# **Optimal Price Scheduling by Augmented Lagrangian in Deregulated Power Market**

V. Kalyanasundaram, K. Vijayakumar and PradeepVishnuram

#### ABSTRACT

Deregulated electricity markets use a market clearing price mechanism to pay market participants for energy and service products. Lagrangianmethod is used for solving the minimization of offer cost in a deregulated electricity market. A general formulation of the market clearing price (MCP) and offer cost minimization approach is presented. The most independent system operators (ISO) determine MCP using that minimizes the offered costs. The solution algorithm for offer cost minimization has been developed based on Lagrangian methodology. Using this method is tested with 30-bus IEEE test system and numerical testing results demonstrate that the method is effective, and the resulting costs are significantly lower in pay at MCP costs when compare to actual purchase cost using MCP.

Keywords: Price, offercost, payment cost, actual cost, market clearing price (MCP), independent system operator (ISO).

#### I. INTRODUCTION

Independent system operators (ISOs) generally adopt a market clearing price (MCP) mechanism to pay market participants and charge consumers for energy and ancillary service products[1]. Under this mechanism, market participants submit supply offers and demand bids for energy and ancillary services to minimize the total bid price and determine the MCPs for each product.Get paid based on the It is crucial for the ISOs to have a proper objective function and to set the MCPs correctly, since market participants are charged or MCPs. The electricity market consists of a network which is open to wholesalers, retailers and the consumers [2-4]. The pool operator is independent of the transmission owners and the consumers and hence determines the market clearing price by combining the facilities of several transmission owners into a single system and fixing it at a single lower price than the combined charges of each utility that may be located between the buyer and seller. The lagrangian approach has been tested on a 30 bus system[5]. From that results offer cost are minimized.A understanding has been made and how various market conditions have an impact on the MCP is discussed.

The electric power industry has over the years been dominated by large utilities that had an overall authority over all activities in generation, transmission and distribution of power within its domain of operation. such utilities have often been referred to as vertically integrated utilities. such utilities served as the only electricity provider in the region and were obliged to provide electricity to everyone in the region[6].

The utilities are vertically integrated, it was often difficult to segregate the costs incurred in generation, transmission or distribution[7-8]. Therefore, the utilities often charged their customers an average tariff rate depending on their aggregated cost during a period. The price setting was done by an external regulatory agency and often involved considerations other than economics. Genetic algorithm based market clearing was performed with minimum cost and maximize social welfare [9]. Wind power marketing scheme was developed with maximum efficiency and low under pay as bid scheme [10]. Price forecast based build hourly offer curves are discussed in [11].

Department of EEE, SRM University, Kattankulathur-603203, Chennai, India, E-mail: kalyan.srm@gmail.com

The above literatue does not deals with the Augmented lagrangian relaxation method based market price forecasting in which this work putforth to develop the optimal algorithm by considering the generation cost, fuel cost and operating cost.

### **II. PROBLEM FORMULATION**

The objective of a power pool operator is to maximize the social welfare function which is determined as the deviation between the cumulative consumer and generator bids. Consider a system with N generators and M consumers. Let the generator bid function for the i<sup>th</sup> generator be

$$C_i(Pg_i) = a_i Pg_i^2 + b_i Pg_i + C_i$$

and the consumer benefit function for the jth load be

$$Bf_j(Pd_j) = \alpha_j Pd_j^2 + \beta_j Pd_j + \gamma_j$$

Now, the objective of the pool market operator is to maximize the social welfare function

$$\sum_{j=1}^{M} Bf_j(Pd_j) - \sum_{i=1}^{N} C_i(Pg_i)$$

Subject to the power balance constraint

$$\sum_{i=1}^{N} (Pg_i) = \sum_{j=1}^{M} (Pd_j)$$

The schedules for each of the generators and the demand of each consumer that can be met is obtained as,

