International Review of Pure and Applied Mathematics
Volume 15 • Number 1 • Jan-June 2019

EDGE DOMINATION NUMBER OF SIERPINSKI CYCLE GRAPH OF ORDER N

Kalanidhi Kalyan[1] & Mohit James[2]

Abstract: Edge domination in Graph is a growing area for researcher and mathematician. In graph theory, an edge dominating set for a graph G(V,E) is a subset S⊆E such that every edge not in S is adjacent to at least one edge in S. The edge domination number is the number of edges in a smallest dominating set for graph G(V,E). Generalized Sierpinski Graphs is a geometrical figure in which each part has the same patterns as the whole. They are useful in modelling structures. In this paper we have examined the edge domination number of Sierpinski Cycle graph of order n≥3.

Keywords: Edge Domination Number, Generalised Sierpinski Graph, Cycle Graph and their Sierpinski graph.

1. INTRODUCTION

In 1958, C. Berge introduced the concept of co-efficient of external stability which was the born of domination number and denoted it $\beta(G)$. In 1962 Ore used the term domination number when he solved a queens problem in the game of chess. He said that the minimum number of queens which dominate all the squares of the chessboard is domination number. Cockayne and Hedetniemi[2] further worked on the domination number in Graph. They used the notation $\gamma(G)$ to denote domination number.

Mitchell and Hedetniemi[13] introduced the concept of edge domination in graph. *A* subset *S* of *E* is called an edge dominating set of *G* if every edge not in *S* is adjacent to some edge in *S*. The edge domination number γ' (G) of *G* is the minimum cardinality taken over all edge dominating sets of *G*.

Sierpinski Sierpinski graphs are never ending pattern that are self similar across different scales. The introduction of Sierpinski graphs were first motivated by topological studies of Lipscomb's space [14,15]. In 1997 Klavzar and Miltunovic[19] introduced Sierpinski Graphs $S(n,G)$which are isomorphic to the graphs of Tower of Hanoi with n disk.They gave the method of construction of Sierpinski Graph of Complete Graph K_k of order 'n' which is obtained after finite number of iteration and denoted it $S(n, K_k)$. In the stage one, they took simply complete Graph and denoted it $S(1, K_k)$. In the stage two, they copied $S(1, K_k)$ Graph k times and add one edge between each pair of $S(1, K_k)$ for forming Graph $S(2, K_k)$. Repeating this process they defined $S(3, K_k)$, $S(4, K_k)$, $S(5, K_k)\cdots S(n, K_k)$. In 2011, Gravier, Kovse and Aline[18] introduced new Graphs known as Generalised Sierpinski Graphs. They replaced Complete Graph K_k by any Graph. In this paper, we have taken Cycle Graph C_n where $n\geq 3$ and construct their Sierpinski graph.

2. PRELIMINARIES

2.1 GENERALISED SIERPINSKI GRAPH

Klavžar and **Milutinovic[19]** introduced Sierpinski graph in 1997. Later Gravier, Kovse and Aline introduced Generalised Sierpinski Graphs[18]. The Generalised Sierpinski graph of G of dimension "n" denoted by $S(n,G)$ is the graph with vertex set $\{1,2,3,\ldots\ldots,n\}^n$ and edge set defined by : $\{u,v\}$ is an edge if and only if there exists $i\in\{1,2,3,\ldots\ldots,n\}$ such that:

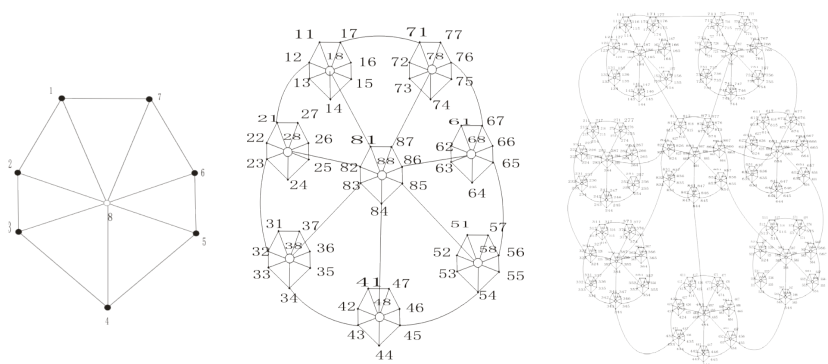
(i) $u_j = v_j$ *if* $j<i$

(ii) $u_i \neq v_i$ and $(u_i,v_i)\in E(G)$

(iii) $u_j = v_i$ and $v_j = u_i$ if $j>i$

In other words, if $\{u,v\}$ is an edge of $S(n,G)$, there is an edge $\{x,y\}$ of G and a word "w" such that $u = wxy\ldots\ldots\ldots y$ and $v = wyx\ldots\ldots\ldots x$. We say that edge $\{u,v\}$ is using edge $\{x,y\}$ of G. Graphs $S(n,G)$ is can be constructed recursively from G with the following process: $S(1,G)$ is isomorphic to G. To construct $S(n,G)$ for $n>1$, copy k times $S(n\text{-}1,G)$ and add to labels of vertices in copy x of $S(n\text{-}1,G)$ the letter x at the beginning. Then for any edge $\{x,y\}$ of G, add an edge between vertex $xy\cdots y$ and vertex $yx\cdots x$. For any word u of length d, with $1 \leq d \leq n$, the subgraph of $S(n,G)$ induced by vertices with label beginning by u, is isomorphic to $S(n\text{-}d,G)$. For a vertex x of G, we call extreme vertex x of $S(n,G)$ the vertex with label $x\cdots x$.

Example.



S(1,W$_7$) **S(2,W$_7$)** **S(3,W$_7$)**

Fig 2.1: Sierpinski wheel Graph of order 7.

2.2 Domination number: A dominating Set for a graph G=(V,E) is a set D⊆V(G) of vertex set such that every vertex not in D is adjacent to at least one member of D. The domination number γ(G) is the minimum size of a dominating set of vertices in G.

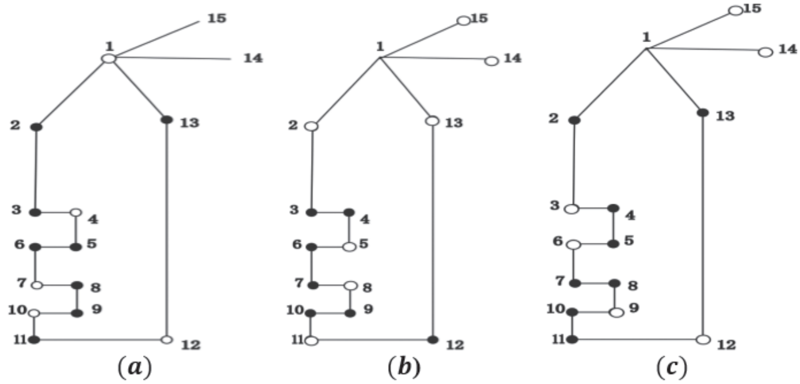
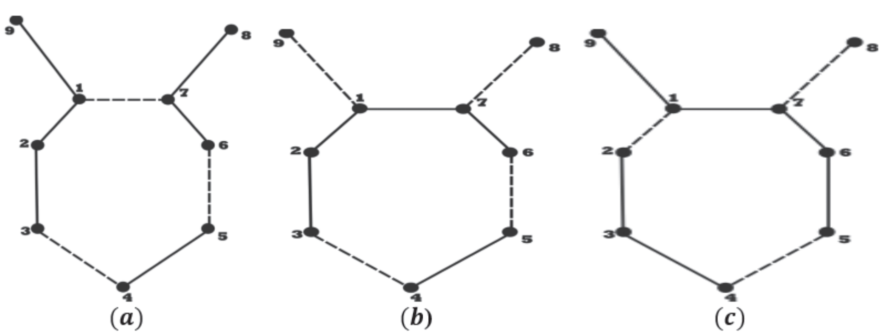


Fig 2. 2:.

In this example, vertex set V(G)={1,2,3,4,5,6,7,8,9,10,11,12,13,14,15} and dominating set for Graph (a), (b) and (c) are {1,4,7,10,12}, {2,5,8,10,13,14,15} and {3,6,,9,12,14,15},. So its domination number γ(G)= 5.

2.3 Edge domination number: An 2.3 Edge domination number: An edge dominating set for a graph G(V,E) is a subset S⊆Esuch that every edge not in S is adjacent to at least one edge in S. The edge domination number is the number of edges in a smallest dominating set for graph G(V,E). An edge dominating set is also known as a line dominating set.



In this example, edge set E(G)={1,2,3,4,5,6,7,8,9} and edge dominating set for Graph (a), (b) and (c) are {(1,7),(3,4),(5,6)}, {(1,9),(3,4),(5,6),(7,8)} and {(1,2),(4,5),(7,8)},. So its edge domination number γ′ (G)= 3..

2.4 Cycle Graph:

The Cycle Graph C_n ($n \geq 3$), of length n is a connected graph which consists of n vertices $v_1, v_2, \cdots v_3$ and n edges $\{v_1, v_2\}$, $\{v_2, v_3\}$, $\{v_3, v_4\} \cdots \{v_{(n-1)}, v_n\}$, $\{v_n, v_1\}$.

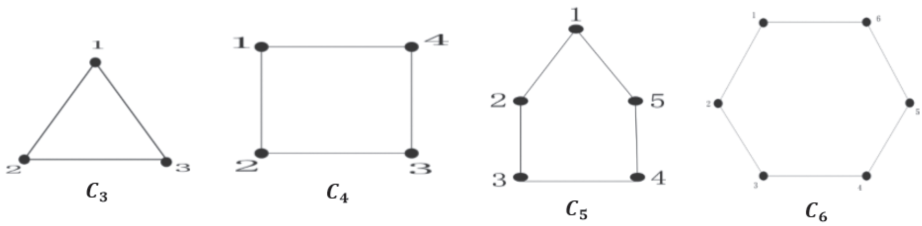
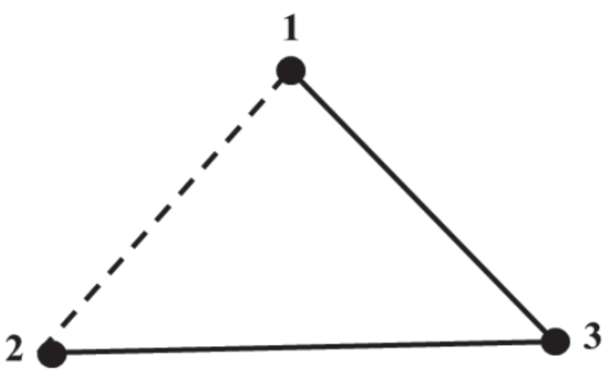


