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Analytical, Numerical Treatment and Complex Dynamics of Jungck Ishikawa Iteration Scheme using Different Functions

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Abstract: In this paper, we have introduce an algorithm and proved theorems for Jungck-Ishikawa iteration for non-self – mappings pair, study its sconvergence property, stability and dependency of data . It is seen that this iterative scheme has much better convergence rate than those of Jungck–Mann, Jungck–Noor in case of complex numbers. Numerical examples are also given in support of validity and applications of our results taking complex values. We introduce in this paper the complex dynamics of various functions like increasing functions, decreasing functions, oscillating functions and biquadratic functions and compared their convergence speeds and applied Jungck Ishikawa iteration to generate Relative Superior Mandelbrot set and Relative Superior Julia set. Only mathematical explanations are derived by using Jungck Ishikawa Iterative scheme for the above functions but in this paper we have generated Mandelbrot sets and its Relative Julia sets. Our results are generalization and extensions of those of various authors in the literature.

Keywords: *Complex dynamics, Relative Superior Mandelbrot set, Relative Julia set, Jungck Ishikawa Iteration.*

1. INTRODUCTION

"Fractal" came from Latin word "fractus" which means "broken". The word fractal was first time used by a mathematician Gaston Julia [Benoit B. Mandelbrot 1983], when he was studying the behavior of Newton's method in complex plane also known as Cayley's problem. Julia derived Julia set in 1919 by introducing the concept of iterative function system (IFS).In 1975, Benoit Mandelbrot after extending the ideas of Julia, introduced the Mandelbrot set by using the complex function $z^2 + c$, where z is used as a complex function and c as a complex parameter [Ashish Negi et al. 2008].

Benoit B. Mandelbrot , mathematician introduced Fractals in 1979 for describing irregular and chaotic natural phenomenon as lunar landscapes, mountains, trees branching and coastlines etc. The object Mandelbrot set and its relative object Julia set due to their complex nature and beauty have become superior in areas of research nowadays. These graphics are obtained by "coloring" the escape speed of the seed points within the certain regions of the complex plane that give rise to the unbounded orbits.

Julia and Mandelbrot sets are always studied under the effect of noises [M Rani 2010] arising in the objects. In 2004, Rani and Kumar [M Rani et al. 2009], introduced superior iterates (a two-step feedback process) in the study of fractal theory and created superior Julia and Mandelbrot sets. Later on, in a series of papers Rani et al. have generated and analyzed superior Julia and superior Mandelbrot sets for quadratic, cubic, biquadratic and nth degree [J.O. Olaleru et al. 2010] introduced Julia and Mandelbrot sets in Jungck Mann and Jungck Ishikawa orbits.

1.1. Preliminaries

1.1.2. Jungck Ishikawa Iteration[8]

Let (X, ||.||) be a Banach space and Y an arbitrary set. Let S, T: Y $\rightarrow$ X be two non-self-mappings such that T(Y) $\subseteq$ S(Y), S(Y) is a complete subspace of X and S is injective. Then for $x_o \in$ Y, define the sequence

$\{Sx_n\}$ iteratively by

$$Sx_{n+1} = \alpha_n Ty_n + (1 - \alpha_n) Sx_n$$
$$Sy_n = \beta_n Tx_n + (1 - \beta_n) Sx_n$$

where $0 \leq \beta_n \geq 1$ and $0 \leq \alpha_n \geq 1$ and α_n & β_n both convergent to non-zero number.

We will be using the contractive definition[20] which Olatinwo has used to prove the strong convergence results for the Jungck-Ishikawa Iteration, that there exists a real number $a \in (0,1)$ and a monotone increasing function $\Phi\ R^+ \rightarrow R^+$ such that

$$\Phi(0) = 0 \text{ and for } x, y \in Y, \text{we have}$$
$$Tx - Sy \leq \Phi(Sx - Tx) + a(Sx - Sy) \quad (1)$$

This condition will be used to prove the theorems.

2. MAIN RESULTS

2.1. Algorithm to Find the Roots or Fixed Points of Different Functions Having Complex Values using Jungck Ishikawa Iteration

Read x

Where x is the initial complex value found out from Julia set (*i.e* c), n is the numbers iterations.

1. $Tx = f(x)$
2. $Sx = f_1(x)$
3. $Sy = (1 - a_n)* Sx + a_{n*}Tx$
4. $f_1(y) = Sy.$

   Find out the value of y by equating the above equation.