$$Pg_i = \lambda - b_i/2a_i$$
$$Pd_i = \lambda - \beta_j/2\alpha_j$$

Objective function,

Minj =  $\sum_{t=1}^{T} \sum_{i=1}^{I} \{ C_i(p_i(t), t) \} i$  = no of offers

Demand constrain

$$P_{d}(t) - \sum_{i=1}^{I} p_{i}(t) = 0, \qquad t = 1, 2, \dots, T$$

Pay at mcp formulation

$$J = \sum_{t=1}^{T} \sum_{i=1}^{I} \{MCP(t)p_i(t)\}$$

Pay as offer cost

 $J=\min\sum_{i=1}^{I} (Pg(i) * Ci)$ 

Actual cost using MCP

 $J = \sum_{i=1}^{l} \{ MCP * p_g(i) \}$ 

#### **III. SOLUTION METHODOLOGY**

Test section dealts with the solution for the problem listed in section II. Augmented lagrangian relaxation method is used to solve the listed problem. It has been formulated by considering the equality and in equality contrainsby summing the quadratic penality terms. Thus the overall convergence has been improved with fast rate. Let  $\{\lambda(t)\}$  be the multipliers of the relax system demand constraints,  $\{\eta_i(t)\}$  be the MCP-offer inequality constraints. The augmented Lagrangian is given by,

$$\begin{split} L_{c}(\lambda,\eta,MCP,P) &\equiv \sum_{i=1}^{I} \sum_{t=1}^{T} \{MCP(t)P_{i}(t) + S_{i}(t)\} + \sum_{t=1}^{T} \{\lambda(t) \left(P_{d}(t) - \sum_{i=1}^{I} P_{i}(t)\right) \\ &+ \sum_{i=1}^{I} \eta_{i}(t)(O_{i}^{r}(t) - MCP(t) + z_{i}(t)^{2})\} \\ &+ \frac{u}{2} \sum_{t=1}^{T} \left\{ (P_{d}(t) - \sum_{i=1}^{I} P_{i}(t))^{2} + \sum_{i=1}^{I} (O_{i}^{r}(t) - MCP(t) + z_{i}(t)^{2})^{2} \right\} \end{split}$$

Where u is the positive penality coefficient. The augmented lagrangian function with minimizing  $\{z_i(t)^2\}$  is given by

$$\begin{split} L_{c}(\lambda,\eta,MCP,P) &\equiv \sum_{i=1}^{I} \sum_{t=1}^{T} \{MCP(t)P_{i}(t) + S_{i}(t)\} + \sum_{t=1}^{T} \{\lambda(t) \left(P_{d}(t) - \sum_{i=1}^{I} P_{i}(t)\right) \\ &+ \sum_{i=1}^{I} \sum_{t=1}^{T} \frac{1}{2u} \max\{0,\eta_{i}(t) + u(O_{i}^{r}(t) - MCP(t))^{2} - \eta_{i}(t)^{2}\} \\ &+ \sum_{t=1}^{T} \frac{u}{2} \left\{ (P_{d}(t) - \sum_{i=1}^{I} P_{i}(t))^{2} \right\} \end{split}$$

# **IV. SIMULATION RESULTS**

The pay-at-MCP is developed with argumented using MATLAB. The obtained results are presented here. Simulation has carried out for 6 unit system, the cost co-efficient and bid coefficient is in a table 1.

			Cost co-effe	Table 1   cient and bid	co-efficient			
GEN.NO	Pmin (MW)	Pmax (MW)	a <sub>i</sub>	$b_{i}$	$c_{i}$	$\alpha_{_i}$	$\beta_{i}$	$\gamma_i$
1	50	200	0.0038	200	0	0.0126	-1.1	22.983
2	20	80	0.0175	1.75	0	0.02	-0.1	25.313
3	15	50	0.0625	1	0	0.027	-0.01	25.505
4	10	35	0.0083	3.25	0	0.0291	-0.005	24.9
5	10	30	0.025	3	0	0.029	-0.004	24.7
6	12	40	0.025	3	0	0.0271	-0.0055	25.3

The system demand assumed as the valley pattern which is given in the table 2.

		Valle	y pattern		
Hour	DEMAND	МСР	Hour	DEMAND	МСР
1	285	81.59866	13	187	67.75561
2	275	80.18611	14	192	68.46188
3	263	78.49103	15	198	69.30943
4	258	77.78476	16	210	71.00449
5	245	75.94844	17	240	75.24215
6	235	74.53587	18	247	76.23095
7	200	69.59194	19	255	77.36099
8	195	68.88565	20	260	78.06727
9	190	68.17937	21	261	78.20852
10	185	67.4731	22	270	79.47982
11	180	66.76682	23	275	80.18611
12	180	66.7668	24	283	81.31615

Table 2 Vallev pattern

# STEP BY STEP PROCEDURE FOR PROPOSED METHOD

Step1: MCP is determined from cost coefficient and bid coefficient as shown in Figure 1.