Fig 2. 3: Cycle Graph of different order

3. EDGE DOMINATION NUMBER OF SIERPINSKI CYCLE GRAPH OF ORDER

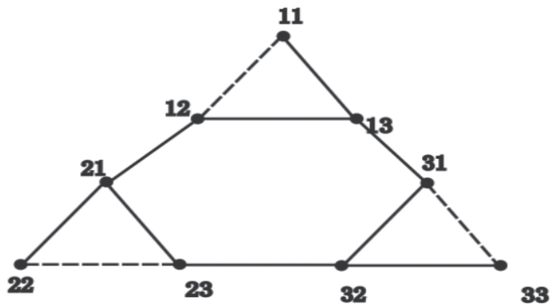
3.1 Edge domination number of Sierpinski Cycle Graph of order '3' i.e. $\gamma'(S(k, C_3))$

Table 3.1

Notation	Graph
$S(1, C_3)$	

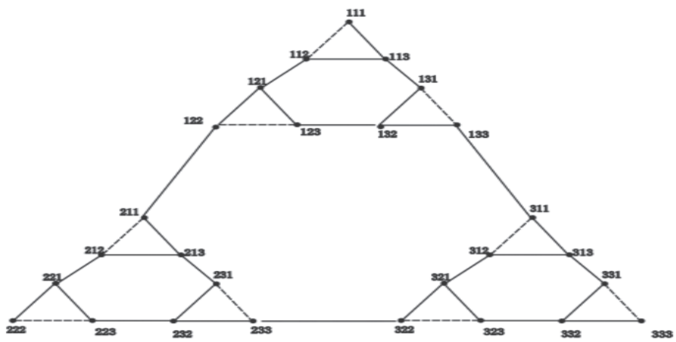
In $S(1, C_3)$, the edge Set is E={(1, 2), (2, 3), (1, 3)}. Here we have taken a subset D={(1, 2)}={dotted line} of edge set E{$S(1, C_3)$}. D has minimum number of cardinality which dominate all edges of $S(1, C_3)$. Therefore, the edge domination number of $S(1, C_3)$ is 1.

$S(2, C_3)$



Similarly, in S(2, C_3), here we have taken a subset D = {(11, 12), (22, 23), (31, 33)} ={All dotted lines of S(2, C_3)} of edge set E{S(2, C_3)}. D has minimum number of cardinality which dominate all edges of S(2, C_3) .Therefore, the edge domination number of S(2, C_3) is 3.

$S(3, C_3)$



Again, in S(3, C_3), here we are taking a subset D = {All dotted edges of S(3, C_3)} of edge set E{S(3, C_3)}. D has minimum number of cardinality which dominate all edges of S(3, C_3). Here each S(2, C_3) contains 3 dotted edges. Therefore, the total number of dotted edges of S(3, C_3) will be 3×3=9 which is the edge domination number.

When we construct S(3, C_3) by copying three times of S(2, C_3) and connect them by edges, there is no need to choose any extra edges for dominating all edges. Because all three edges which connect to all three copies of S(2, C_3) are adjacent to three edges (11, 12), (22, 23) and (31, 33) which are choosed in minimum edge dominating set D in S(2, C_3). For example, in figure S(3,C_3), there are three edges {(122, 211), (233, 322), (133, 311)} which connects to all three copies of S(2, C_3) are adjacent to three edges (11, 12), (22, 23) and (31, 33) which are already choosed in minimum edge dominating set D in S(2, C_3), so here, there is no need to choose any extra edges in S(3, C_3).The edge domination number of S(3, C_3) will be 3×edge domination number of S(2, C_3) = 3×3=9. Similarly, we can find the edge domination number of S(4, C_3) and it will be 3×edge domination number of S(3,

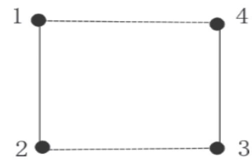
C_3)=3×9=27. Continung this process we can find the edge domination number S(5, C_3), S(6, C_3)....... S(k, C_3). Therefore, the edge domination number of S(k, C_3)=1×$3_{(k-1)}$ where $k \geq 1$. The edge domination number of S(1, C_3) to S(k, C_3) has been tabulated in Table No. - 4.1.

Table 3.2

3.2 Edge domination number of Sierpinski Cycle Graph of order '4' i.e. γ' (S(k, C_4))

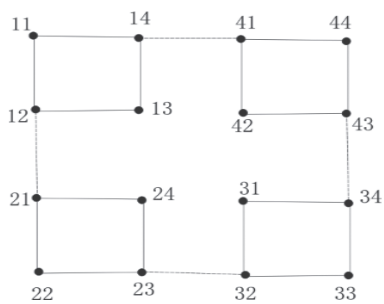
Notation | **Graph**

$S(1, C_4)$



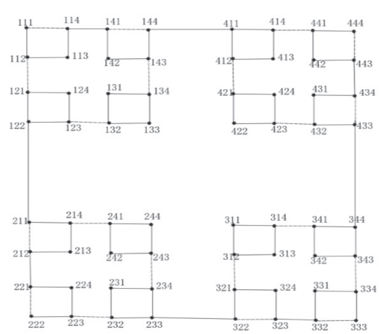
In S(1, C_4), the edge Set is E={(1, 2), (2, 3), (3, 4), (1, 4)}. Here we have taken a subset D={(1, 4), (2, 3}={dotted lines} of edge set E{S(1, C_4)}. D has minimum number of cardinality which dominate all edges of S(1, C_4). Therefore, the edge domination number of S(1, C_4) is 2.

$S(2, C_4)$



Similarly, in S(2, C_4), here we have taken a subset D={(12, 21), (23, 32), (34, 43),(14, 41)}={All dotted lines of S(2, C_4)} of edge set E{S(2, C_4)}. D has minimum number of cardinality which dominate all edges of S(2, C_4) .Therefore, the edge domination number of S(2, C_4) is 4.

$S(3, C_4)$



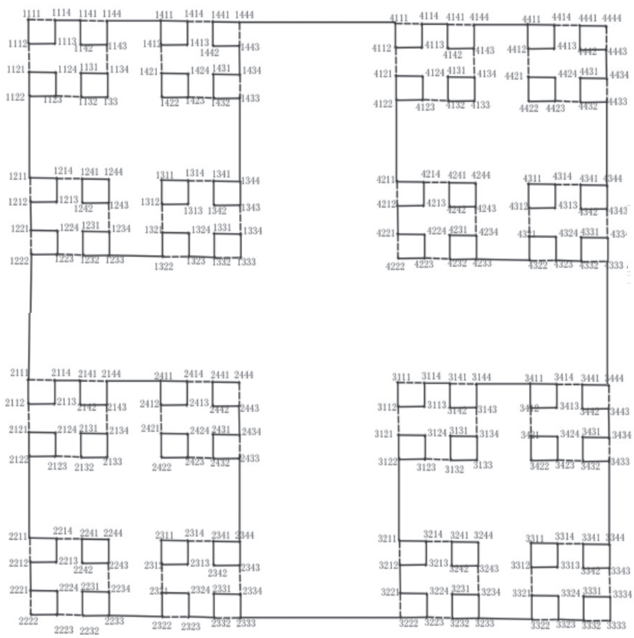
Again, in $S(3, C_4)$, here we are taking a subset $D =$

$$\left\{\begin{matrix}(111,114),(141,144),(112,121),(123,\ 132),(134,143),\\(211,212),(214,241),(221,222),(223,232),(234,243),\\(314,341),(312,321),(322,323),(332,333),(334,343),\\(414,441),(412,421),(423,432),(433,434),(443,444)\end{matrix}\right\} =$$

$\{All\ dotted\ edges\ of\ S(3, C_4)\}$

of edge set $E\{S(3, C_4)\}$. D has minimum number of cardinality which dominate all edges of $S(3, C_4)$. Here each $S(2, C_4)$ contains 5 dotted edges. Therefore, the total number of dotted edges of $S(3, C_4)$ will be 20 which is the edge domination number.

$\mathbf{S(4, C_4)}$



When we construct $S(4, C_4)$ by copying four times of $S(3, C_4)$ and connect them by edges, there is no need to choose extra edges for dominating all edges. Because all four edges which connect to all copies of $S(3, C_4)$ will be adjacent to the four edges (111, 114), (221, 222), (332, 333) and (443, 444), which are choosed in minimum edge dominating set D of $S(3, C_4)$. For example, we can see in figure S(4,C_4), There are four edges (1222, 2111), (2333, 3222), (3444, 4333) and (1444, 4111) which connects to all copies of $S(3, C_4)$ are adjacent to edges (1221, 1222) or (2111, 2114), (2332, 2333) or (3221, 3222), (3443, 3444) or (4332, 4333) and (4114, 4111) or (1444, 1443) respectively. All these edges we have already choosed in $S(3, C_4)$, so here, there is no need to choose extra edges in $S(4, C_4)$.The edge domination number of $S(4, C_4)$ will be 4×edge domination number of $S(3, C_4)$ = 4×20=80. Similarly, we can find the edge domination number of $S(5, C_4)$

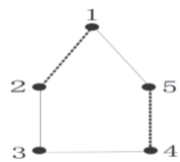
and it will be 4×edge domination number of S(4,C_4)=4×80=320. Continung this process we can find $S(6, C_4)$, $S(7, C_4)$....... $S(k, C_4)$. Therefore, the edge domination number of $S(k, C_4)=20\times 4^{k-3}=5\times 4^{k-2}$ where $k \geq 3$. The edge domination number of $S(1, C_4)$, $S(2, C_4)$ and the edge domination number of $S(3, C_4)$ to $S(k, C_4)$ have been tabulated in Table No. – 4.2.

3. 3 Edge domination number of Sierpinski Cycle Graph of order '5' i.e. $\gamma'(S(k, C_5))$.

Table 3.3

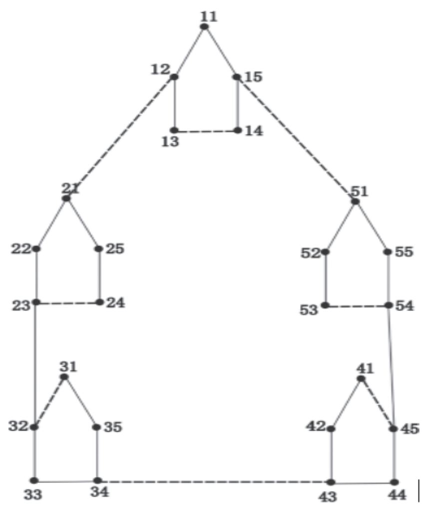
Notation **Graph**

$\boldsymbol{S(1, C_5)}$



In $S(1, C_5)$, the edge Set is E={(1, 2), (2, 3), (3, 4), (4, 5), (5, 1)}. Here we have taken a subset D={(1, 2), (4, 5}={All dotted lines} of edge set E{$S(1, C_5)$}. D has minimum number of cardinality which dominate all edges of $S(1, C_5)$. Therefore, the edge domination number of $S(1, C_5)$ is 2.

$\boldsymbol{S(2, C_5)}$



Similarly, In $S(2, C_5)$, Here we have taken a subset $D =$

$$\begin{Bmatrix}(12,21),(15,51),(13,14),(23,24),\\(31,32),(34,43),(41,45),(53,54)\end{Bmatrix} = \{All\ dotted\ lines\ of\ S(2, C_5)\}$$

of edge set E{$S(2, C_5)$}. D has minimum number of cardinality which dominate all edges of $S(2, C_5)$ .Therefore, the edge domination number of $S(2, C_5)$ is 8.