5. $Ty = f(y)$
6. $Sx = (1 - b_n)* Sx + b_{n*}Ty$
7. $f_1(x) = Sx$

Find out the value of x by equating the above equation.

8. Repeat step 4 to 8 until Tx = Sx = p

where p is the point where Sx , Tx sequences converges to a fixed point $i.e$ p.

2.2. Theorem

Let (X, ||.||) be an arbitrary Banach space, and Let Q, P: Y → X be non-self-operators on an arbitrary set satisfying contractive Condition. Assuming that P (Y) ⊆ Q (Y), Q(Y) is a complete subspace of X and Qz = Pz = p(say). For ex- x_0 € Y, let $\{Qxn\}^{¥}_{n=0}$ be Jungck Ishikawa iterative scheme,w here α_n & β_n are the sequences of positive numbers in [0,1] with α_n satisfying $å^{¥}_{n}\ \alpha_{n=0=\infty.}$ Then Jungck Ishikawa iterative scheme $\{Qxn\}^{¥}_{n=0}$ converges strongly to p. Also, p will be the unique common fixed point of Q, T provided that Y = X, and Q, P are weakly compatible.

Proof : First we prove that Jungck Ishikawa iterative scheme $\{Qxn\}^{¥}_{n=0}$ converges strongly to p.

It follows from Jungck Ishikawa iterative scheme and contractive condition (7) that

$$\begin{aligned}\|Qx_{n+1}-p\| &= \|(1-a_n)Qx_n + a_n Py_n - (1-a_n+a_n)p\| \text{ £ } (1-a_n)\|Qx_n - p\| + a_n\|Py_n - p\| \\ &= (1-a_n)\|Qx_n-p\| + a_n\|Py_n - Pz\| \\ &\leq (1-a_n)\|Qx_n-p\| + a_n\|Pz-Py_n\| \\ &\leq (1-a_n)\|Qx_n-p\| + a_n\{\phi(\|Qz-Pz\|) + a(\|Qz - Qy_n\|)\} \\ &= (1-a_n)\|Qx_n-p\| + \alpha_n\{\phi * 0) + a(\|Qy_n-p\|)\} \\ &= (1-a_n)\|Qx_n-p\| + aa_n\|Qy_n-p\| \end{aligned} \quad (2)$$

Similarly

$$\begin{aligned}\|Qy_n-p\| &= \|(1-\beta_n)Qx_n + \beta_n Px_n - (1-\beta_n+\beta_n)p\| \leq (1-\beta_n)\|Qx_n-p\| + b_n\|Px_n-p\| \\ &= (1-\beta_n)\|Qx_n-p\| + \beta_n\|Px_n - Pz\| \\ &\leq (1-\beta_n)\|Qx_n - p\| + b_n\|Pz-Px_n\| \\ &\leq (1-\beta_n)\|Qx_n-p\| + \beta_n\{\phi(\|Qz-Pz\|) + a(\|Qz-Qx_n\|)\} \\ &= (1-\beta_n)\|Qx_n-p\| + \beta_n\{f * 0) + a(\|Qx_n - p\|) \\ &= (1-\beta_n)\|Qx_n-p\| + a\beta_n\|Qx_n-p\| \\ &\leq (1-b_n(1-a))\|Qx_n-p\| \end{aligned} \quad (3)$$

It follows from (1) that

$$\|Qx_{n+1}-p\| = (1-\alpha_n)\|Qx_n-p\| + a\alpha_n(1-\beta_n(1-a))\|Qx_n-p\| \quad (4)$$

Using $a\alpha_n(1-\beta_n(1-a)) \leq a\alpha_n$ inequality (4) yields

$$\begin{aligned}\|Qx_{n+1}-p\| &\leq (1-\alpha_n)\|Qx_n-p\| + a\alpha_n\|Qx_n-p\| \\ &\leq (1-\alpha_n+a\alpha_n)\|Qx_n-p\| \\ &\leq (1-\alpha_n(1-a))\|Qx_n-p\| \\ &\leq \prod_{k=0}^{n}(1-\alpha_k(1-a)(Qx0-p)-(1-a\textstyle\sum^{\infty} k=0\,\alpha k) \end{aligned} \quad (5)$$

Hence it follows from (5) that $\lim_{n \to \infty} \| Qx_n - p \| = 0$ .Therefore $\{Qxn\}_{n=0}^{\infty}$ converges strongly to p.

