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# A STUDY ON THE EFFICIENCY OF OPTION MARKETS: SHORT-TERM VERSUS LONG-TERM: AN IMPLIED VOLATILITY APPROACH

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***Abstract:*** *Existing studies on the efficiency of option markets rarely discuss the separation between short-term and long-term markets. The theory of efficient markets is uniformly applied to both short- and long-term strategies. In this paper, we use informational content of implied volatility to examine the efficiency of both short-term and long-term S&P 500 index options based on the fact that implied volatility is the market expectation of underlying return volatility, provided that option markets are efficient, option pricing models are correctly specified, and the maturity effect on the sensitivity of the implied volatility estimation. From the theoretical perspective, we expect that implied volatility from long-term options contains more information about future volatility if both long- and short-term option markets have the same levels of efficiency. From both theoretical and empirical results we find that implied volatilities from short-term option markets contain more information about future return volatility of the S&P 500 index and the short-term index option markets have a higher level of efficiency. Our results are consistent with the high liquidity of short-term option markets.*

***Keywords:*** *Option, Market efficiency, Implied volatility*

***JEL Classification: C18, G17***

## 1. INTRODUCTION

Index options have been one of the most successful innovative financial instruments as indicated by its high trading volume. They enable investors to gain exposure to the market as a whole or to specific segments of the market with one trading decision and frequently with one transaction, and they permit portfolio managers to limit downside risk. Given their prominence and functions, the pricing efficiency of these markets is of great importance to academics, practitioners, and regulators. To detect inefficient pricing (often called mispricing) requires computing a theoretically efficient price or price range and comparing it with prices of options traded in financial markets. However valuing an index option in theory is complicated and challenging.

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Even though the Black-Scholes model has been heavily criticized since its inception, empirical results (e.g. Jackwerth (1999), Alexander & Nogueira (2004), among others), however, show that many of the sophisticated alternative option pricing models (e.g., Heston (1993), Dupire (1994), Hagan *et al* (2002), among others) are not only hard to implement but also do not fit the observed option prices either and some even perform worse than Black-Scholes. Furthermore, the sophisticated alternative models have some common drawbacks: the parameters are not always linked to observable and/or intuitive financial concepts, the calibration process must be run frequently (e.g.daily), and the pricing "simple" formulas are generally available only for the European-style plain payoffs. Einstein (1989) demonstrated that the Black-Scholes valuation framework can recover unbiased estimates of implied volatility from stochastic volatility models. Good financial models must be clean and simple as pointed out by Derman and Wilmott (2009). Due to its relatively simple structure compared to the alternatives, its robustness and its clear engineering mean, the Black-Scholes Model can still be considered the most accurate and convenient available option pricing model, especially for pricing European-style options.

Implied volatility is widely interpreted as the market expectation of underlying return volatility over the remaining life of the option provided that option pricing models are correctly specified and option markets are efficient. If the option pricing model is reasonable and correctly specified, the informational content of implied volatility can infer the market efficiency. Thus implied volatility approach becomes one of the most popular approaches to examining the efficiency of option markets. Examples of other efficiency tests are Black and Scholes (1972), Bodurtha and Courtadon (1986), Poon and Pope (2000), among others.

However the implied volatility actual computation process can have two relevant classes of difficulties. The first one is that to estimate the volatility surface is itself a very hard problem, where "degrees of freedom" may result in very different outputs. The second feature, as pointed out by Hentchel (2003) and many others, is that the presence of measurement errors like finite quote precision, bid-ask spread, non-synchronous pricing, or infrequent trading (for options away from money) can have a significant effect on the implied volatility estimation. We will discuss the implied volatility computation issues in the next section and demonstrate analytically the maturity effect on the implied volatility estimation in the presence of measurement errors: at shorter (longer) maturities, small pricing error can have greater (smaller) impact on implied volatilities. We thus infer that implied volatility from long-term options contain more information about future volatility if both long- and short-term option markets have the same level of efficiency; we thus then infer short-term markets are more efficient if short-term implied volatility contains more information about future volatility.

