all } j = 1, 2, \ldots, p.$$

Example 3.2.2. For the Adjacency matrix A(G) in Example 3.1.8. row operations are illustrated as follows

(i) If the Row operation $V_5 \oplus V_6$ is performed then the corresponding entries are

$$1 \quad 0 \quad 1 \quad 1 \quad - \quad -$$

(ii) If the Row operation $V_2 \oplus V_5$ is performed then the corresponding entries are

1 – 1 1 1

(iii) If the Row operation $V_2 \oplus V_3 \oplus V_4$ is performed then the corresponding entries are

1 – – – 1 1

(iv) If the Row operation $V_1 \oplus V_2 \oplus V_3$ is performed then the corresponding entries are

– – – 1 0 1

Remark 3.2.3. After performing the row operation the entry at the position of the column will be '–' corresponding to the row labels included in the row operations. Hence after performing the row operations the number of entries with '–' are equal to the number of vertices included in the row operation.

Theorem 3.2.4. The vertices included in the finite number of row operations of an Adjacency matrix will denote a dominating set if and only if there is no zero entry after performing the row operations.

Proof. By the Definition 3.2.1, after performing the row operations there will be 3 entries namely 0, 1, –. The number of entries '–' will be equal to the number of rows included in the operation and these rows are the vertex labels included in the row operation. The entry '1' will denote that the vertex at the position of the column is adjacent to any one of the vertex included in the row operation. The entry '0' will denote that the vertex at the position of the column is not adjacent to any vertex included in the row operation. If after performing the row operations there is no zero entry then it will indicate that the vertices are either adjacent to the vertices in the row operation or included in the row operation. Hence the vertices included in the finite number of row operations of an Adjacency matrix with no zero entry will form a dominating set of the graph G.

Conversely, assume that there is atleast one zero entry after performing the row operations then it will indicate that vertex at the position of the column is not adjacent to any of the vertex included in the row operation, that is, there exist a vertex which is not dominated by any vertex in the dominating set. It is a contradiction. Hence the theorem follows.

Illustration 3.2.5. For the Row operations discussed in Example 3.2.2. (i). and (iv). does not form a dominating set, since the second and fifth entries are zero respectively. Whereas the Row operations (ii). and (iii). will form a dominating set.

In (ii). the rows included in the row operation with labels are V_2 and V_5 and the corresponding vertices are v_2 and v_5. Therefore the set $\{v_2, v_5\}$ is a dominating set of the graph G in Figure 3.1.6.

In (iii). the rows included in the row operation with labels are V_2, V_3 and V_4 and the corresponding vertices are v_2, v_3 and v_4. Therefore the set $\{v_2, v_3, v_4\}$ is also a dominating set of the graph G in Figure 3.1.6.

The minimum number of rows included in the Row operations required to form all the entries to be either '1' or '–' will form a minimum dominating set.

In the above row operations, (ii). is the minimum number of operations required. Therefore the set $\{v_2, v_5\}$ is a minimum dominating set.

Definition 3.2.6. Let G be a connected graph and S be a dominating set of G. **Domination matrix** is a matrix with columns labels are the vertices in the set S and the row labels are the vertices in the complement of the set S. **Domination matrix** is a submatrix of the adjacency matrix with the given column and row labels. The domination matrix is denoted by DM(G).

The domination matrix corresponding to the minimum dominating set will form a **Minimum Domination matrix.** It is denoted by $DM_\gamma(G)$.

Remark 3.2.7. All the entries in the domination matrix are either '0' or '1', there is no '-' entry since the labels in the rows and columns are different.

Example 3.2.8. In Example 3.2.2. the row operations (ii). and (iii). form the dominating set and the corresponding domination matrix is given below.

$$DM_1(G) = \begin{array}{c} \\ \mathbf{V}_1 \\ \mathbf{V}_3 \\ \mathbf{V}_4 \\ \mathbf{V}_6 \end{array} \begin{array}{c} \begin{array}{cc} \mathbf{V}_2 & \mathbf{V}_5 \end{array} \\ \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \end{array}$$

$$DM_2(G) = \begin{array}{c} \\ \mathbf{V}_1 \\ \mathbf{V}_5 \\ \mathbf{V}_6 \end{array} \begin{array}{c} \begin{array}{ccc} \mathbf{V}_2 & \mathbf{V}_3 & \mathbf{V}_4 \end{array} \\ \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \end{array}$$

Both the matrices $DM_1(G)$ and $DM_2(G)$ are dominating matrices since there is atleast a '1' in each row. The dominating matrix $DM_1(G)$ consists of minimum number of vertices namely v_2 and v_5. Therefore $DM_1(G)$ is a minimum dominating matrix. Hence $DM_\gamma(G) = DM_1(G)$.

3.3. Matrix as a Domination Matrix

Theorem 3.3.1. A given (0, 1) matrix is a domination matrix if and only if there is atleast a '1' in each row of the matrix.

Proof. Given a (0,1) matrix then the label entries in the column are mapped into the vertices in the dominating set S of the graph G and the label entries in the row are mapped into the vertices in the complement of a dominating set namely, V – S. Therefore the vertices in the row and column are different. The number '1' in the matrix will indicate that there is an edge between the vertex in the row and the vertex in the column. The number '0' in the matrix will indicate that there is no edge between the vertex in the row and the vertex in the column. If there is atleast a '1' in each row then that vertex is dominated by atleast one vertex in the column, that is the dominating set. Hence (0, 1) matrix will be a domination matrix.

Conversely, assume that (0,1) matrix as a domination matrix. Then by the definition, each vertex in the row must be dominated by atleast one vertex of the column. On the contrary, if all the entries in a row are '0' then it will indicate that vertex is not dominated by any vertex in the column, a contradiction. This completes the proof of the theorem.

