INTERNATIONAL JOURNAL OF

ELECTRONICS ENGINEERING

ISSN : 0973-7383

Volume 11 • Number 1 • 2019

An Analysis of Spatial Temporal Data

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Abstract: Many dynamic spatial data in regional or transportation analysis contains network features; the spatial weights which reflect the strength of the interaction characterize the corresponding network structure. The degree of interaction for each pair of nodes and corresponding spatial weight could be varied by time and location. This paper studies a general model with spatial temporal dependence and interaction. We propose a general spatial temporal model describing the features of spatial and time series data, and generate the model based on network features. This model is modified to include spatial autoregressive, spatial lag of independent variable, spatial error, temporal autoregressive and moving average. The spatial diffusion and interactive effects in network system is applied to derive time varying weights and coefficients.

Keywords: Weighted network, Spatial temporal dependence

1. INTRODUCTION

There is a common statistical characteristic for spatio-temporal data that nearby (space and time) units tend to be more interacted than those far apart described as spatial and temporal dependence. The spatial dependence characteristic is captured by the spatial weight matrix modified in spatial analysis, which allows for endogenous interaction effect, interaction effects among the error terms and exogenous interaction effects. In the regional analysis, spatial spillover is commonly described by a spatial weights matrix. The form of spatial weights is essential in the estimation of spatial dependence models. Most spatial research replies on the assumption the spatial weight matrix is exogenous. Getis and Aldstadt (2010) construct a spatial weights matrix based on the local statistics model which depends on the designation of distance. The simulation experiments show the flexibility of the model compared to the rigidity of the global models.

Many researches assume the spatial interaction to be endogenously determined. Qu and Lee (2015) present the SAR model with an endogenous spatial weight matrix. They establish the consistency and asymptotic normality of estimators by three estimation methods: two-stage instrumental variable method, quasi-maximum likelihood estimation approach, and generalized method of moments. Koroglu and Sun (2016) considers a spatial Durbin model with nonparametric spatial weights to estimate the unknown spatial weighting functions via a nonparametric two-stage least squares method. Sun, Y. (2016) applies local linear regression to estimate spatial weighting function by a nonparametric GMM estimation method. A consistency result is derived to support the method.

Many spatial data interact not only spatially but also temporally. The temporal dependence characteristic is commonly presented by time series analysis such as autoregressive and moving average models. The space-time autoregressive moving average and its various extensions have been proposed for describing spatio-temporal processes. Pace et al. (1998, 2000) analyze the spatial as well as the temporal dependence of the data by the spatio-temporal auto regression model. The interaction effects are modeled by linearly combining both spatial and temporal dependence in the weights matrix. Smith and Wu (2009) proposed a spatial model with temporally auto-correlated residuals. The weights matrix is a Hadamard product between spatial and temporal distance. Cheng et al. (2014) describe autocorrelation in network data with a dynamic spatial weight matrix. The result shows that the performance of estimation and prediction is improved compared with standard models that are widely used for space–time modeling. Thanos et al. (2015, 2016) put the temporal dimension into spatial Hedonic models through a spatio-temporal data analysis. They found that ignoring the temporal dimension leads to the underestimation of environmental disamenities, possibly due to overestimation of spatial dependence. The parameters which specificity the spatial and temporal correlations in these model are mostly assumed to be fixed globally, which is inadequate to describe data with dynamic and heterogeneous dependence and exogenous dependence.

Besides considering the temporal and spatial interactions in spatial analysis, many spatial data such as regional or transportation data could be characterized by a dynamic network structure. The feature of the underlie network effect could be applied into the spatial analysis. LeSage, P. and Llano (2016) introduce a Bayesian hierarchical regression model to estimate spatial interaction relations involving origin-destination flows. Land and Deane (1992) estimate linear models with spatial-or social network-effects. They did not consider temporal and spatial network structure. In a traditional network system, the degree of each node is the sum of the links to all other nodes. Each link weights the same. However, the connections among nodes in many real-world spatial networks are not merely binary choice, but rather have different strength. In region or transportation networks, the degree of flows among nodes is different. The spatial weights reflect the strength of the interaction or dependence of pairs of nodes, it characterizes the corresponding network. The degree of interaction for each pair of nodes could be time and location various. The form and the feature of the spatial weights matrix affect the generating process of the network and consequently will affect the resulting size distribution of the region.

