IJMMS, Vol. 13, No. 1, (January-June 2017):167-176

ISSN: 0973-3329

ANGULAR DISPLACEMENT IN A SHAFT III AND $\bar{H}$-FUNCTION WITH GENERAL CLASS OF POLYNOMIALS

Parul Gupta*[*] and Ashok Singh Shekhawat[**]

Abstract

The importance of using special functions and their applications in space research and other technical application has given rise to several problem of special function.

The object of present paper is to discuss the application of certain products involving $\bar{H}$-functions ([3],[4]), a general class of polynomial [10] and generalized polynomial [(9)] in obtaining a solution of partial differential equation

$$\frac{\partial^2 \psi}{\partial t^2} = R^2 \frac{\partial^2 \psi}{\partial x^2}$$

concerning to a problem of angular displacement in a shaft. The established result may be found useful in several interesting situation related to applied mathematics, mathematical analysis and physics.

Classification code: *33EXX, 33E30*

Keywords: *$\bar{H}$-functions, general class of polynomial, partial differential Equations, generalized polynomial.*

1. INTRODUCTION

We consider the problem of determining the twist $\psi(x, t)$ in a shaft of circular section with its axis along the x-axis. This displacement $\psi(x, t)$ due to initial twist must satisfy the boundary value problem, if we assume that both ends $x = 0$ and $x = \lambda$ of shaft are free.

* Department of Mathematics, Suresh Gyan Vihar University, Jaipur, Rajasthan, India. *Email: parul.gupta@live.in*

** Department of Mathematics, Suresh Gyan Vihar University , Jaipur, Rajasthan, India. *Email: csmaths2004@yahoo.com*

$$\frac{\partial^2\psi}{\partial t^2} = R^2\frac{\partial^2\psi}{\partial x^2} \tag{1.1}$$

where, R is constant.

$$\frac{\partial\psi}{\partial x}(0,t) = 0, \frac{\partial\psi}{\partial t}(x,0) \text{ and } \psi = (x,0) = f(x) \tag{1.2}$$

Let

$$f(x) = \left(\sin\frac{\pi x}{2\lambda}\right)^{2\rho-\mu-1}\left(\cos\frac{\pi x}{2\lambda}\right)^{\mu-1} S_{N_1,\dots,N_s}^{M_1,\dots,M_s}\left[y_1\left(\tan\frac{\pi x}{2\lambda}\right)^{2h_1},\dots,y_s\left(\tan\frac{\pi x}{2\lambda}\right)^{2h_s}\right]$$

$$S_{N'}^{M'}\left[y'\left(\tan\frac{\pi x}{2\lambda}\right)\right]^{2h'} S_{N''}^{M''}\left[y''\left(\tan\frac{\pi x}{2\lambda}\right)\right]^{2h''} \bar{H}_{P,Q}^{M,N}\left[z\left(\tan\frac{\pi x}{2\lambda}\right)\right]. \tag{1.3}$$

(a) The generalized polynomial defined by Srivastava [9] is given by

$$S_{N_1,\dots,N_s}^{M_1,\dots,M_s}[x_1,\dots,x_s] = \sum_{\alpha_1=0}^{[N_1/M_1]}\cdots\sum_{\alpha_s=0}^{[N_s/M_s]}\frac{(-N)_{M_1\alpha_1}}{\alpha_1!}\cdots\frac{(-N_s)_{M_s\alpha_s}}{\alpha_s!}$$

$$.\ B[N_1\alpha_1;\dots;N_s\alpha_s]x_1^{\alpha_1}\dots x_s^{\alpha_s} \tag{1.4}$$

where, $N_{i'} = 0,1,2,\dots\forall i' = (1,\dots,s), M_1,\dots,M_s$ are arbitrary positive integers and coefficient $[N_1\alpha_1;\dots;N_s\alpha_s]$ are arbitrary constant, real or complex.

