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Numerical Investigation of Simplified Two Channel Model of a Nuclear Reactor Core using Single Term Haar Wavelet Series Method

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Abstract: This paper presents a comparison of Single Term Haar Wavelet Series (STHWS) method and the classical fourth order Runge-Kutta (RK) method to solve the simplified two channel model of a nuclear reactor core from fluid dynamics. The results obtained using RK method and the STHWS methods are compared with the exact solutions of the problem. To illustrate the effectiveness of the STHW, four cases in non-linear singular systems from fluid dynamics have been considered and compared with the classical fourth order Runge-Kutta, and are found to be very accurate. Solutions for the non-linear singular systems from fluid dynamics are presented in the tables.

Keywords: *Haar wavelet, single-term Haar wavelet series, linear singular systems, non-linear singular systems.*

1. INTRODUCTION

This chapter establishes a clear procedure for solving non-linear system in fluid dynamics using classical fourth order RK and STHW along with the exact solutions. The approximate solutions obtained are compared with the exact solutions of the non-linear system in fluid dynamics, and are found to be very accurate.

Most of the realistic singular non-linear systems do not admit any analytical solution and hence a numerical procedure has to be used. In the last few years substantial progress has been made in finding the numerical solution of special classes of nonlinear singular systems of differential equations. A general numerical procedure for their solution has not previously existed. Hence it is important to understand the structure of such systems and develop efficient methods for solving them. The conventional methods such as Euler, Runge-Kutta and Adams-Moulton are restricted to very small step size in order that the solution is stable.

Runge-Kutta methods have become very popular, both as computational techniques as well as subject for research, which were discussed by Butcher [2-4] and Shampine [21]. This method was derived by Runge about the year 1894 and extended by Kutta a few years later. They developed algorithms to solve differential equations efficiently and yet are the equivalent of approximating the exact solutions by matching '*n*' terms of the Taylor series expansion. The beauty of the RK pair is that it requires no extra function evaluations, which is the most time consuming aspect of all ODE solvers. This breakthrough initiated a search for RK algorithms of higher and higher order and better error estimates.

Nandhakumar et al. [7] introduce Haar Wavelet Series to numerical investigation of an industrial robot arm control problem. Sekar et al. [8-9] introduced the STHW to study the nonlinear singular systems and second order mechanical vibratory systems. Murugesan et al. [6] discussed the nonlinear singular systems from fluid dynamics using the RK-methods based on variety of means. In this paper, we consider the same non-linear singular systems from fluid dynamics (discussed by Murugesan et al [6]) but presenting a different approach by the STHW with more accuracy.

2. SINGLE TERM HAAR WAVELET SERIES (STHWS) METHOD

The orthogonal set of Haar wavelets $h_i(t)$ is a group of square waves with magnitude of ±1 in some intervals and zeros elsewhere [2]. In general,

$$\begin{aligned} h_n(t) &= h_1(2^j t - k), \\ n &= 2^j + k, j \geq 0, 0 \leq k < 2^j, n, j, k \in \mathrm{Z}\} \end{aligned}$$

$$h_1(t) = \begin{cases} 1, 0 \leq t < \dfrac{1}{2} \\ -1, \dfrac{1}{2} \leq t < 1 \end{cases}$$

Namely, each Haar wavelet contains one and just one square wave, and is zero elsewhere. Just these zeros make Haar wavelets to be local and very useful in solving stiff systems. Any function $y(t)$, which is square integrable in the interval [0,1). Can be expanded in a Haar series with an infinite number of terms

$$\begin{aligned} y(t) &= \sum_{i=0}^{\infty} c_i h_i(t) \\ i &= 2^j + k, j \geq 0, 0 \leq k < 2^j, n, j, t, \in [0, 1]\} \end{aligned} \tag{1}$$

where the Haar coefficients $c_i = 2^j \int_0^1 y(t) h_i(t) dt$ are determined such that the following integral square error ε is minimized:

$$\varepsilon = \int_0^1 \left[y(t) - \sum_{i=0}^{m-1} c_i h_i(t) \right]^2 dt, m = 2^j, j \in \{0\} \cup \mathrm{N} \}$$

Usually, the series expansion (1) contains an infinite number of terms for a smooth $y(t)$. If $y(t)$ is a piecewise constant or may be approximated as a piecewise constant, then the sum in (1) will be terminated after m terms, that is

$$\begin{aligned} y(t) &\approx \sum_{i=0}^{m-1} c_i h_i(t) = c_{(m)}^{\mathrm{T}} h_{(m)}(t), t \in [0,1] \\ c_{(m)}(t) &= [c_0 c_1 \ldots c_{m-1}]^{\mathrm{T}}, \\ h_{(m)}(t) &= [h_0(t)] h_1(t) \ldots h_{m-1}(t)]^{\mathrm{T}}, \end{aligned} \tag{2}$$

where "T" indicates transposition, the subscript m in the parantheses denotes their dimensions. The integration of Haar wavelets can be expandable into Haar series with Haar coefficient matrix P[3].