The dynamic spatial network system consists of spatio-temporal data; there is lack of research analyzing the spatio-temporal data base on the network structure. The propose of this work is to modify a spatio-temporal model based on a dynamic spatial network system, to investigate appropriate parameterization and the role of each parameter in the model, to develop the corresponding hypothesis testing for spatial or temporal dependences and network effects, to examine the form and estimation method of the corresponding spatial weight matrix, and to empirically examine the feature of the model by calibrating the parameter and simulating the model.

2. THE SPATIAL MODEL

This work plan to theoretically analyze the general spatial model, spatial panel data and time series data model to generate a spatial-temporal model based on the weighted network structure;

A general spatial model

$$Y = \rho WY + \alpha i_N + X\beta + WX\theta + \varepsilon, \qquad \varepsilon = \lambda W\varepsilon + u$$

where *Y* represents an $N \times 1$ vector of the dependent variable, *X* denotes an $N \times K$ matrix of explanatory variables associated with the $K \times 1$ parameter vector β, and ε is a vector of independently and identically distributed disturbance terms with zero mean and variance σ^2. The spatial weights matrix *W* is a positive $N \times N$ matrix that describes the structure of dependence between units in the sample. i_N is an $N \times 1$ vector of ones. The variable *WY* denotes the endogenous interaction effects among the dependent variables; the model becomes autoregressive model with this term. The variable *WX* is the exogenous interaction effects among the explanatory variables. The variable *Wu* denotes the interaction effects among the disturbance terms; the model becomes spatial error model with this term. The scalar parameters ρ and λ measure the strength of dependence between units. The model becomes ordinary least squares model when the parameters ρ , λ and θ equal zero. The reduced form is the follows:

$$Y = (I - \rho W)^{-1} \alpha i_N + (I - \rho W)^{-1}(X\beta + WX\theta) + (I - \rho W)^{-1}\varepsilon$$

$$\left[\frac{\partial E(Y)}{\partial x_{1k}} \cdots \frac{\partial E(Y)}{\partial x_{Nk}} \right] = (I - \rho W)^{-1}[I\beta_k + W\theta_k]$$

The direct effect of the explainable variable is denoted by the diagonal elements of the matrix; the spillover effects are represented by the off diagonal elements of the matrix[1]. In simple case of geographical units, the spatial weights matrix *W* could define elements $w_{ij} = 1$ if two unites share a common border and zero otherwise. Generally, closer unites has more weight in the matrix. In regional science, the spatial spillover effect presenting the spatial interaction or spatial influence between unites is a main interest. The strength of the spatial interaction is denoted by the spatial weights matrix *W*; each element in the matrix represents the degree of interaction between each pairs of units which could be measured by functions of spatial or temporal distances.

The spatial weight matrix:

A spatial weight matrix summarizes spatial relations or influence between pairs of spatial unites, which is conventionally nonnegative and excludes self-influence by assuming all diagonal elements of *W* are zero. There are various forms of spatial weight matrices used in practice, mostly based on distance. Neumayer and Plumper (2013) uses parametric approach to define the spatial weight matrix as negative power function of distance:

$w_{ij} = d_{ij}^{-\gamma}$, where γ denotes the distance decay parameter.

The negative exponential function of distance is another alternative:

$w_{ij} = e^{-\alpha d_{ij}}$, where α is any positive exponent.

If the weights are normalized to have unit sum in each row, the spatial weights are row-normalized by the power distance. The normalized weight defines the fraction of all spatial influence of the corresponding pair of units.

$$\varpi_{ij} = \frac{d_{ij}^{-\gamma}}{\sum_{i \neq k} d_{ik}^{-\gamma}}$$

Another alternative of normalized weights are row-normalized by the exponential distance:

$$\varpi_{ij} = \frac{\exp(-\gamma d_{ij})}{\sum_{i \neq k} \exp(-\gamma d_{ik})}$$

Thanos et al (2016) combine temporal and spatial approach to Hedonic pricing model to analyze the housing market. The spatio-temporal distance weight is the product of a function of spatial distance d_{ij} and a function of the temporal distance τ_{ij} :

$$w_{ij} = s_{ij} f(\tau_{ij}) = f(d_{ij}) f(\tau_{ij}),$$

where s_{ij} represents the degree of spatial connection which is a function of the spatial distance; and τ_{ij}

represents the degree of temporal connection. Their approach considers past, current and future independent variables. This model does not contain the feature of time series data, and it is designed for testing specific effects. It is not a general modification.

