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Analysis of speed control of DC motor using LQR method

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ABSTRACT

DC motors are widely used in various industries because they cater to applications of high and low power and in both fixed and variable speed electric drives. This paper analyses the transient parameters of the speed of separately excited DC motors using the Linear Quadratic Regulator (LQR) method. The speed of the DC motor is analyzed by measuring the output. The LQR method has been applied on three different motors of different specifications. The speed of the three motors is compared on the basis of the transient specifications using the LQR method. Simulation of a motor in MATLAB has shown that the performance of a motor with a low power application has a significant and improved quality response with the LQR method. Properties such as minimized deviation in speed, easy design and cost reduction make it efficient for use in controlled and balanced systems. These properties enhance speed control of a DC motor by achieving the desired performance for the systems where different power ratings occur.

Keywords: Optimal control, Linear Quadratic regulator, transient response

1. INTRODUCTION

There are various uses of direct current machines in the industry. DC motors are used in both high and low power applications as well as fixed and variable speed electric drives. For example their applications range from low power toys, spinning and weaving machines, vacuum cleaners, elevators, electric traction and so on [1]. The speed of a DC motor can be adjusted easily, by changing voltage and current depending on the type of the DC motor used. This ability to control the speed ensures good performance of the system [4]-[6]. In this paper we use three different types of DC motors. The supply given to the system is a unit step response. We first analyze the speed of the DC Motor by its output. In order to obtain optimal results for the speed we use an LQR controller. The parameters for the values of the variables Q and R are set by trial and error method to obtain improved results. Hence this paper aims at analyzing the LQR controller effect on different power rated motors for obtaining ideal output results of a DC Motor. Here a comparison of the output response of the motors has been made on the basis of transient parameters such as rise time, maximum overshoot and settling time. Feedback, need for fluctuations and optimization are considered to be the concepts of modern control theory. This method has been used in many applications as mentioned in the previous paragraph. LQR controller method holds properties such as robustness, reliability, generation of static gain, etc. Using this optimal control method in large systems with multiple inputs, multiple outputs efficient control is attained reliably and economically. Considerable attention has been gained by this Linear Quadratic Regulator method and is used often.

The linear quadratic regulator technique is used to design an optimal con-troller that minimizes a given cost function, the performance index [2]. Among the many properties that makes DC motor preferred to other motors, the simplicity, reliability and minimal cost makes it appealing. For the various applications, desired

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performance of the motor is attained by the precise control of motor [3]. In recent years many advanced improvements in controlling the systems have been developed to meet the requirement of users in the field of robotics, process control and many other domains. Optimal state space control of DC motor by the designing of Linear Quadratic Regulator has been proposed, by including feed-forward friction compensation in the feedback controller. The augmentation of feed forward compensation in the controller has considerably reduced the error across the array of reference speed [8]. The determination of the optimal speed control of DC motors using the design methods such as LQR method and Genetic algorithm proposes a comparison to find the guaranteed response to satisfy its specification for maximum overshoot and accommodation time. This is done by taking into consideration the difficulties that can occur with the control designing process of defining weight matrices in the LQR method [9]. The comparative analysis of the performance and simulation of the separately excited DC motors speed control has been studied by using a Fractional order Proportional- Integral-Derivatives (FOPID) controller and a Linear Quadratic Regulator (LQR). This analysis has shown superior results by implementing the LQR method based on settling time, steady state error and overshoot [10]. The determination for a better control strategy for controlling the speed of DC mo-tor using PID controller technique and LQR method presented better transient parameters with the LQR method, verified by simulation [11]. Because the robustness of LQR method provides a more accurate dynamic response in terms of transient parameters it is preferred to traditional PID controllers [12]. The system response by LQR method displays better performance with flexibility in high range and control when compared to Ziegler-Nichols PID-MZN controller and PIPSO controller [13].