$$\int h_{(m)}(\tau) d\tau = \mathrm{P}_{(m \times n)} h_{(m)}(t), t \in [0, 1]$$

where the m-square matrix P is called the operational matrix of integration and single-term $P_{(1\times1)} = \frac{1}{2}$. Let us define [12]

$$h_{(m)}(t)h^{T}_{(m)}(t) \approx M(m \times m)(t), \tag{3}$$

and $$M_{(1\times1)}(t) = h_0(t) \text{ (3) staisfies}$$

$$M_{(m\times m)}(t)c_{(m)} = C_{(m\times m)}h_{(m)}(t),$$

where $c_{(m)}$ is defined in (2) and $$C_{(1\times1)} = c_0$$

3. REPRESENTATION OF EQUATIONS OF FLOW AS A NON-LINEAR SYSTEM

The simplified model consists of two connected sub channels filled with a steadily flowing fluid. Control volumes and flow variables for the system are shown in Figure 1. Here, m_i, represents the axial mass flow rate in sub channel i and w represents the cross-flow rate per unit length, assumed positive if the flow is from sub channel 1 to sub channel 2.

An application of the principles of conservation of mass, momentum and energy to the control volumes yields the following set of equations for sub channel 1.

Continuity : $$\frac{dm_i}{dx} = -w \tag{4}$$

Axial momentum:

$$\frac{d}{dx}(m_1u_1) + w.\left[H(w)u_1 + H(-w)u_2\right] = -F_1 - A_1\frac{dp_1}{dx} \tag{5}$$

Energy : $$\frac{d}{dx}(m_1h_1) = q_1 - w.\left[H(w)h_1 + H(-w)h_2\right] \tag{6}$$

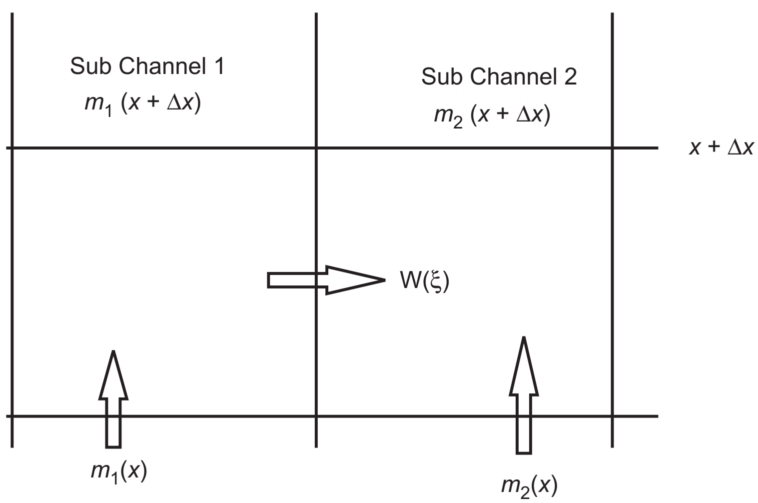

Figure 1: Flow variables

Analogous equations for sub channel 2 can be obtained from these by substituting $-w$ for w and by interchanging subscripts 1 and 2. In this equation set, H is the Heavy side unit step function. F represents pressure loss per unit length due to friction, A is the cross-sectional area, q represents the heat energy added per unit length, and the variables, u, p and h stand for particle velocity, pressure and enthalpy respectively.

In analogy with the pressure drop due to friction in a long pipe, a lateral momentum balance may be taken as $p_1 - p_2 = C_w|W|$, where C is a cross-flow friction factor.

To simplify the above equation, the following assumptions are made. Cross-sectional area is constant; the coolant is incompressible; there is no enthalpy change; and the frictional pressure loss function is of the form $F_1 = m_1 u_1 F$, where F is a constant.