Now we are proving that p is unique common fixed point of S and T. Let there exist another pair of coincidence say p^* .then, there exists $r^* \in$ X such that $Qr^* = Pr^* = p^*$. But from contractive condition (1), we have

$$0 \le \| p - p^* \| = \| Pr - Pr^* \| \le \phi(\| Qr - Pr \|) + a \| Qr - Pr^* \|$$
$$= a \| p - p^* \|$$

which implies that $p = p^*$ as $0 \le a < 1$.

Now as Q and P are weakly compatible and $p = Pr = Qr$, so $Pp = P(Pr) = P(Qr) = Q(Pr)$ and hence $Pp = Qp$. Therefore,Tp is a point of coincidence of Q,P and since the point of coincidence is unique then $p = Pp$. Thus, $Pp = Qp = p$, and therefore p is fixed point of Q and P and which is unique.

3. FIXED POINTS

We have discussed here four cases to find out the fixed points.

Decreasing Function $(1-x)^6 + c = 0$: In order to solve this function by Jungck Ishikawa Iterative scheme, we write it in the form of Sx = Tx where the function T, S is defined as $T(x) = (1-x)^9 + c$ and S$x = x$, respectively.

Increasing Function $x^2 - 2x - 3 + c = 0$: In order to solve this function by Jungck Ishikawa Iterative scheme, we write it in the form of Sx = Tx where the function T, S is defined as $Tx = (x^2 - 3) + c$ and $Sx = 2x$, respectively.

Oscillating Function $1/x + c = 0$: In order to solve this function by Jungck Ishikawa Iterative scheme, we write it in the form of Sx = Tx where the function T, S is defined as $Tx = 1/x + c$ and $Sx = x$, respectively.

Biquadratic Function $x^4 - 36x^2 - 52x + 87 + c = 0$: In order to solve this function by Jungck Ishikawa Iterative scheme, we write it in the form of Sx = Tx where the function T, S is defined as $T(x) = x^4 - 36x^2 + 87 + c$ and $Sx = 52x$, respectively.

3.1. Fixed Points of Decreasing Function

Table 1
Orbit of F (z) for (z_0 = 1.325 – 0.1375*i*) at α = β = 0.5 and *c* = 0.1

Number of iterations	*z*	|*Tz*|	|*Sz*|
11	0.2633	0.2599	0.2633
12	0.2627	0.2609	0.2627
13	0.2623	0.2611	0.2623
14	0.2621	0.2614	0.2621
15	0.2620	0.2616	0.2620
16	0.2619	0.2617	0.2619
17	0.2619	0.2617	0.2619
18	0.2619	0.2618	0.2619
19	0.2618	0.2618	0.2618
20	0.2618	0.2618	0.2618
21	0.2618	0.2618	0.2618
22	0.2618	0.2618	0.2618
23	0.2618	0.2618	0.2618
24	0.2618	0.2618	0.2618
25	0.2618	0.2618	0.2618

Here we skipped 10 iterations and observe that the value converges to a fixed point 0.2618 after 18 iterations.

Table 2
Orbit of F(z) for (z_0 = 0.83125 + 0.61875i) at α = 0.8, β = 0.6, c = 0.1

Number of iterations	*z*	|*Tz*|	|*Sz*|
11	0.2622	0.2610	0.2622
12	0.2621	0.2613	0.2621
13	0.2620	0.2615	0.2620
14	0.2619	0.2616	0.2619
15	0.2619	0.2617	0.2619
16	0.2618	0.2618	0.2618
17	0.2618	0.2618	0.2618
18	0.2618	0.2618	0.2618
19	0.2618	0.2618	0.2618
20	0.2618	0.2618	0.2618

Here we skipped 10 iterations and observe that the value converges to a fixed point 0.2618 after 15 iterations.

3.2. Fixed Points of Increasing Function

Table 3
Orbit of F(z) for (z_0= 1.21875 – 0.425i) at α = β = 0.5 and c = 0.1

Number of iterations	*z*	|*Tz*|	|*Sz*|
11	-0.9738	-1.9537	-1.9476
12	-0.9743	-1.9487	-1.9517
13	-0.9746	-1.9492	-1.9507
14	-0.9748	-1.9494	-1.9502
15	-0.9748	-1.9496	-1.9499
16	-0.9748	-1.9496	-1.9498
17	-0.9748	-1.9497	-1.9497
18	-0.9748	-1.9497	-1.9497
19	-0.9748	-1.9497	-1.9497
20	-0.9748	-1.9497	-1.9497

Here we skipped 10 iterations and observed that the value converges to fixed point -0.9748 after 16 iterations.