In the past years many advanced improvements in controlling systems have been developed to meet the requirement of users in robotics, process control, etc. domains. For example because of the improved system stability, effective control and balancing properties, the LQR method is suitable for robotic applications such as two wheeled self balancing robots [14]. The linear quadratic regulator technique is an algorithm used to design an optimal controller for the minimization of a cost function given. The performance index is parameterized by two matrices, Q and R. These matrices weight the state vector and system input respectively. These matrices are used for regulating the penalties on changing the state variable and the control signal. The cost function contributes to the values of element used in the matrices such as Q and R. Algebraic Riccati Equation (ARE) is solved initially to find the control law. The defined cost function is solved to obtain an optimal feedback gain matrix, this will result in optimal result evaluation [9][10]. The paper studies the speed control of DC motors with different power ratings and the amount of power required by implementing the Linear Quadratic Regulator (LQR) technique. The rest of the paper has been organized as follows: First, in section 2, the description of system model and Linear Quadratic Regulator - the optimal controller is explained. Then in section 3, the Implementation is described. The results and discussion section are in Section 4. Conclusion and future scope have been laid out in the last section

2. SYSTEM MODEL

2.1. DC Motor Mathematical Model

A separately excited DC motor is used in this work. DC motor's adaptability to adjustable speed drives makes it preferred to ac motors which are associated with a constant speed rotating fields. Since DC motors can be adjusted over a wide range of operating speed, a variety of methods are used. Armature voltage control method is used here. In this method the armature current is controlled by armature voltage Va, keeping the field current (if) constant.

The linear model of a simple DC motor consists of an electrical equation and a mechanical equation. Applying Kirchhoff's Voltage Law (KVL) and Newton's second law, equation (1) and (2) is obtained.

$$\frac{di_a}{dt} = -\frac{R_a}{L_a} i_a - \frac{K_b}{L_a} i_\omega + \frac{V_a}{L_a} i_a \tag{1}$$

$$\frac{d\omega}{dt} = -\frac{K_T}{J} i_a - \frac{B_m}{J} i_\omega \quad (2)$$

The steady state representation is obtained from the above equation

$$\begin{bmatrix} \frac{di_a}{dt} \\ \frac{d\omega}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-R_a}{L_a} & \frac{-K_b}{L_a} \\ \frac{K_T}{J} & \frac{-B_m}{J} \end{bmatrix} \begin{bmatrix} i_a \\ \omega \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} \\ 0 \end{bmatrix} [V_a] \quad (3)$$

$$y = [0\ 1] \begin{bmatrix} i_a \\ \omega \end{bmatrix} \quad (4)$$

Using the state space model transfer function of the motor has been obtained using the formula

$$G(s) = C(sI - A)^{-1} B + D$$

This formula has been used in equation (3), thereby equation (4) is obtained. The armature controlled transfer function of the DC motor can be written as follows

$$G(s) = \frac{w(s)}{E_a(s)} = \frac{K_T}{(L_a s + R_a)(Js + B) + K_b K_T} \quad (5)$$

Linear Quadratic Regulator Linear Quadratic Regulator (LQR) is a method used in modern control theory. The analysis of such a system is done using the state-space approach. Owing to the simplistic approach of the state space method, multi-input multi-output system used this method. Wide applications make use of linear quadratic regulator design technique. Fig. 2 represents the block diagram of DC motor control system with the LQR Controller. The speed of motor is subjected to change on exposure to change in load and other external disturbances. The reduction of minimization of deviation in the speed of motor is the function of LQR. The output of the system, speed of the motor is compared with the input applied to the system that is the voltage of the motor. Reduction of deviation of the motor speed is performed by the LQR. The state space equation for the system is written in general as

$$X^* = AX + Bu \quad (6)$$

A represents the state matrix of order n*n, B the control matrix of order n m respectively. n is the number of state variable and m is the number of input variable. The state matrix and control matrix pairs are assumed