(b) The general class of polynomials defined by Srivastava [10] is given by

$$S_N^M[x] = \sum_{\alpha=0}^{[N/M]}\frac{(-N)_{M\alpha}}{\alpha!}B[N,\alpha]x^\alpha \tag{1.5}$$

(c) The $\bar{H}$-function defined by Inayat-Hussain [(3),(4)] is

$$\bar{H}_{P,Q}^{M,N}[z] = \bar{H}_{P,Q}^{M,N}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right],$$

$$= \frac{1}{2\pi i}\int_{-i\omega}^{+i\omega}\varphi(\xi)z^\xi d\xi, \tag{1.6}$$

where, $i = \sqrt{-1}$ and

$$\varphi(\xi)=\frac{\prod_{j=1}^{M}\Gamma(b_j-B_j\xi)\prod_{j=1}^{N}\{\Gamma(1-a_j+\alpha_j\xi)\}^{A_j}}{\prod_{j=M+1}^{Q}\{\Gamma(1-b_j+\beta_j\xi)\}^{B_j}\prod_{j=N+1}^{P}\Gamma(a_j-\alpha_j\xi)} \quad (1.7)$$

where, $a_j\,(j=1,...,P)$ and $b_j\,(j=1,...,Q)$ are complex parameters, $a_j \geq 0$ (j = 1, …, P) and the exponents A_j (j = 1, …, N) and B_j(j = M + 1, …, Q) can take on non-integer values, when these exponents take integer value.

The behaviour of $\overline{H}$-function for small values of $|z|$ follows easily from a result recently given by Rathie [7], we have

$$\overline{H}_{P,Q}^{M,N}[z]=0(|z|^{\alpha}),\ \alpha=\min_{1\leq j\leq m}[\mathrm{Re}(b_j/\beta_j)],\ |z|\to 0 \quad (1.8)$$

2. MAIN INTEGRAL

The integral to be established here is:

$$\int_0^{\lambda}\left(\cos\frac{\pi\rho x}{\lambda}\right)\left(\sin\frac{\pi x}{2\lambda}\right)^{2\rho-\mu-1}\cos\left(\frac{\pi x}{2\lambda}\right)^{\mu-1}S_{N'}^{M'}\left[y'\left(\tan\frac{\pi x}{2\lambda}\right)\right]^{2h'}$$

$$S_{N''}^{M''}\left[y''\left(\tan\frac{\pi x}{2\lambda}\right)\right]^{2h''}S_{N_1,\ldots,N_s}^{M_1,\ldots,M_s}\left[y_1\left(\tan\frac{\pi x}{2\lambda}\right)^{2h_1}\ldots y_s\left(\tan\frac{\pi x}{2\lambda}\right)^{2h_s}\right]$$

$$\overline{H}_{P,Q}^{M,N}\left[z\left(\tan\frac{\pi x}{2\lambda}\right)^{2k}\right]dx$$

$$=\frac{\lambda.2^{(2\rho-\mu+2\mu'\alpha'+2\mu''\alpha''+\alpha\sum_{i=1}^{s}h_i\alpha_i)}}{\Gamma 2\rho\sqrt{\pi}}\sum_{\alpha'=0}^{N'/M'}\cdot\sum_{\alpha''=0}^{N''/M''}\sum_{\alpha_1=0}^{[N_1/M_1]}\cdots\sum_{\alpha_s=0}^{[N_s/M_s]}$$

$$B'[N',\alpha']B''[N'',\alpha'']B[N_1,\alpha_1;...;N_s,\alpha_s](y')^{\alpha'}(y'')^{\alpha''}y_1^{\alpha_1}\ldots y_s^{\alpha_s}.\overline{H}_{P+2,Q+1}^{M+1,N+1}$$