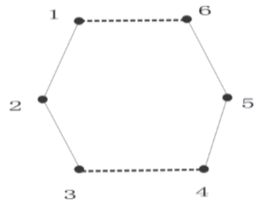
$S(3, C_5)$

Again, in S(3, C_5), here we are taking a subset D={All dotted edges of S(3, C_5)} of edge set E{S(3, C_5)}. D has minimum number of cardinality which dominate all edges of S(3, C_5). Here each S(2, C_5) contains 7 dotted edges and we take five edges which connects all five copies of (2, C_5) . Therefore, the total number of dotted edges of S(3, C_5) will be 5×7+5=40 which is the edge domination number. Note:- Here, figure S(3, C_5) is very large. If we zoom S(3, C_5), we can see all dotted edges which are taken.

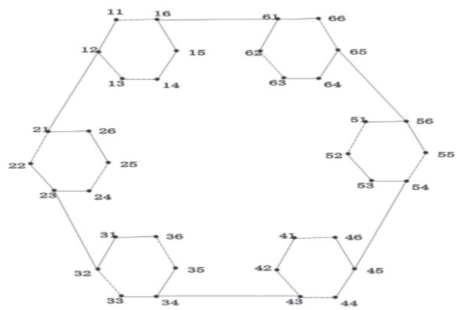
When we construct S(4, C_5) by copying five times of S(3, C_5) and connect them by five edges, there is no need to choose any extra edges for dominating all edges. Because all five edges which connect to all copies of S(3, C_5) will be adjacent to the five edges (111, 115), (222, 223), (333, 334), (443, 444) and (554, 555) which are choosed in minimum edge dominating set D in S(3, C_5). Therefore, the edge domination number of S(4, C_5) will be 5×edge domination number of S(3, C_5) = 5×40=200. Similarly, we can find the edge domination number of S(5, C_5) and it will be 5×edge domination number of S(4, C_5)=5×200=1000. Continung this process we can find edge domination number S(6, C_5), S(7, C_5)....... S(k, C_5). Therefore, the edge domination number of S(k, C_5)=40× 5^{k-3}=8×5^{k-2} where k ≥ 2. The edge domination number of S(1, C_5) and the edge domination number of S(2, C_5) to S(k, C_5) have been tabulated in Table No. – 4.4.

3.4 Edge domination number of Sierpinski Cycle Graph of order '6' i.e. γ' (S(k, C_6)).

Table 3.4

Notation	Graph
$S(1, C_6)$	

In $S(1, C_6)$, the edge Set is E={(1, 2), (2, 3), (3, 4), (1, 4), (4, 5), (5, 6)}. Here we have taken a subset D={(1, 6), (3, 4)}={All dotted lines} of edge set E{$S(1, C_6$)}. D has minimum number of cardinality which dominate all edges of $S(1, C_6)$. Therefore, the edge domination number of $S(1, C_6)$ is 2.

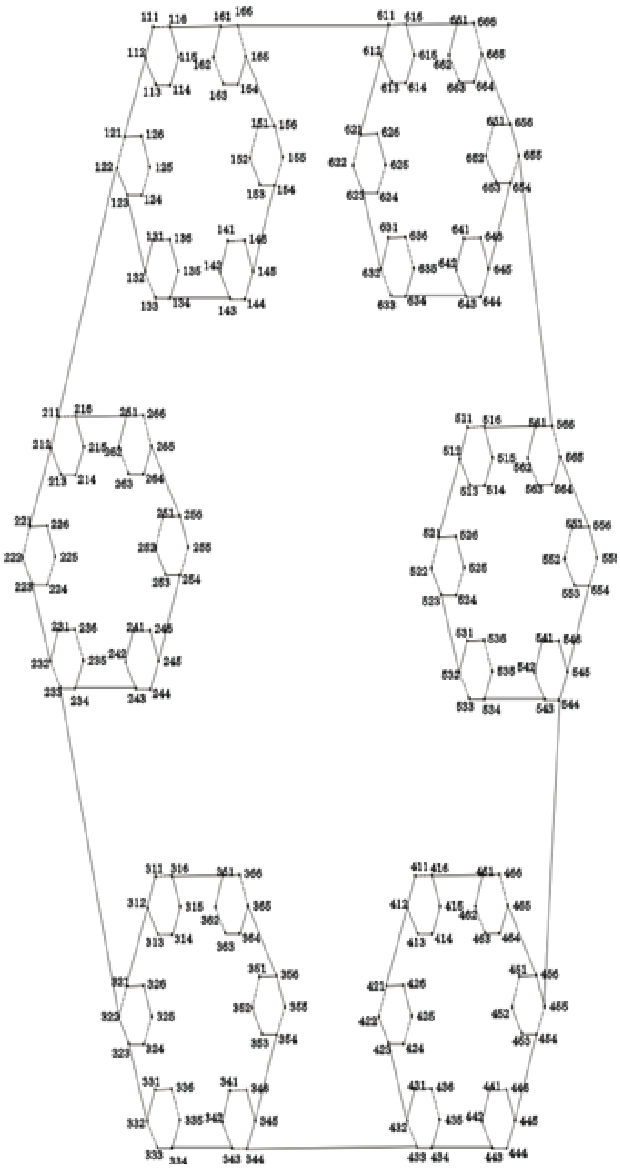
$S(2, C_6)$	

Similarly, in $S(2, C_6)$, here we have taken a subset $D =$

$$\begin{Bmatrix} (11,16),(13,14),(21,22),(24,25) \\ (32,33),(35,36),(41,46),(43,44) \\ (51,52),(54,55),(62,63),(65,66) \end{Bmatrix} = \{All\ dotted\ lines\ of\ S(2, C_6)\}$$

of edge set E{$S(2, C_4)$}. D has minimum number of cardinality which dominate all edges of $S(2, C_6)$ .Therefore, the edge domination number of $S(2, C_6)$ is 12.

S(3, C_6)



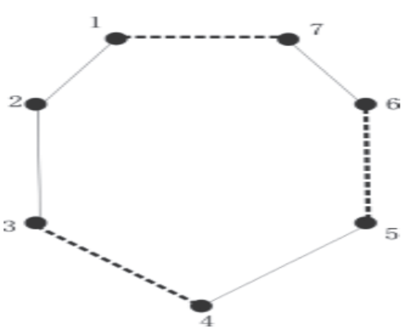
Again, in S(3, C_6), here we are taking a subset D={All dotted edges of S(3, C_6)} of edge set E{S(3, C_6)}. D has minimum number of cardinality which dominate all edges of S(3, C_6) . Here each S(2, C_6) contains 12 dotted edges. Therefore, the total number of dotted edges of S(3, C_6) will be 6×12=72 which is the edge domination number.

Note:- Here, figure S(3, C_6) is very large. If we zoom S(3, C_6), we can see all dotted edges which are taken.

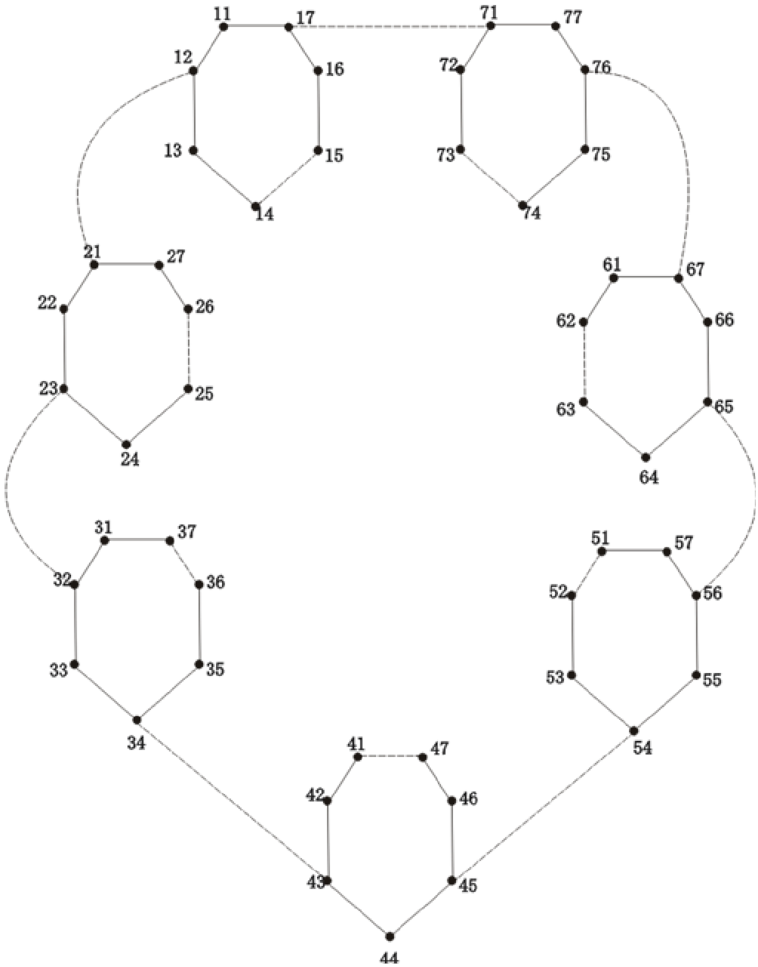
When we constructed $S(3, C_6)$ by copying six times of $S(2, C_6)$ and connect them by edges, there is no need to choose any extra edges for dominating all edges. Because all six edges which connect to all six copies of $S(2, C_6)$ are adjacent to the six edges (11, 16), (21, 22), (32, 33), (43, 44), (54, 55) and (65, 66) which are choosed in minimum edge dominating set D in $S(2, C_6)$. For example, in figure $S(3, C_6)$, there are six edges which connects to all six copies of $S(2, C_6)$ are adjacent to edges which are already choosed in $S(2, C_6)$, so here, there is no need to choose any extra edges for dominating all edges in $S(3, C_6)$.The edge domination number of $S(3, C_6)$ will be 6×edge domination number of $S(2, C_6) = 6\times12=72$.Similarly, we can find the edge domination number of $S(4, C_6)$ and it will be 6× edge domination number of $S(3, C_6)=6\times72=432$. Continung this process we can find the edge number $S(5, C_6)$, $S(6, C_6)$....... $S(k, C_6)$. Therefore, the edge domination number of $S(k, C_6)=12\times 6^{k-2}=2\times6^{k-1}$ where $k \geq 1$. The edge domination number of $S(1, C_6)$ to $S(k, C_6)$ has been tabulated in Table No. – 4.1.

