SOME PROPERTIES OF (1,2)S_P G-CLOSED SETS IN BITOPOLOGICAL SPACES

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Abstract:

In this paper, a new kind of generalized closed sets called (1,2)$S_p g$-closed sets in bitopological spaces are introduced and studied some of their properties.

Key words:

(1,2)semi-open sets, (1,2)pre-open sets, (1,2)S_p-open sets, (1,2)S_p-closed sets, (1,2)$S_p g$-open sets, (1,2)$S_p g$-closed sets.

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1.INTRODUCTION

A topological space occurs for every metric space but the bitopological spaces occurs for quasi-metric spaces. A space X equipped with two arbitrary topologies τ_1 and τ_2 is defined as the bitopological space by Kelly [3] in the year 1963 denoted it by a triplet (X, τ_1, τ_2). Every bitopological space (X, $\tau_{1,}$ τ_2) can be regarded as a topological space (X, τ) if $\tau_1 = \tau_2 = \tau$.

The generalized closed sets, simply g-closed sets in a topological space were introduced and studied by Levine [6] in the year 1970. In bitopological spaces, Fukutake [1] introduced and investigated the concept of g-closed in the year 1985. There were many different kinds of generalized closed sets on bitopological spaces introduced by different authors.

In this paper, a new kind of generalized closed sets in bitopological spaces are introduced and compare with some of the corresponding generalized closed sets and studied their properties.

2. PRELIMINARIES

Throughout this paper X and Y will be denote the topological spaces. If A is a subset of X, then the closure and interior of A in X are denoted by cl(A) and int(A) respectively.

Definition 2.1 [4] A subset A of a bitopological space X is called a

(i) (1,2)semi-open if $A \subseteq \tau_1\tau_2\mathrm{Cl}(\tau_1\mathrm{Int}(A))$.

(ii) (1,2)pre-open if $A \subseteq \tau_1\text{Int}(\tau_1\tau_2\text{Cl}(A))$.

(iii) (1,2)regular-open if A = $\tau_1\text{Int}(\tau_1\tau_2\text{Cl}(A))$.

The collection of all (1,2)semi-open, (1,2)pre-open and (1,2)regular-open sets are denoted by (1,2)SO(X), (1,2)PO(X) and (1,2)RO(X) respectively.

Definition 2.2. [4] A subset A of a bitopological space X is called a

(i) (1,2)α-closed if $\tau_1\text{Cl}(\tau_1\tau_2\text{Int}(\tau_1\text{Cl}(A))) \subseteq A$.

(ii) (1,2)semi-closed if $\tau_1\tau_2\text{Int}(\tau_1\text{Cl}(A)) \subseteq A$.

(iii) (1,2)pre-closed if $\tau_1\text{Cl}(\tau_1\tau_2\text{Int}(A)) \subseteq A$.

(iv) (1,2)regular-closed if $A = \tau_1\text{Cl}(\tau_1\tau_2\text{Int}(A))$.

The set of all (1,2)α-closed, (1,2)semi-closed, (1,2)pre-closed and (1,2)regular-closed sets are denoted as (1,2)αCL(X), (1,2)SCL(X), (1,2)PCL(X) and (1,2)RCL(X) respectively.

Also, for any subset A of X, the (1,2)α-closure, (1,2)semi-closure, (1,2)pre-closure and (1,2)regular-closure of A is denoted as (1,2)αCl(A), (1,2)SCl(A), (1,2)PCl(A) and (1,2)RCl(A) respectively.

Definition 2.3. [2] A (1,2)semi-open set A of a bitopological space X is called (1,2)S_p-open set if for each $x \in A$, there exists a (1,2)pre-closed set F such that $x \in F \subseteq A$.

Remark 2.4. [2] Any intersection of (1,2)S_p-closed sets of a bitopological space X is (1,2)S_p-closed.

Definition 2.5. [5] A subset A of a space (X, τ) is called

(i) generalized-closed (briefly g-closed) [2], if cl(A) $\subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) and the complement of a g-closed set is called g-open.

3. (1,2)S_pG-CLOSED SET

Definition 3.1. A subset A of bitopological space X is called a (1,2)S_p-generalized-closed (briefly (1,2)S_pg-closed) set if (1,2)S_pCl(A) $\subseteq U$ whenever $A \subseteq U$ and $U \in$ (1,2)S_pO(X).

The family of all (1,2)S_pg-closed sets is denoted by (1,2)S_pGCL(X).

Remark 3.2. Every (1,2)S_p-closed set is a (1,2)S_pg-closed set but the converse need not always be true and is shown in the following example.

Example 3.3. Let $X = \{a,b,c,d\}$ with two topologies τ_1 = {ϕ, X, {a}, {b}, {a, b}} and τ_2 = {ϕ, X, {a, b, d}}. Then (1,2)S_pO(X) = {ϕ, X, {a, c, d}, {b, c, d}} and (1,2)S_pCL(X) = {X, ϕ, {b}, {a}}. Also, (1,2)S_pGCL(X) = { X, ϕ, {a}, {b}, {a, b}, {a, b, c}, {a, b, d}}. Here, {a, b}, {a, b, c}, {a, b, d} are in (1,2)$S_p g$-closed sets but not (1,2) S_p-closed set.