$$\left[z.4^k\left|\begin{matrix}\left(1-\rho+\frac{\mu}{2}-h'\alpha'-h''\alpha''-\sum_{i=1}^{s}h_i\alpha_i,k;1\right),(a_j,\alpha_j;A_j)_{1,N},\\(a_j,\alpha_j)_{N+1,P},\left(\mu-h'\alpha'-h''\alpha''-\sum_{i=1}^{s}h_i\alpha_i,2k\right)\\\left(\frac{1}{2}-\rho+\frac{\mu}{2}-h'\alpha'-h''\alpha''-\sum_{i=1}^{s}h_i\alpha_i,k\right),(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right] \quad (2.1)$$

where, $h_i > 0, h', h'' > 0 (i = 1,...,s), h > 0$, $2\rho > \text{Re}\left(\mu - 2h\frac{b_j}{\beta_j}\right) > 0 (j = 1,...,\text{M})$, M is an arbitrary integer and the coefficient $\text{B}(\text{N}_1,\alpha_1;...;\text{N}_s,\alpha_s]$, $\text{B}'(\text{N}',\alpha'),\text{B}''(\text{N}'',\alpha'')$ are arbitrary constant, real or complex.

3. EVALUATION OF (2.1)

The integral in (2.1) can be established by making use of the $\overline{\text{H}}$ -functions in terms of Mellin-Barnes contour integral given by (1.6) and the definition of generalized polynomials given by (1.4). The general class of polynomial given by (1.5), then inter changing the order of summation and integration, evaluate the inner integral with the help of a result given by Chaurasia and Gupta ([12], p.59), we arrive at desired result.

4. SOLUTION OF PROBLEM

The solution of problem to be established is

$$\psi(x,t) = \frac{1}{2^{\mu}\sqrt{\pi}} \sum_{\alpha_1=0}^{[\text{N}_1/\text{M}_1]} \cdots \sum_{\alpha_s=0}^{[\text{N}_s/\text{M}_s]} \sum_{\alpha'=0}^{[\text{N}'/\text{M}']} \sum_{\alpha''=0}^{[\text{N}''/\text{M}'']} \frac{(-\text{N}')_{\text{M}'\alpha'}}{\alpha'!} \frac{(-\text{N}'')_{\text{M}''\alpha''}}{\alpha''!}$$

$$\frac{(-\text{N}_1)_{\text{M}_1\alpha_1}}{\alpha_1!},...,\frac{(-\text{N}_s)_{\text{M}_s\alpha_s}}{\alpha_s!} \text{B}'(\text{N}',\alpha')\text{B}''(\text{N}'',\alpha'')\text{B}(\text{N}_1,\alpha_1;...;\text{N}_s\alpha_s)(y')^{\alpha'}(y'')^{\alpha''}$$

$$y_1^{\alpha_1},...,y_s^{\alpha_s} \frac{2^{\left(2\rho+2\sum_{i=1}^{s} h_i\alpha_i + h'\alpha' + h''\alpha''\right)}}{\Gamma(2\rho)} \overline{\text{H}}_{\text{P}+2,\text{Q}+1}^{\text{M}+1,\text{N}+1}$$

$$\left[z.4^k \middle| \begin{matrix} \left(1-\rho+\frac{\mu}{2}-h'\alpha'-h''\alpha''-\sum_{i=1}^{s} h_i\alpha_i, k; 1\right), (a_j,\alpha_j;\text{A}_j)_{1,\text{N}}, \\ (a_j,\alpha_j)_{\text{N}+1,\text{P}}, \left(\mu - h'\alpha' - h''\alpha'' - \sum_{i=1}^{s} h_i\alpha_i, 2k\right) \\ \left(\frac{1}{2}-\rho+\frac{\mu}{2}-h'\alpha'-h''\alpha''-\sum_{i=1}^{s} h_i\alpha_i, k\right), (b_j,\beta_j)_{1,\text{M}}, (b_j,\beta_j;\text{B}_j)_{\text{M}+1,\text{Q}} \end{matrix} \right]$$

$$\left(\cos\frac{\pi\eta x}{\lambda}\right)\left(\cos\frac{\pi\eta \text{R}'t}{\lambda}\right), \tag{4.1}$$

which holds true under the same conditions needed for (2.1).