3.5 Edge domination number of Sierpinski Cycle Graph of order '7' i.e. $\gamma'(S(k, C_7))$.

Table3.5

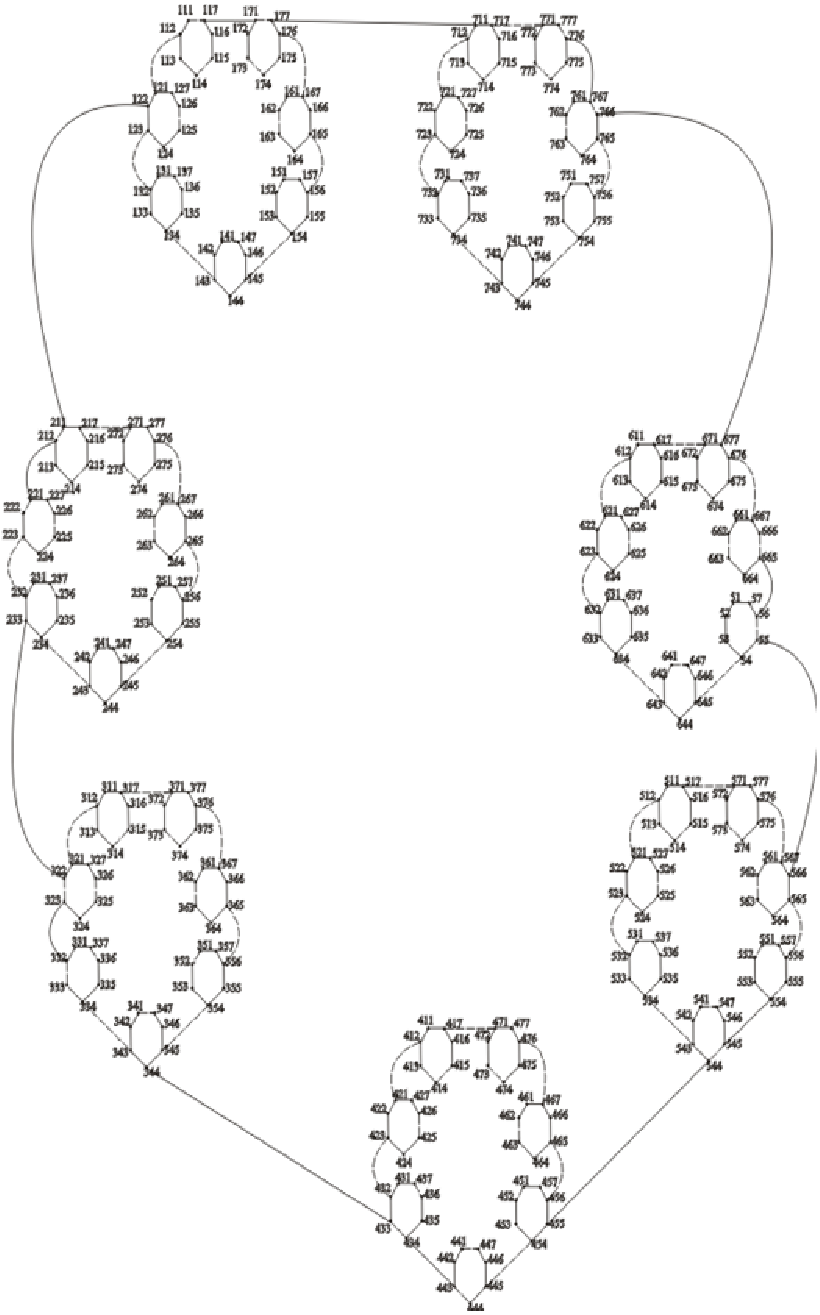
Notation	Graph
$S(1, C_7)$	

In $S(1, C_7)$, the edge Set is E={(1, 2), (2, 3), (3, 4), (4, 5), (5, 6),(6, 7),(1, 7)}. Here we have taken a subset D={(1, 7), (3, 4), (5, 6)}={All dotted lines} of edge set E{$S(1, C_7)$}. D has minimum number of cardinality which dominate all edges of $S(1, C_7)$. Therefore the edge domination number of $S(1, C_7)$ is 3.



S(2, C_7)

Similarly, in S(2, C_7), here we have taken a subset D={All dotted lines of S(2, C_7)} of edge set E{S(2, C_7)}. D has minimum number of cardinality which dominate all edges of S(2, C_7) .Therefore, the edge domination number of S(2, C_4) is 14.



S(3, C_7)

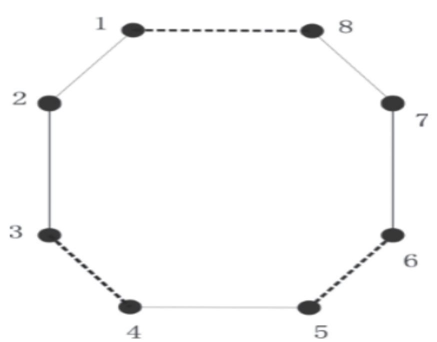
Again, in $S(3, C_7)$, here we are taking a subset D={All dotted edges of $S(3, C_7)$} of edge set E{$S(3, C_7)$}. D has minimum number of cardinality which dominate all edges of $S(3, C_7)$. Here each $S(2, C_7)$ contains 15 dotted edges. Therefore, the total number of dotted edges of $S(3, C_7)$ will be 15×7=105 which is the edge domination number.

Note:- Here, figure $S(3, C_7)$ is very large. If we zoom $S(3, C_7)$, we can see all dotted edges which are taken.

When we construct $S(4, C_7)$ by copying seven times of $S(3, C_7)$ and connect them by edges, , there is no need to choose any extra edges for dominating all edges. Because all seven edges which connect to all seven copies of $S(3, C_7)$ will be adjacent to the seven edges (111, 117), (221, 222), (332, 333), (443, 444),(554, 555), (665, 666), (776, 777) which are choosed in minimum dominating set D in $S(3, C_7)$. Therefore, the edge domination number of $S(4, C_7)$ will be 7×edge domination number of $S(3, C_7)$ = 7×105=735. Similarly, we can find the edge domination number of $S(5, C_7)$ and it will be 7×edge domination number of $S(4, C_7)$=7×735=5145. Continung this process we can find $S(6, C_7)$, $S(7, C_7)$....... $S(k, C_7)$. Therefore, the edge domination number of $S(k, C_7)=15\times7^{k-2}$ where $k \geq 3$. The edge domination number of $S(1, C_7)$, $S(2, C_7)$ and the edge domination number of $S(3, C_7)$ to $S(k, C_7)$ have been tabulated in Table No. – 4.2.

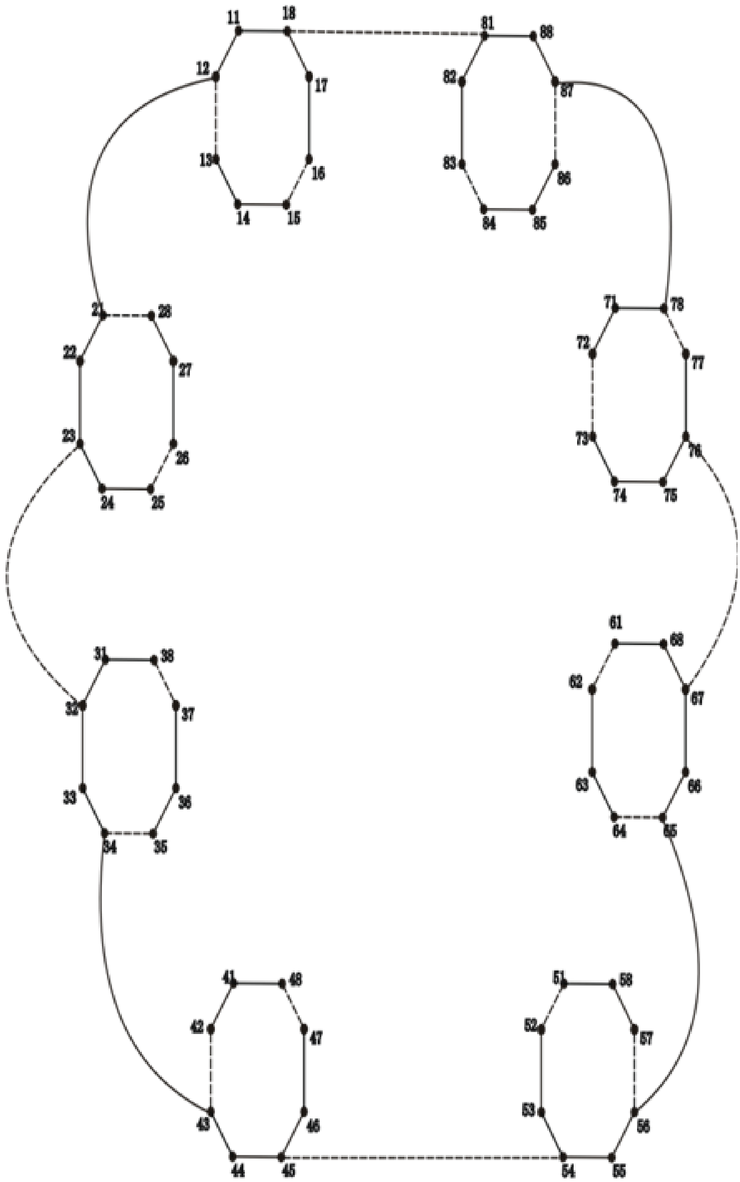
3.6 Edge domination number of Sierpinski Cycle Graph of order '8' i.e. γ' {$S(k, C_8)$}

Table 3.6

Notation	Graph
$S(1, C_8)$	

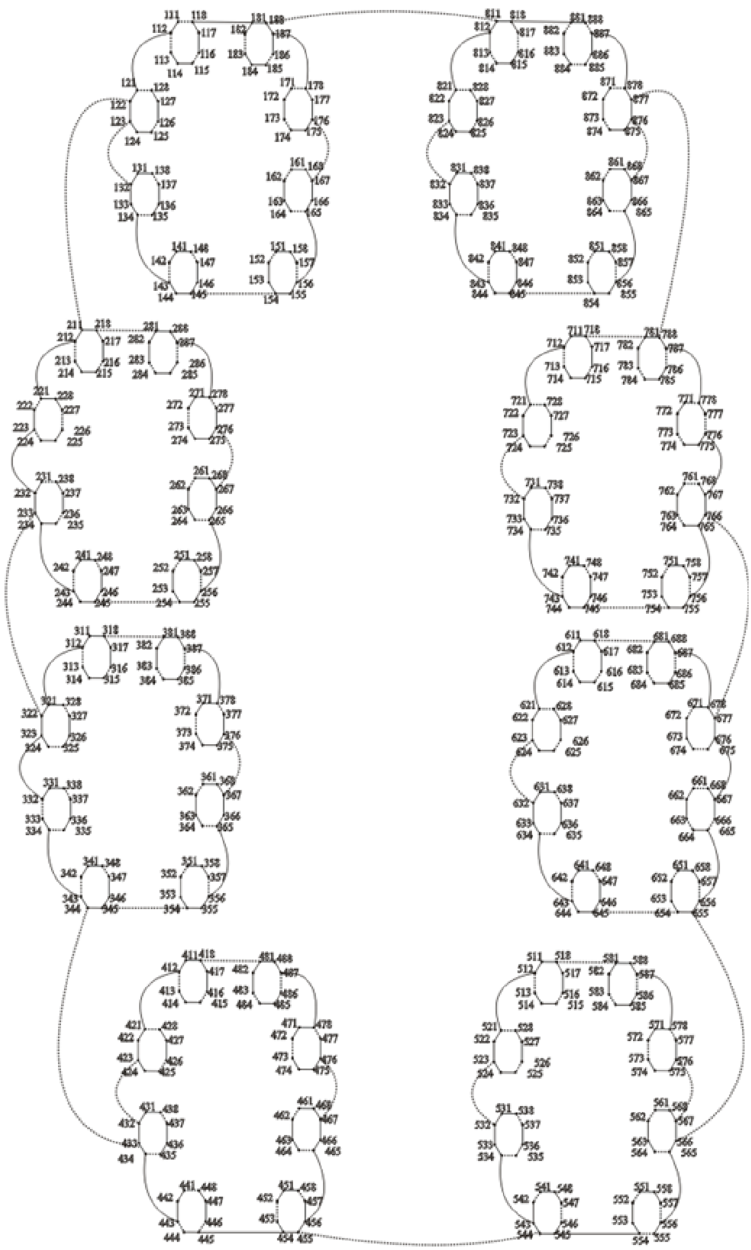
In$S(1, C_8)$, the edge Set is E={(1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7), (7, 8), (8, 1)}. Here we have taken a subset D={(1, 8), (3, 4), (5, 6)}={All dotted lines} of edge set E{$S(1, C_8)$}. D has minimum number of cardinality which dominate all edges of $S(1, C_8)$. Therefore, the edge domination number of $S(1, C_8)$ is 3.

