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SMART ENHANCED ASSUMED STRAIN ELEMENT OF SMART LAMINATED PLATES

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ABSTRACT

In the presentation, generalized variational principle of Mindlin type laminated plates with piezoelectric sensor/actuator layer has been studied. It shown that piezoelectric actuator is playing as an active damper and smart finite element formulation can be easily done by adding one electric d.o.f on the normal element. The principle has been applied to constructing enhanced assumed strain element.

Keywords: smart materials, plates, finite element

1. INTRODUCTION

As a result of rapid advance in smart structures, the research thrust today is towards using the piezoelectric materials as distributed sensors or actuators in control applications. Lots of problems have been investigated with emphasis on the laminated Mindlin plates manufactured with piezoelectric sensor and actuator layers [1-10, 11, 14, 15]. Due to the complicate of final governing equations of the smart structures, finite element method as a dominated approach has being used to simulate the process of deformation and control of the structures. Since then develop of the first plate bending finite elements, a very large number of elements has been proposed. In recent years, considerable attention has been devoted to the development of lower order, locking free quadrilateral elements (typically bilinear) which exhibit *high accuracy in coarse meshes*; particularly in *bending dominated situations*. Unfortunately, for plates or shells shear locking occurs when pure bending modes are spoiled by parasitic shear strains so that the kinematics Kirchhoff condition cannot be represented. For nearly incompressible conditions, volumetric locking may also occur; in this case, deviatoric modes always come along undesirable with volumetric strains [11, 13].

To overcome locking, a number of techniques have been proposed. Quadrilateral elements of Mindlin plates by using EAS methods, the element proposed by Simo & Rifai (1990) is still locking. It is clearly to have a need to design a new EAS element for Mindlin plates, which is locking free and high accuracy in coarse meshes.

In this paper, in order to able to use the development of Sun [13], a two filed generalized variational principle has been constructed for smart laminated Mindlin plates with feedback of sensor's signal under some assumptions to both sensor/actuator layers and ordinary layers and their interfaces. Based on the principle, smart finite element (SFE) concept on element level has been proposed, i.e., the SFE can be constructed by adding one feedback electric d.o.f. on the normal element. This understanding will make the analysis of smart structures more easily with the possibility of using all results of FEM development [12].

2. BRIEF CONTENT OF NEW EAS CONCEPT [13]

The central idea is as follows: We start with a two-field modified Hu-Washizu variational formulation of elasticity, and consider strain fields of the form $\varepsilon = Lu + \tilde{\varepsilon}$. In which Lu is the symmetric gradient of the displacement field. In a finite element context, $\tilde{\varepsilon}$ is refereed as the enhanced part of the strain field. Design of the enhanced strain follows Simo & Rifai (1990). Within the content the orthogonality condition subjected to the **Q** and **E:** $\iint \mathbf{Q}^T \mathbf{E} d\xi d\eta = \mathbf{0}$ **,** is superfluous.

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The two-field modified Hu-Washizu formulation is

$$\Pi(\varepsilon,\mathbf{u}) = \int [\varepsilon^T \mathbf{D}(\mathbf{Lu}) - \tfrac{1}{2}\varepsilon^T D\varepsilon] dV - \int \mathbf{f}^T \mathbf{u} dV - \int \mathbf{t}_e^T \mathbf{u} dS_\sigma \tag{1}$$

Introducing enhanced strain field, we have

$$\Pi(\tilde{\varepsilon},\mathbf{u}) = \tfrac{1}{2}\int (\mathbf{Lu} + \tilde{\varepsilon})^T \mathbf{D}(\mathbf{Lu} - \tilde{\varepsilon})] dV - \int \mathbf{f}^T \mathbf{u} dV - \int \mathbf{t}_e^T \mathbf{u} dS_\sigma \tag{2}$$

Clearly, the advantage of this method is that the interpolation of the enhanced field $\tilde{\varepsilon}$ is not subject to any interelement continuity requirement and, in particular, can be derived consistent with any even 'incompatible mode' field, not to be subjected any other condition, such as an orthogonality condition. The recovery of stress field can be in the classical way, i.e., $\sigma = \mathbf{D}(\mathbf{Lu} + \tilde{\varepsilon})$.

3. SMART LAMINATED MINDLIN TYPE PLATES

The smart laminated plates integrated by fibber reinforced composite lamina and distributed piezoelectric sensors and actuators; that are assumed to be perfectly bonded together with the reinforced lamina. The sensors are used to monitor the deformation of the structures and the actuators are used to control and enhance the deformation.