Table 4
Orbit of F(z) for (z_0= – 2.35 – 0.1*i*) at α = 0.8, β = 0.8 and *c* = 0.1

Number of iterations	*Z*	*|Tz|*	*|Sz|*
16	-0.9766	-1.9462	-1.9532
17	-0.9766	-1.9474	-1.9520
18	-0.9756	-1.9482	-1.9512
19	-0.9753	-1.9487	-1.9507
20	-0.9752	-1.9491	-1.9503
21	-0.9751	-1.9493	-1.9501
22	-0.9750	-1.9494	-1.9500
23	-0.9749	-1.9495	-1.9499
24	-0.9749	-1.9496	-1.9498
25	-0.9749	-1.9496	-1.9497
26	-0.9749	-1.9496	-1.9497
27	-0.9749	-1.9497	-1.9497
28	-0.9749	-1.9497	-1.9497
29	-0.9749	-1.9497	-1.9497
30	-0.9749	-1.9497	-1.9497

Here we skipped 15 iterations and observe that the value converges to a fixed point -0.9749 after 26 iterations.

3.3. Fixed Points Of Oscillating Function

Table 5
Orbit of F(*z*) for (*z*0 = 0.075 + 0.1125*i*)at α = β = 0.5, *c* = 0.1

Number of iterations	*z*	*|Tz|*	*|Sz|*
11	1.0483	1.0539	1.0483
12	1.0498	1.0525	1.0498
13	1.0506	1.0519	1.0506
14	1.0509	1.0515	1.0509
15	1.0511	1.0514	1.0511
16	1.0512	1.0513	1.0512
17	1.0512	1.0513	1.0512
18	1.0512	1.0513	1.0512
19	1.0512	1.0513	1.0512
20	1.0512	1.0513	1.0512
21	1.0512	1.0513	1.0512
22	1.0512	1.0512	1.0512

Here we skipped 10 iterations and observed that the value converges to fixed point 1.0512 after 21 iterations.

Table 6
Orbit of F(z) for ($z0 = 0.0625 – 0.375i$) at α = β = 0.8 and c = 0.1

Number of iterations	*z*	*|Tz|*	*|Sz|*
21	1.0513	1.0512	1.0513
22	1.0513	1.0512	1.0513
23	1.0513	1.0512	1.0513
24	1.0513	1.0512	1.0513
25	1.0513	1.0512	1.0513
26	1.0513	1.0512	1.0513
27	1.0513	1.0512	1.0513
28	1.0513	1.0512	1.0513
29	1.0513	1.0512	1.0513
30	1.0512	1.0512	1.0512
31	1.0512	1.0512	1.0512
32	1.0512	1.0512	1.0512
33	1.0512	1.0512	1.0512
34	1.0512	1.0512	1.0512
35	1.0512	1.0512	1.0512

Here we skipped 20 iterations and observed that the value converges to fixed point 1.0512 after 29 iterations.

3.4. Fixed Points Of Biquadratic Function

Table 7
Orbit of F(z) for ($z_0 = 0.3375 + 0.18125i$) at α = β = 0.5, c = 0.1

Number of iterations	*z*	*|Tz|*	*|Sz|*
21	1.6624	86.4469	86.4466
22	1.6624	86.4469	86.4468
23	1.6624	86.4470	86.4469
24	1.6624	86.4470	86.4469
25	1.6624	86.4470	86.4470
26.	1.6624	86.4470	86.4470
27.	1.6624	86.4470	86.4470
28.	1.6624	86.4470	86.4470
29.	1.6624	86.4470	86.4470
30.	1.6624	86.4470	86.4470

Here we skipped 20 iterations and observed that the value converges to a fixed point 1.6624 after 24 iterations.

Table 8
Orbit of F(z) for (z_0 = 0.18125 – 0.1*i*) at α = 0.8, β = 0.8, *c* = 0.1

Number of Iterations	*z*	|*Tz*|	|*Sz*|
6	1.6614	86.4383	86.3935
7	1.6622	86.4449	86.4340
8	1.6624	86.4465	86.4438
9	1.6624	86.4468	86.4462
10	1.6624	86.4469	86.4438
11	1.6624	86.4470	86.4462
12	1.6624	86.4470	86.4468
13	1.6624	86.4470	86.4469
14	1.6624	86.4470	86.4470
15	1.6624	86.4470	86.4470

Here we skipped 5 iterations and observed that the value converges to a fixed point 1.6624 after 13 iterations.