S(2, C_8)



Similarly, in S(2, C_8), here we have taken a subset D=={All dotted lines of S(2, C_5)} of edge set E{S(2, C_8)}. D has minimum number of cardinality which dominate all edges of **S**(2, C_5) . Each S(2, C_8) contains two dotted edges and we take four edges which connect copies of S(2, C_8). Therefore, the edge domination number of S(2, C5) is 8×2+4=20.

S(3, C_8)

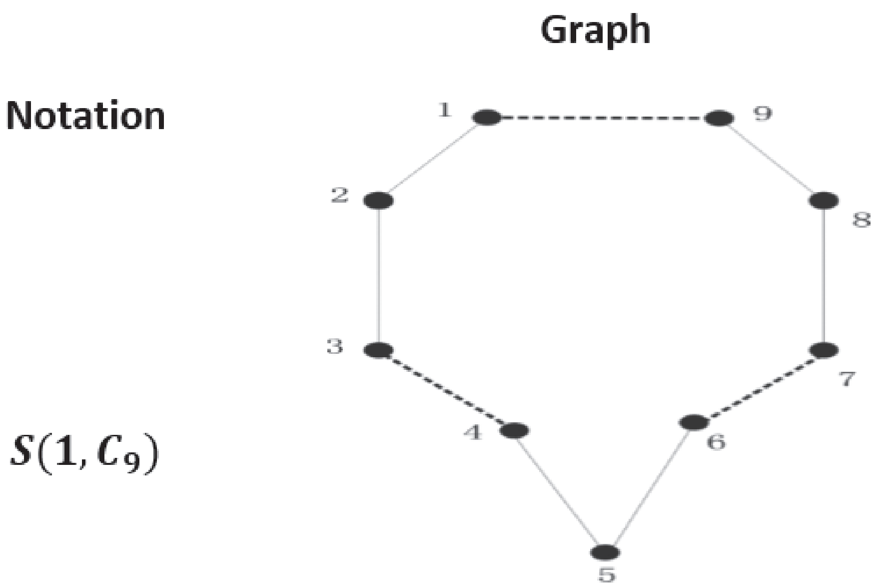


Again, in S(3, C_8), here we are taking a subset D={All dotted edges of S(3, C_8)} of edge set E{S(3, C_8)}. D has minimum number of cardinality which dominate all edges of S(3, C_8) . Here each S(2, C_8) contains 20 dotted edges and we take eight edges which connects all eight copies of (2, C_8) . Therefore, the total number of dotted edges of S(3, C_8) will be $8\times20+8=168$ which is the edge domination number.

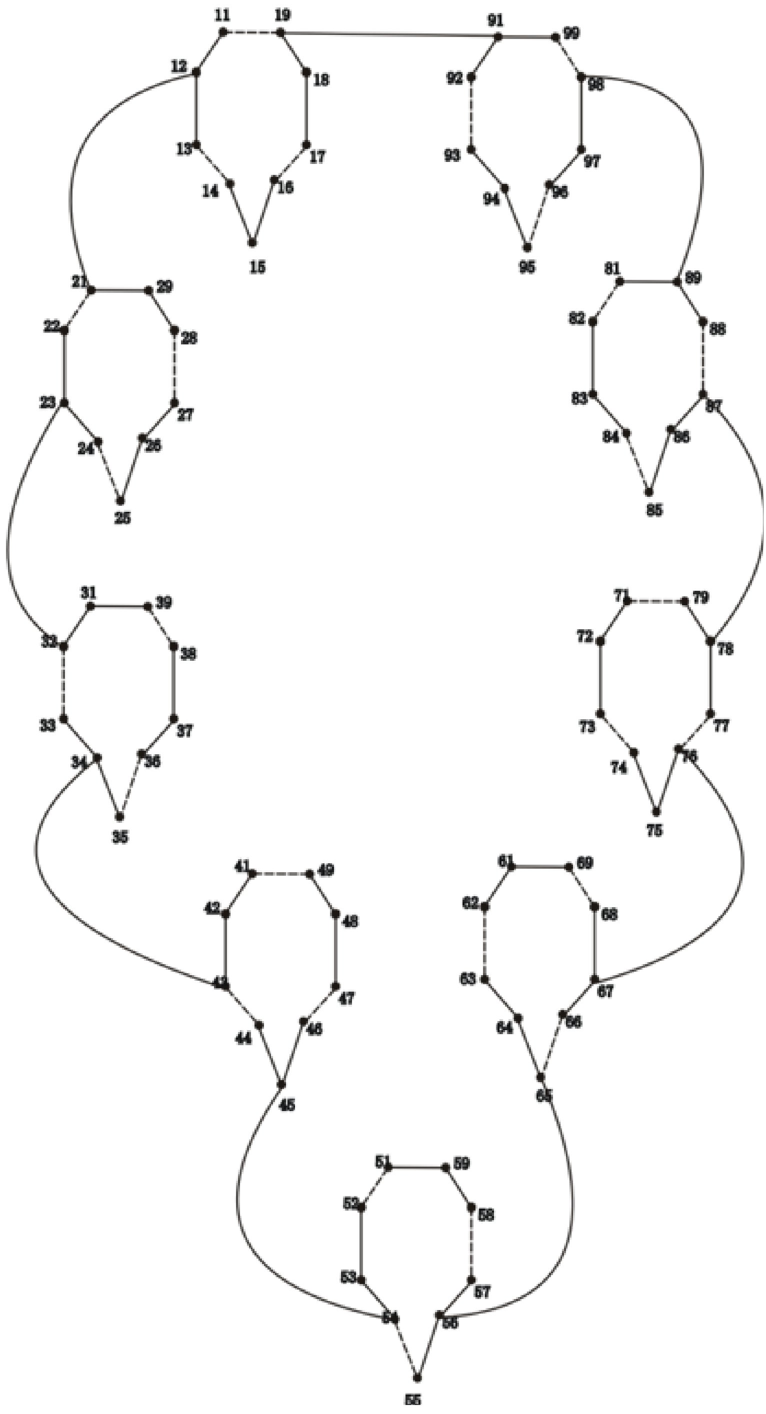
Note:- Here, figure S(3, C_8) is very large. If we zoom S(3, C_8), we can see all dotted edges which are taken.

When we construct S(4, C_8) by copying eight times of S(3, C_8) and connect them by edges, there is no need to choose any extra edges for dominating all edges. Because all eight edges which connect to all eight copies of S(3, C_8) will be adjacent to the eight edge (111, 118), (221, 222), (332, 333), (443, 444),(554, 555), (665, 666), (776, 777) and (887, 888), which are choosed in minimum edge dominating set D in S(3, C_8). Therefore, the edge domination number of S(4, C_8) will be 8×edge domination number of S(3, C_8) = 8×168=1344. Similarly, we can find the edge domination number of S(5, C_8) and it will be 8 × edge domination number of S(4, C_8)= 8×=10752. Continung this process we can find S(6, C_8), S(7, C_8)....... S(k, C_8). Therefore, the edge domination number of S(k, C_8)=168× 8^{k-3} =21×8^{k-2} where k ≥ 3. The edge domination number of S(1, C_8), S(2, C_8) and the edge domination number of S(3, C_8) to S(k, C_8) have been tabulated in Table No. – 4.3.

3.7 Edge domination number of Sierpinski Cycle Graph of order '9' i.e. γ' (S(k ,C_9)).



In S(1, C_9), the edge Set is E={(1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7), (7, 8), (8, 9), (9, 1)}. Here we have taken a subset D={(1, 9), (3, 4), (6, 7)}={All dotted lines} of edge set E{S(1, C_9)}. D has minimum number of cardinality which dominate all edges of S(1, C_9). Therefore, the edge domination number of S(1, C_9) is 3.


11
19
91
99
12
18
92
98
13
17
93
97
14
16
94
96
15
95
21
29
81
89
22
28
82
88
23
27
83
87
24
26
84
86
25
85
31
39
71
79
32
38
72
78
33
37
73
77
34
36
74
76
35
75
41
49
61
69
42
48
62
68
43
47
63
67
44
46
64
66
45
65
51
59
52
58
53
57
54
56
55