The deformation of piezoelectric structures will let sensor generate charge; the total charge developed on the surface of the sensor layer is $q(t)$. When piezoelectric sensors are used as strain sensors, the output charge can be transformed to sensor current $i(t) = \dfrac{dq(t)}{dt}$, and current is converted into the open circuit sensor voltage output of controller $V(t) = -G_c i(t)$, which is actually the applied load on the actuators of smart structures given by controller. The actuator layers will deform and are result as damper of the smart structures.

Therefore, the simulation of smart structures consists of following steps.

Sensor analysis - this step is to formulate sensor equation and get current $I(t)$.

Since no external electric field is applied to the sensor layer, the electric displacement developed on the sensor surface is directly proportional to the strain acting on the sensor. If the poling direction is z for sensors with the electrodes on the upper and lower surfaces, the electric displacement is $D_z = e_{31}\varepsilon_x + e_{32}\varepsilon_y + e_{36}\gamma_{xy}$.

The total charge on the sensor layer will be $q(t) = \iint_A D_z dxdy$, where A is the domain of the sensor. The current can be obtained by $i(t) = \dfrac{dq(t)}{dt}$, or $i(t) = \iint_A (e_{31}\dot{\varepsilon}_x + e_{32}\dot{\varepsilon}_y + e_{36}\dot{\gamma}_{xy}) dxdy$. It can be further expressed in terms of velocity.

Controller analysis - this step is to design a control mechanism, or called as feedback analysis, usually, this step is simulated by introducing a series of gain G_c and negative velocity feedback control.

The input for controller is i(t); the output of the controller will be in voltage $V(t) = -G_c i(t)$. The gain G_c plays a role as amplifier and "-" present the negative feedback control. In other word, "$-G_c$" represents a controller in practice.

Actuator analysis - this step is to formulate actuator equation, that is, the reaction forces and moment supplied by the actuator layers.

Since the actuator layers are thin, so that the electric field intensity E will be uniform for each layer. For the kth layer, $E_z^k = V^k/h^k$, where V^k is applied voltage across the kth layer and h^k is the thickness of the kth layer. According the definition of resultant forces and moment of laminated structures (beams, plates and shells), the contribution of actuator layers on resultant forces is $N^p = \sum_{k=1}^{n} (E_z^k e_{31}^k)(z_k - z_{k-1})$, and resultant moment is

$M^p = \frac{1}{2}\sum_{k=1}^{n}(E_z^k e_{32}^k)(z_k^2 - z_{k-1}^2)$, where z_k is the distance of k[th] layer to the reference surface of the smart structures, and number n denotes the total number of layers.

Smart structure analysis - this last step is to combine the above all results to formulate a governing equations by Hamilton principle, called smart structures equations.

By using Hamilton principle and taking into account shear strain, the two field generalized weak form of smart structure governing equations can be expressed as

$$\begin{aligned}&\int_S [I_1\dot{\mathbf{u}}^{\mathrm{T}}\delta\dot{\mathbf{u}} + I_2\delta(\dot{u}^T\dot{\theta}) + I_3\dot{\theta}^T\delta\dot{\theta}]dS + \int q\delta w dS -\\ &\int_S [(A_s\gamma)^T\delta(\nabla w - \theta) + (A_s\delta\gamma)^T(\nabla w - \theta - \gamma)]dS -\\ &\int_S [(\mathbf{N} - \mathbf{N}^{\mathrm{p}})^T\delta\varepsilon + (\mathbf{M} - \mathbf{M}^{\mathrm{p}})^T\delta\kappa + \mathbf{Q}^T\delta\gamma]dS = 0.\end{aligned} \tag{3}$$

$\dot{u} = \{\dot{u}\ \dot{v}\}^T$, $\dot{\theta} = \{\dot{\theta}_x\ \dot{\theta}_y\}^T$; and moments of area are

$$\begin{aligned}&I_1 = \sum \rho_k(z_k - z_{k-1}), \quad I_2 = \tfrac{1}{2}\sum \rho_k(z_k^2 - z_{k-1}^2),\\ &I_3 = \tfrac{1}{3}\sum \rho_k(z_k^3 - z_{k-1}^3).\end{aligned} \tag{4}$$

The laminate constitutive equation are expressed as

$N = A\varepsilon + B\kappa$, $M = B\varepsilon + D\kappa$, where A, B, C and A_s are the elastic coefficients matrices based on the matric transformation [4, 5, 12].

