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The Efficiency of Curve Regression Using Linier and Quadratic Truncated Spline for Estimated Growth Curve of Green Beans

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Abstract: In the case of green bean plant growth patterns unknown data, the data pattern predictor variables of unknown shape can be estimated with a spline function approach. Best approach for nonparametric regression spline for being able to handle the behavior patterns of data relationships vary over a certain sub-interval. The method is often used in nonparametric regression is truncated spline. The purpose of this research is Analyze the suitability of the growth model in describing the pattern growth curve of green beans (*Vigna Radiata L.*) with a growth model Lundqvist-Korf and Hossfeld, gain a better growth model in explaining the growth of green bean plants (*Vigna Radiata L.*) and estimate the time of maximum growth of bean plants green (*Vigna Radiata L.*). From the results of analysis show that the optimal knots on the growth of green beans (*Vigna Radiata L.*) with a quadratic polynomial order of 2 knots in point 4 and 8 with minimum GCV value of 0.424. Based on the chart the best model estimation. There were changes in the pattern of growth curve of green beans (*Vigna Radiata L.*) with 3 treatment at the age of 4 and 8 (MST).

Keywords: Truncated Spline, Growth Curve, Green Beans

1. INTRODUCTION

Growth is one of the main characteristics of living beings and is one of concrete process. The models of growth may take form into weight or height changes of living beings as the time passed by. Moreover, growth models have been applied to various fields such as biology, botany, forestry, zoology, and ecology and have been agreed that growth occurred in any living beings (humans, animals, and plants). In chemistry or chemistry engineering, growth is a product of various chemical's chain reactions.

According to (Gille [6]), growth model is classified into three classes: exponential growth, sigmoid growth, and bell-shaped growth. The exponential growth is frequently found on the growth of skull and brain which possess no inflection points in the curve. The sigmoid model is one of S-shaped growth and possesses inflection points in which the points indicate the maximum growth. This model of growth is frequently found in the growth of weight or height. The bell-shaped model can be distinguished by increased initial growth until it reaches the maximum points and finally declined. This kind of model is frequently found in bacteria and organ.

Those three models of growth proposed by (Gille [6]) belong to parametric regression model group which assumes that the relationship pattern between respond and predictor should be initially recognized (exponential, sigmoid, and bell-shaped). However, a number of researches show that the known patterns are prone to significant miscalculation so that more flexible model that needs no initial recognition of the relationship between respond and predictor is urged. One that answers the need of more flexible model is nonparametric truncated spline regression model group. The approach uses a truncated spline with more emphasis on the parts (regime) of the regression curve. In the approach, using truncated spline also includes degree of truncated polynomial regression which determines the shape of the curve itself; there are several forms of degree, such as linear and quadratic polynomials. The use of Ramsey's Regression Specification Error Test (RESET test) to test the linearity is important, if the assumptions are not fulfilled and the relations among the variables are unknown, then the best alternative is to use nonparametric regression

Plants growth is aimed at nothing but to the pace of growth. Plants' pace of growth can be differentiated into two factors: absolute and relative. Absolute plants' pace of growth is the height changes of the plants as the time changes while the relative plants' pace of growth is the decreased speed of growth of the plants as the time changes (France [5]). Based on the previous explanations, this research is aimed at scrutinizing the efficiency of linier and quadratic truncated regression curve in predicting the growth rate of green bean plants. Thereby, the objectives of the research are: (1) estimating the function of quadratic and linier truncated spline-based regression, (2) applying the quadratic and linier truncated spline regression function as well as calculating the efficiency curve the growth of green bean plants in Indonesia.

2. THEORITICAL REVIEW

2.1. The Efficiency of Quadratic and Linier Nonparametric Truncated Spline Regression

The model of curve of growth of this research is based on the growth of green bean plants with three treatment of planting media composition, namely P1 (alluvial soil + cow manure + sand), P2 (alluvial soil + cow manure + sawdust), and P3 (kompo alluvial soil + cow manure + wood charcoal). Secondary data from research conducted by (Riadi [10]) is aimed at obtaining the growth curve of green bean plants on three planting media compositions. The research is conducted at experimental garden of Faculty of Agriculture, University of Tanjungpura, Pontianak.