Theorem 3.4. In a bitopological space X, a subset A of X is called a (1,2)$S_p g$-closed set iff [(1,2)S_pCl(A) −A] contains no non-empty (1,2) S_p-closed set.

Proof. Let A be a (1,2)S_pg-closed set. Then there exists a (1,2)S_p-open set U such that $A \subseteq U$ and (1,2)S_pCl(A) ⊆ U. Let F be a non-empty (1,2) S_p-closed set such that $F \subseteq$ [(1,2)S_pCl(A) − A]. Then $F^c \supseteq$ [(1,2)S_pCl(A)− A]^c, which implies $A \subseteq F^c$. Hence (1,2)S_pCl(A) ⊆ F^c which implies $A \subseteq$ [(1,2)S_pCl(A)]^c. Therefor,e $A \subseteq$ (1,2)S_pCl(A) ∩ [(1,2)S_pCl(A)]^c. Thus F = ϕ. Hence [(1,2)S_pCl(A) – A] contains no non-empty (1,2)S_p-closed set.

Also, Let $A \subseteq U$ and $U \in$ (1,2)S_pO(X) such that A is not a (1,2)$S_p g$-closed set. Then (1,2)S_pCl(A) is not a subset of U, which implies that (1,2)S_pCl(A) ⊆ U^c. Then (1,2)S_pCl(A) ∩ U^c is a non-empty (1,2)S_p-closed subset of [(1,2)S_pCl(A) – A], which is a contradiction. Hence A is (1,2)$S_p g$-closed set.

Theorem 3.5. A (1,2)$S_p g$-closed set is (1,2) S_p-closed iff [(1,2)S_pCl(A) – A] is (1,2)S_p-closed.

Proof. If A is (1,2)S_p-closed, then [(1,2)S_pCl(A)– A] = ϕ. By Theorem 3.4, [(1,2)S_pCl(A) – A] is (1,2)S_p-closed. Also [(1,2)S_pCl(A) – A] itself is a subset of it. By Theorem 3.4, [(1,2)S_pCl(A)– A] = ϕ. Hence A is (1,2)S_p-closed.

Remark 3.6. The intersection of two (1,2)$S_p g$-closed sets need not always be a (1,2)$S_p g$-closed set and is shown in the following example.

Example 3.7 Let X = {a, b, c, d} with two topologies τ_1 = {ϕ, X, {a}, {c}, {a, c}, {a, d}, {a, c, d}} and τ_2 = {ϕ, X}. Then (1,2)S_pO(X) = {ϕ, X, {b, c}, {a, b, d}} and (1,2)S_pCL(X) = { X, ϕ, {a, d}, {c}}. Also, (1,2)S_pGCL(X) = { X, ϕ, {a}, {c}, {d}, {a, c}, {a, d}, {c, d}, {a, b, c}, {a, c, d}, {b, c, d}}. Here {a, b, c} and {b, c, d} ∈ (1,2)S_pGCL(X) but {a, b, c} ∩ {b, c, d} = {b, c} ∉ (1,2)S_pGCL(X).

Definition 3.8. If (1,2)S_p-open sets are closed with respect to finite intersection then X is called the (1,2)S_p-topology.

Theorem 3.9. If A and B are (1,2)$S_p g$-closed sets, then $A \cup B$ is also (1,2)$S_p g$-closed set only if the space is (1,2)S_p-topology.

Proof. Let A and B are (1,2)$S_p g$-closed sets. Also, let $A \cup B \subseteq U$, where $U \in$ (1,2)S_pO(X). Since X is an (1,2)S_p-topological space, (1,2)S_pCl($A \cup B$) = (1,2)S_pCl(A) ∪ (1,2)S_pCl(B). As A and B are (1,2)$S_p g$-closed sets, implies (1,2)S_pCl(A) $\subseteq U$ and (1,2)S_pCl(B) $\subseteq U$. Thus (1,2)S_pCl($A \cup B$) $\subseteq U$. Hence, $A \cup B$ is also (1,2)$S_p g$-closed only if the space is (1,2)S_p-topology.

Remark 3.10. The condition "(1,2)S_p-topology" cannot be removed in Theorem 3.9 and is justified by the following example.

Example 3.11. Let X = {a, b, c, d} with two topologies τ_1 = {ϕ, X, {c}, {a, b}, {a, b, c}, {a, b, d}} and τ_2 = {ϕ, X}. Then (1,2)S_pO(X) = {ϕ, X, {c}, {a, b}, {a, b, c}, {a, b, d}} and (1,2)S_pCL(X) = { X , ϕ, {a, b, d}, {c, d}, {d}, {c}}. Also, (1,2)S_pGCL(X) = {ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}}. Here {a}, {c} ∈ (1,2)S_pGCL(X). But {a} ∪ {c} = {a, c} ∉ (1,2)S_pGCL(X).