4. GENERATION OF RELATIVE SUPERIOR MANDELBROT SETS FOR DIFFERENT FUNCTIONS

We generated the Relative Superior Mandelbrot sets. We present here some beautiful filled Relative Superior Mandelbrot sets for quadratic, cubic and biquadratic function.

4.1. Relative Superior Mandelbrot Sets For Decreasing Function

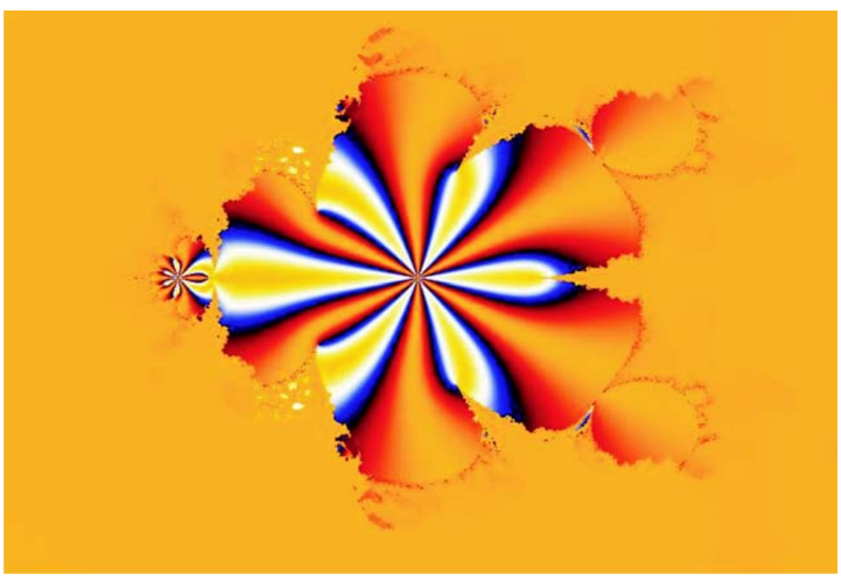

Figure 1: Relative Superior Mandelbrot Set for α = β = 0.5, *c* = 1.325 – 0.1375*i*

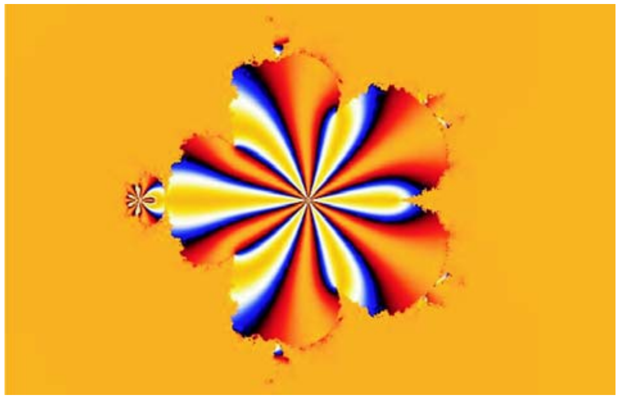

Figure 2: Relative Superior Mandelbrot Set for α = 0.8, β = 0.6, *c* = 0.83125 + 0.61875*i*

4.2. Relative Superior Mandelbrot Sets For Increasing Function

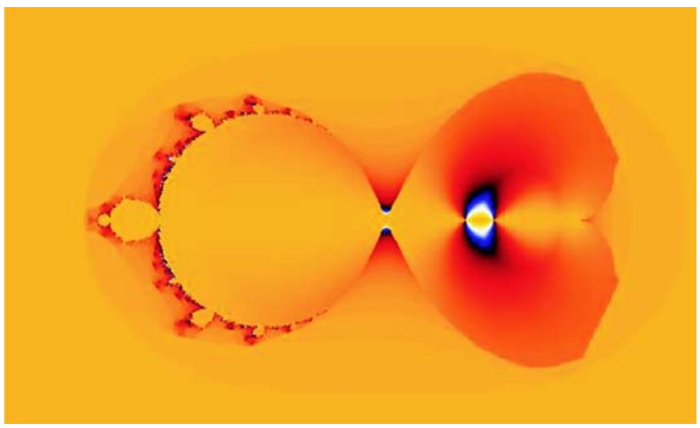

Figure 3: Relative Superior Mandelbrot Set for α = β = 0.5 and *c* = 1.21875 – 0.425