With these assumptions, the equations may be combined and written in the following form:

$$\frac{dm_1}{dx} = -w$$

$$\frac{d}{dx}\left(w|w|\right) = \in^{-1}\left\{\frac{1-2m_1}{2} + 2w\left[1 - \mathrm{H}(w)m_1 + \mathrm{H}(-w)(m_1 - 1)\right]\right. \quad (7)$$

To make the above system (3.4) into the symmetric form, take

$$x = m_1 - \frac{1}{2}$$

$$y = \frac{w}{2}$$

and $t = x$

Hence we get

$$\frac{dx}{dt} = -2y$$

$$\frac{d}{dt}\left(y|y|\right) = (4\varepsilon)^{-1}\left[-x + 2\left(y - 2x|y|\right)\right].$$

Replacing x by x_1 and y by x_2, we have

$$\dot{x}_1 = -2x_2$$

$$\frac{d}{dt}\left(x_2|x_2|\right) = (4\varepsilon)^{-1}\left[-x_1 + 2\left(x_2 - 2x_1|x_2|\right)\right] \quad (8)$$

An analysis is carried out in four different ways depending upon the values of x_2 and ε as given below :

1. $x_2 > 0$ and $\varepsilon \neq 0$
2. $x_2 < 0$ and $\varepsilon \neq 0$
3. $x_2 > 0$ and $\varepsilon \neq 0$
3. $x_2 < 0$ and $\varepsilon \neq 0$

In the first two cases the parameter ε has been varied from $10^0, 10^1, 10^2, \ldots, 10^7$ and in the last two cases, ε has been set to zero.

Case(*i*) : When $x_2 > 0$ and $\varepsilon \neq 0$

In this case Eq.(3.5) becomes

$$\dot{x}_1 = -2x_2$$

$$8\varepsilon x_2 \dot{x}_2 = -x_1 + 2x_2 - 4x_1 x_2$$

The above two equations can be considered as a system of equations of the form

$$\begin{bmatrix} 1 & 0 \\ 0 & 8\varepsilon x_2 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -4x_1 x_2 \end{bmatrix} \quad (9)$$

This is of the form (Evans et al. [5])

$$\mathrm{K}(x(t))\dot{x}(t) = \mathrm{A}x(t) + f(x(t)) \quad (10)$$

The first order non-linear system (3.6), representing the highly simplified two channel model of a nuclear reactor core from fluid dynamics, when $x_2 > 0$ and $\varepsilon \neq 0$, can be converted into a second order equation in order to reduce the number of equations, as well as the number of unknowns, and is given as

$$\ddot{x}_1 = \frac{1}{2\varepsilon}\left[\frac{-x_1}{\dot{x}_1} - 1 + 2x_1\right], \tag{11}$$

where $x_2 = \dfrac{-\dot{x}_1}{2}$

Hence Eq. (3.8) is of the form $\ddot{x}_2 = \varphi(\varepsilon) f(t, x_1, \dot{x}_1)$,

where $\varphi(\varepsilon) = \dfrac{1}{2\varepsilon}$

Case(*ii*) : When $x_2 < 0$ and $\varepsilon \neq 0$

In this case Eq. (3.5) becomes

$$\dot{x}_1 = 2x_2$$
$$8\varepsilon x_2\dot{x}_2 = x_1 + 2x_2 + 4x_1x_2$$

i.e.,

$$\begin{bmatrix} 1 & 0 \\ 0 & 8\varepsilon x_2 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 4x_1x_2 \end{bmatrix} \tag{12}$$

This is also of the form (3.7). The first order non-linear system (3.9), representing the highly simplified two channel model of a nuclear reactor core from fluid dynamics, when $x_2 < 0$ and $\varepsilon \neq 0$, can be converted into a second order equation in order to reduce the number of equations, as well as the number of unknowns, and is given as

$$\ddot{x}_1 = \frac{1}{2\varepsilon}\left[\frac{x_1}{\dot{x}_1} + 1 + 2x_1\right], \tag{13}$$

where $x_2 = \dfrac{\dot{x}_1}{2}$

Hence Eq. (3.10) is of the form $\ddot{x}_1 = \varphi(\varepsilon) f(t, x_1, \dot{x}_1)$,

where $\varphi(\varepsilon) = \dfrac{1}{2\varepsilon}$

Case(*iii*) : When $x_2 > 0$ and $\varepsilon = 0$

In this case Eq.(3.5) becomes

$$\dot{x}_1 = -2x_2$$
$$0 = -x_1 + 2x_2 - 4x_1x_2$$

i.e.,

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -4x_1x_2 \end{bmatrix}$$

The system (11) is a singular non-linear system and it is of the form (3.7). This system can be written as

$$\dot{x}_1 = \frac{x_1}{2x_1 - 1}$$

and $\ddot{x}_2 = \dfrac{-\dot{x}_1}{2}$

The above equations has been converted into a second order equations as

$$\ddot{x}_1 = \frac{-\dot{x}_1}{(2x_1 - 1)^2}, \tag{14}$$

where $x_2 = \dfrac{-\dot{x}_1}{2}$

Hence Eq. (12) is of the form $\ddot{x}_1 = f(t, x_1, \dot{x}_1)$,

Case(*iv*) : When $x_2 < 0$ and $\varepsilon = 0$

In this case Eq. (3.5) becomes

$$\dot{x}_1 = 2x_2$$
$$0 = -x_1 - 2x_2 - 4x_1x_2$$

i.e.,

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -4x_1x_2 \end{bmatrix} \quad (15)$$