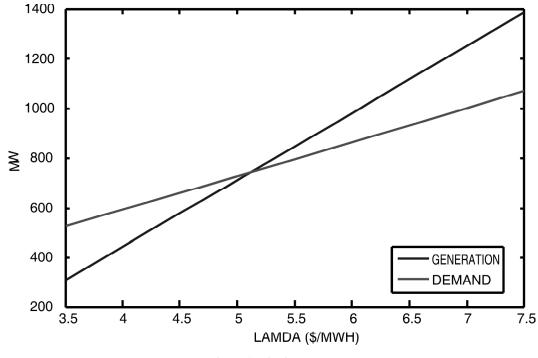


Figure 1: Bidding graph

Step2: Calculate individual offer cost and find total pay as offer cost

Hour	Demand	offer1 \$/MW	OFFER2 \$/MW	Offer 3 \$/mw	OFFER 4\$/MW	OFFER5 \$/MW	OFFER6 \$/MW
1	285	81.59866	81.59869	81.5988	82.0032	84	86.4
2	275	80.18611	80.18606	80.1861	82.0032	84	86.4
3	263	78.49103	78.49103	78.4911	82.0032	84	86.4
4	258	77.78476	77.78476	77.7849	82.0032	84	86.4

5   245   75.948     6   235   74.533     7   200   69.593     8   195   68.883	58774.535899469.5919	75.9483 74.5359 69.5919 68.8857	82.0032 82.0032 82.0032	84 84 84	86.4 86.4
7 200 69.59	94 69.5919	69.5919			
			82.0032	84	
8 195 68.884	68.88563	68 8857			86.4
0 175 00.000		00.0057	82.0032	84	86.4
9 190 68.179	68.17936	68.1795	82.0032	84	86.4
10 185 67.47	67.47308	67.473	82.0032	84	86.4
11 180 66.766	66.76681	66.7668	82.0032	84	86.4
12 180 66.76	66.76681	66.7668	82.0032	84	86.4
13 187 67.755	67.75558	67.7556	82.0032	84	86.4
14 192 68.462	68.46185	68.4618	82.0032	84	86.4
15 198 69.309	69.30941	69.3093	82.0032	84	86.4
16 210 71.004	49 71.00444	71.0046	82.0032	84	86.4
17 240 75.242	75.24216	75.2421	82.0032	84	86.4
18 247 76.230	76.23092	76.2309	82.0032	84	86.4
19 255 77.360	77.36098	77.361	82.0032	84	86.4
20 260 78.067	78.06725	78.0672	82.0032	84	86.4
21 261 78.208	78.20854	78.2085	82.0032	84	86.4
22 270 79.479	79.47979	79.4799	82.0032	84	86.4
23 275 80.186	611 80.18606	80.1861	82.0032	84	86.4
24 283 81.310	81.31612	81.3162	82.0032	84	86.4

Step3: Find actual pay as offer cost

pay as offer	hour					
cost6	cost5	cost4	cost3	cost2	cost1	
1036.8	840	820.032	1566.664	3846.668	15231.13985	1
1036.8	840	820.032	1501.781	3645.226	14338.21393	2
1036.8	840	820.032	1425.689	3409.784	13295.95629	3
1036.8	840	820.032	1394.551	3313.701	12871.11457	4
1036.8	840	820.032	1315.128	3069.441	11792.44561	5
1036.8	840	820.032	1255.58	2887.013	10988.18976	6
1036.8	840	820.032	1057.609	2285.92	8347.914601	7
1036.8	840	820.032	1030.661	2204.802	7992.898641	8
1036.8	840	820.032	1004.045	2124.871	7643.425316	9
1036.8	840	820.032	977.7512	2046.128	7299.503319	10
1036.8	840	820.032	951.8008	1968.573	6961.115121	11
1036.8	840	820.032	951.8008	1968.573	6961.106568	12
1036.8	840	820.032	988.229	2077.481	7436.408786	13
1036.8	840	820.032	1014.645	2156.699	7782.547752	14
1036.8	840	820.032	1046.785	2253.332	8205.244914	15
1036.8	840	820.032	1112.514	2451.72	9074.565023	16
1036.8	840	820.032	1285.188	2977.633	11387.54637	17
1036.8	840	820.032	1327.203	3106.494	11955.95516	18
1036.8	840	820.032	1376.02	3256.619	12618.86134	19
1036.8	840	820.032	1406.958	3351.989	13040.38834	20
1036.8	840	820.032	1413.188	3371.21	13125.35235	21
1036.8	840	820.032	1469.846	3546.293	13900.05916	22
1036.8	840	820.032	1501.781	3645.226	14338.21393	23
1036.8	840	820.032	1553.579	3805.993	15050.78107	24