5. DERIVATION ON (4.1):

The solution of problem can be written as {[6], Churchill 1941, p.125(4)}.

$$\psi(x,t)=\frac{1}{2}a_0+\sum_{\eta=1}^{\infty} a_\eta\left(\cos\frac{\pi\eta x}{\lambda}\right)\left(\cos\frac{\pi\eta R't}{\lambda}\right), \tag{5.1}$$

where, $a_\eta(\eta=0,1,2,...)$ are the coefficients in Fourier cosine series for $f(x)$ in the interval $(0,\lambda)$ if $t=0$, then by virtue of (1.3), we get

$$\left(\sin\frac{\pi x}{2\lambda}\right)^{2\rho-\mu-1}\left(\cos\frac{\pi x}{2\lambda}\right)^{\mu-1} S_{N_1,...,N_s}^{M_1,...,M_s}\left[y_1\left(\tan\frac{\pi x}{2\lambda}\right)^{2h_1},...,y_s\left(\tan\frac{\pi x}{2\lambda}\right)^{2h_s}\right]$$

$$S_{N'}^{M'}\left\{y'\left(\tan\frac{\pi x}{2\lambda}\right)^{2h'}\right\}S_{N''}^{M''}\left\{y''\left(\tan\frac{\pi x}{2\lambda}\right)^{2h''}\right\}\bar{H}_{P,Q}^{M,N}\left[z\left(\tan\frac{\pi x}{2\lambda}\right)^{2k}\right]$$

$$=\frac{1}{\alpha}a_0+\sum_{\eta=1}^{\infty}a_\eta\left(\cos\frac{\pi x}{\lambda}\right), \tag{5.2}$$

Multiplying both sides of (5.2) by $\left(\cos\frac{\pi\rho x}{\lambda}\right)$ and integrating with respect to x from 0 to λ, we have

$$\int_0^{\lambda}\left(\cos\frac{\pi\rho x}{\lambda}\right)\left(\sin\frac{\pi x}{2\lambda}\right)^{2\rho-\mu-1}\cos\left(\frac{\pi x}{2\lambda}\right)^{\mu-1}$$

$$S_{N_1,...,N_s}^{M_1,...,M_s}\left[y_1\left(\tan\frac{\pi x}{2\lambda}\right)^{2h_1}...y_s\left(\tan\frac{\pi x}{2\lambda}\right)^{2h_s}\right]S_{N'}^{M'}\left[y\left(\tan\frac{\pi x}{2\lambda}\right)\right]^{2h'}$$

$$S_{N''}^{M''}\left[y''\left(\tan\frac{\pi x}{2\lambda}\right)\right]^{2h''}\bar{H}_{P,Q}^{M,N}\left[z\left(\tan\frac{\pi x}{2\lambda}\right)^{2k}\right]=\frac{1}{\alpha}a_0\int_0^{\lambda}\left(\cos\frac{\pi\rho x}{\lambda}\right)dx$$

$$+\sum_{\eta=1}^{\omega}a_\eta\int_0^{\lambda}\left(\cos\frac{\pi\eta x}{\lambda}\right)\left(\cos\frac{\pi\rho x}{\lambda}\right)dx \tag{5.3}$$