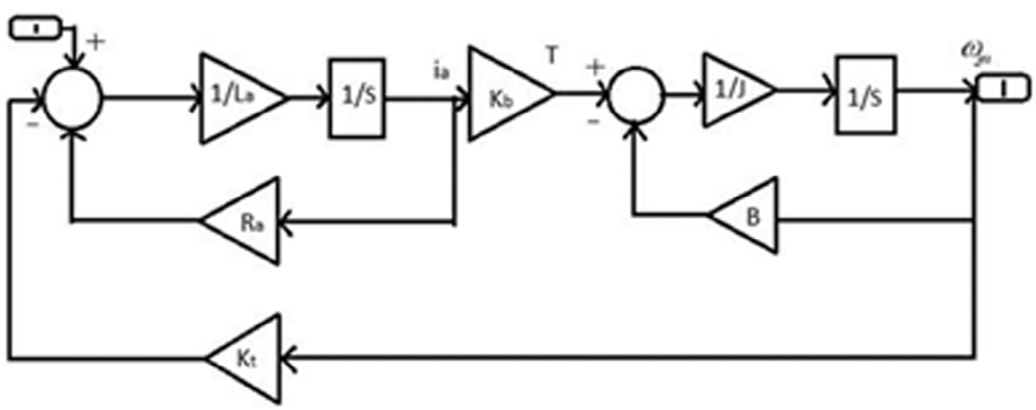

Figure 1: Simulink Model of armature controlled DC motor

such that the system is controllable. The linear quadratic regulator controller is designed such that the performance index is reduced to a minimized value. This is meant for achieving the desired performance of the system. The performance index (J) for a linear quadratic regulator controller is designed as

$$J = \int \left(x^T Q_x + u^T R_u \right) dt \tag{7}$$

Equation (7) represents the quadratic performance index. This is represented as a weighted integral. The square of errors of the states and inputs contribute the weighted integral. The symmetric positive semi-definite state weighting and control weighting matrices are Q and R respectively. The relative weighting of individual state variables and control inputs are controlled by the element Q and R chosen. This also affects the relative weighting of state vector and control vector against each other. In LQR optimization process weighting matrices Q and R are important components. The freedom of selection of the matrices Q and R is up to the designer, since these matrices have great influence on the system behavior. An iterative procedure is applied for the selection of matrices Q and R. The understanding of problems experienced with concerned system is the basis of this procedure. Trial and error methods have been used commonly to determine the Q and R matrices. This simple method makes it easy to apply in linear quadratic regulator application. Time consumption is high as this depends on the selection of choosing values for Q and R for obtaining the optimum results. The numbers of state and control matrix elements are equivalent to the state variable and input variable elements respectively.

For simplicity the matrix values of diagonal-o elements are taken as zero. Ir-respective of the size of Q and R matrices, performance index will always be a scalar quantity in case the diagonal matrix is selected. The conventional lin-ear quadratic regulator finds the optimal control input law u^*. The constraints caused by the matrices Q and R minimize the performance index. The closed loop of dc motor control system with linear quadratic regulator is shown in the Fig.4. The closed loop optimal control law is defined as:

$$u^* = -Kx \tag{8}$$

Where K represents the optimal feedback gain matrix. The gain matrix works to minimize the performance index in equation (8). It determines the proper placement of closed loop poles for the minimization. The feedback gain matrix K is dependent on A, B, Q and R matrices. The feedback gain matrix k is obtained by solving the Algebraic Riccati Equation (ARE), P defines a symmetric and positive definite matrix which is obtained by solution of the ARE is defined as:

$$A^T P + PA - PBR^{-1} B^T P + Q = 0 \tag{9}$$

$$K = AX - BKx = \left(A - BK \right) x \tag{10}$$

Substituting the equation (9) in equation (7) gives:

$$x = AX - BKx = \left(A - BK \right) x \tag{11}$$

The presence of negative real parts in Eigen values of the matrix (A-BK), represents the existence of positive definite solution.