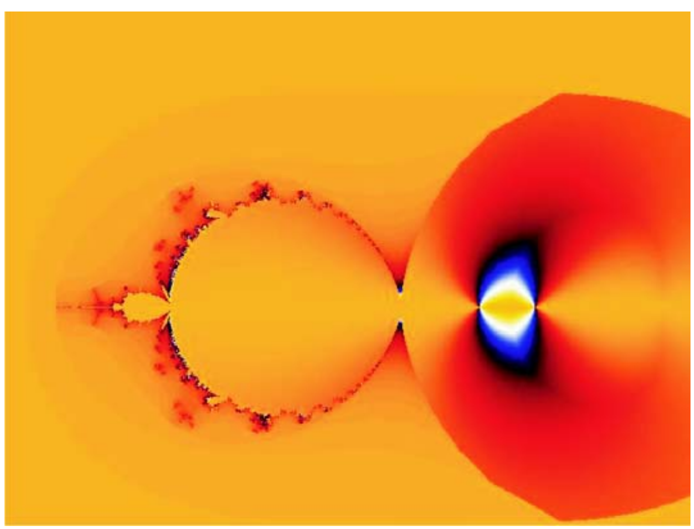

Figure 4: Relative Superior Mandelbrot Set for α = β = 0.8 , *c* = –2.35 – 0.1*i*

4.3. Relative Superior Mandelbrot Sets For Oscillating Function

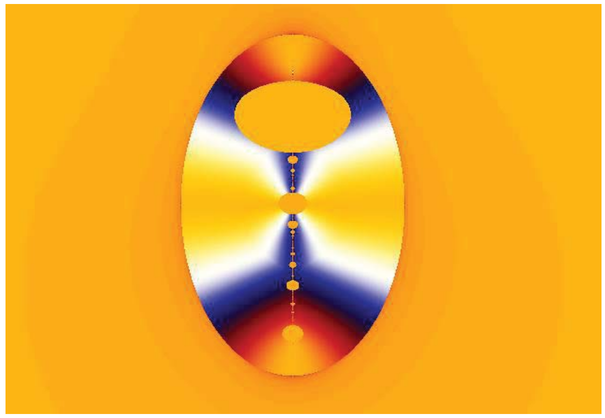

Figure 5: Relative Superior Mandelbrot Set for α = β = 0.5, *c* = 0.075 + 0.1125*i*

Figure 6 : Relative Superior Mandelbrot Set for α = β = 0.8, *c* = 0.0625 – 0.375*i*

4.4. Relative Superior Mandelbrot Sets For Biquadratic Function

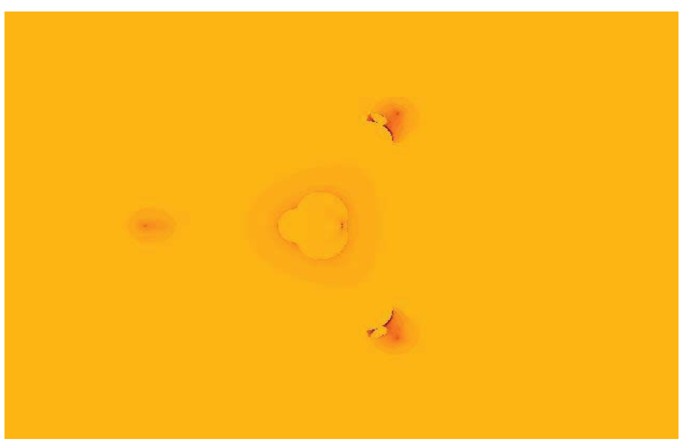

Figure 7: Relative Superior Mandelbrot Set for α = β = 0.5, *c* = 0.3375 + 0.18125*i*

Figure 8: Relative Superior Mandelbrot Set for α = β = 0.8, *c* = 0.18125 – 0.1*i*

5. GENERATION OF RELATIVE SUPERIOR JULIA SETS FOR DIFFERENT FUNCTIONS

We generated the Relative Superior Julia sets. We have presented here some beautiful filled Relative Superior Julia sets for quadratic, cubic and biquadratic function.

