

Predictive Modeling for Biological Oxygen Demand in Ground Water

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Abstract : In this article we describe a scientific method to predict one of the most significant water quality characteristic-the biological oxygen demand (BOD) using advanced stochastic modeling techniques. The study is based on data about 21 variables on water quality and related soil and location characteristics. The data on some of the variables are directly measured, while the others are measured through the laboratory analysis of the soil and water samples collected from 167 locations selected at random in the district of Kasaragod. We use multiple regression models to predict the level of BOD in ground water in terms of soil and water source location characteristics. The significant feature of the model is that it enables to predict the BOD of water in a location based on a set of soil and location characteristics. One of the important uses of the model is that it helps in fixing the locations for waste dump area, septic tank, digging well etc. in town planning, designing residential layouts, industrial layouts, hospital/hostel construction etc.

Keywords : Biological Oxygen Demand (BOD), Generalized Linear Model, Multiple Regression Model, Prediction, Stochastic Model, Water Quality.

1. INTRODUCTION

The quality of water is described in terms of about a dozen characteristics identifiable by means of lab tests of water samples. As per WHO standard the quality of water is described in three categories microbial, physical and chemical. The microbial parameters include Escherichia coli (E. coli) and coliform organisms, physical parameters include temperature, smell, taste, colour and turbidity and chemical parameters include pH, dissolved oxygen(DO), biological oxygen demand (BOD), total dissolved solids (TDS), total hardness (TH), chlorides (Cl), sulphates (SO₄), fluorides (F), nitrates (NO₃), calcium(Ca), magnesium(Mg), iron (Fe), etc. Each of these characteristics has different impact on water quality. In this paper we consider estimation of biological oxygen demand (BOD) in water that would be available in a location in terms of several related soil and location characteristics. BOD is described in terms of the amount of oxygen that is required in one liter of water for decomposing the organic waste in the water. If the water contains more organic waste it requires more BOD. Thus the level of water pollution due to organic waste can be identified based on the level of BOD.

Only a few studies were reported so far on BOD in water. Mehta et al.[1975]^[1] conducted a study to identify the time dependence of BOD, dissolved oxygen and temperature and built autoregressive integrated moving average (Arima) models. However they found that Arima model for BOD is not capable of giving satisfactory results. Brill Jr. et al.[1984]^[2] studied the importance of potential water quality impact in terms of BOD of transferable discharge permit(TDP) program. Carmichael et al.[2000]^[3] conducted a study to understand the usefulness of multiple pollutant management techniques by comparing single and multiple-pollutant least cost approach. Singh et al.[2004]^[4], Jha et al.[2012]^[5] and Chen et al.[2014]^[6] described the temporal and spatial variations of water quality using

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cluster analysis, factor analysis and principal component analysis. Yang et al.[2009]^[7] used the Near Infrared Region Spectrum to study the simultaneous determination of BOD and chemical oxygen on demand(COD). Yahya et al.[2012]^[8] developed simple linear regression model for water quality characteristics including BOD and studied the seasonal variation in water quality characteristics. Hur et al.[2012]^[9] studied the correlation between the water quality parameters BOD, COD and total nitrogen and explanatory variables UV absorption values at 220nm, protein-like organic substances and microbial humic group. Walega et al.[2014]^[10] developed a simple linear regression model to predict the level of BOD based on time of infiltration through a soil bed. Obaid et al.[2015]^[11] developed multiple regression model for predicting BOD during festival and rainy days based on 34 years rain fall, temperature and sewer flow data. Hsieh et al.[2016]^[12] developed a multiple regression model for BOD determination using density of the biosensor microbial fuel cell.

It is clear that water pollution due to BOD is mainly because of lack of scientific management of industrial and domestic waste, increased use of pesticides and chemical fertilizers in agriculture, poor sanitation facilities etc. Many factors can affect the quality of the water in an ecosystem including discharges of industrial and agricultural wastes, domestic and institutional wastes, contamination from leakages from septic tanks, proximity to garbage disposal sites etc. The existing practice for fixing the location for drinking water collection is mainly based on the surface distance of the water source location from the location of waste disposal. Field observations and soil testing allow us to assess the links between land use and its effects on water quality. Thus there is sufficient reason to suspect strong correlation between soil structure and water quality characteristics. Survey of literature indicates that no scientific study were done so far for the estimation of BOD in ground water using advanced stochastic models based on location and soil characteristics at national or international levels. In this study our main objective is to measure the correlation of BOD with several soil and water source location characteristics and exploit the same to estimate the level of BOD in ground water in a location.