Using (2.1) along with orthogonality property of cosine functions, we get

$$a_\eta = \sum_{\alpha'=0}^{[N'/M']} \sum_{\alpha''=0}^{[N''/M'']} \sum_{\alpha_1=0}^{[N_1/M_1]} \cdots \sum_{\alpha_s=0}^{[N_s/M_s]} \frac{(-N')_{M'\alpha'}}{\alpha'!} \frac{(-N'')_{M''\alpha''}}{\alpha''!}$$

$$\frac{(-N_1)_{M_1\alpha_1}}{\alpha_1!},\ldots,\frac{(-N_s)_{M_s\alpha_s}}{\alpha_s!} B(N_1,\alpha_1;\ldots;N_s\alpha_s)(y')^{\alpha'}(y'')^{\alpha''}$$

$$y_1^{\alpha_1},\ldots,y_s^{\alpha_s} \frac{2^{2\eta-\mu+h'\alpha'+h''\alpha''+\sum_{i=1}^{s} h_i\alpha_i+1}}{\Gamma 2\eta\sqrt{\pi}} \overline{H}_{P+2,Q+1}^{M+1,N+1}$$

$$\left[z.4^k \left| \begin{array}{l} \left(1-\rho+\frac{\mu}{2}-h'\alpha'-h''\alpha''-\sum_{i=1}^{s} h_i\alpha_i, k;1\right),(a_j,\alpha_j;A_j) \\ \left(\frac{1}{2}-\eta+\frac{\mu}{2}-h'\alpha'-h''\alpha''-\sum_{i=1}^{s} h_i\alpha_i, k\right),(b_j,\beta_j)_{1,M},(a_j,\alpha_j)_{N+1,P} \\ \left(\mu-h'\alpha'-h''\alpha''-\sum_{i=1}^{s} h_i\alpha_i, 2k.\right),(b_j,\beta_j;B_j)_{M+1,Q} \end{array} \right. \right] \quad (5.4)$$

Using (5.1) and (5.4), we arrive at the desired solution in (4.1).

6. SPECIAL CASES

1. Taking M = 1, N = 3, P = Q and replacing z by – z and using

$$g(\gamma,\eta,\nu,p;z) = \frac{a_{d-1}\,\Gamma(p+1)\,\Gamma(\frac{1}{2}+\frac{\nu}{2})}{(-1)^p\, 2^{2+p}\sqrt{\pi}\,\Gamma(\gamma)\Gamma(\gamma-\frac{\nu}{2})}$$

$$.\overline{H}_{3,3}^{1,3}\left[-z \left| \begin{array}{l} (1-\gamma,1;1),(1-\gamma+\frac{\nu}{2},1;1),(1-\eta,1;1+p) \\ (0,1),(-\frac{\gamma}{2},1;1),(-\delta,1;1+p) \end{array} \right. \right],$$

where, $a_d = \frac{2^{1-d}\,\pi^{-d/2}}{\Gamma(d/2)}$ [9,p.4121, eq.(5)]

The above function is connected with certain class of Feynman integrals.

We obtain from (2.1)

(a) $$\int_0^{\lambda}\left(\cos\frac{\pi\rho x}{\lambda}\right)\left(\sin\frac{\pi x}{2\lambda}\right)^{2\rho-\mu-1}\left(\cos\frac{\pi x}{2\lambda}\right)^{\mu-1} S_{N'}^{M'}\left[y'\left(\tan\frac{\pi x}{2\lambda}\right)^{2h'}\right]$$

$$S_{N''}^{M''}\left[y_1\left(\tan\frac{\pi x}{2\lambda}\right)^{2h''}\right] S_{N_1,\ldots,N_s}^{M_1,\ldots,M_s}\left[y_1\left(\tan\frac{\pi x}{2\lambda}\right)^{2h_1},\ldots,y_s\left(\tan\frac{\pi x}{2\lambda}\right)^{2h_s}\right]$$

$$g\left[\gamma,\eta,\nu,p;z\left(\tan\frac{\pi x}{2\lambda}\right)^{2k}\right]$$

$$=\frac{a_{d-1}\Gamma(p+1)\,\Gamma\left(\frac{1}{2}+\frac{\nu}{2}\right)}{(-1)^p\,2^{2+p}\sqrt{\pi}\,\Gamma(\gamma)\Gamma\left(\gamma-\frac{\nu}{2}\right)}\;\frac{\lambda 2^{2\rho-\mu+2h'\alpha'+2h''\alpha''+2\sum_{i=1}^{s}h_i\alpha_i}}{\Gamma 2\rho}$$