Example 3.3.2. Consider the following matrices

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

The matrix A is a Domination matrix and the dominating set $S = \{v_1, v_2, v_3\}$ and the set $V – S = \{u_1, u_2, u_3\}$ and $E(G) = \{u_1v_1, u_1v_3, u_2v_3, u_3v_1\}$.

The matrix B is not a Domination matrix since all the entries in the last row is zero.

Remark 3.3.3. The graph obtained from the (0, 1) matrix need not be a connected graph. If all the entries in any row/column are zero in (0, 1) matrix then the corresponding vertex in that row/column will be an isolated vertex.

In Example 3.2.2, the graph obtained from the (0, 1) matrix is not a connected graph, since all the entries in the second column are zero.

3.4. Locating Domination Matrix

Theorem 3.4.1. A domination matrix is a locating domination matrix if and only no two rows are identical.

Proof. Let G be a connected graph and S be a dominating set of G. If in a domination matrix DM(G) no two rows are identical then it will represent that the vertices corresponding to the two rows does not have the same neighbor in S. By the definition of a locating dominating set for any two vertices u, $v \in V - S$, $N(u) \cap S \neq N(v) \cap S$. Hence DM(G) will be a locating domination matrix.

Conversely, assume that DM(G) be a locating domination matrix of the graph G. On the contrary, if two rows with row labels V_i and V_j are identical then it will indicate that for the corresponding vertices v_i and v_j, $N(v_i) \cap S = N(v_j) \cap S$, which is a contradiction. Hence the theorem follows.

Remark 3.4.2. The locating domination matrix is denoted by LDM(G).

The locating domination matrix which includes minimum number of entries in the column will form a **Minimum Locating Domination matrix.** It is denoted by $LDM_\gamma(G)$.

In Example 3.2.8. both the matrices $DM_1(G)$ is not a Locating Domination matrix. Since in $DM_1(G)$ the two rows namely V_1 and V_3 as well as V_4 and V_6 are identical.

But the domination matrix $DM_2(G)$ is a Locating Domination Matrix since no two rows are identical. Hence $LDM(G) = DM_2(G)$.

Example 3.4.3. For the Adjacency matrix A(G) in Example 3.1.8. corresponding to the graph G in Figure 3.1.6. If the Row operation $V_1 \oplus V_3 \oplus V_4 \oplus V_6$ is performed then the corresponding entries are

$$- \quad 1 \quad - \quad - \quad 1 \quad -$$

Therefore the set $\{v_1, v_3, v_4, v_6\}$ is a Dominating set of the graph G. The corresponding domination matrix is given by

$$DM_3(G) = \begin{array}{c c} & \begin{array}{cccc} \mathbf{V}_1 & \mathbf{V}_3 & \mathbf{V}_4 & \mathbf{V}_6 \end{array} \\ \begin{array}{c} \mathbf{V}_2 \\ \mathbf{V}_5 \end{array} & \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \end{array}$$

The Domination matrix $DM_3(G)$ is a locating dominating matrix since no two rows are identical. Hence $LDM(G) = DM_3(G)$.

The domination matrix $DM_2(G)$ contains the minimum number of vertices in the column therefore, it is a minimum locating domination matrix. Hence $LDM_\gamma(G) = DM_2(G)$.

3.5. Co–isolated Locating Domination Matrix

Theorem 3.5.1. A locating domination matrix will be a co–isolated locating domination matrix if and only if there exists atleast one row with same number of 1's in the Adjacency matrix A(G) and in the Locating Domination Matrix LDM(G).

Proof. Let G be a connected graph and S be a locating dominating set of G. If in a locating domination matrix LDM(G) there exist atleast one row with same number of 1's as in the Adjacency matrix A(G), then it will indicate that the corresponding vertex in the row has all its neighbor only in the column of LDM(G). Hence there exist an isolated vertex in the row namely, V – S. This implies that, S is a co–isolated locating dominating set. Therefore LDM(G) will be a co–isolated locating domination matrix.

Conversely, assume that LDM(G) be a co–isolated locating domination matrix of the graph G. On the contrary, if there does not exist any row with same number of 1's in A(G) and LDM(G) then it indicates that there is no isolated vertex in LDM(G), a contradiction. This completes the proof of the theorem.

Remark 3.5.2. The Co–isolated Locating Dominating matrix is denoted by CLDM(G). The Co–isolated locating domination matrix which includes minimum number of entries in the column will form a **Minimum Co–isolated Locating Domination matrix.** It is denoted by $CLDM_{\gamma}(G)$.

Example 3.5.3. In Example 3.4.3., the locating domination matrix $DM_3(G)$ is also a co–isolated locating domination matrix since the vertices v_2 and v_5 have the same number of 1's in the adjacency matrix A(G) as well as in the domination matrix $DM_3(G)$ namely 2 and 2 respectively. Therefore v_2 and v_5 are isolated vertices. Hence $CLDM(G) = DM_3(G)$. Also, $DM_3(G)$ is a minimum co–isolated locating domination matrix. Therefore $CLDM_{\gamma}(G) = DM_3(G)$

4. CONCLUSION

In this paper, the concept of some new parameters namely Domination matrix, Locating domination matrix and co–isolated locating domination matrix are introduced and studied with examples and illustrations. A necessary and sufficient condition for a domination matrix to be locating domination matrix and co–isolated locating domination matrix are also established.

5. OPEN PROBLEMS

- To define some more types of domination matrices namely, degree equitable domination matrix, total domination matrix, etc.
- To find the domination number of standard graphs namely path, cycle, etc using Domination matrix.

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