The outline of the remainder of the paper is as follows. In section 2, we discuss the issues incurred in volatility surface computation. In section 3, we redemonstrate the sensitivity of the price error to the implied volatility estimation at different maturities as seen in Wang (2008). In section 4, we introduce the data and empirical testing methodology to test the efficiency of index option markets at different maturities. In section 5, we compare the level of efficiency inferred in the implied volatilities from both short-term and long-term index options. Section 6 concludes.

## 2. THE VOLATILITY SURFACE COMPUTATION

We first want to explain some theoretical reasons and some market practices that can lead to quite a different "freeze" of the implied volatility surface at the same time $t$ for the same index $I$. Let $X$ be the strike and $T$ the time to maturity of the option. To compute implied volatility, we need option market prices. The first question is: from where do we get the option prices? Some providers or operators prefer to take the prices from the stock exchange, such as CBOE, EUREX, IDEM, where the options are listed with standard conditions and a well-defined "grid" for $(T,X)$. The weakness of this approach is that for several underlying factors, the markets do not quote long maturity (i.e. > 2 years) options, hence it is difficult to extrapolate the longer volatility structure needed to price or hedge long-term OTC options. Some other providers prefer to take the implied volatility directly from a panel of large investment banks or broker. Once the contributors have given the data some cleaning and rejected outliers, smoothing procedures are applied. Another problem is related to the selection of the available options for the surface is that some very deep-out-of-the-money options are illiquid and the prices are not reliable: what rule or threshold to apply in order to detect and reject the information? To this extent, we also recall that call and put options with the same strike and maturity are linked by the put-call parity: due to even infinitesimal arbitrage opportunities, to take into account both may imply some redundant dangerous information. Another myth in the implied volatility computation is that the Black-Scholes inversion depends fully on "given" parameters. We recall that for the free-risk rate, the market operators may adopt two different approaches: the best practice is now to build the zero coupon curve through a bootstrap process, by merging the money market rates (e.g. EURIBOR rates) with swap rates. The old classical approach uses the *T-bill* and *T-bond* rates. For the dividends, the differences may be even larger: only in the short term are dividends deterministic.

Another issue relates to the representation of the surface. In the standard theoretical notation, the surface is indexed by the $(T,X)$ couple. But now consider some requirements that arise from evaluation or hedging processes. From time $t$ to the next $(t+1)$, how does the volatility change? Given the same time maturity and the same moneyness, how should the surfaces of two different indexes be compared? In other words, we are speaking about a *longitudinal time series* analysis or a *cross section* analysis. If we think that the volatility depends on the moneyness of the option, it is then better to index the surface by some moneyness measure, such as the put moneyness $m = X/S$.

Some other features may be dealt but the above points lead to an important conclusion: different types of setup, parameters, and methodologies in computing the implied volatility surface may determine different outputs, so it has to be thought as an estimation process, not as a "deterministic" one. Since, however, our empirical test focuses on at-the-money or near-the-money implied volatilities from options with maturities of three months or less, most of the above mentioned problems can be avoided in our study.

## 3. OPTION MATURITY IMPACT ON THE IMPLIED VOLATILITY ESTIMATION

Existing study, such as Bodurtha and Courtadon (1986) and Poon and Pope (2000), on the efficiency of option markets rarely discuss the division between short-term and long-term markets and the efficient market results are commonly applied uniformly across the board to both short- and long- term strategies. In this section, we will focus on the option maturity effect on the implied volatility estimation from a theoretical point of view.

As shown in Wang (2012), like stock options, the call prices of index options, denoted as $C$, are a function of the following six underlying parameters: the index level $S$, the exercise price of the option $X$, the time to maturity of the option $T$, the risk free rate of interest $r$, the volatility (annual standard deviation of the index return) $\sigma$, and the annual dividend $D$:

$$C = f(S, X, T, r, \sigma, D) \tag{1}$$

Among the above six variables, the exercise price and time to maturity of the option are stated in the terms of option contract; the stock index level and the risk-free rate of interest are market-determined values. Since most firms tend to pay stable quarterly dividends during the calendar year, little uncertainty exists about the dividend for short-term (three months or less) index options. The only unknown parameter is the future market volatility of the index return. Since the option index level is observable, the future market implied volatility then can be backed out through option pricing model. However, as pointed out in previous research, such as Hentschel (2003), that estimation of the implied volatility, that is, by inverting the option pricing model, is subject to considerable errors when option characteristics are observed with plausible errors stemming from finite quote precision, bid-ask spreads, non-synchronous observations, and other measurement errors.