The system (3.13) is a singular non-linear system and it is of the form (3.7). This system can be written as

$$\dot{x}_1 = \frac{-x_1}{1+2x_1}$$

and

$$\ddot{x}_2 = \frac{\dot{x}_1}{2}$$

Further, the above equations has been converted into a second order equations as

$$\ddot{x}_1 = \frac{-\dot{x}_1}{(2x_1-1)^2}, \quad (16)$$

where

$$x_2 = \frac{-\dot{x}_1}{2}$$

Hence Eq. (3.14) is of the form $\ddot{x}_1 = f(t, x_1, \dot{x}_1)$,

4. RESULTS

The objective of this section is to find discrete solutions to the simplified two channel model of a nuclear reactor core from fluid dynamics under all four cases discussed in the section 3. In the case of (*i*) and (*ii*), the system has been reduced to a singular system, which is further converted into a second order equation.

It is very difficult to obtain the exact solution of this non-linear equation. Hence it has been analysed by the following numerical methods by the way of determining the discrete solutions at different time intervals:

1. Classical fourth order Runge-Kutta method (RK(4)).
2. Single-Term Haar Wavelet Series (STHW).

1. **Solution by Classical Fourth Order Runge-Kutta Method:** The classical fourth order Runge-Kutta method RK(4) is given by

$$k_1 = hf(x_n, y_n)$$
$$k_2 = hf\left(x_n, y_n + \frac{1}{2}k_1\right)$$
$$k_3 = hf\left(x_n, y_n + \frac{1}{2}k_2\right)$$
$$k_4 = hf(x_n, y_n)$$
$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

The above method has been applied to determine the approximate solutions for all four cases of the nuclear reactor core problem discussed in previous section.

The discrete solutions of a two channel model of a nuclear reactor core problem for the cases (*i*) when $x_2 > 0$ and $\varepsilon \neq 0$ (*ii*) when $x_2 < 0$ and $\varepsilon \neq 0$ [*i.e.*, Eq.(6) and Eq. (8)] have been determined using the method RK(4) by varying the parameter ε from 10^0 to 10^7 with $x_1(0) = \dot{x}_1(0) = 1$ and the results are given in the Tables 4.1, 4.2, 4.3 and 4.4 and the discrete solution for the cases (*iii*) when $x_2 > 0$ and $\varepsilon = 0$ (*iv*) when $x_2 < 0$ and $\varepsilon = 0$, [*i.e.*, singular systems] have been determined using RK(4) with $x_1(0) = 1$, $\dot{x}_1(0) = 1$ and the results are given in the Tables 4.5 and 4.6.

2. **Solution by STHWS Method:** The STHWS has been applied to determine the approximate solutions for all four cases of the two channel model of a nuclear reactor core problem for the cases (*i*) when $x_2 > 0$ and $\varepsilon \neq 0$ (*ii*) when $x_2 < 0$ and $\varepsilon \neq 0$ [*i.e.*, Eq.(3.6) and Eq. (3.8)] have been determined using the method STHW by varying the parameter ε from 10^0 to 10^7 with $x_1(0) = 1$, $\dot{x}_1(0) = 1$ and the results are given in the Tables 4.7, 4.8, 4.9 and 4.10 and the discrete solution for the cases (*iii*) when $x_2 > 0$ and $\varepsilon = 0$ (*iv*) when $x_2 < 0$ and $\varepsilon = 0$, [*i.e.*, singular systems] have been determined using STHW with $x_1(0) = 1$, $\dot{x}_1(0) = 1$, and the results are given in the Tables 11 and 12.

Table 1
Solutions of equation (6) by RK for x_1

S. No.	Time	*Discrete solutions of x_1 in case (i) using RK*							
		$\varepsilon = 10^0$	$\varepsilon = 10^1$	$\varepsilon = 10^2$	$\varepsilon = 10^3$	$\varepsilon = 10^4$	$\varepsilon = 10^5$	$\varepsilon = 10^6$	$\varepsilon = 10^7$
1.	0	1	1	1	1	1	1	1	1
2.	0.5	0.5	1.50105	1.5001	1.5	1.5	1.5	1.5	1.5
3.	1	2.103502	2.00852	2.00084	2.00008	2.00001	2	2	2
4.	1.5	2.907169	2.5293	2.50282	2.50028	2.50003	2.5	2.5	2.5
5.	2	4.13519	3.07107	3.00671	3.00067	3.00007	3	3	3
6.	2.5	6.119246	3.64267	3.51314	3.5013	3.50013	3.50001	3.5	3.5
7.	3	9.381907	4.25438	4.02279	4.00225	4.00022	4.00002	4	4
8.	3.5	14.767	4.91829	4.53632	4.53632	4.50036	4.50003	4.50001	4.50001
9.	4	23.65592	5.6486	5.05446	5.05446	5.00053	5.00004	5.00002	5.00002
10.	4.5	38.32171	6.46194	5.57791	5.57791	5.50076	5.50006	5.50003	5.50003
11.	5	62.5103	7.37769	6.10744	6.10744	6.00104	6.0001	6.00004	6.00004