This study is conducted in the district of Kasaragod of Kerala State. Data has been collected by the method of stratified random sampling by dividing the area into 6 strata namely Agricultural land (Pesticide using area), Coastal area, Forest area, Hospital area, Market places and Waste dump area depending on the possible levels of contamination of ground water. Since well water is the main source of drinking water in the district, we confined our study on well water. As many as 167 well water sources were selected at random ranging 25 to 35 from the neighborhood of each stratum. Soil and water samples were collected from these locations. The location characteristics observed are distance of well from septic tank, slope of well from septic tank, number of septic tanks near to the well, distance of well from nearest stratum boundary, slope of well from nearest stratum boundary and depth of the well. The soil characteristics measured are porosity, pH, electrical conductivity, organic carbon, texture, iron, sulphur, phosphorus, calcium, magnesium, copper, zinc, manganese and potassium. All these characteristics combined together we call as water source location characteristics. BOD in water and soil characteristics is identified by testing water and soil samples in the laboratory.

In this article we describe the procedure of estimating BOD in ground water using multiple regression models. The model utilizes effectively the benefit of the relationship of BOD with water source location characteristics. The whole analyses were done using SPSS.

The remaining part of this article is organized as follows: section 2 describes briefly some of the advanced statistical concepts based on which the study is made. Basic statistical characteristics of the data under study are presented in the first sub section of section 3 and the model building process and its features are discussed in the next sub section. The article closes with a conclusion, acknowledgement and the list of references.

2. STATISTICAL/MATHEMATICAL BACKGROUND

Following the terminology and convention in general linear modeling we regard the BOD in ground water as the response variable and water source location characteristics as the explanatory variables. First of all we identified the statistical properties satisfied by the response variable and explanatory variables so as to decide the kind of model and type of analysis to be performed. For this purpose descriptive statistical analysis (mean, standard deviation, skewness and kurtosis) were done. Constructing the correlation matrix between the response variable

and set of explanatory variables, we identified the highly correlated water source location characteristics. Since response variable is continuous scale variable, multiple regression model is appropriate in this case.

A multiple regression model is a model belonging to the class of general linear models. A detailed description of the model choice, fitting and the various features of the models are available in standard text books such as Yule and Kendall[1922]^[13], Koutsoyiannis[1973]^[14], Draper and Smith[1998]^[15], Hosmer and Lemeshow[2000]^[16], Montgomery, Peck and Vining[2003]^[17], etc.,.

The functional form of a multiple regression model (general linear model) is,

$$Y = X'\beta + \varepsilon ; \quad (1)$$

where Y is the response variable, X is the column vector of explanatory variables, β is the column vector of the regression coefficients and ε is the random error.

For the validity of the multiple regression models the variables in the model are to satisfy the following assumptions:

The error must follow normal distribution with zero mean and constant variance (Gaussian and homoscedastic). Explanatory variables must be non correlated (no multicollinearity).

The regression coefficients are estimated using method of least squares. The least squares estimates are

$$\hat{\beta} = (X'X)^{-1} X'Y ; \quad (2)$$

provided that $(X'X)^{-1}$ exists.

In (2), X denotes the observation matrix on explanatory variables and Y the column vector of observations on the response variable.

In fitting the above model one important element is selecting the explanatory variables for inclusion in the model. Normally all those explanatory variables that are highly correlated with the response variable are to be included. There are several statistical procedures for selecting variables for inclusion in the model such as stepwise regression, forward selection, backward elimination etc. Adequacy of the fitted model is assessed using Analysis of Variance (ANOVA) techniques which performs a F- test for the overall adequacy of the model and provides the coefficient of determination R^2 (higher the value of R^2 more appropriate the fitted model). The significance of each regression coefficients can also be tested using individual t-test provided the model assumptions are satisfied. Even after fitting the model as described above there are several techniques to verify the model adequacy as well as the extent to which the model assumptions are satisfied (post-hoc tests). Finally the model adequacy can be validated using about 10% of the real data. Further details on estimates and their properties can be had from the standard references.