By its definition, the *vega* of a call option $\left(\frac{\partial C}{\partial \sigma}\right)$ measures the sensitivity of the option price to the underlying stock's return volatility and the reciprocal of the *vega* $\left(\frac{\partial \sigma}{\partial C}\right)$ measures the sensitivity of the implied volatility estimation to the option price change. Though it is known that the *vega* of an option varies with time-to-maturity (see, for instance, Hull (2008)), the following **theorem** shows exactly how the sensitivity of option price to the volatility varies with maturities in the Black-Scholes model setting (the proof is similar to what shown in Wang (2012)).

***Theorem:*** *Within the Black-Scholes model setting, the option vega* $\frac{\partial C}{\partial \sigma}$ *(as a function of maturity T) increases as maturity increases up to the point* $\overline{T}$ *(defined below), then declines as the maturity continues to increase from* $\overline{T}$.

$$\overline{T} = \frac{1 + \sqrt{1 + 4.\frac{\left(\ln \frac{S}{X}\right)^2}{\sigma^2}\left(.\left(\frac{r-d}{\sigma} + \frac{\sigma}{2}\right)^2 + 4d\right)}}{2\left(\frac{r-d}{\sigma} + \frac{\sigma}{2}\right)^2 + 4d} \tag{2}$$

where *S* denotes underlying stock price, *d* as dividend yield, and *r* being risk-free interest rate.

***Note***: *For at-the-money option,* $\overline{T}$ *reaches its minimum and reduces to*

$$\overline{T}_{\min} = \frac{1}{\left(\frac{r-d}{\sigma} + \frac{\sigma}{2}\right)^2 + 2d} \tag{3}$$

$\overline{T}_{\min}$ *is typically larger than 0.25 years (3 months) (typically,* <10% and $5\% < \sigma < 100\%$).

From the above **theorem** we know immediately that $\frac{\partial \sigma}{\partial C}$ declines as T increases from 0 to $\overline{T}$, holding strike constant, which indicates that as T increases up to $\overline{T}$ (typically larger than 3 months), the sensitivity of the price error to the implied volatility estimation declines. When $T < \overline{T}$, the shorter the maturity, the more sensitive the price error is to the implied volatility estimation.

The implication of the above **theorem** has three folds. First, unless short term options markets are fairly efficient, that is, options are fairly priced, their implied volatilities contain little information about the future true volatility due to the high sensitivity of the implied volatilities to the price errors. Secondly, if option markets are reasonably efficient, the longer term (but less than $\overline{T}$ ) option implied volatilities should better represent future true volatility due to the lower sensitivity of long-term option's implied volatility to the price errors. Thirdly, if there is evidence shows that short-term implied volatility contains more information about future volatility than long-term implied volatility does, one can infer that short-term markets are more efficient. In Sections 4 and 5, we empirically test the informational content of both short-term and long-term implied volatility to infer the option market efficiency.

## 4. DATA AND METHODOLOGY

In this section, we empirically test the informational content of the implied volatility of short-term and long-term options to infer the efficiency level of both short-term and long-term option markets. Following conventional testing method, we regress the subsequent realized volatility against at-the-money implied volatility (ATMIV) (as shown in Wang, *et al.* (2012), near-the-money implied volatilities provide better measures of future volatilities).