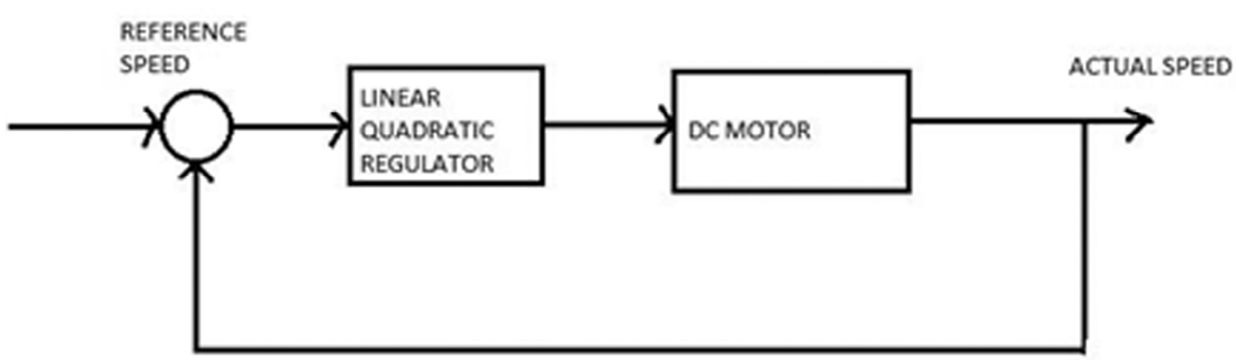

Figure 2: Block diagram of DC motor control system using LQR optimal Controller

3. IMPLEMENTATION

The LQR method has been implemented in the system model developed in MATLAB. The performance and tuning using the LQR Optimal Control has been compared with three different power rated DC motors. Simulations are done using MATLAB 8.2 Fig. 1 shows the simulink model of DC motor using armature control method. The specifications of the three DC motors are given in Table 1. By using the electrical equation and the mechanical equation (1) and (2) respectively, the armature current and the angular velocity is measured which describes the DC motor system. The specifications presented in Table 1 gives the different parameters of the separately excited DC motor which are used to find the matrices A, B, C and D. The equations (3) and (4) are used for this purpose. The three motors with different parameter is simulated for the unit step response, as per the parameters shown in table 1. The Transfer function of the three motors is given by the following equations.

$$G_1(s) = \frac{0:015}{0:01s^2 + 0:014s + 0:40015} \tag{12}$$

$$G_2(s) = \frac{0:2}{0:1s^2 + 2:54s + 0:44} \tag{13}$$

$$G_3(s) = \frac{0:023}{0:005s^2 + 0:010015s + 0:000559} \tag{14}$$

Test Case Motor 1: The test system dealing with motor 1, the data of DC motor is shown in table 1. The transfer functions of DC motor equations 5 and 12 are used to find out the response of the system while applying the step function as an input. The tuning of different points such as the LQR parameter Q and R is