3. RESULTS AND DISCUSSION

4. PRELIMINARY ANALYSIS

Table (1) and Table (2) present the basic statistical characteristics of the observed variables.

Table 1. Biological Oxygen Demand in mg/l

Mean	3.74
95% Confidence Interval for Mean	(3.47, 4.01)
Median	3.50
Variance	3.16
Minimum	0.55
Maximum	7.80
Skewness	0.26
Kurtosis	-0.74

Table 2. Water Source Location Characteristics

<i>Variable</i>	<i>Minimum</i>	<i>Maximum</i>	<i>Mean</i>	<i>Variance</i>
Distance of well from nearest stratum boundary in meters	0.50	500.00	122.86	18045.72
Slope of well from nearest stratum boundary angle in degrees	-50.50	63.40	-1.95	209.29
Depth of well in meters	4.00	20.00	10.65	13.21
Distance of well from nearest septic tank in meters	5.00	110.00	13.74	143.11
Slope of well from nearest Septic tank in degrees	-2.80	46.80	3.60	77.62
Number of septic tanks near to the well	1.00	10.00	2.00	5.92
Soil pH	3.08	7.56	5.93	0.84
Soil EC in micro Siemens	1.00	50.00	15.33	86.69
Soil organic carbon in %	0.06	5.81	2.92	2.93
Soil Potassium in kg/hector	49.28	1736.00	274.83	48678.73
Soil Sulphur in kg/hector	0.00	42.65	3.54	62.47
Soil Phosphorus in kg/hector	0.11	159.60	34.84	1362.77
Soil Porosity in %	20.00	61.55	35.68	96.05
Soil Manganese in kg/hector	0.00	146.50	33.98	1653.09
Soil Zinc in kg/hector	0.00	102.56	27.85	579.73
Soil Copper in kg/hector	0.00	99.68	14.45	255.33
Soil Iron in kg/hector	8.00	611.52	110.26	8068.28
Soil Calcium in kg/hector	25.76	22512.00	3199.93	11823201.08
Soil Magnesium in kg/hector	45.14	206.45	104.23	1005.06

As per the WHO guidelines the water quality is good when BOD is less than 2 mg/l. Table (3) gives the classification of wells in terms of WHO standard.

Table 3. Classification of Wells in Terms of WHO Standard

<i>Biological Oxygen Demand</i>	<i>Frequency</i>	<i>Percentage</i>
2	37	22.16%
> 2	130	77.84%

As indicated by Table (3) more than 77% of the well water sources in the study area are contaminated.

The average BOD level at each stratum and their diagrammatic representations are given in Table (4) and Figure (1) respectively. The results indicate that only water in the wells around the forest area meet the WHO standard.

Table 4. Comparison of Mean BOD in Various Locations

<i>Location</i>	<i>Average BOD</i>
Forest Area	1.32
Pesticide Using Area	2.53
Coastal Area	4.10
Market Places	4.54
Waste Dump Area	4.51
Hospital Area	4.83