Based on the type of growth data previously explained, the data is categorized as longitudinal data, i.e. three subjects based on three treatment (P1,P2,P3), and each green bean plants is measured during 1-12 of after planting period (MST). Longitudinal data is the result of observation conducted on n of independent subjects in which each subject is observed separately and repeatedly on certain span of time, Wu and Zhang [12]. Hedeker [8], the advantage of longitudinal analysis is the additional information of the value of respond variable towards time of each object. Non-parametric regression model which includes *n* observation is as follows (Fernandes [3], Eubank [2]):

$$y_{it} = f_{it}(x_i) + \varepsilon_{it}; i = 1, 2, ..., n \tag{1}$$

truncated spline uses *truncated* power basis with K knot, and function $\left(x_{it} - k_{it}\right)_+^P$ and defined as follows.

$$\left(x_t - k_{1i}\right)_+ = \begin{cases} \left(x_t - k_{1i}\right), & x_t \geq k_{1i} \\ 0, & x_t < k_{1i} \end{cases}$$

Nonparametric Truncated Spline regression model of equation (1) is rewritted in the following formula:

$$f_{it}(x_i) = \alpha_{it0} + \sum_{p=1}^{3} \alpha_{it} x_i^p + \sum_{k=1}^{3} \beta_{it} (x_i - k_{1i})_+^p \tag{2}$$

Elsewhere :

i: 1,2,…, n with n is the quantity of observations
k: the quantity of knot point $\quad k$: 1,2,3
p : order of polynomial $\quad p$: 1,2,3
t : observation time $\quad t$: 1,2,...,t

the matrix of non-parametric regression is expressed as follows.

$$\begin{bmatrix} y_{11} \\ y_{12} \\ \vdots \\ y_{1t} \\ y_{21} \\ y_{22} \\ \vdots \\ y_{2t} \\ \vdots \\ y_{n1} \\ y_{n2} \\ \vdots \\ y_{nt} \end{bmatrix}_{ntx1} = \begin{bmatrix} f_1(x_1) \\ f_1(x_2) \\ \vdots \\ f_1(x_t) \\ f_2(x_1) \\ f_2(x_2) \\ \vdots \\ f_2(x_t) \\ \vdots \\ f_n(x_1) \\ f_n(x_2) \\ \vdots \\ f_n(x_t) \end{bmatrix}_{ntx1} + \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{12} \\ \vdots \\ \varepsilon_{1t} \\ \varepsilon_{21} \\ \varepsilon_{22} \\ \vdots \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{n1} \\ \varepsilon_{n2} \\ \vdots \\ \varepsilon_{nt} \end{bmatrix}_{ntx1} \tag{3}$$

Above equation could be simplified into:

$$\underset{\sim}{y} = \underset{\sim}{f} + \underset{\sim}{\varepsilon} = \mathbf{X}\underset{\sim}{\beta} + \underset{\sim}{\varepsilon} \tag{4}$$

Elsewhere:

$$\underset{\sim}{y} = (\underset{\sim}{y}^t)^t$$
$$\underset{\sim}{\beta} = (\underset{\sim}{\beta}_1^t, \underset{\sim}{\beta}_2^t, \ldots, \underset{\sim}{\beta}_p^t)^t$$

$$\underset{\sim}{\varepsilon} = (\varepsilon_{11}, \ldots, \varepsilon_{1n},\ \varepsilon_{21}, \ldots, \varepsilon_{2n},\ \varepsilon_{31}, \ldots, \varepsilon_{3n},\ \varepsilon_{41}, \ldots, \varepsilon_{4n})^t$$

Matrix **X** is expressed as:

$$\mathbf{X} = \begin{pmatrix} \mathbf{X}_{1n\times(1+p+m)} & \mathbf{O}_{n\times(1+p+m)} & \mathbf{O}_{n\times(1+p+m)} & \mathbf{O}_{n\times(1+p+m)} \\ \mathbf{O}_{n\times(1+p+m)} & \mathbf{X}_{2n\times(1+p+m)} & \mathbf{O}_{n\times(1+p+m)} & \mathbf{O}_{n\times(1+p+m)} \\ \mathbf{O}_{n\times(1+p+m)} & \mathbf{O}_{n\times(1+p+m)} & \mathbf{X}_{3n\times(1+p+m)} & \mathbf{O}_{n\times(1+p+m)} \\ \mathbf{O}_{n\times(1+p+m)} & \mathbf{O}_{n\times(1+p+m)} & \mathbf{O}_{n\times(1+p+m)} & \mathbf{X}_{4n\times(1+p+m)} \end{pmatrix}_{(4n)\times 4(1+p+m)}$$