$$\sum_{\alpha'=0}^{[N'/M']}\sum_{\alpha''=0}^{[N''/M'']}\sum_{\alpha_1=0}^{[N_1/M_1]}\cdots\sum_{\alpha_s=0}^{[N_s/M_s]}\frac{(-N')_{M'\alpha'}}{\alpha'\,!}\cdots\frac{(-N'')_{M''\alpha''}}{\alpha''!}\frac{(-N_1)_{M_1\alpha_1}}{\alpha_1\,!},\ldots,$$
$$\frac{(-N_s)_{M_s\alpha_s}}{\alpha_s\,!}$$

$$B'(N'\alpha')\,B''(N''\alpha'')\,B[N_1\alpha_1;\ldots;N_s\alpha_s](y')^{\alpha'}(y'')^{\alpha''}y_1^{\alpha_1}\ldots y_s^{\alpha_s}$$

$$\bar{H}_{5,4}^{2,4}\left[-z.4^k\left|\begin{matrix}\left(1-\eta+\frac{\mu}{2}-h'\alpha'-h\alpha''-\sum_{i=1}^{s}h_i\alpha_i,k;1\right),\left(1-\gamma+\frac{\nu}{2},1;1\right),\\(1-\delta,1;1+p),\left(\mu-h'\alpha'-h''\alpha''-\sum_{i=1}^{s}h_i\alpha_i,2k\right)\\\left(\frac{1}{2}-\eta+\frac{\mu}{2}-h'\alpha'-h''\alpha''-\sum_{i=1}^{s}h_i\alpha_i,k\right),(0,1),\left(-\frac{\nu}{2},1;1\right),\\(-\delta,1;1+p)\end{matrix}\right.\right] \quad (6.1)$$

valid under same conditions as those required for (2.1).

And from (4.1), we obtain

(b) $$\psi(x,t)=\frac{1}{2^{\mu}\sqrt{\pi}}\sum_{\alpha'=0}^{[N'/M']}\sum_{\alpha''=0}^{[N''/M'']}\sum_{\alpha_1=0}^{[N_1/M_1]}\cdots\sum_{\alpha_s=0}^{[N_s/M_s]}\frac{(-N')_{M'\alpha'}}{\alpha'\,!}\cdots\frac{(-N'')_{M''\alpha''}}{\alpha''!}$$

$$\frac{(-N_1)_{M_1\alpha_1}}{\alpha_1!},\ldots,\frac{(-N_s)_{M_s\alpha_s}}{\alpha_s!}B'(N'\alpha')\,B''(N''\alpha'')\,B[N_1\alpha_1;\ldots;N_s\alpha_s]$$

$$(y')^{\alpha'}(y'')^{\alpha''}\,y_1^{\alpha_1}\ldots y_s^{\alpha_s}$$

$$\frac{a_{d-1}\,\Gamma(p+1)\,\Gamma\left(\frac{1}{2}+\frac{\nu}{2}\right)}{(-1)^p\,2^{2+p}\,\Gamma(\gamma)\,\Gamma\left(\gamma-\frac{\nu}{2}\right)}\,\frac{2^{2\eta+2+h'\alpha'+h''\alpha''+\sum_{i=1}^{s}h_i\alpha_i+1}}{2\eta!}\,\bar{H}^{2,4}_{5,4}$$

$$\left[-z.4^k\left|\begin{array}{c}\left(1-\eta+\frac{\mu}{2}-h'\alpha'-h''\alpha''-\sum_{i=1}^{s}h_i\alpha_i,k;1\right),(1-\gamma,1;1),\left(1-\gamma+\frac{\nu}{2},1;1\right)\\(1-\delta,1;1+p),\left(\mu-h'\alpha'-h''\alpha''-\sum_{i=1}^{s}h_i\alpha_i,\alpha k\right)\\\left(\frac{1}{2}-\eta+\frac{\mu}{2}-h'\alpha'-h''\alpha''-\sum_{i=1}^{s}h_i\alpha_i,k\right),(0,1),\left(-\frac{\nu}{2};1\right),(-\delta,1;1+p)\end{array}\right.\right]$$