Lemma 3.12. For a bitopological space X, either every singleton set $\{x\}$ is (1,2)S_p-closed or its complement $\{x\}^c$ is (1,2)S_pg-closed.

Proof. If $\{x\}$ is not (1,2)S_p-closed, then the only (1,2)S_p-open set containing $X - \{x\}$ is X. Hence, $\{x\}^c$ is (1,2)S_p-closed.

Definition 3.13. A subset of A of a bitopological space X is called a (1,2)S_p-generalized-open (briefly (1,2)$S_p g$-open) set if A^c is (1,2)$S_p g$-closed. Then, the family of all (1,2)$S_p g$-open sets is denoted by (1,2)S_pGO(X).

Lemma 3.14. For any subset A of a bitopologicl space X, (1,2)S_pInt[(1,2)S_pCl(A) − A] = ϕ.

Theorem 3.15. For a subset A of X, (1,2)$S_p Cl(A^c)$ = [(1,2)$S_p Int(A)]^c$.

Proof. Let A be a subset of X. Then $A^c = X - A$. Also, let E be a (1,2)S_p-open set such that $E \subset A$. By definition (1,2)S_pInt(A) = ∪ {E / $E \subset A$ and $E \in$ (1,2)S_pO(X)}. That is, (1,2)S_pInt(A) = ∪ {$X - F$ / $X - A \subset F$ and $F = X - E$}. Hence, (1,2)S_pInt(A) = X − ∩ {F / F is (1,2)S_p-closed and $X - A \subset F$}. Therefore, (1,2)S_pInt(A) = X – (1,2)$S_p Cl(X - A)$. That is, (1,2)S_pInt(A) = X– (1,2)$S_p Cl(A^c)$ which implies (1,2)$S_p Cl(A^c)$ = X – (1,2)S_pInt(A). Hence (1,2)$S_p Cl(A^c)$= [(1,2)$S_p Int(A)]^c$.

Theorem 3.16. A subset A of a bitopological space X is (1,2)$S_p g$-open iff $F \subseteq$ (1,2)S_pInt(A) whenever F is (1,2)$S_p g$-closed and $F \subseteq A$.

Proof. Let A be (1,2)$S_p g$-open. Then A^c is (1,2)$S_p g$-closed implies (1,2)$S_p Cl(A^c) \subseteq F^c$. By Theorem 3.15, (1,2)$S_p Cl(A^c)$ = $[(1,2)S_p\text{Int(A)}]^c$ implies $[(1,2)S_p\text{Int(A)}]^c \subseteq F^c$. Hence, $F \subseteq (1,2)S_p$Int(A) whenever F is (1,2)S_pg-closed and $F \subseteq A$.

Also, Let $F \subseteq$ (1,2)S_pInt(A), where F is (1,2)S_pg-closed and $F \subseteq A$. Let $G = X - F$ be a (1,2)$S_p g$-open such that $A^c \subseteq G$. Then by assumption, $G^c \subseteq$ (1,2)S_pInt(A) which implies that $[(1,2)S_p\text{Int}(A)]^c \subseteq G$ implies (1,2)$S_p Cl(A^c) \subseteq G$. Therefore A^c is (1,2)$S_p g$-closed. Hence, A is (1,2)$S_p g$-open.

Theorem 3.17. A subset A of a bitopological space X is (1,2)$S_p g$-closed, if [(1,2)S_pCl(A) − A] is (1,2)$S_p g$-open.

Proof. Let $A \subseteq U$ and $U \in$ (1,2)S_pO(X). Now (1,2)S_pCl(A) ∩ ($X - U$) ⊆ (1,2)S_pCl(A) ∩ ($X - A$) = [(1,2)S_pCl(A) − A] and by Remark 2.4, (1,2)S_pCl(A) ∩ ($X - U$) is (1,2)S_p-closed and by assumption, [(1,2)S_pCl(A) − A] is (1,2)$S_p g$-open. By Theorem 3.16, (1,2)S_pCl(A) ∩ ($X - A$) ⊆ (1,2)S_pInt(A)[(1,2)S_pCl(A) − A] = ϕ which implies (1,2)S_pCl(A) ∩ ($X - U$) = ϕ Then, (1,2)S_pCl(A) ⊆ U. Hence A is (1,2)$S_p g$-closed.

Theorem 3.18. If a subset A of X is (1,2)$S_p g$-open, then $G = X$, whenever G is (1,2)S_p-open and (1,2)S_pInt(A) ∪ $A^c \subseteq G$.

Proof. Let G is (1,2)S_p-open and (1,2)S_pInt(A) ∪ $A^c \subseteq G$. Then $G^c \subseteq$ [(1,2) S_pCl(A^c) − A^c]. Now G is (1,2)S_p-open implies G^c is (1,2)S_p-closed and A is (1,2)$S_p g$-open implies A^c is (1,2)$S_p g$-closed. Hence by Theorem 3.4, G^c = ϕ, which implies $G = X$.

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