5.1. Relative Superior Julia Sets For Decreasing Function

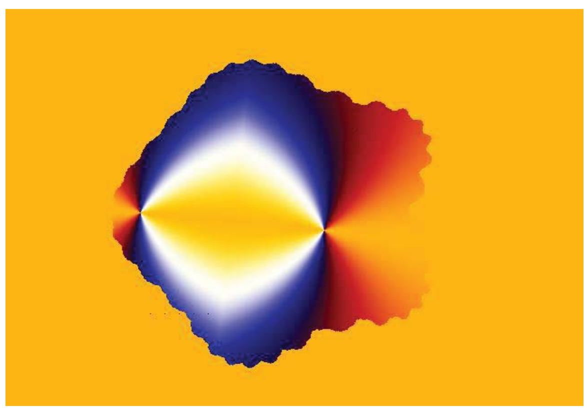

Figure 9: Relative Superior Julia Set for α = β = 0.5, *c* = 1.325 – 0.1375*i*

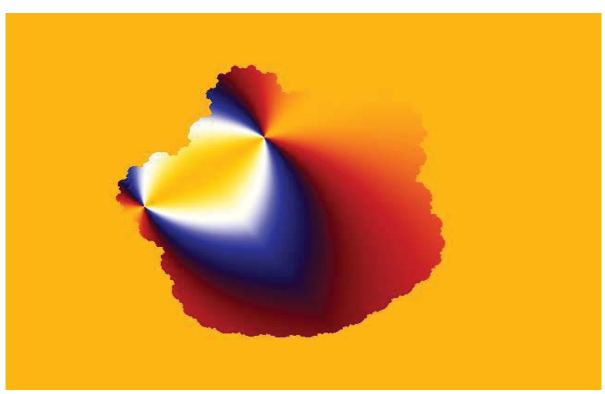

Figure 10: Relative Superior Julia Set for α = 0.8, β = 0.6, *c* = 0.83125 + 0.61875*i*

5.2. Relative Superior Julia Sets For Increasing Function

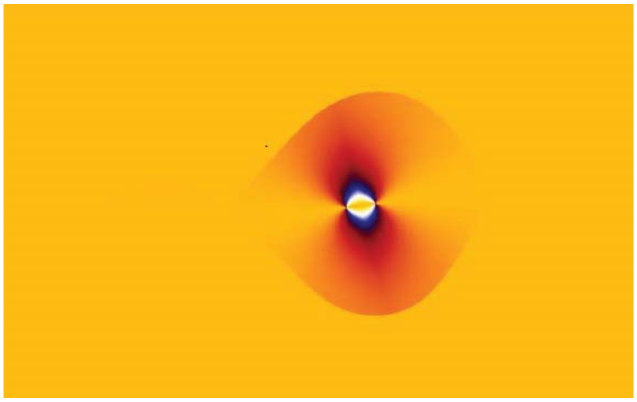

Figure 11: Relative Superior Julia Set for α = β = 0.5 and *c* = 1.21875 – 0.425*i*