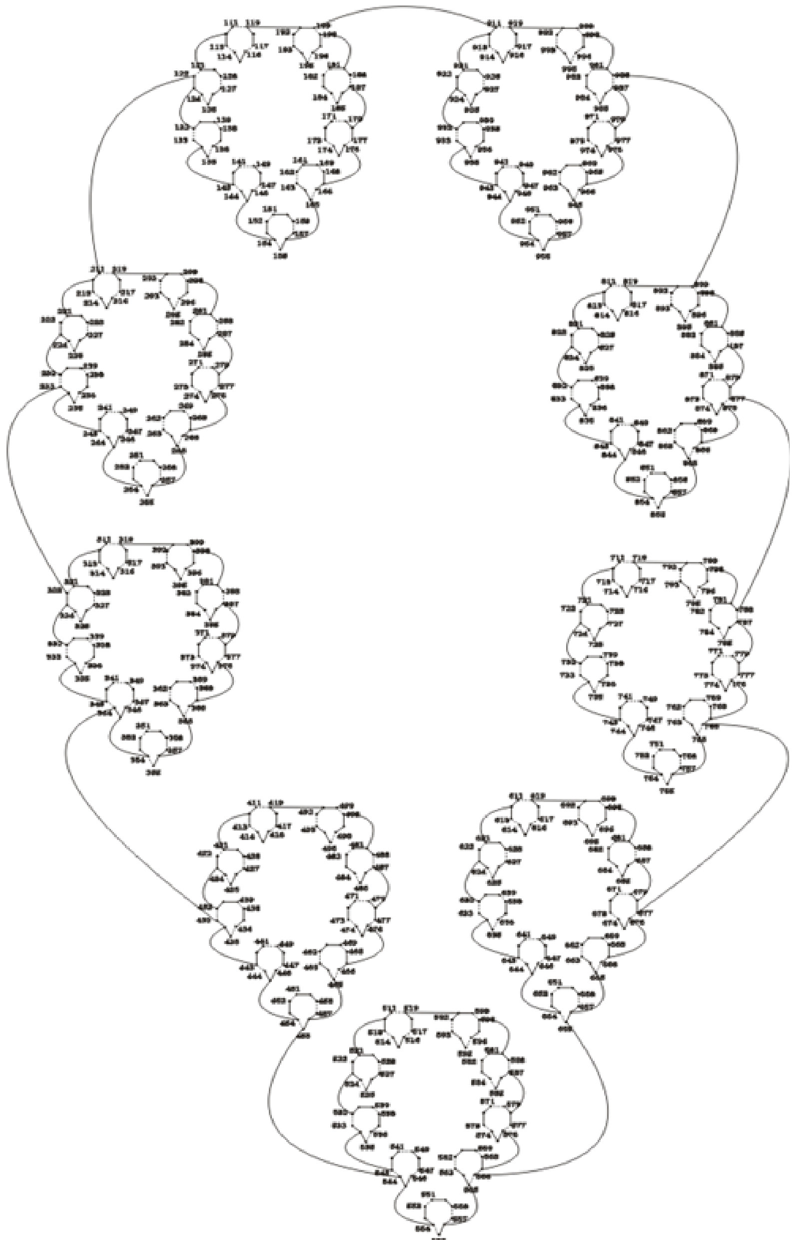
$S(2, C_9)$

Similarly, in S(2, C_9), here we have taken a subset D=

$$\left\{\begin{matrix}(11,19),(13,14),(16,17),\\(21,22),(24,25),(27,28),\\(32,33),(35,36),(38,39),\\(41,49),(43,44),(46,47),\\(51,52),(54,55),(57,58),\\(62,63),(65,66),(68,69),\\(71,79),(73,74),(76,77),\\(81,82),(84,85),(87,88),\\(92,93),(95,96),(98,99)\end{matrix}\right\} = \{All\ dotted\ lines\ of\ S(2, C_9)\} \quad \text{of edge set}$$

E{S(2, C_9)}. D has minimum number of cardinality which dominate all edges of S(2, C_9) .Therefore, the edge domination number of S(2, C_9) is 27.

Note:- Here we choosed nine edges (11, 19), (21, 22), (32, 33), (43, 44), (54, 55), (65, 66), (76, 77), (87, 88) and (98, 99) in minimum dominating set D in S(2, C_9) . When we construct S(3, C_9) by copying nine times S(2, C_9) and connect them by nine edges, there is no need to choose any extra edges for dominating all edges. Because the edges which connect to nine copies of S(2, C_9) are adjacent to these edges which are choosed in minimum dominating set D in S(2, C_9).



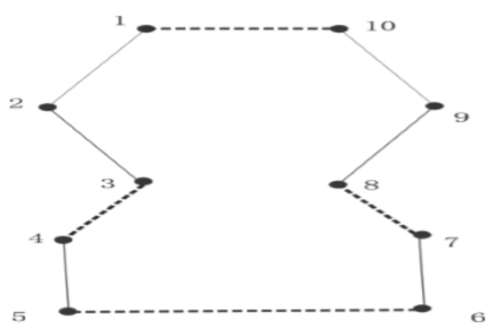
S(3, C_9)

Again, in S(3, C_9), here we are taking a subset D={All dotted edges of S(3, C_9)} of edge set E{S(3, C_9)}. D has minimum number of cardinality which dominate all edges of S(3, C_9) . Here each S(2, C_9) contains 27 dotted edges. Therefore, the total number of dotted edges of S(3, C_9) will be 27×9=243 which is the edge domination number.

Note:- Here, figure S(3, C_9) is very large. If we zoom S(3, C_9), we can see all dotted edges which are taken.

When we construct S(3, C_9) by copying nine times of S(2, C_9) and connect them by edges, there is no need to choose any extra edges for dominating all edges. All nine edges which connect to all nine copies of S(2, C_9) will be adjacent to the nine edges (11, 19), (21, 22), (32, 33), (43, 44), (54, 55), (65, 66), (76, 77), (87, 88) and (98, 99)which are choosed in minimum edge dominating set D of S(2, C_9).For example, in figure S(3, C_9),there are nine edges which connects to all nine copies of S(2, C_9) are adjacent to edges which we have already choosed in S(2, C_9), so here, there is no need to choose any extra edges for dominating all edges in S(3, C_9).The edge domination number of S(3, C_9) will be 9×edge domination number of S(2, C_9) = 9×27=243. Similarly, we can find the edge domination number of S(4, C_9) and it will be 9×edge domination number of S(3, C_9)=9×243=2187. Continung this process we can find the edge domination number of S(5, C_9), S(6, C_9),....... S(k, C_9). Therefore, the edge domination number of S(k, C_9)=$27\times 9^{k-2} = 3\times 9^{k-1}$ where $k \geq 1$. The edge domination number of S(1, C_9) to S(k, C_9) has been tabulated in Table No. – 4.1.

4(B).8 Edge domination number of Sierpinski Cycle Graph of order '10' i.e. γ' (S(k, C_{10})).

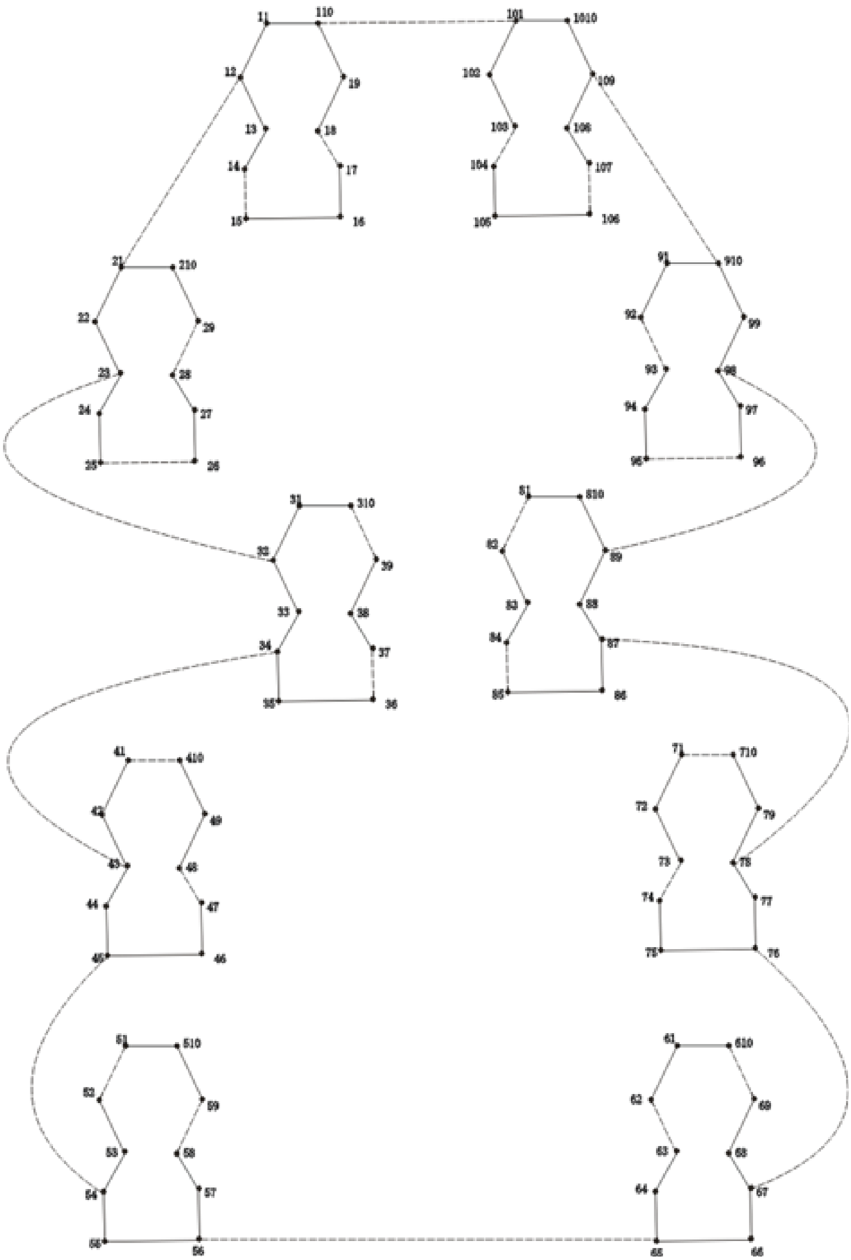
Notation	Graph
$S(1, C_{10})$	

In S(1, C_{10}), the edge Set is E={(1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7), (7, 8), (8, 9), (9, 10), (1, 10) }.

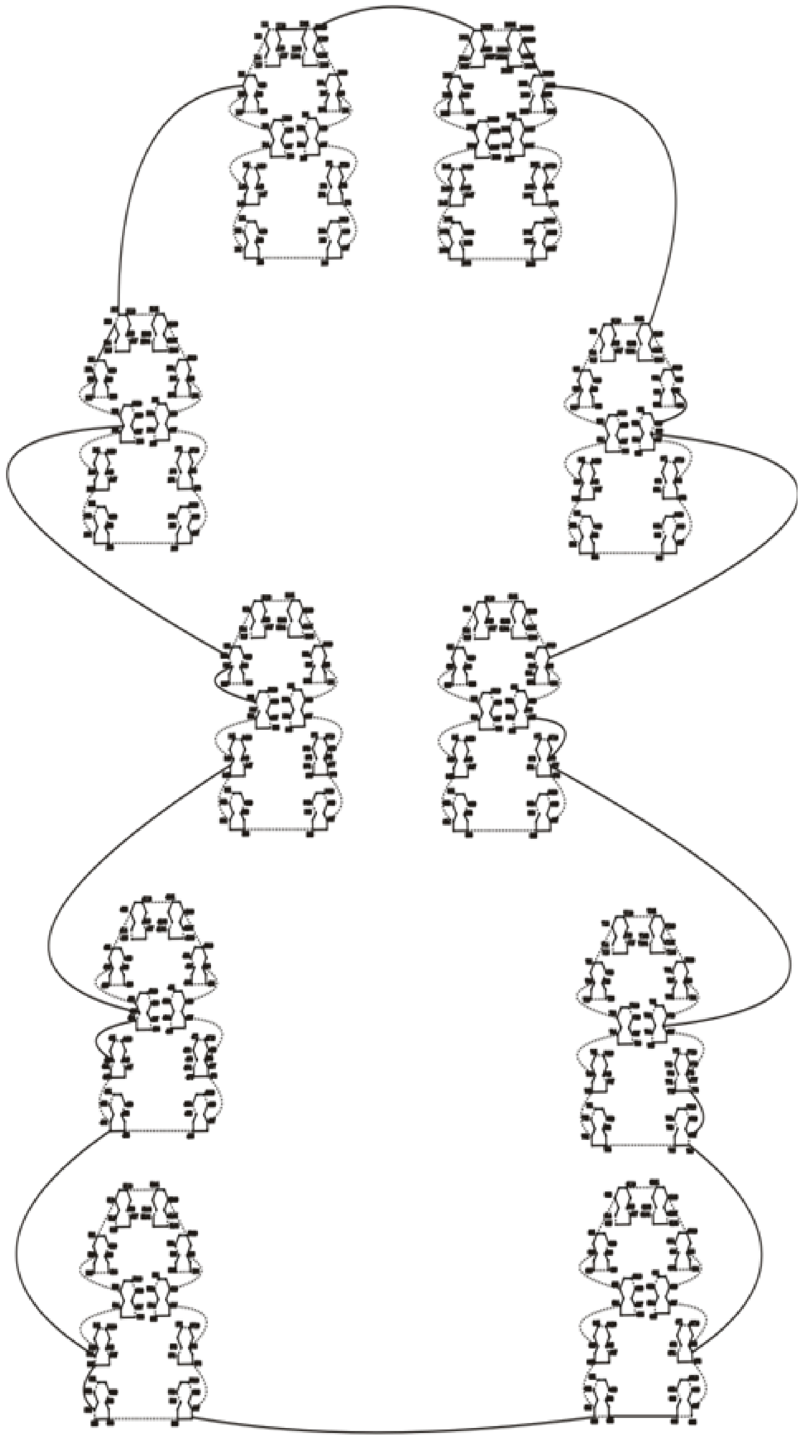
Here we have taken a subset D={(1, 10), (3, 4), (5, 6), (7, 8)}={All dotted lines}