Table 2
Solutions of equation (6) by RK for x_2

S. No	Time	*Discrete solutions of x_2 in case (i) using RK*							
		$\varepsilon = 10^0$	$\varepsilon = 10^1$	$\varepsilon = 10^2$	$\varepsilon = 10^3$	$\varepsilon = 10^4$	$\varepsilon = 10^5$	$\varepsilon = 10^6$	$\varepsilon = 10^7$
1.	0	1	1	1	1	1	1	1	1
2.	0.5	-0.535368	-0.50316	-0.50031	-0.500003	-0.50003	-0.5	-0.5	-0.5
3.	1	-0.670201	-0.51292	-0.50125	-0.500125	-0.50001	-0.5	-0.5	-0.5
4.	1.5	-0.972082	-0.52991	-0.50283	-0.500281	-0.50003	-0.5	-0.5	-0.5
5.	2	-1.538084	-0.5551	-0.50505	-0.500501	-0.50005	-0.50005	-0.5	-0.5

S. No	Time	Discrete solutions of x_2 in case (i) using RK							
		$\varepsilon = 10^0$	$\varepsilon = 10^1$	$\varepsilon = 10^2$	$\varepsilon = 10^3$	$\varepsilon = 10^4$	$\varepsilon = 10^5$	$\varepsilon = 10^6$	$\varepsilon = 10^7$
6.	2.5	-2.515008	-0.5898	-0.50793	-0.500782	-0.50008	-0.50001	-0.5	-0.5
7.	3	-4.147465	-0.63563	-0.51148	-0.501127	-0.50011	-0.50001	-0.5	-0.5
8.	3.5	-6.847252	-0.69456	-0.51572	-0.501535	-0.50015	-0.50001	-0.5	-0.5
9.	4	-11.30009	-0.76884	-0.52067	-0.502007	-0.5002	-0.50002	-0.5	-0.5
10.	4.5	-18.64044	-0.86107	-0.52636	-0.502542	-0.50025	-0.50003	-0.5	-0.5
11.	5	-30.74066	-0.97417	-0.53282	-0.503141	-0.50031	-0.50003	-0.5	-0.5

Table 3

Solutions of equation (9) by RK for x_1 x_1

S. No.	Time	Discrete solutions of x_1 in case (ii) using RK							
		$\varepsilon = 10^0$	$\varepsilon = 10^1$	$\varepsilon = 10^2$	$\varepsilon = 10^3$	$\varepsilon = 10^4$	$\varepsilon = 10^5$	$\varepsilon = 10^6$	$\varepsilon = 10^7$
1.	0	1	1	1	1	1	1	1	1
2.	0.5	1.770253	1.527938	1.50281	1.500281	1.500028	1.500003	1.5	1.5
3.	1	3.227253	2.123737	2.012484	2.00125	2.000125	2.000012	2	2
4.	1.5	5.743905	2.806125	2.530888	2.503093	2.50031	2.500031	2.5	2.5
5.	2	9.967638	3.595289	3.059891	3.005599	3.0006	3.00006	3	3
6.	2.5	16.98114	4.51351	3.601371	3.510154	3.501016	3.500101	3.5	3.5
7.	3	28.578028	5.585829	4.15721	4.015746	4.001575	4.000157	4	4
8.	3.5	47.720726	6.840786	4.729303	4.522964	4.502297	4.500229	4.50001	4.50001
9.	4	79.296951	8.311201	5.319559	5.031993	5.0032	5.000318	5.00002	5.00002
10.	4.5	131.36757	10.035072	5.929907	5.543022	5.504303	5.5000427	5.50003	5.50003
11.	5	217.22435	12.056558	6.562268	6.056237	6.005626	6.000559	6.00004	6.00004