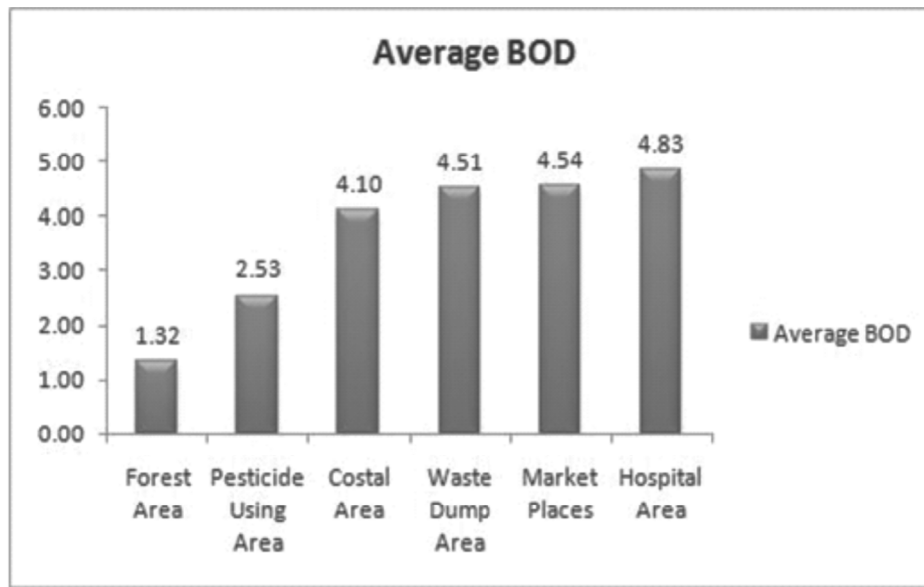


Fig. 1. Comparison of Mean BOD in Various Locations.

For comparing average BOD in various stratum locations we used ANOVA technique. Before conducting the ANOVA we have to check the normality of BOD and this is being done using Normal P-P Plot.

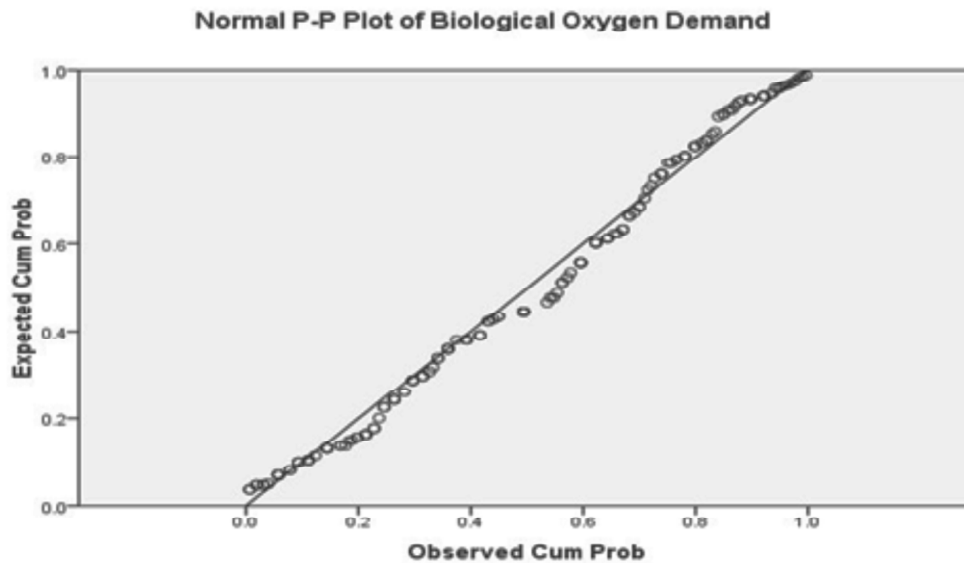


Fig. 2. Normal P-P Plot.

Figure (2), reveals that BOD data satisfies normality assumption approximately so that ANOVA can be performed.

Table 5. ANOVA Table for Comparing the Mean BOD in Various Strata

Source of Variation	SS	df	MS	F	P-Value.
Strata	2240.90	5	48.18	27.30	0.00
Error	284.11	161	1.77		
Total	525.01	166			

The P-value corresponding to the F-test in the ANOVA being highly significant we conclude that the average BOD level differs significantly over the strata.

Advanced model based analysis are performed as described below.

3.2. Model Based Analysis

Let us first examine the simple correlations between BOD level and each of the explanatory variables under study.

Table 6. Correlation Matrix

<i>Explanatory Variables</i>	<i>Water BOD</i>
Distance of well from nearest stratum boundary in meters	-0.47**
Slope of well from nearest stratum boundary angle in degrees	0.20*
Depth of well in meters	-0.16
Distance of well from nearest septic tank in meters	-0.11
Slope of well from nearest Septic tank in degrees	-0.12
Number of septic tanks near to the well	0.50**
Distance of well from nearest flowing water body in meters	0.38**
Soil pH	0.19*
Soil EC	0.33**
Soil Organic Carbon	0.37**
Soil Potassium	-0.03
Soil Sulphur	0.40**
Soil Phosphorus	0.37**
Soil Porosity	-0.16
Soil Manganese	0.33**
Soil Zinc	0.26**
Soil Copper	0.09
Soil Iron	0.16
Soil Calcium	0.23**
Soil Magnesium	0.25**

** indicates significant correlation with BOD at 5% level of significance.

* indicates significant correlation with BOD at 1 % level of significance.