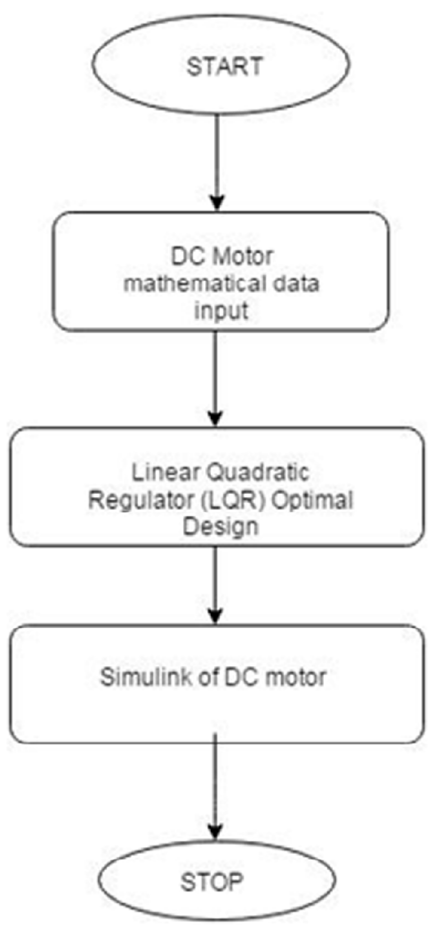

Figure 3: Algorithm for LQR method

done with various values. The matrices A, B, C and D obtained from the specification of DC motor are A = [-4 -0.02 ; 0.5 -10] B = [4 ; 0] C = [0 1] D = [0] respectively. The Q and R values, R = 0.000000009 Q = [0.000000009; 15] are used in motor 1, giving a dynamic response.

Test Case Motor 2: This test case system is similar to Motor 1. In this test system the data of DC motor is also shown in table 1. The transfer functions of DC motor equations 5 and 13 are used to find out the response of the system while applying the step function as an input. The tuning of different points such as the LQR parameter Q and R are done with various values. The matrices A,B,C and D obtained from the specification of DC motor are A = [-4 -0.4 ; 0.1666 -0.1666] B = [4 ; 0] C = [0 1] D = [0] respectively. The Q and R values, R = 0.000000009 Q = [0.000000009; 15] are used in motor 2, giving a dynamic response.

Test Case Motor 3: The test done on motor 3 is similar to those of test cases motor 1 and 2. Using the transfer function equations of DC motor 5 and 14, the response of the system is determined. The matrices A,B,C and D, obtained from the specification of DC motor, are A = [-2 -0.046 ; 2.3 -0.003] B = [2 ; 0] C = [0 1] D = [0] respectively. The Q and R values, R = 0.000000009 Q = [0.000000009; 15] are used in motor 3, giving the desired response.

Table 1
DC motor parameters

DC Motor Parameters	Motor 1	Motor 2	Motor 3
Armature Resistance Ra (Ohm)	2	2	1
Armature Inductance La (H)	0.5	0.5	0.5
Moment of inertia J (Kgm2)	0.02	1.2	0.01
Friction constant B (Nms)	0.2	0.2	0.00003
Torque constant KT (Nm/A)	0.015	0.2	0.023
EMF constant Kb (Vs/rad)	0.01	0.2	0.023

4. RESULTS AND DISCUSSIONS

The performance and tuning using the Linear-Quadratic Regulator (LQR) con-troller Optimal Control has been compared with three different power rated DC motors. Simulations were carried out using MATLAB