From the definition of N^p and M^p, which contain the integration form of velocity of displacement, and the weak form of SSE, the piezoelectric actuators will play as a velocity type of damping, have nothing to do with mass and stiffness. In other words, the smart finite element can be formulated by adding one electric d.o.f. on the normal element. This understanding will be very useful to the researcher who prefer to use commercial FEM package with user's interface, since they only need to write damping subroutine. This also brings a great business opportunity to FEM package developer to develop a programme with a feature of solving smart structures, which has not be in market so far.

4. NEW SMART EAS FINITE ELEMENT

Then we have enhanced shear strain field as follows

$$\begin{Bmatrix}\tilde{\gamma}_{zx}\\ \tilde{\gamma}_{yz}\end{Bmatrix} = -\frac{j}{j_0}\begin{bmatrix}\xi\eta & 0\\ 0 & \xi\eta\end{bmatrix}\begin{Bmatrix}\alpha_1\\ \alpha_2\end{Bmatrix} + \frac{j}{j_0}\mathbf{J}^{-1}\begin{bmatrix}-\xi & \xi\eta^2\\ -\eta & \xi^2\eta\end{bmatrix}\begin{Bmatrix}\alpha_3\\ \alpha_4\end{Bmatrix} \tag{5}$$

This field is the lowest bilinear element for Mindlin type plates.

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References

[1] Crawley E. F., Intelligent Structures for Aerospace: A Technology Overview and Assessment, *AIAA Journal*, **32**(8), 1689-1699, (1994).

[2] Crawley E. F., Lazarus K. B., Induced Strain Actuation of Isotropic and Anisotropic Plates, *AIAA Journal*, 29(6), 944-951, (1991).

[3] Gandhi, M. V. and Thompson, B. S., *Smart Materials and Structures*, Chapman & Hall, 1992.

[4] Huang D. and Sun B., Nonlinear Vibration Analysis of Smart Composite Beams, *Proceedings of the 2nd South Africa Conference on Applied Mechanics, Cape Town*, **1**, 481-492, (1998).

[5] Huang, D. and Sun B., Vibration Analysis of Laminated Composite Beams with Piezoelectric Layers using a Higher order Theory, ICCT/2, Durban, 1998.

[6] Lee C. K., Theory of Laminated Piezoelectric Plates for the Design of Distributed Sensors/Actuators. Part I: Governing Equations and Reciprocal Relationships, *Journal of Acoustical Society of America*, **87**(3), 1144-1158, (1990).

[7] Lee, C. K. and Moon, F. C., Modal Sensors/actuators, *J. Appl. Mech., ASME*, 434 / Vol. 57, June 1990.

[8] Mitchell J. A., Reddy J. N., A Refined Hybrid Plate Theory for Composite Laminates with Piezoelectric Laminae, *International Journal of Solids Structures*, **32**(16), 2345-2367, (1995).

[9] Pai P. F., Nayfeh A. H., Oh K. and Mook D. T., A Refined Nonlinear Model of Composite Plates with Integrated Piezoelectric Actuators and Sensors, *International Journal of Solids Structures*, **30**(12), 1603-1630, (1993).

[10] Shen Y. and Yin L., Strain Sensor of Composite Plates Subjected to Low Velocity Impact with Distributed Piezoelectric Sensors: A Mixed Finite Element Approach, *Journal of Sound and Vibration*, **199**(1), 17-31, (1997).

[11] Simo, J. C. and Rifai, M. S., A Class of Mixed Assumed Strain Methods and the Method of Incompatible Models, *Int. J. Numer. Methods Eng.*, **29**, 1595-1638, (1990).

[12] Sun B, Some Problems of Piezoelectric Sensor Mechanics, ICCT/2, Durban, 1998.

[13] Sun, B, New EAS Element Theoretical Formulation based on two Field Hu-Washizu Variational Principle, *1st SA Conf. Appl. Mech.* **96**, 106-115.

[14] Tzou, H. S. *et al*., Sensor Mechanics of Distributed Shell Convolving Sensors Applied to Flexible Rings, *ASME J. Appl. Mech*., 40/Vol. 115, January 1993.

[15] Wang B. T., Rogers C. A., Laminated Plate Theory for Spatially Distributed Induced Strain Actuators, *Journal of Composite Materials*, 25, 43433-452, (1991).