$$\left(\cos\frac{\pi\eta x}{\lambda}\right)\left(\cos\frac{\pi\eta R't}{\lambda}\right). \qquad (6.2)$$

valid under same conditions as those required for (4.1).

2. Taking M = 1, N = 3, P = 3, Q = 2 and replacing z by $\{-(1+\omega)^{-2}$ and using

$$\beta F(d;\omega)=-\frac{1}{4\sqrt{\pi}(1+\omega)^2}\bar{H}^{1,3}_{3,2}\left[-(1+\omega)^2\left|\begin{array}{l}(0,1;1),(0,1;1),(-1/2,1;d)\\(0,1),(-1,1;(1+d)\end{array}\right.\right]$$

[9, p.4121, eq.(5)]

the above function is the exact partition function of Gaussian model in statistical mechanics.

We obtain from (2.1)

(a) $$\int_0^{\lambda}\left(\cos\frac{\pi\rho x}{\lambda}\right)\left(\sin\frac{\pi x}{2\lambda}\right)^{2\rho-\mu-1}\left(\cos\frac{\pi x}{2\lambda}\right)^{\mu-1}S^{M'}_{N'}\left[y'\left(\tan\frac{\pi x}{2\lambda}\right)^{2h'}\right]$$

$$S^{M''}_{N''}\left[y_1\left(\tan\frac{\pi x}{2\lambda}\right)^{2h''}\right]S^{M_1,\ldots,M_s}_{N_1,\ldots,N_s}\left[y_1\left(\tan\frac{\pi x}{2\lambda}\right)^{2h_1},\ldots,y_s\left(\tan\frac{\pi x}{2\lambda}\right)^{2h_s}\right]$$

$\beta.F\,(d;w)\,dx$

$$= -\frac{\lambda e^{2\rho-\mu+2h'\alpha'+2h''\alpha''+\alpha\sum_{i=1}^{s} h_i\alpha_i-2}}{\Gamma 2\rho\sqrt{\pi}\left(\frac{d+1}{2}\right)(1+\omega)^2}$$

$$\sum_{\alpha'=0}^{[N'/M']}\sum_{\alpha''=0}^{[N''/M'']}\sum_{\alpha_1=0}^{[N_1/M_1]}\cdots\sum_{\alpha_s=0}^{[N_s/M_s]}\frac{(-N')_{M'\alpha'}}{\alpha'\,!}\cdots\frac{(-N'')_{M''\alpha''}}{\alpha''!}\frac{(-N_1)_{M_1\alpha_1}}{\alpha_1\,!},\ldots,$$

$$\frac{(-N_s)_{M_s\alpha_s}}{\alpha_s\,!}B'(N'\alpha')B''(N''\alpha'')B[N_1\alpha_1;\ldots;N_s\alpha_s](y')^{\alpha'}(y'')^{\alpha''}y_1^{\alpha_1}\ldots y_s^{\alpha_s}$$

$$\bar{H}_{5,3}^{2,4}\left[-(1+\omega)^{-2}4^k\left|\begin{matrix}\left(1-\rho+\frac{\mu}{2}-h'\alpha'-h\alpha''-\sum_{i=1}^{s}h_i\alpha_i;1\right),(0,1;1),\\(0,1,1),(-1,1;d),\left(\mu-h'\alpha'-h''\alpha''-\sum_{i=1}^{s}h_i\alpha_i,2k\right)\\\left(\frac{1}{2}-\rho+\frac{\mu}{2}-h'\alpha'-h''\alpha''-\sum_{i=1}^{s}h_i\alpha_i,k\right),(0,1),\\(-1,1;1+d)\end{matrix}\right.\right] \tag{6.3}$$

valid under the same conditions as those required for (2.1).