Table (6) reveals that out of the 20 explanatory variables under study, 13 exhibit significant simple correlations with the response variable. This provides justification to use a multiple regression model, however, it does not x the explanatory variables to be included in the model. This is due to the fact that rather than the individual influence some of the variables may have joint influence on the response. We describe below the appropriate model selection and its features.

3.2.1. Multiple Regression Model Fitting

Stepwise regression technique is used to develop the model for BOD as it allows a free choice for each variable for inclusion or exit from the model depending on the corresponding influence. Out of the 20 explanatory variables used in the study 10 are included in the final model for predicting BOD. The fitted model information is presented in Table (7). Col.(1) of Table (7) contains the 10 explanatory variables finally included in the model, col.(2) the corresponding regression coefficients and their standard errors. Col. (3) and (4) provide the t-test statistics and the corresponding p-values testing the significance of each regression coefficient. Note that the

stepwise regression method retains only those variables with significant regression coefficient (p-value < .05), no matter whether the variable individually have significant correlation with the response or not. The entries in the last column are the variance inflation factor (VIF) which reflects the extent of multicollinearity present among the explanatory variables.

Table 7. Variables and Their Coefficients in the Regression Model

<i>Variables</i>	<i>Coefficients</i>		<i>t</i>	<i>P-value</i>	<i>VIF</i>
		<i>Std. Error</i>			
(Constant)	5.787	0.464	12.483	0.000	
Number of septic tanks near to the well (X_1)	0.093	0.055	1.683	0.095	2.435
Distance of well from nearest stratum boundary in meters (X_2)	-0.004	0.001	-6.074	0.000	1.201
Soil Porosity in % (X_3)	-0.042	0.010	-4.231	0.000	1.222
Slope of well from nearest stratum boundary angle in degrees (X_4)	0.015	0.004	3.812	0.000	1.164
Soil Organic Carbon in % (X_5)	0.274	0.058	4.763	0.000	1.372
Soil Sulphur in kg/hector (X_6)	0.059	0.015	3.988	0.000	1.718
Soil Manganese in kg/hector (X_7)	0.007	0.002	4.204	0.000	1.530
Soil Potassium in kg/hector (X_8)	-0.001	0.000	-2.912	0.004	1.170
Soil Magnesium in kg/hector (X_9)	-0.010	0.004	-2.842	0.005	1.745
Soil Phosphorus in kg/hector (X_{10})	0.007	0.003	2.510	0.013	1.366

Since VIF is less than 10 (very close to 1) there is probably no cause for multicollinearity concern. Substituting the values of regression coefficients in equation (1) we will get the regression model as,

$$Y = 5.787 + 0.093X_1 - 0.004X_2 - 0.042X_3 + 0.015X_4 + 0.274X_5 + 0.059X_6 + 0.007X_7 - 0.001X_8 - 0.010X_9 + 0.007X_{10} + \varepsilon$$

where Y – Biological Oxygen Demand in mg/l

The model accuracy measure R^2 and the overall significance test results of the model are presented in Tables (8) and (9) respectively.

Table 8. Model Summary

<i>R Square</i>	<i>Adjusted R Square</i>	<i>Std. Error of the Estimate</i>
0.69	0.66	0.94

Table 9. ANOVA Table for Testing Overall Significance of Regression Model

<i>Source of Variation</i>	<i>SS</i>	<i>df</i>	<i>MS</i>	<i>F</i>	<i>P-value</i>
Regression	206.80	10	20.68	34.44	0.00
Residual	93.66	156	0.60		
Total	300.46	166			

Table 10. Model Validation Using Randomly Selected Location Values for Well Water Sources