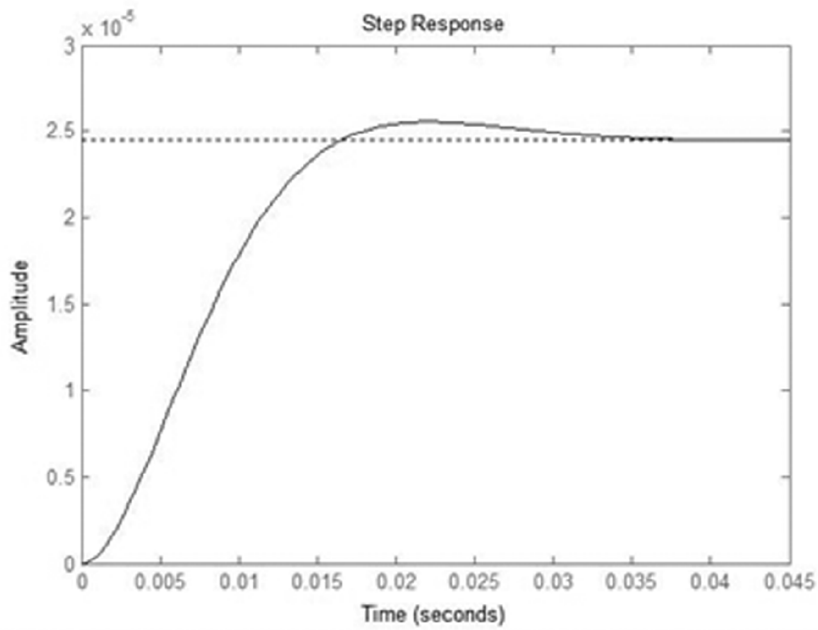

Figure 4: Step response of motor 1 with LQR controller

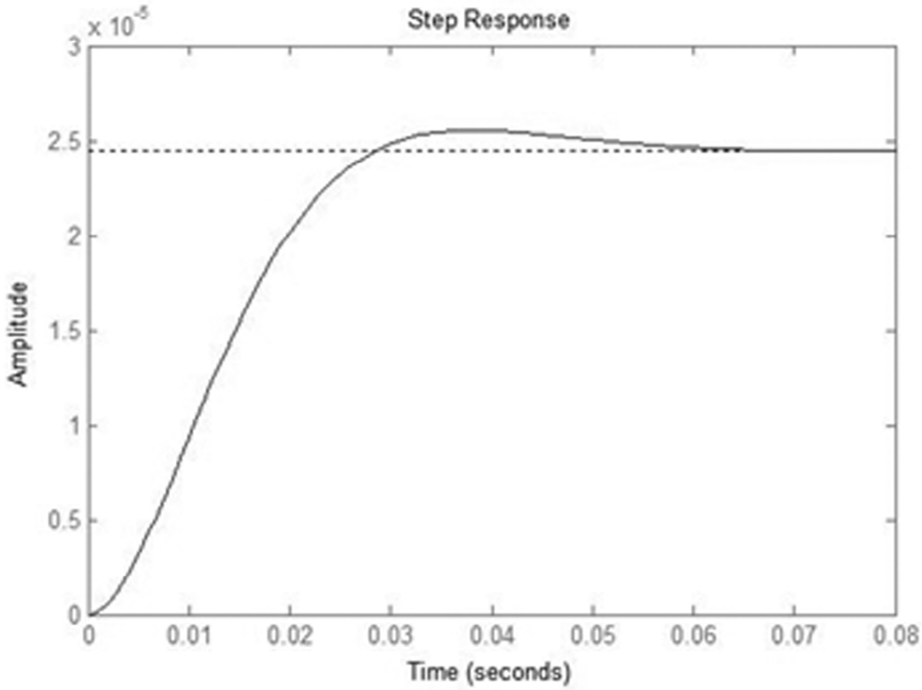

Figure 5: Step response of motor 2 with LQR controller

Table 2
Transient response of motor 1,motor 2 and motor 3

Motor	*Rise time (s)*	*Max. Overshoot (percent)*	*Settling time (s)*
Motor 1	0.016	4.30	0.0295
Motor 2	0.0184	4.31	0.0511
Motor 3	0.00496	4.32	0.0138