Table 4

Solutions of equation (9) by RK for x_2

S. No	Time	Discrete solutions of x_2 in case (ii) using RK							
		$\varepsilon = 10^0$	$\varepsilon = 10^1$	$\varepsilon = 10^2$	$\varepsilon = 10^3$	$\varepsilon = 10^4$	$\varepsilon = 10^5$	$\varepsilon = 10^6$	$\varepsilon = 10^7$
1.	0	1	1	1	1	1	1	1	1
2.	0.5	1.069232	0.55884	0.505932	0.500594	0.500059	0.500006	0.5	0.5
3.	1	1.905654	0.6333587	0.513728	0.501375	0.500138	0.500014	0.5	0.5
4.	1.5	3.232944	0.732259	0.523392	0.502343	0.500234	0.5000234	0.5	0.5
5.	2	5.390528	0.849773	0.534929	0.503499	0.50035	0.500035	0.5	0.5
6.	2.5	8.928052	0.99083	0.548344	0.504843	0.500484	0.500049	0.5	0.5
7.	3	14.747292	1.158551	0.563649	0.506373	0.500637	0.500064	0.500002	0.5
8.	3.5	24.332659	1.356814	0.580855	0.508092	0.500809	0.500081	0.500005	0.5
9.	4	40.130196	1.590331	0.599979	0.509998	0.501	0.5001	0.500008	0.5
10.	4.5	66.171783	1.86475	0.621044	0.512091	0.501209	0.500121	0.500011	0.5
11.	5	109.10431	2.186782	0.644074	0.514372	0.501437	0.500144	0.500014	0.5

Table 5
Solutions of equation (11) by RK

S. No	*Time*	*Discrete solutions of x_1 and x_2 in case (iii) using RK*	
		x_1	x_2
1.	0	1	1
2.	0.5	1.428211	-0.38467
3.	1	1.791537	-0.34678
4.	1.5	2.127465	-0.32681
5.	2	2.447541	-0.31418
6.	2.5	2.757087	-0.30538
7.	3	3.059052	-0.29885
8.	3.5	3.355264	-0.29378
9.	4	3.646944	-0.2872
10.	4.5	3.934948	-0.28639
11.	5	4.219906	-0.2836

Table 6
Solutions of equation (3.13) by RK

S. No	*Time*	*Discrete solutions of x_1 and x_2 in case (iv) using RK*	
		x_1	x_2
1.	0	1	1
2.	0.5	1.488729	0.479521
3.	1	1.961758	0.467443
4.	1.5	2.424938	0.459403
5.	2	2.881311	0.453635
6.	2.5	3.332675	0.449281
7.	3	3.780187	0.445871
8.	3.5	4.224638	0.443124
9.	4	4.666598	0.44086
10.	4.5	5.106484	0.438962
11.	5	5.544618	0.437346

Table 7
Solutions of equation (6) by STHW for x_1

S. No.	Time	Discrete solutions of x_1 in case (i) using STHW							
		$\varepsilon = 10^0$	$\varepsilon = 10^1$	$\varepsilon = 10^2$	$\varepsilon = 10^3$	$\varepsilon = 10^4$	$\varepsilon = 10^5$	$\varepsilon = 10^6$	$\varepsilon = 10^7$
1.	0	1	1	1	1	1	1	1	1
2.	0.5	0.500001	1.50105	1.50019	1.50009	1.50009	1.5	1.5	1.5
3.	1	2.103502	2.00852	2.00084	2.00008	2.00001	2	2	2
4.	1.5	2.907169	2.52939	2.50282	2.50028	2.50003	2.5	2.5	2.5
5.	2	4.135199	3.07107	3.00671	3.00067	3.00007	3	3	3
6.	2.5	6.119246	3.64267	3.51314	3.5013	3.50013	3.50001	3.5	3.5
7.	3	9.381907	4.25438	4.02279	4.00225	4.00022	4.00002	4	4
8.	3.5	14.76799	4.91829	4.53632	4.53632	4.50036	4.50003	4.50001	4.50001
9.	4	23.65592	5.64869	5.05446	5.05446	5.00053	5.00004	5.00002	5.00002
10.	4.5	38.32171	6.46194	5.57791	5.57791	5.50076	5.50006	5.50003	5.50003
11.	5	62.51039	7.37769	6.10744	6.10744	6.00104	6.00019	6.00004	6.00004