Matrix **O** is distinguished by all zero valued aspects of the matrix, elsewhere n is the quantity of observation predicted from the matrix $\mathbf{X}_i$, with symbolizes the observation subject as follows.

$$\mathbf{X_i} = \begin{pmatrix} 1 & x_1 & \cdots & x_1^p \, (x_1 - k_{i1})_+^p & \cdots & (x_1 - k_{im})_+^p \\ 1 & x_2 & \cdots & x_2^p \, (x_1 - k_{i1})_+^p & \cdots & (x_1 - k_{im})_+^p \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 1 & x_n & \cdots & x_n^p \, (x_1 - k_{i1})_+^p & \cdots & (x_1 - k_{im})_+^p \end{pmatrix}_{n\times(1+p+m)}$$

The above truncated spline model is illustrated in two lowest forms of ordo namely spline linier orde and quadratic. The curve of linier model is illustrated in the following:

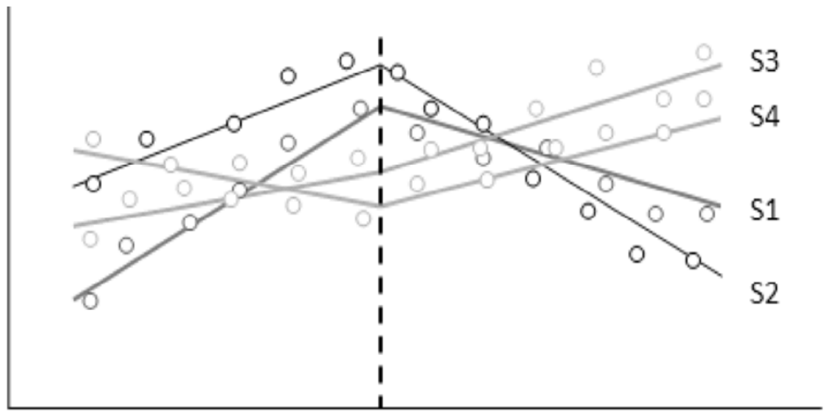

Figure 1: Linear Model of Truncated Spline

The equation of polynominal linear ordo of truncated spline model is formulated as follows.

$$y_{it} = \alpha_{0i} + \alpha_{1i} x_t + \beta_{1i}\left(x_t - k_{1i}\right)_+ + \beta_{2i}\left(x_t - k_{2i}\right)_+ + \cdots + \beta_{mi}\left(x_t - k_{mi}\right)_+ + \varepsilon_{it} \tag{5}$$

The curve of quadratic model is formulated in the following:

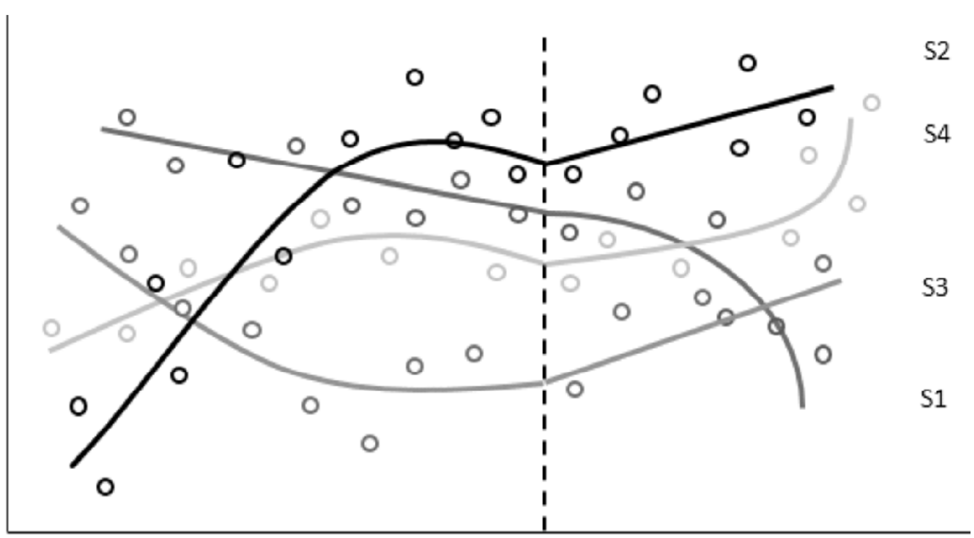

Figure 2: Quadratic Model of Truncated Spline

Knot points are found in the data which no patterned shape of cutve. The best spline is highly depended on the location of knot k_1, k_2,... k_3,. In this research, determining the knot points could be done using Generalized Cross Validation (GCV). The advantage of this method is not need to concerning σ^2 and variance toward transformation are also not needed (Wahba [11]). Generalized Cross Validation is a method of choosing relatively few knot, as stated in the following (Budiantara [1]):

$$GCV(\underset{\sim}{\mathbf{K}}) = \min\left[\frac{MSE(\underset{\sim}{\mathbf{K}})}{\left[(nt)^{-1}tr\left(\mathbf{I}-H(\underset{\sim}{\mathbf{K}})\right)\right]^2}\right] \tag{6}$$

The efficiency of regression curve would be analyzed if there are two predictors involved. This research focuses on the analysis of the curve of linier and quadratic truncated spline. If two predictors obeyed the general condition of Crammer-Rao, Relative Efficiency (ER) of GCV_{linear} and $GCV_{quadratic}$ are defined as the ratio of error variance as follows.

$$ER_\theta(\mathrm{GCV}_{linear}, \mathrm{GCV}_{quadratic}) = \frac{\mathrm{GCV}_{linear}(\theta)}{\mathrm{GCV}_{quadratic}(\theta)} \tag{7}$$

Jika $ER_\theta(\mathrm{GCV}_{linear}, \mathrm{GCV}_{quadratic}) < 1$ then GCV_{linear} is more efficient than $GCV_{quadratic}$ (Zacks, [13]).

3. RESULT AND DISCUSSION

The first objective of the research is estimating the function of truncated spline-based nonparametric regression as stated in the following Theorem 1:

Theorem 1: The function of non-parametric regression as presented by equation (2), by the assumption of $\mathrm{E}(\underset{\sim}{\varepsilon}) = \underset{\sim}{0}$ and $\mathrm{Var}(\underset{\sim}{\varepsilon}) = \mathbf{\Sigma}$, then the spline estimator that minimizes Weighted Least Square

$$\min\{\underset{\sim}{\varepsilon}^t\mathbf{\Sigma}^{-1}\underset{\sim}{\varepsilon}\} = \min\left\{\left(\underset{\sim}{y}-\mathbf{X}\underset{\sim}{\beta}\right)^t\mathbf{\Sigma}^{-1}\left(\underset{\sim}{y}-\mathbf{X}\underset{\sim}{\beta}\right)\right\}$$

Is $\hat{\underset{\sim}{f}} = H(\underset{\sim}{\mathbf{K}})\underset{\sim}{y}$ with :

$$H(\underset{\sim}{\mathbf{K}}) = \mathbf{X}\left(\mathbf{X}^t\Sigma^{-1}\mathbf{X}\right)^{-1}\mathbf{X}^t\mathbf{X}^{-1} \tag{8}$$

Proof of Theorem 2: Considering the equation (3) namely the function $\underset{\sim}{f} = \mathbf{x}\underset{\sim}{\theta}$ by Fernandes [4], then the multi-responses truncated spline non-parametric regression model could be formulated to:

$$\underset{\sim}{y} = \underset{\sim}{f} + \underset{\sim}{\varepsilon} \text{ or } \underset{\sim}{\varepsilon} = \underset{\sim}{y} - \underset{\sim}{f} \text{ or } \underset{\sim}{\varepsilon} = \underset{\sim}{y} - \mathbf{x}\underset{\sim}{\theta}$$