$$LRV_t = \alpha_0 + \alpha_i \, LATMIV_t + e_t \tag{4}$$

where $LRV_t$ denotes the log realized volatility for period $t$ and $LATMIV_t$ denotes the log at-the-money implied volatility at the beginning of period $t$ and $e_t$ estimation noise. We use logarithmic transformed data to better conform the normality assumption. We test the following two hypotheses. Firstly, we test whether implied volatility measure (ATMIV) contains any information about future volatility; if it does not, the slope coefficient $\alpha_i$ should be zero and this leads to the first testable hypothesis $H_0 : \alpha_i = 0$. Next, we test whether volatility forecast is an unbiased estimate of future realized volatility; if this is so, the slope coefficient $\alpha_i$ should be 1 and the intercept $\alpha_0$ should be zero. This testable hypothesis can be formulated as $H_0 : \alpha_0 = 0$ and $\alpha_i = 1$. We use S&P 500 index option data sample running from January 3, 2000 to January 7, 2005. We obtained historical S&P 500 index options data and daily dividends from OptionMetrics and daily Treasury bill yields (the risk-free rates) from the Federal Reserve Bulletin. Dividend adjusted daily returns are from CRSP. We test the informational content of implied volatilities from options with maturities of one month, two months, and three months. We did not include longer maturity options for the following reasons. (1) Samples obtained from longer maturities exhibit higher degree of overlap which can render the OLS test statistics invalid. (2) A constant volatility assumption over a long period of time is not appropriate since volatility does change. (3) Short maturity reduces the possible variation in implied volatility estimation due to the methodologies and data sources as mentioned in Section 3.

## 5. INFORMATION CONTENT OF IMPLIED VOLATILITY

We report the testing results in the following table. Results shown here strongly reject both hypotheses in all cases. The fact that the slope coefficients are significantly different from zero at all conventional significant levels implies that implied volatility measures contain some information about future volatility. The extremely low F-tests values also reject the joint hypothesis $H_0 : \alpha_0 = 0$ and $\alpha_i = 1$ f at 1% significant level in all three cases, which indicates that the implied volatility forecasts are biased forecasts of future volatility. The Durbin-Watson (DW) statistics for the one-month case are not significantly different from two, indicating that the regression residuals are not auto-correlated for one-month data. The Durbin-Watson (DW) statistics for both two- and three-month cases are lower due to the overlapping data used. We expected that longer maturity option implied volatilities contain more information about the future volatility over their corresponding forecast horizons than one-month implied volatility due to lower sensitivity of longer term option implied volatility estimation to the price error if both short and long-term markets have same level of efficiency. However, our expectation is not supported by the empirical testing results. In fact, as shown in the table, the regression goodness-of-fit (the adjusted $R^2$) appears to decline as maturity increases from one month to two months to three months, indicating that the price errors for longer term options are larger, thus indicating lower efficiency of longer term option markets.

**Table**
**Realized Volatility Regressed on Implied Volatility**

| | *One Month* | *Two Months* | *Three Months* |
|---|---|---|---|
| Intercept | 0.0808 | 0.10381 | 0.06788 |
| | (0.42) | (0.48) | (0.27) |
| Implied Volatility | 1.13468 | 1.14446 | 1.12338 |
| | (9.61) | (8.54) | (7.28) |
| Adj. R-Square | 0.6114 | 0.5578 | 0.4767 |
| DW | 1.802 | 1.071 | 0.579 |

The data consisst of 59 (58 for two-month and three-month cases) implied volatilities and realized volatilities observations from January 3, 2005 through January 7, 2005. The two-month and three-month implied volatility sample used in the test contains a minor degree overlap problem. The numbers in parentheses are t-statistics and the F-statistic for joint hypotheses and $H_0 : \alpha_0 = 0$ and $\alpha_i = 1$, $F_{one\text{-}month}$ (2,57) = 0.0003, $F_{two\text{-}month}$ (2,56) = 0.0007, $F_{three\text{-}month}$ (2,56) = 0.0015.

## 6. CONCLUSION

The common assumption often drawn from research of market efficiency is that this theory applies uniformly across the board to both short- and long-term markets. In practice this is far from the case because of the many structural factors that interfere with making markets efficient at all levels. In this paper we use informational content of implied volatility to examine the efficiency of option markets. We use the theoretical evidence of the maturity effect on the implied volatility estimation in the presence of measurement errors and examine efficiency in both short and long-term option markets. Our study shows that the sensitivity of implied volatility estimation with an equivalent magnitude of price error is supposed to decline as option maturity increases and thus implied volatility is expected to contain more information about future volatility for the long–term option markets. Contrary to what is expected, the empirical test results show that the implied volatility contains less information as option maturity increases, indicating the increased inefficiency of long term option markets. We attribute such results to the lower liquidity of longer term option markets.

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