Table 8
Solutions of equation (6) by STHW for x_2

S. No.	Time	Discrete solutions of x_2 in case (i) using STHW							
		$\varepsilon = 10^0$	$\varepsilon = 10^1$	$\varepsilon = 10^2$	$\varepsilon = 10^3$	$\varepsilon = 10^4$	$\varepsilon = 10^5$	$\varepsilon = 10^6$	$\varepsilon = 10^7$
1.	0	1	1	1	1	1	1	1	1
2.	0.5	-0.535368	-0.50316	-0.50031	-0.500003	-0.50003	-0.5	-0.5	-0.5
3.	1	-0.670201	-0.51292	-0.50125	-0.500125	-0.50001	-0.5	-0.5	-0.5
4.	1.5	-0.972082	-0.52991	-0.50283	-0.500281	-0.50003	-0.5	-0.5	-0.5
5.	2	-1.538084	-0.55519	-0.50505	-0.500501	-0.50005	-0.50005	-0.5	-0.5
6.	2.5	-2.515008	-0.58989	-0.50793	-0.500782	-0.50008	-0.50001	-0.5	-0.5
7.	3	-4.147465	-0.63563	-0.51148	-0.501127	-0.50011	-0.50001	-0.5	-0.5
8.	3.5	-6.847252	-0.69456	-0.51572	-0.501535	-0.50015	-0.50001	-0.5	-0.5
9.	4	-11.30009	-0.76884	-0.52067	-0.502007	-0.50029	-0.50002	-0.5	-0.5
10.	4.5	-18.64044	-0.86107	-0.52636	-0.502542	-0.50025	-0.50003	-0.5	-0.5
11.	5	-30.74066	-0.97417	-0.53282	-0.503141	-0.50031	-0.50003	-0.5	-0.5

Table 9
Solutions of equation (9) by STHW for x_1

S. No	Time	*Discrete solutions of x_1 in case (ii) using STHW*							
		$\varepsilon = 10^0$	$\varepsilon = 10^1$	$\varepsilon = 10^2$	$\varepsilon = 10^3$	$\varepsilon = 10^4$	$\varepsilon = 10^5$	$\varepsilon = 10^6$	$\varepsilon = 10^7$
1.	0	1	1	1	1	1	1	1	1
2.	0.5	1.7702536	1.5279384	1.50281	1.500281	1.500028	1.5000035	1.5	1.5
3.	1	3.2272538	2.1237375	2.012484	2.001255	2.000125	2.0000122	2	2
4.	1.5	5.7439054	2.8061257	2.530888	2.503093	2.500319	2.5000317	2.5	2.5
5.	2	9.9676389	3.5952898	3.059891	3.005599	3.000699	3.0000649	3	3
6.	2.5	16.981142	4.5135187	3.601371	3.510154	3.501016	3.5001018	3.5	3.5
7.	3	28.578028	5.5858293	4.15721	4.015746	4.001575	4.0001572	4	4
8.	3.5	47.720726	6.8407866	4.729303	4.522964	4.502297	4.5002298	4.50001	4.50001
9.	4	79.296951	8.3112018	5.319559	5.031993	5.003299	5.0003189	5.00002	5.00002
10.	4.5	131.36757	10.035072	5.929907	5.543022	5.504303	5.5000427	5.50003	5.50003
11.	5	217.22435	12.056558	6.562268	6.056237	6.005626	6.0005599	6.00004	6.00004

Table 10
Solutions of equation (9) by STHW for x_2

S. No	Time	*Discrete solutions of x_2 in case (ii) using STHW*							
		$\varepsilon = 10^0$	$\varepsilon = 10^1$	$\varepsilon = 10^2$	$\varepsilon = 10^3$	$\varepsilon = 10^4$	$\varepsilon = 10^5$	$\varepsilon = 10^6$	$\varepsilon = 10^7$
1.	0	1	1	1	1	1	1	1	1
2.	0.5	1.0692324	0.5588483	0.505932	0.5005946	0.500059	0.5000066	0.5	0.5
3.	1	1.9056584	0.6333587	0.513728	0.5013750	0.500138	0.5000141	0.5	0.5
4.	1.5	3.2329445	0.7322594	0.523392	0.5023438	0.500234	0.5000234	0.5	0.5
5.	2	5.3905289	0.8497732	0.534929	0.5034992	0.500356	0.5000355	0.5	0.5
6.	2.5	8.9280521	0.9908322	0.548344	0.5048437	0.500484	0.5000491	0.5	0.5
7.	3	14.747292	1.1585519	0.563649	0.5063731	0.500637	0.5000646	0.500002	0.5
8.	3.5	24.332659	1.3568148	0.580855	0.5080929	0.500809	0.5000811	0.500005	0.5
9.	4	40.130196	1.5903316	0.599979	0.5099982	0.501384	0.5001683	0.500008	0.5
10.	4.5	66.171783	1.8647593	0.621044	0.5120917	0.501209	0.5001218	0.500011	0.5
11.	5	109.10431	2.1867828	0.644074	0.5143724	0.501437	0.5001449	0.500014	0.5

Table 11
Solutions of equation (11) by STHW

S. No	*Time*	*Discrete solutions of x_1 and x_2 in case (iii) using STHW*	
		x_1	x_2
1.	0	1	1
2.	0.5	1.428211	-0.38467
3.	1	1.791537	-0.34678
4.	1.5	2.127465	-0.32681
5.	2	2.447541	-0.31418
6.	2.5	2.757087	-0.30538
7.	3	3.059052	-0.29885
8.	3.5	3.355264	-0.29378
9.	4	3.646944	-0.2872
10.	4.5	3.934948	-0.28639
11.	5	4.219906	-0.2836