of edge set E{S(1, C_{10})}. D has minimum number of cardinality which dominate all edges of S(1, C_{10}). Therefore the edge domination number of S(1, C_{10}) is 4.

S(2, C_{10})



Similarly, in S(2, C_{10}), here we have taken a subset D={All dotted lines of S(2, C_{10})} of edge set E{S(2, C_{10})}. D has minimum number of cardinality which dominate all edges of S(2, C_{10}). Therefore, the edge domination number of S(2, C_4) is 30.



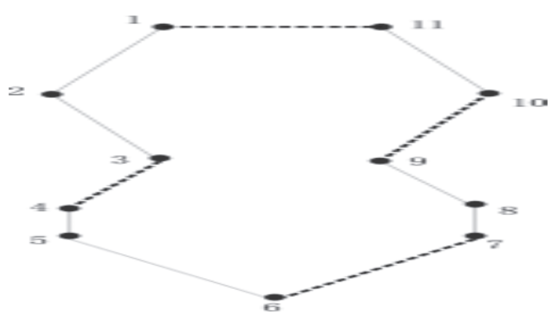
Again, in $S(3, C_{10})$, here we are taking a subset D={All dotted edges of $S(3, C_{10})$} of edge set E{$S(3, C_{10})$}. D has minimum number of cardinality which dominate all edges of $S(3, C_{10})$. Here each $S(2, C_{10})$ contains 31 dotted edges. Therefore, the total number of dotted edges of $S(3, C_{10})$ will be 31×10=310 which is the edge domination number.

Note:- Here, figure $S(3, C_{10})$ is very large. On zooming $S(3, C_{10})$, all dotted edges are visible.

When we construct $S(4, C_{10})$ by copying ten times of $S(3, C_{10})$ and connect them by edges, there is no need to choose any extra edges for dominating all edges. All ten edges which connect to all copies of $S(3, C_{10})$ will be adjacent to the ten edges (111, 1110), (221, 222), (332, 333), (443, 444), (554, 555), (665, 666), (776, 777),(887, 888), (998, 999), (10109, 101010), which are choosed in minimum dominating set D in $S(3, C_{10})$. Therefore, the edge domination number of $S(4, C_{10})$ will be 10×edge domination number of $S(3, C_{10})$=10×310=3100. Similarly, we can find the edge domination number of $S(5, C_{10})$ and it will be 10×edge domination number of $S(4, C_{10})$=10×3100=31000. Continung this process we can find $S(6, C_{10})$, $S(7, C_{10})$....... $S(k, C_{10})$. Therefore, the edge domination number of $S(k, C_{10}) = 31\times10^{k-2}$ where $k \geq 3$. The edge domination number of $S(1, C_{10})$, $S(2, C_{10})$ and the edge domination number of $S(3, C_{10})$ to $S(k, C_{10})$ have been tabulated in Table No. – 4.2.

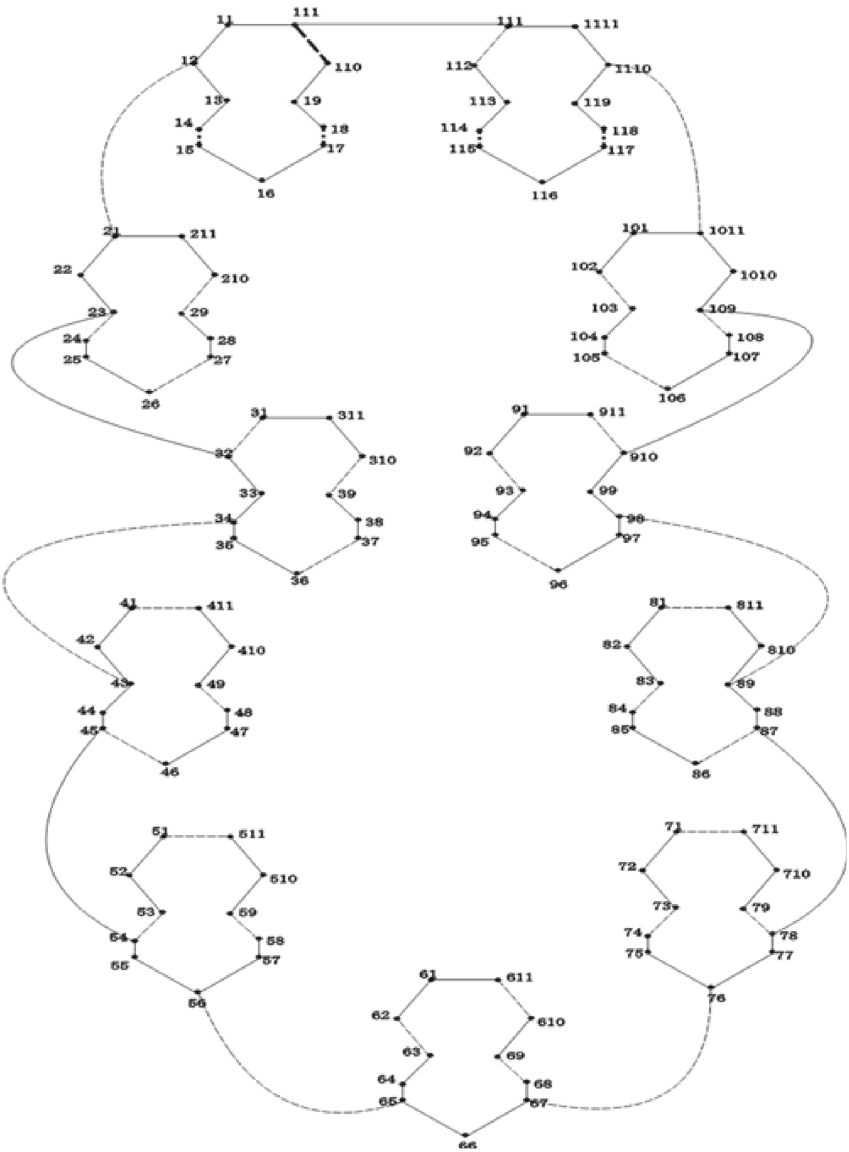
3.9 Edge domination number of Sierpinski Cycle Graph of order '11' i.e. $\gamma'(S(k, C_{11}))$.

Table 3.9

Notation	Graph
$S(1, C_{11})$	

In $S(1, C_{11})$, the edge Set is E={(1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7), (7, 8), (8, 9), (9, 10), (10, 11), (1, 11)}.

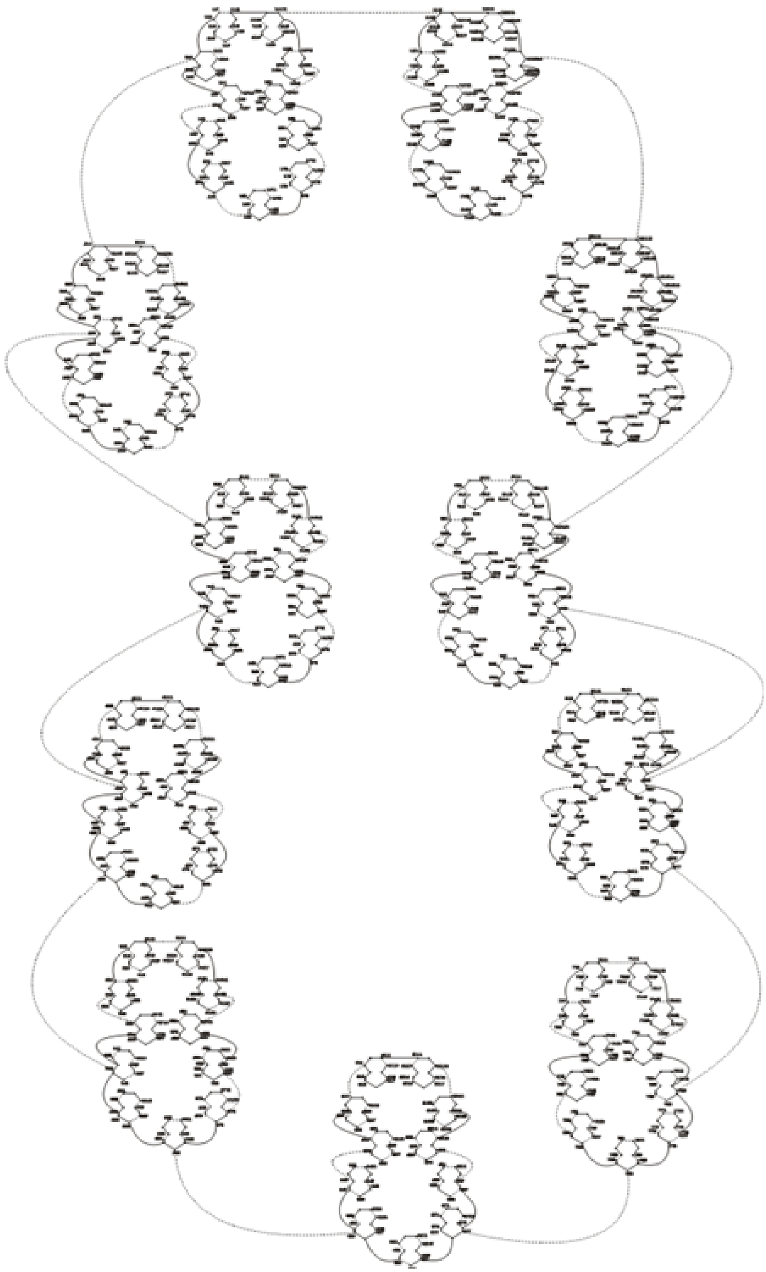
Here we have taken a subset D={(1, 11), (3, 4), (6, 7), (9, 10)}={All dotted lines} of edge set E{$S(1, C_{11})$}. D has minimum number of cardinality which dominate all edges of $S(1, C_{11})$. Therefore, the edge domination number of $S(1, C_5)$ is 4.