The Spatial Panel Data Models:

The spatial panel model describes spatial specific effects and spatial interaction effects between units; the model may contain a spatially lagged dependent variable or a spatially lagged independent variable known as the spatial lag model; or the model may contain a spatial autoregressive process in the error term known as the spatial error model. When the spatial interaction effect occurs in the independent variable that they are correlated across space, the model is the spatial lag model:

$$y_{it} = \delta \sum_j w_{ij} y_{jt} + x_{it}\beta + \mu_i + \varepsilon_{it},$$

where *i* is an index for the spatial units, and *t* is an index for time periods. δ denotes the spatial autoregressive coefficient and w_{it} is an element of a spatial weights matrix ***W***.[2] μ_i is a spatial effect which captures all space-specific time-invariant variables.

The spatial weights matrix is assumed to be a non-negative matrix of order *N*. The value of the dependent variable of certain unit is jointly determined by that of the neighboring units.[3] When the spatial interaction effect is in error term that they are correlated across space, the model becomes the spatial error model:

$$y_{it} = x_{it}\beta + \mu_i + \phi_{it},$$

$$\phi_{it} = \rho \sum_j w_{ij}\phi_{jt} + \varepsilon_{it},$$

ε_{it} is an independently and identically distributed error term with zero mean and variance σ^2, μ_i denotes a spatial specific effect. When the spatial interaction effects exist in dependent variable, independent variable and the error term that these variables may be correlated across space correspondingly, the model become a general spatial panel model:

$$y_{it} = \delta \sum_j w_{ij} y_{jt} + x_{it}\beta + \alpha \sum_j w_{ij} x_{jt} + \mu_i + \phi_{it},$$

$$\phi_{it} = \rho \sum_j w_{ij}\phi_{jt} + \varepsilon_{it},$$

The general model includes a spatially lagged independent variable, a spatially lagged dependent variable and a spatially autocorrelated error term. Where *i* is an index for the spatial units, and *t* is an index for time periods. δ is the spatial autoregressive coefficient describing the spatially dependence of the independent variable; and w_{it} is an element of a spatial weights matrix ***W***. α is the spatial autoregressive coefficient describing the spatially dependence of the dependent variable; ρ is the spatial autocorrelation coefficient describing the spatially dependence of the error term. μ_i is a spatial specific effect as fixed effect, which is a random variable with independently and identically distributed with zero mean and variance σ_μ^2 . ϕ_{it} denotes a spatially autocorrelated error term and ε_{it} is an independently and identically distributed error term with zero mean and variance σ^2, and it is assumed that the μ_i and ε_{it} are independent of each other.

The interaction between spatial unites are captured by the spatially lagged dependent variable and the spatially lagged independent variable (the spatial lag model) and a spatial autoregressive process in the error term (the spatial error model). The main approach to estimate the model is based on the maximum likelihood principle. Row normalization of the spatial weight matrix W made the impact on each unit by all other units sum to one, while column normalization made the impact of each unit on all other units sum to one. This model reflects time period of data; however, these are lack of the possible feature of time series data. Thanos et al. (2016) put the temporal dimension into spatial Hedonic models through a spatio-temporal data analysis. Their work considers the time period of the data and variables without taking into account the possible time serious feature: autoregressive and moving average. The general spatial panel model add the spatial interaction features into the panel model by adding the spatial weights matrix. These researches not take into account the time series features, they only indicate the time period in the data.

The autoregressive moving average time series model:

$$y_t = c + \sum_{i}^{p} \nu_i y_{t-i} + \varepsilon_t + \sum_{i}^{q} \theta_i \varepsilon_{t-i},$$

where c is a constant and t is an index for time periods. ε_{it} is an independently and identically distributed error term with zero mean and variance σ^2.