Table 12
Solutions of equation (13) by STHW

S. No	*Time*	*Discrete solutions of x_1 and x_2 in case (iv) using STHW*	
		x_1	x_2
1.	0	1	1
2.	0.5	1.488729	0.479521
3.	1	1.961758	0.467443
4.	1.5	2.424938	0.459403
5.	2	2.881311	0.453635
6.	2.5	3.332675	0.449281
7.	3	3.780187	0.445871
8.	3.5	4.224638	0.443124
9.	4	4.666598	0.44086
10.	4.5	5.106484	0.438962
11.	5	5.544618	0.437346

5. CONCLUSIONS

The nuclear reactor core problem has been studied under four different cases by way of determining the discrete solutions for different time't' using the classical fourth order Runge-Kutta method and STHW. In [1], for the same problem, the approximate solution was determined using STHW method and it was mentioned that the classical RK method failed to obtained approximate solutions when the parameter $\varepsilon \geq 10^3$. But in this chapter, it has been established that the Runge-Kutta methods are adequate enough to determine approximate solutions for all values of ε (*i.e.*, $\varepsilon = 0, 10^0, 10^1, ..., 10^7$). In cases (*iii*) and (*iv*), when $\varepsilon = 0$, the system reduces to a singular system for both $x_2 > 0$ and $x_2 < 0$. It is observed that, for a singular system, the discrete solutions obtained by the classical Runge-Kutta method and STHW are found to be similar (refer Tables 1-12). However, for the cases (*i*) when $x_2 > 0$ and $\varepsilon \neq 0$ (*ii*) when $x_2 < 0$ and $\varepsilon \neq 0$, it has been noted that the discrete solutions, obtained by employing the discussed the classical forth order Runge-Kutta and STHW, coincide with each other (refer Tables 1-12). When $\varepsilon \geq 10^6$, the discrete solution obtained for the nuclear reactor core problem converges and remains stable.

The researcher has successfully introduced STHW which has been exclusively developed for solving non-linear system in fluid dynamics. Finally, in this chapter, it is concluded that from the table and figures, which indicate the error to be almost, less with the stiff system of higher dimension using STHW. Hence, by comparing the results obtained for the nuclear reactor core problem discussed under four cases; the STHW is more suitable for studying the nuclear reactor core problem.

REFERENCES

[1] Balachandran, K. and Murugesan, K. (1991) "Numerical Solution of a Singular Nonlinear System from Fluid Dynamics", *International* Journal *of Computer Mathematics*, **38**, pp. 211-218.

[2] Butcher, J. C. (1964) "On Runge-Kutta Processes of Higher Order", *Journal of the Australian Mathematical Society*, **4**, pp. 179.

[3] Butcher, J. C. (1987) *The Numerical Analysis of Ordinary Differential Equations*, John Wiley & Sons, U.K.

[4] Butcher, J. C. (2003) *The Numerical Methods for Ordinary Differential Equations*, John Wiley & Sons, U.K.

[5] Evans, D. J., Murugesan, K., Sekar, S. and Hyun-Min Kim. (2006) "Non-linear Singular Systems using RK-Butcher Algorithms", *International Journal of Computer Mathematics*, **83**, No. 1, pp. 131-142.

[6] Murugesan, K., Paul Dhayabaran, D. and Evans, D. J. (2000) "Analysis of Non-linear Singular System from Fluid Dynamics using Extended Runge-Kutta Methods", *International Journal of Computer Mathematics,* **76**, pp. 239-266.

[7] Nandhakumar, S., Selladurai, V. and Sekar, S. (2009) "Numerical Investigation of an Industrial Robot Arm Control Problem using Haar Wavelet Series", *American Journal of Engineering and Applied Sciences***,** **2**, No. 4, pp. 584-589.

[8] Sekar, S. and Manonmani, A. (2009) "A Study on Time-Varying Singular Nonlinear Systems Via Single-Term Haar Wavelet Series", *International Review of Pure and Applied Mathematics***,** **5**, No. 2, pp. 435-441.

[9] Sekar, S. and Manonmani, A. (2009) "A Study on Second Order Mechanical Vibratory Systems Using Single-Term Haar Wavelet Series", *International Journal of Logic Based Intelligent Systems*, **3**, No. 2, pp. 197-202.

[10] Shampine, L. F. (1985) "Some Practical Runge-Kutta Formulas", *Mathematics of Computations,* **46**, pp. 135-150.