And from (4.1), we have

(b) $$\psi(x,t) = \frac{1}{2^{\mu+1}\sqrt{\pi}\frac{(d+1)}{2}(1+\omega)^2}\sum_{\alpha'=0}^{[N'/M']}\sum_{\alpha''=0}^{[N''/M'']}\sum_{\alpha_1=0}^{[N_1/M_1]}\cdots\sum_{\alpha_s=0}^{[N_s/M_s]}B'(N'\alpha')$$

$$B''(N''\alpha'')B[N_1\alpha_1;\ldots;N_s\alpha_s]y_1^{\alpha_1}\ldots y_s^{\alpha_s}(y')^{\alpha'}(y'')^{\alpha''}\frac{2^{2\eta+2h'\alpha'+2h''\alpha''+2\sum_{i=1}^{s}h_i\alpha_i}}{\sqrt{2\eta}}$$

$$\bar{H}_{5,3}^{2,4}\left[-(1+\omega)^{-2}4^k\left|\begin{matrix}\left(1-\eta+\frac{\mu}{2}-h'\alpha'-h\alpha''-\sum_{i=1}^{s}h_i\alpha_i;1\right),(0,1;1),(0,1,1),\\(-1,1;d),\left(\mu-h'\alpha'-h''\alpha''-\sum_{i=1}^{s}h_i\alpha_i,2k)\right.\\\left(\frac{1}{2}-\eta+\frac{\mu}{2}-h'\alpha'-h''\alpha''-\sum_{i=1}^{s}h_i\alpha_i,k\right),(0,1),(-1,1;1+d)\end{matrix}\right.\right]$$

$$\left(\cos\frac{\pi\eta x}{\lambda}\right)\left(\cos\frac{\pi\eta R't}{\lambda}\right) \tag{6.4}$$

valid under same conditions as for (4.1).

7. CONCLUSION

The results obtained in this paper are of a general character and may prove to be useful in several interesting situations appearing in the literature on applied mathematics and mathematical physics.

References

[1] A.A.Inayat-Hussain., "New properties of Hypergeometrical series derivable from Feynman integrals:I", Transformation and reduction formula, J. Phys. A. Math. Gen., 20, 4109-4117 (1987).

[2] A.A.Inayat-Hussain., "New properties of Hypergeometrical series derivable from Feynman integrals : II", A generalization of the H-function, J. Phys. A. Math. Gen., 20, 4119-4128 (1987).

[3] A.K.Rathie., "A new generalization of generalized hypergeometric functions", Le mathematiche fasc. II, 52, 297-310 (1997).

[4] A.M. Mathai and R.K.Saxena., "The H-function with applications in statistics and other disciplines", John Wiley and Sons, New York (1978).

[5] C.Fox., "The G and H-functions as symmetrical Fourier kernels", Trans. Amer. Math. Soc. 98, 395-429 (1961).

[6] H.M.Srivastava., "A multilinear generating function for the Konhauser sets of biorthogonal polynomials suggested by the Laguerre polynomials", Pacific J. Math. 117, 183-191 (1985).

[7] H.M.Srivastava and N.P.Singh., "The integration of certain products of multivariable H-function with general class of polynomials", Rend. Circ. Mat. Palermo(2) 32, 157-187 (1983).

[8] H.M.Srivastava., "A contour integral involving Fox's H-function, Indian J. Math.", 14 (1972), 1-6.

[9] R.G. Buschman and H.M.Srivastava., "The $\bar{H}$-function associated with a certain class of Feynman integrals", J. Phys. A: Math. Gen. 23, 4707-4710 (1990).

[10] R.V.Churchill., "Fourier Series and boundary value problems", McGraw-Hill Book Co., Inc, New York (1941).