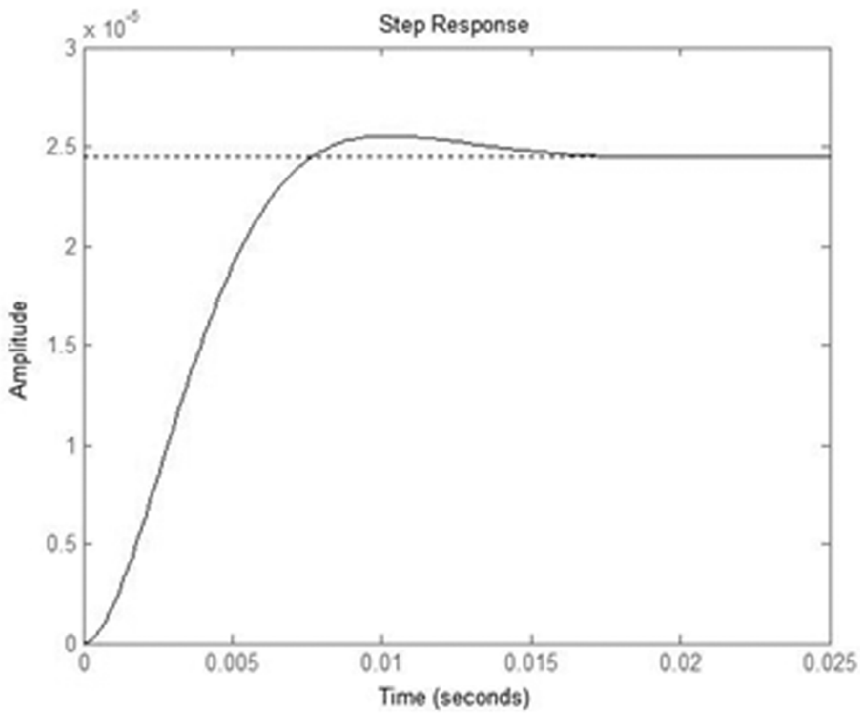

Figure 6: Step response of motor 3 with LQR controller

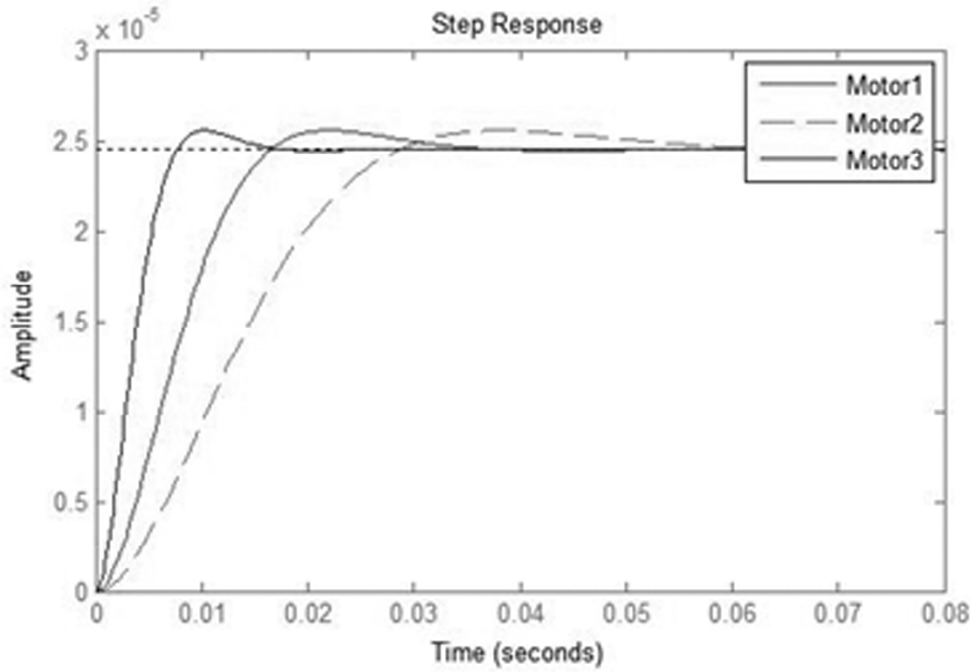

Figure 7: Step response of motor 1, motor 2 and motor 3

8.2. The step response of motor 1, motor 2, and motor 3 is shown in Fig. 4, 5, 6 respectively. The results based on transient response have been shown in table 2.

5. CONCLUSION AND FUTURE SCOPE

In this paper, a comparative study of transient response of DC motors of di fferent power rating has been done using LQR optimal controller. The method has been implemented on the three motors in MATLAB software. The simulation results prove that the LQR controller offers an improved dynamic performance for low power application DC motor i.e. motor 3 in terms of rise time and settling time. Based on the results, this can lead to significant improvement in the effect of jitter on the closed loop response and the transportation delay on applications like cruise control, process control etc. More research on the selection of Q and R in the LQR method could prove to be significant in the understanding of a generic system transient response.

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