Sl. N.	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	BOD		
											Actual	Predicted	Relative Variation
1	1	2	58.69	1.75	1.81	0.84	0.00	80.64	55.66	2.69	3.50	3.36	3.96%
2	1	20	49.65	-34.60	0.38	0.98	10.01	172.48	82.32	0.11	2.40	2.43	1.37%
3	1	6	57.20	-0.06	1.85	0.90	79.97	448.00	104.27	125.89	4.20	3.96	5.67%
4	10	5	43.04	-0.05	1.09	30.24	107.74	257.60	174.72	102.03	7.50	6.43	14.21%
5	10	5	33.33	0.05	2.58	42.65	109.48	448.00	206.45	112.68	7.80	7.56	3.03%
6	10	10	35.61	-0.59	2.54	38.75	62.89	474.88	112.45	87.49	7.60	7.61	0.11%
7	8	15	42.41	12.50	2.08	26.04	28.45	380.8	174.72	87.47	5.25	5.67	7.94%
8	2	120	37.89	62.50	1.06	3.58	45.02	430.08	122.08	73.25	4.60	4.52	1.81%
9	1	35	41.87	-10.50	1.54	0.14	34.50	884.80	89.15	9.86	2.65	2.79	5.21%
10	1	400	47.68	-5.50	1.04	0.00	0.00	91.84	86.24	4.26	1.48	1.55	5.07%
11	1	140	28.57	3.50	3.90	0.42	0.00	224.00	106.18	57.68	4.00	4.38	9.60%
12	1	250	32.11	-17.40	0.65	2.44	22.48	71.68	85.46	14.56	2.50	2.93	17.02%
13	1	12	57.25	-1.25	0.91	0.25	13.80	109.76	87.81	0.11	2.80	2.78	0.65%
14	1	300	36.73	-10.60	1.54	2.35	3.56	120.96	156.80	12.45	1.85	1.96	6.05%
15	1	60	25.41	-6.20	4.90	0.17	0.00	204.60	80.19	42.87	5.00	5.13	2.52%
16	1	125	45.12	40.70	1.18	0.21	59.40	392.00	110.45	4.50	3.65	3.38	7.37%
17	2	10	33.80	1.60	5.81	0.42	72.13	259.84	82.88	26.43	5.65	5.76	1.88%

The tables (7)-(9) together reveal that among the 20 explanatory variables used only the 10 included in the fitted model have significant influence on BOD and the included variables do not exhibit any serious multicollinearity. The fitted model is capable of explaining up to 69% of variation in BOD and the model is very highly significant. Table (10) presents the illustration of model validation. The relative percentage variation entries in the last column of this table are computed as,

$$\text{Relative Variation} = (|\text{Actual Value} - \text{Predicted}| / (\text{Actual Value})) * 100$$

The relative variation in the last column of Table (10) reveals that except in 2 cases the value is less than 10% and 2 cases exceeding 10% have relative variation of magnitude 14% and 17% only. Admitting about 10% variation we conclude that 15 out of 17 cases agree with the model as indicated in Table (11).

Table 11. Model Validation Using Relative Variation

Relative Variation (%)	Frequency	Percentage
10%	15	88.24
> 10%	2	11.76

5. CONCLUSION

Though everybody will admit that the water quality very much depends on the soil and location characteristics this is the first scientific study to highlight their actual impacts. Starting from 20 suspected soil and location characteristics we establish here the significant influence of 10 of them on BOD, one of the very significant determinants of water quality. The magnitudes of the regression coefficients in the fitted model reveal that among the 10 influential characteristics, organic carbon, number of septic tanks around and sulphur content in the soil have relatively major positive influence on BOD. While three more components, slope of the region, manganese and phosphorus contents

in the soil also have positive contribution, all other components, distance from stratum boundary (market place, hospital area, waste dump area, etc.), soil porosity, potassium and magnesium contents in soil have negative influence. The most significant feature of the model developed here is that using it one can determine the quality (in terms of BOD) of the water in a location by observing the set of 10 water source characteristics. This knowledge can be used to fix the locations for digging wells, waste dumping, market places etc. while planning residential and industrial lay outs, town planning etc. Similar models can also be derived for several other important water quality characteristics.

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