Hour	Actual costs using MCP1	Actual costs using MCP 2	Actual costs using MCP 3	actual costs using MCP 4	actual costs using MCP 5	actual costs using MCP 6
1	23295.07	5883.234	2396.11	1248	1248	1497.6
2	22315.7	5673.358	2337.342	1248	1248	1497.6
3	21140.45	5421.524	2266.83	1248	1248	1497.6
4	20650.77	5316.592	2237.452	1248	1248	1497.6
5	19377.58	5043.767	2161.049	1248	1248	1497.6
6	18398.2	4833.903	2102.293	1248	1248	1497.6
7	14970.41	4099.368	1896.623	1248	1248	1497.6
8	14480.72	3994.436	1867.245	1248	1248	1497.6
9	13991.03	3889.504	1837.867	1248	1248	1497.6
10	13501.35	3784.572	1808.477	1248	1248	1497.6
11	13011.66	3679.641	1779.099	1248	1248	1497.6
12	13011.65	3679.641	1779.099	1248	1248	1497.6
13	13697.22	3826.543	1820.233	1248	1248	1497.6
14	14186.9	3931.475	1849.611	1248	1248	1497.6
15	14774.54	4057.398	1884.867	1248	1248	1497.6
16	15949.78	4309.232	1955.391	1248	1248	1497.6
17	18887.89	4938.835	2131.671	1248	1248	1497.6
18	19573.46	5085.737	2172.805	1248	1248	1497.6
19	20356.95	5253.631	2219.818	1248	1248	1497.6
20	20846.64	5358.563	2249.196	1248	1248	1497.6
21	20944.57	5379.554	2255.074	1248	1248	1497.6
22	21826.01	5568.426	2307.964	1248	1248	1497.6
23	22315.7	5673.358	2337.342	1248	1248	1497.6
24	23099.19	5841.252	2384.354	1248	1248	1497.6

Step 4 : Calculate cost using payment cost minimization technique.

The comparision of pay at MCP, total pay as offer and total actual cost for each hour is lised in Table 2.

	Table 2
Comparision of pay at MCP,	, total pay as offer and total actual cost

Total actual cost using MCP	Total pay as offer	Pay at MCP costs	Demand	Hour
35568	23341.3	23255.4	285	1
34320	22182.1	22051.2	275	2
32822.4	20828.3	20643.1	263	3
32198.4	20276.2	20068.3	258	4
30576	18873.8	18607.3	245	5
29328	17827.6	17515.7	235	6
24960	14388.3	13918.2	200	7
24336	13925.2	13432.6	195	8
23712	13469.2	12954	190	9
23088	13020.2	12297.5	185	10
22464	12578.3	12017.9	180	11
22464	12578.3	12017.9	180	12

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Total actual cost using MCP	Total pay as offer	Pay at MCP costs	Demand	Hour
23337.6	13199	12670.2	187	13
23961.6	13650.7	13144.5	192	14
24710.4	14202.2	13723.2	198	15
26208	15335.6	14910.8	210	16
29952	18347.2	18058.1	240	17
30825.6	19086.5	18828.8	247	18
31824	19948.3	19726.8	255	19
32448	20496.2	20297.4	260	20
32572.8	20606.6	20412.3	261	21
33696	21613	21459.3	270	22
34320	22182.1	22051.2	275	23
35318.4	23107.2	23012.4	283	24
\$695011	\$425063	\$417074.045		

# **II. CONCLUSION**

The augmented Lagrangian relaxation method within an advanced has been presented for solving the payat-MCP problem. The MCP using the "right" Pay-at-MCP formulation, Numerical testing shows that the method and its use can lead to significant savings in the cost of purchasing power. Currently most ISO'sminimize the purchase cost under a Pay-as-Offer formulation but pay selected participants at MCP's.It can lead to much higher overall purchase costs than the costs that can be achieved by using the Pay-at-MCP formulation. The successful and use of the Pay-at-MCP formulation produces the result in lower overall purchase costs and subsequently lower prices for consumers.

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