$S(2, C_{11})$

Similarly, in $S(2, C_{11})$, here we have taken a subset D={All dotted lines of $S(2, C_{11})$} of edge set E{$S(2, C_{11})$}. D has minimum number of cardinality which dominate all edges of $S(2, C_{11})$. Each $S(1, C_{11})$ contains three dotted lines and we take six edges which connect copies of $S(1, C_{11})$. Therefore, the edge domination number of $S(2, C_5)$ is 11×3+6=39.

S(3, C_{11})



Again, in S(3, C_{11}), here we are taking a subset D={All dotted edges of S(3, C_{11})} of edge set E{S(3, C_{11})}. D has minimum number of cardinality which dominate all edges of S(3, C_{11}) . Here each S(2, C_{11}) contains 38 dotted edges and we take eleven edges which connects all eleven copies of (2, C_{11}) . Therefore, the total number of dotted edges of S(3,C_11) will be 11×38+11=429 which is the edge domination number. Note:- Here, figure S(3,C_11) is very large. If we zoom S(3,C_11), we can see all dotted edges which are taken.

When we construct S(4, C_{11}) by copying eleven times of S(3, C_{11}) and connect them by edges, there is no need to choose any extra edges for dominating all edges. All eleven edges which connect to eleven copies of S(3, C_{11}) will be adjacent to the eleven edges (111, 111), (221, 222), (332, 333), (443, 444), (554, 555), (665, 666), (776, 777), (887, 888), (998, 999), (10109, 101010), (111110, 111111), which are choosed in minimum edge dominating set D in S(3, C_{11}). The edge domination number of S(4, C_{11}) will be 11 × edge domination number of S(3, C_{11}) = 11×429=4719. Similarly, we can find the edge domination number of S(5, C_{11}) and it will be 11×edge domination number of S(4, C_{11})=11×4719=51909. Continung this process we can find S(6, C_{11}), S(7, C_{11})....... S(k, C_{11}). Therefore, the edge domination number of S(k, C_{11})=$39\times11^{k-2}$ where k ≥ 2. The edge domination number of S(1, C_{11}) and the edge domination number of S(2, C_{11}) to S(k, C_{11}) have been tabulated in Table No. – 4.4.

4. RESULTS & CONCLUSION

From Table 3.1, 3.4 & 3.7, we conclude that

Table-4.1

Edge domination number	Cycle Graph		
	C_3	C_6	C_9
$\gamma'(S(1,G))$	1	2	3
$\gamma'(S(2,G))$	3	12	27
$\gamma'(S(3,G))$	9	72	243
$\gamma'(S(4,G))$	27	432	2187
...	...	...	...
...	...	...	...
...	...	...	...
$\gamma'(S(k,G))$	$1\times3^{k-1}$	$2\times6^{k-1}$	$3\times9^{k-1}$

From given this table, we conclude that the edge domination Number of Sierpinski Cycle Graph of order , where , is given by

$$\gamma'(S(k, C_{3n})) = n\,(3n)^{k-1}, where\ n = 1, 2, 3, 4, 5, \cdots\ and\ k \geq 1.$$

From Table 3.2, 3.5 & 3.8, we conclude that

Table-4.2

Edge domination number	Cycle Graph		
	C_4	C_7	C_{10}
$\gamma'(S(1,G))$	2	3	4
$\gamma'(S(2,G))$	4	14	30
$\gamma'(S(3,G))$	20	105	310
$\gamma'(S(4,G))$	80	735	3100
....	....	....	....
....	....	....	....
....	....	....	....
$\gamma'(S(k,G))$	$5 \times 4^{k-2}$	$15 \times 7^{k-2}$	$31 \times 10^{k-2}$

From given this table, we conclude that the edge domination Number of Sierpinski Cycle Graph C_{4+3n} of order 'n', where n=0,1,2,3,4,5⋯, is given by

- $\gamma'(S(1, C_{4+3n})) = n + 2,\ where\ n = 0, 1, 2, 3, 4, 5, \cdots.$
- $\gamma'(S(2, C_{4+3n})) = (n + 1) \times (4 + 3n),\ where\ n = 0, 1, 2, 3, 4, 5, \cdots$
- $\gamma'(S(k, C_{4+3n})) = [(n + 1) \times (4 + 3n) + 1](4 + 3n)^{k-2},\ where\ n = 0, 1, 2, 3, \cdots\ and\ k \geq 3.$

From Table 3.6, we conclude that

Table-4.3

Edge domination number	C_8
$\gamma'(S(1,G))$	3
$\gamma'(S(2,G))$	20
$\gamma'(S(3,G))$	168
$\gamma'(S(4,G))$	1344
...	...
...	...
...	...
$\gamma'(S(k,G))$	$21 \times 8^{n-2}$

From given this table, we conclude that the edge domination Number of Sierpinski Cycle Graph C_{5+3n} of order 'n', where n=1,3,5,7⋯, is given by

- $\gamma'(S(1, C_{5+3n})) = n + 2, \; where\; n = 1, 3, 5, \cdots$
- $\gamma'(S(2, C_{5+3n})) = (n + 1) \times (5 + 3n) + \frac{(5+3n)}{2}, \; where\; n = 1, 3, 5, 7 \cdots$
- $\gamma'(S(k, C_{5+3n})) = \left[(n + 1) \times (5 + 3n) + \{\frac{5+3n}{2} + 1\}\right] \times (5 + 3n)^{k-2}$, $where\; n = 1, 3, 5, 7 \cdots \;\&\; k \geq 3.$

From Table 3.3 & 3.9, we conclude that

Table-4.4

Edge domination number	Cycle Graph	
	C_5	C_{11}
$\gamma'(S(1, G))$	2	4
$\gamma'(S(2, G))$	8	39
$\gamma'(S(3, G))$	40	429
…	…	…
…	…	…
…	…	…
$\gamma'(S(k, G))$	$8 \times 5^{k-2}$	$39 \times 11^{k-2}$

From given this table, we conclude that the edge domination Number of Sierpinski Cycle Graph C_{5+3n} of order 'n', where n=0,2,4,6⋯, is given by

- $\gamma'(S(1, C_{5+3n})) = n + 2, \; where\; n = 0, \; 2, 4, 6 \cdots$
- $\gamma'(S(2, C_{5+3n})) = (n + 1) \times (5 + 3n) + \frac{\{(5+3n)+1\}}{2}, \; where\; n = 0, 2, 4, 6, \cdots$
- $\gamma'(S(k, C_{5+3n})) \left[(n + 1) \times (5 + 3n) + \frac{\{(5+3n)+1\}}{2}\right] \times (5 + 3n)^{k-2}$, $where\; n = 0, 2, 4, 6, \cdots \;\&\; k \geq 3.$

REFERENCE

[1] Michael D. Plummer, "*Some recent result on Domination in Graphs" Journal of Graph Theory***, vol. 26, 457-474, 2006.**

[2] T. W. Haynes, S. T. Hedetniemi and P.J. Slater, "*Fundamentals of Domination in Graphs***", Marcel Dekker, Inc. New York, 1998.**

[3] Mohit James , "*Domination number in Sierpinski 4-cycle Graph***", International Journal of Mathematics Trends and Technology, Vol.52, 562-569, 2017.**

[4] O. T. Manjusha and M. S. Sunitha, "*Strong Domination in Fuzzy Graph***", Science Direct, vol. 7, 369-377, 2015.**

[5] A. M. Hinz, S. Klavzar, U. Milutinovic and C. Petr, "*The Tower of Hanoi Myths and Maths***", Birkh• auser/Springer, Basel, 2013.**

[6] A. Vijayan, J. Sherina Beul, "*Edge-Vertex Dominating Sets and Edge- Vertex Domination Polynomial of cycles***", Open Journal of Discrete Mathematics, vol. 5, 74-8, 2014.**

[7] A. M. Hinz, A. Schief, "*The average distance on the Sierpriński gasket***",Probab. Theory Related Fields 87, 129-138, 1990.**

[8] **Alberto M. Teguia and Anant P. Godbole, "***Sierprinski Gasket Graphs and Some of their properties***", www.arxiv:math/0509259vl, 1-15, 2005.**

[9] **FuXulianga, Yang Yuanshenga, Jiang Baoqia, "***Dominating set Generalized Peterson graph", Science direct discrete mathematics***, vol.309, 2445-2451, 2009.**

[10] **N. Siddiqui and Mohit James, "***Domination and Chromatic Number of Pan Graph and Lollipop Graph***", International Journal of Technical Innovation in Modern Engineering & Science(IJTIMES), Vol.4, 932-939, 2018.**

[11] **B. L. Hartnell, L. K. Jørgensen, P. D. Vestergaard, and C. A. Whitehead, "***Edge stability of the k-domination number of trees***," Bulletin of the Institute of Combinatorics and Its Applications, vol. 22, pp. 31–40, 1998.**

[12] **S. T. Hedetniemi and R. C. Laskar, "***Bibliography on domination in graphs and some basic definitions of domination parameters***," Discrete Mathematics, vol. 86, no. 1–3, pp. 257–277, 1990.**

[13]. **S. Mitchell and S. T. Hedetniemi, "***Edge domination in trees***",** *Congr. Numer***. 19, pp. 489-509, 1977.**

[14] **S.L. Lipscomb, "***A universal one-dimensional metric space***", TOPO 72–General Topology and its Applications, Second Pittsburgh Internat. Conf., Lecture Notes in Math. 378, pp. 248–257, 1974.**

[15] **U. Milutinovic, "***Completeness of the Lipscomb space***", Glas. Mat. Ser. III 27(47) , pp. 343–364, 1992.**

[16] **S. Arumugam and S. Velammal, "***Edge Domination In Graphs***", Taiwanese Journal of Mathematics , Vol. 2, No. 2, pp. 173-179, June 1998.**

[17] **Araya Chaemchan, "***The edge domination number of connected graphs***", Australasian Journal Of Combinatorics, Volume 48, pp. 185–189, 2010.**

[18] **S. Gravier, M. Kovse and A. Parreau, "***Generalized Sierpinski graph***", 2011, https://pdfs.semanticscholar.org**

[19] **S. Klavžar and U. Milutinovic, "***Graphs S(n; k) and a variant of the Tower of Hanoï problem***", Czechoslovak Math. J. 47(122) 95-104, 1997.**

Kalanidhi Kalyan[1]
Kalanidhi.kalyan@gmail.com
Mohit James[2]
jamesmohit@yahoo.co.in
[1&2]Department of Mathematics and Statistics,
SHUATS, Prayagraj, (U.P), India