3. THE SPATIAL-TEMPORAL MODEL

Considering the time series model and the spatial model, we propose a general spatial-temporal autoregressive moving average model:

$$\begin{aligned} y_{i,t} &= \delta \sum_{j=1}^{N} w_{ij} y_{j,t} + \sum_{j=1}^{N} \sum_{k=1}^{p} f(w_{ij}) \nu_{jk} y_{j,t-k} + x_{i,t}\beta + \alpha \sum_{j=1}^{N} w_{ij} x_{j,t} + \mu_i + \phi_{i,t} \\ \phi_{i,t} &= \rho \sum_{j=1}^{N} w_{ij} \phi_{j,t} + \varepsilon_{i,t} + \sum_{j=1}^{N} \sum_{k=1}^{q} f(w_{ij}) \theta_{jk} \phi_{j,t-k} \end{aligned} \tag{1}$$

where δ denotes the spatial autoregressive coefficient; ρ is the spatial autoregressive coefficient; α is the spatial interaction coefficient of independent variable; ν_{jk} denotes the spatial temporal interaction effect from area j at time t-k; θ_{jk} denotes the spatial temporal interaction effect of error term from area j at time t-k; $f(w_{ij})$ which is a function of spatial weights measures the spatial-temporal interaction effect. N is the number of spatial unites; p and q denotes time period.

This proposed spatial model combines the features of the general spatial and time series data. It consists of spatial autoregressive, spatial lag of independent variable, spatial error, temporal autoregressive and moving average model. The values of the parameters in the model imply the corresponding features of the model. Consider some simple case: The model becomes a general spatial model describes spatial dependence without temporal dependence if $\nu_{jk} = \theta_{jk} = 0, \forall j,k$:

$$\begin{aligned} y_{i,t} &= \delta \sum_{j=1}^{N} w_{ij} y_{j,t} + x_{i,t}\beta + \alpha \sum_{j=1}^{N} w_{ij} x_{j,t} + \mu_i + \phi_{i,t} \\ \phi_{i,t} &= \rho \sum_{j=1}^{N} w_{ij} \phi_{j,t} + \varepsilon_{i,t} \end{aligned} \tag{2.1}$$

The model is spatial lagged in independent variable and spatial error model if $\nu_{jk} = \theta_{jk} = \delta = 0, \forall j,k$:

$$y_{i,t} = x_{i,t}\beta + \alpha\sum_{j=1}^{N} w_{ij}x_{j,t} + \mu_i + \phi_{i,t}$$
$$\phi_{i,t} = \rho\sum_{j=1}^{N} w_{ij}\phi_{j,t} + \varepsilon_{i,t} \tag{2.2}$$

The model can be simplified to the spatial error model if $\nu_{jk} = \theta_{jk} = \delta = \alpha = 0, \forall j,k$:

$$y_{i,t} = x_{i,t}\boldsymbol{\chi} + \boldsymbol{\nu}_i + \boldsymbol{\gamma}_{i,t}$$
$$\boldsymbol{\gamma}_{i,t} = \boldsymbol{\sigma}\sum_{j=1}^{N} w_{ij}\boldsymbol{\gamma}_{j,t} + \boldsymbol{\phi}_{i,t} \tag{2.3}$$

The model becomes the spatial lagged model if $\nu_{jk} = \theta_{jk} = \rho = 0, \forall j,k$:

$$y_{i,t} = \boldsymbol{\varepsilon}\sum_{j=1}^{N} w_{ij}y_{j,t} + x_{i,t}\boldsymbol{\chi} + \boldsymbol{\beta}\sum_{j=1}^{N} w_{ij}x_{j,t} + \boldsymbol{\nu}_i + \boldsymbol{\phi}_{i,t} \tag{2.4}$$

The hypothesis testing of the corresponding coefficients of the model can investigate the feature of the data: the hypothesis $H_0 : p = q = \delta = 0$ can be tested whether the model is the spatial lagged in independent variable and spatial error model. Furthermore, the testing of the values of the time series parameters (*p,q*) determines the temporal feature and order of the model:

The model becomes a spatial-temporal autoregressive model of order one AR(1) if $p = 1$ and $\theta_{jk} = 0, \forall j,k$:

$$y_{i,t} = \delta\sum_{j=1}^{N} w_{ij}y_{j,t} + \sum_{j=1}^{N} f(w_{ij})\nu_{jk}y_{j,t-1} + x_{i,t}\beta + \alpha\sum_{j=1}^{N} w_{ij}x_{j,t} + \mu_i + \phi_{i,t}$$
$$\phi_{i,t} = \rho\sum_{j=1}^{N} w_{ij}\phi_{j,t} + \varepsilon_{i,t} \tag{3.1}$$