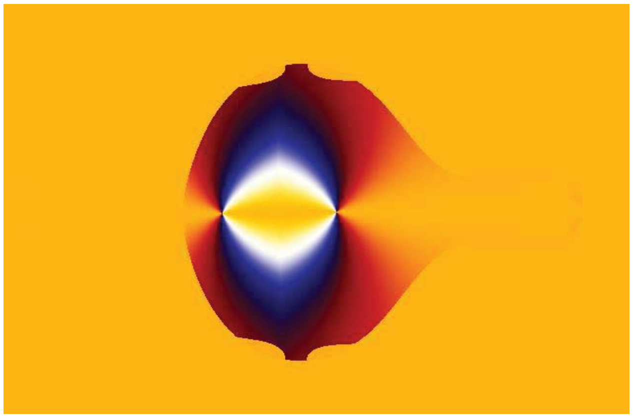

Figure 12: Relative Superior Julia Set for α = β = 0.8 and *c* = –2.35 – 0.1*i*

5.3. Relative Superior Julia Sets For Oscillating Function

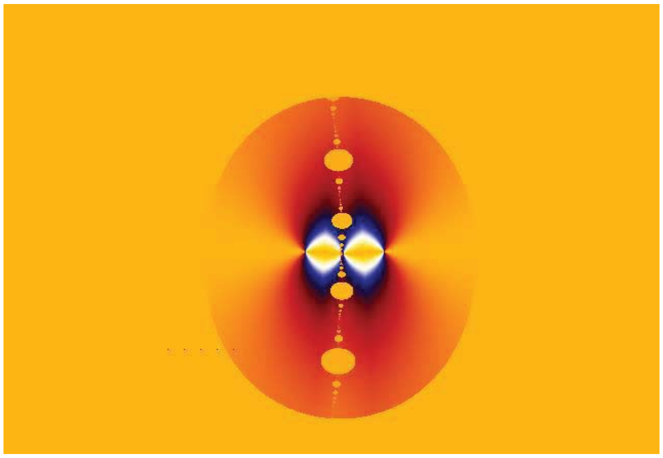

Figure 13: Relative Superior Julia Set for α= β = 0.5, *c* = 0.075 + 0.1125*i*

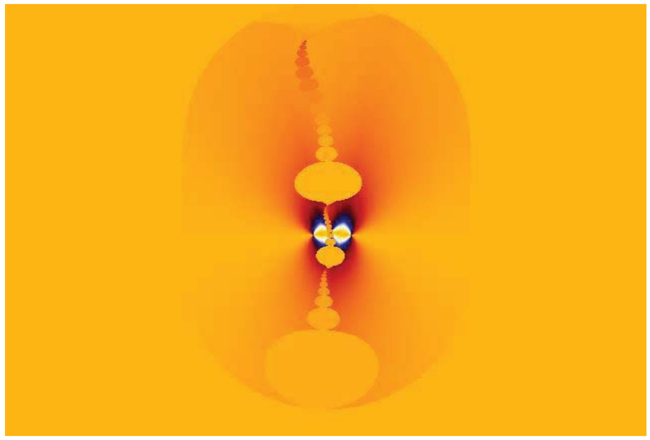

Figure 14: Relative Superior Julia Set for α = β = 0.8, c = 0.0625 – 0.375i

5.4. Relative Superior Julia Sets For Biquadratic Function

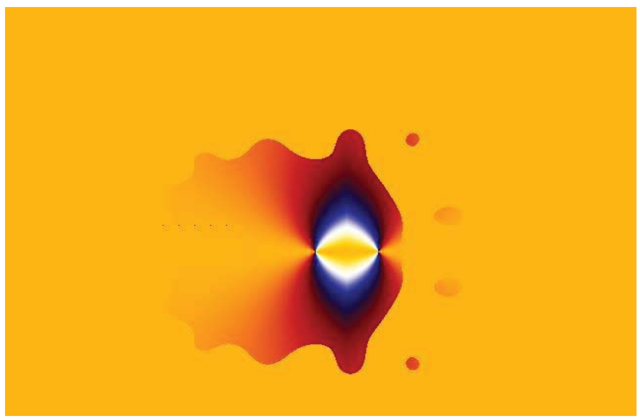

Figure 15: Relative Superior Julia Set for α = β = 0.5, c = 0.3375 + 0.18125i

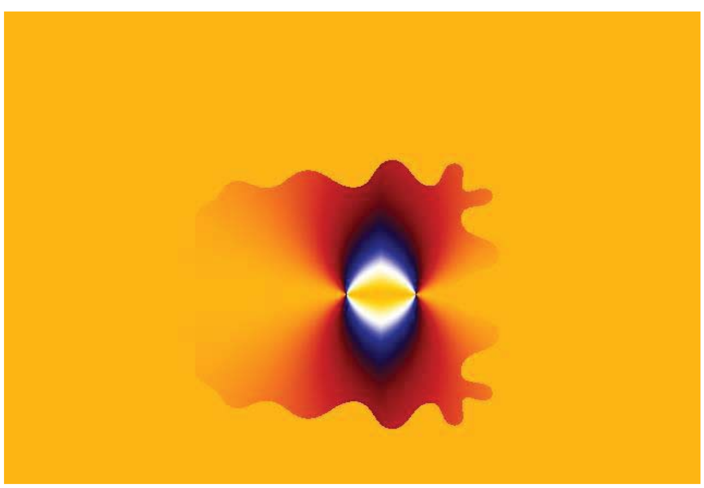

Figure 16 : Relative Superior Julia Set for α = β = 0.8, c = 0.18125 – 0.1i

6. CONCLUSION

In this paper we have given an algorithm to find fixed point of any functions having complex values using Jungck Ishikawa Iteration .We have proved theorem for Jungck Ishikawa Iteration which were till now not proved for Jungck Ishikawa Iteration .We have compared the stability results of decreasing, increasing, oscillating and quadratic functions by taking complex values and found that we are getting the stability point of oscillating function later as compared to other functions and as we are increasing the value of alpha and beta we are getting fixed point later as compared to smaller values .Relative Superior Mandelbrot sets and Relative Superior Julia Sets for different functions appear like beautiful images which follow some symmetry in different functions.

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