After deciding the weighting, the function of truncated spline could be optimized using Weighted Least Square (WLS) by the following equation:

$$\min\{\underset{\sim}{\varepsilon}^t\mathbf{\Sigma}^{-1}\underset{\sim}{\varepsilon}\} = \min\left\{\left(\underset{\sim}{y}-\mathbf{X}\underset{\sim}{\beta}\right)^t\mathbf{\Sigma}^{-1}\left(\underset{\sim}{y}-\mathbf{X}\underset{\sim}{\beta}\right)\right\}$$

Then, conduct the following partial derivative:

$$Q\left(\underset{\sim}{\beta}\right)=\left(\underset{\sim}{y}-\mathbf{X}\underset{\sim}{\beta}\right)^{t}\boldsymbol{\Sigma}^{-1}\left(\underset{\sim}{y}-\mathbf{X}\underset{\sim}{\beta}\right)$$
$$=\left(\underset{\sim}{y}^{t}-\mathbf{X}^{t}\underset{\sim}{\beta}^{t}\right)\boldsymbol{\Sigma}^{-1}\left(\underset{\sim}{y}-\mathbf{X}\underset{\sim}{\beta}\right)$$
$$=\underset{\sim}{y}^{t}\boldsymbol{\Sigma}^{-1}\underset{\sim}{y}-\underset{\sim}{\beta}^{t}\mathbf{X}^{t}\boldsymbol{\Sigma}^{-1}\underset{\sim}{y}-\underset{\sim}{y}^{t}\boldsymbol{\Sigma}^{-1}\mathbf{X}\underset{\sim}{\beta}+\underset{\sim}{\beta}^{t}\mathbf{X}^{t}\boldsymbol{\Sigma}^{-1}\mathbf{X}\underset{\sim}{\beta}$$
$$=\underset{\sim}{y}^{t}\boldsymbol{\Sigma}^{-1}\underset{\sim}{y}-2\underset{\sim}{\beta}^{t}\mathbf{X}^{t}\boldsymbol{\Sigma}^{-1}\underset{\sim}{y}+\underset{\sim}{\beta}^{t}\mathbf{X}^{t}\boldsymbol{\Sigma}^{-1}\mathbf{X}\underset{\sim}{\beta}$$

Then minimize to estimate $\underset{\sim}{\beta}$ and equated to zero so it obtained:

$$\frac{\partial Q(\underset{\sim}{\theta})}{\partial \underset{\sim}{\theta}}=-2\mathbf{x}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\underset{\sim}{y}+2\mathbf{x}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\mathbf{x}\underset{\sim}{\theta}$$

$$0=-2\mathbf{X}^{t}\boldsymbol{\Sigma}^{-1}\underset{\sim}{y}+2\mathbf{X}^{t}\boldsymbol{\Sigma}^{-1}\underset{\sim}{y}\underset{\sim}{\beta}$$
$$2\mathbf{X}^{t}\boldsymbol{\Sigma}^{-1}\underset{\sim}{y}=2\mathbf{X}^{t}\boldsymbol{\Sigma}^{-1}\mathbf{X}\underset{\sim}{\beta}$$
$$\mathbf{X}^{t}\boldsymbol{\Sigma}^{-1}\underset{\sim}{y}=\mathbf{X}^{t}\boldsymbol{\Sigma}^{-1}\mathbf{X}\underset{\sim}{\beta}$$
$$\underset{\sim}{\beta}=\left(\mathbf{X}^{t}\boldsymbol{\Sigma}^{-1}\mathbf{X}\right)^{-1}\mathbf{X}^{t}\boldsymbol{\Sigma}^{-1}\underset{\sim}{y}$$

Estimation of the function $\underset{\sim}{\hat{f}}$ is obtained by changing to:

$$\underset{\sim}{\hat{f}}=\mathbf{X}\beta=\mathbf{X}\left(\mathbf{X}^{t}\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{X}\right)^{-1}\mathbf{X}^{t}\hat{\boldsymbol{\Sigma}}^{-1}\underset{\sim}{y}$$
$$=H(\underset{\sim}{\mathbf{K}})\underset{\sim}{\mathrm{y}}$$