The model becomes a spatial-temporal moving average model of order one MA(1) if $q = 1$ and $\nu_{jk} = 0, \forall j,k$:

$$y_{i,t} = \delta\sum_{j=1}^{N} w_{ij}y_{j,t} + x_{i,t}\beta + \alpha\sum_{j=1}^{N} w_{ij}x_{j,t} + \mu_i + \phi_{i,t}$$
$$\phi_{i,t} = \rho\sum_{j=1}^{N} w_{ij}\phi_{j,t} + \varepsilon_{i,t} + \sum_{j=1}^{N} f(w_{ij})\theta_{jk}\phi_{j,t-1} \tag{3.2}$$

The model becomes a spatial-temporal autoregressive moving average model ARMA(1,1) if $p = 1$ and $q = 1$:

$$y_{i,t} = \delta\sum_{j=1}^{N} w_{ij}y_{j,t} + \sum_{j=1}^{N} f(w_{ij})\nu_{jk}y_{j,t-1} + x_{i,t}\beta + \alpha\sum_{j=1}^{N} w_{ij}x_{j,t} + \mu_i + \phi_{i,t}$$
$$\phi_{i,t} = \rho\sum_{j=1}^{N} w_{ij}\phi_{j,t} + \varepsilon_{i,t} + \sum_{j=1}^{N} f(w_{ij})\theta_{jk}\phi_{j,t-1} \tag{3.3}$$

The model becomes a spatial-temporal autoregressive moving average model without spatial-temporal interaction effect if $p = q = 1$, $f(w_{ii}) = 1$ and $\nu_{jk} = \theta_{jk} = 0, \forall j \neq i$:

$$y_{i,t} = \delta\sum_{j=1}^{N} w_{ij}y_{j,t} + v_i y_{i,t-1} + x_{i,t}\beta + \alpha\sum_{j=1}^{N} w_{ij}x_{j,t} + \mu_i + \phi_{i,t}$$
$$\phi_{i,t} = \rho\sum_{j=1}^{N} w_{ij}\phi_{j,t} + \varepsilon_{i,t} + \theta_i\phi_{i,t-1} \tag{3.4}$$

The proposed model is modified to captures spatial dependence, temporal dependence and spatial temporal interactive effect both for dependent variable and error term. The hypothesis testing of the coefficients in this general model can statistically test the feature of the data.

4. Estimation of the proposed spatial temporal model:

When $v_{jk} = \delta, \alpha = \alpha_j, \theta_{jk} = \rho (j = 1,\ldots,N, k = 0,\ldots,q)$, the proposed spatial temporal model (1) can be simplified into the following:

$$y_{i,t} = \delta\sum_{j=1}^{N}\sum_{k=0}^{p} w_{ij}y_{j,t-k} + x_{i,t}\beta + \alpha\sum_{j=1}^{N} w_{ij}x_{j,t} + \mu_i + \phi_{i,t},$$
$$\phi_{i,t} = \varepsilon_{i,t} + \rho\sum_{j=1}^{N}\sum_{k=0}^{q} w_{ij}\phi_{j,t-k}, \tag{4}$$

where t denotes time periods, $t = 1,\ldots,T$ ..

$$\overline{y_i} = \frac{1}{T}\sum_{t=1}^{T} y_{i,t}, \qquad \overline{x_i} = \frac{1}{T}\sum_{t=1}^{T} x_{i,t},$$

$$y_{i,t}^{\ *} = y_{i,t} - \overline{y_i} \quad and \quad x_{i,t}^{\ *} = x_{i,t} - \overline{x_i}.$$

Assume $\rho = 0$ and fixed spatial specific effects and take into account the endogeneity of $\sum_j w_{ij}y_j^{\ t}$, the model become

$$y_{i,t} = \delta\sum_{j=1}^{N}\sum_{k=0}^{p} w_{ij}y_{j,(t-k)} + x_{i,t}\beta + \alpha\sum_{j=1}^{N} w_{ij}x_{j,t} + \mu_i + \phi_{i,t}. \tag{4.1}$$

The log-likelihood function of model is

$$LogL = -\frac{NT}{2}\log(2\pi\sigma^2) + T\left|I_N - \delta W\right| - \frac{1}{2\sigma^2}\sum_{i=1}^{N}\sum_{t=1}^{T}(y_{i,t} - \delta\sum_{j=1}^{N}\sum_{k=0}^{p} w_{ij}y_{j,(t-k)} - x_{i,t}\beta - \alpha\sum_{j=1}^{N} w_{ij}x_{j,t} - \mu_i)^2.$$

Assume $\rho = \alpha = 0$ and fixed spatial specific effects, the model becomes

$$y_{i,t} = \delta\sum_{j=1}^{N}\sum_{k=0}^{p} w_{ij}y_{j,(t-k)} + x_{i,t}\beta + \mu_i + \phi_{i,t} \tag{4.2}$$

The log-likelihood function becomes

$$LogL = -\frac{NT}{2}\log(2\pi\sigma^2) + T\left|I_N - \delta W\right| - \frac{1}{2\sigma^2}\sum_{i=1}^{N}\sum_{t=1}^{T}(y_{i,t} - \delta\sum_{j=1}^{N}\sum_{k=0}^{p} w_{ij} y_{j,(t-k)} - x_{i,t}\beta - \mu_i)^2 .$$

The estimators of β is

$$\beta = (X^{*'}X^{*})^{-1}X^{*'}[Y^{*} - \frac{\delta}{\delta(B)}(I_T \otimes W)Y^{*}],$$

5. The weighted networks

Suppose a system starts with pre-existing m nodes all connected to one another, assume the network is undirected that link's direction does not count. The degree of each node is the sum of all connections which is a binary choice: zero or one. In weighted networks, the weights can be described by a spatial weights matrix W denoting the strength of the edge connecting node i and j. The spatial weight represents any property of the connection: distance, traffic or capacity of the connected node which may be time variable. The probability of node i to be chose at time t is:

$$P_{i,t} = m\frac{\sum_j w_{ij,t}}{\sum_i\sum_j w_{ij,t}} = m\frac{s_{i,t}}{\sum_i s_{i,t}} \tag{5.1}$$

where $w_{ij,t}$ is the spatial weight of the link connecting node i and j at time t; and $\sum_j w_{ij,t} = s_{i,t}$,the degree of each node which is the sum of weights of the certain node, describes the importance of node i. The above probability of certain node to be chose denotes the relative importance or attractiveness of each node. The normalized weights in section 2.2 measures the relative attractiveness of the corresponding link from certain node.

Consider the feature of network into the spatial temporal data, apply the normalized weight ϖ_{ij} as the relative attractiveness of spatial interaction, and apply the time varying probability to be chosen into the proposed general spatial-temporal autoregressive moving average model;

$$\begin{aligned} y_{i,t} &= \delta\sum_{j=1}^{N}\varpi_{ij,t}y_{j,t} + \sum_{j=1}^{N}\sum_{k=1}^{p} f(\varpi_{ij,t})V(P_{i,t-k})y_{j,t-k} + x_{i,t}\beta + \alpha\sum_{j=1}^{N}\varpi_{ij,t}x_{j,t} + \mu_i + \phi_{i,t} \\ \phi_{i,t} &= \rho\sum_{j=1}^{N}\varpi_{ij,t}\phi_{j,t} + \varepsilon_{i,t} + \sum_{j=1}^{N}\sum_{k=1}^{q} f(\varpi_{ij,t})\theta_{jk}\phi_{j,t-k} \end{aligned} \tag{5.2}$$

The dynamic interaction coefficient is assumed to be a function of the time varying probability which depends on spatial weight, $V(P_{i,t-k}) = \nu_{jk}$.. The modified general model (5.2) considers the features of networks into the spatial dynamic data.

6. Concluding remarks

This paper plan to modify a general spatio-temporal model including spatial temporal dependence and interaction from the network point of view, and to generate the model based on the network feature. We propose a general spatial temporal model describing the features of spatial and time series data. The model is modified to consist of spatial autoregressive, spatial lag of independent variable, spatial error, temporal autoregressive and moving average model. The spatially interactive feature of network system is applied into the proposed model to derive a time varying weights and coefficients spatial temporal model.

For future research, the hypotheses testing and estimation of the corresponding parameters of the proposed model could be analyzed.

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