Elsewhere $\underset{\sim}{\hat{f}}=(\hat{f}_1,\hat{f}_2,\hat{f}_3,\hat{f}_4)^{-1}$. Then simplified to $\underset{\sim}{\hat{f}}=H(\underset{\sim}{\mathbf{K}})\underset{\sim}{y}$ with matrix $H(\underset{\sim}{\mathbf{K}})=\mathbf{X}\left(\mathbf{X}^{t}\Sigma^{-1}\mathbf{X}\right)^{-1}\mathbf{X}^{t}\mathbf{X}^{-1}$

3.1. Estimation and Efficiency of Green Bean Plants Growth Curve

The second of objective of the research is to apply the function of longitudinal data of truncated spline regression to the growth of green bean plants (*Vigna Radiata L.*) as presented in the following:

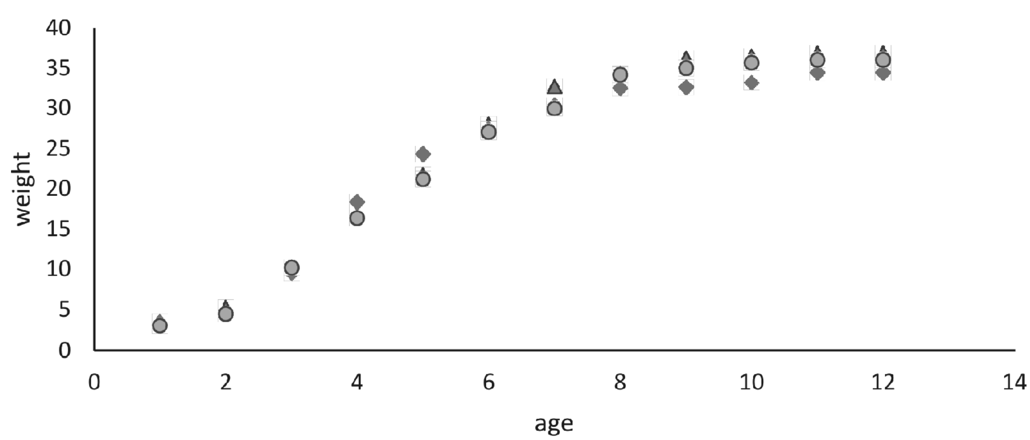

Figure 4: Data pattern of weight (kg) towards age (month)

Figure 4 shoes that the growth of green bean plants (*Vigna Radiata L.*) is constantly increased up to age 12 (MST). Relationship pattern between weight and age (MST) is non-linier so that the researcher uses truncated spline non-parametric regression analysis. To reveal if the data were non-linear, Modified Ramsey Reset Test is conducted with the following result:

Table 1
The result of linearity test

Data	$F_{calculate}$	*P - value*
The growth of green bean plants *(Vigna Radiata L.)*	32.707	< 0.001

Table 1 shows that the growth of green bean plants (*Vigna Radiata L.*) possesses the value P > 0.005 which means that the data of the growth of green bean plants (*Vigna Radiata L.*) are not linear. To reveal that autocorrelation among the predictor variables, the Box Pierce's Test method is used and resulted in the following:

Table 2
The result of autocorrelation test

Data	*Autocorrelation*	t_{hitung}	*P – value*
The growth of green bean plants *(Vigna Radiata L.)*	0,983	31,490	< 0,001

Table 2 shows that the growth of green bean plants (*Vigna Radiata L.*) has the autocorrelation amounted to 0.983, which means that the growth of green bean plants (*Vigna Radiata L.*) at *t*-time is highly influenced by the growth of green bean plants (*Vigna Radiata L.*) at (*t-i*)-time. The stipulation of optimal knot points with regard to minimum value of GCV is resulted in the following knot points:

Table 3
The value of GCV Non-parametric truncated spline regression

Polynomial Degree	*Knot*	*Points*		*GCV minimum*
Linear	2	4	8	2.445
Quadratic	2	4	8	0.424

Table 3 shows the value of GCV of each model. On linier and quadratic poly-nominal ordo, there is obtained the optimal know at the rate of 2 knot and located at point four and eight.

The choosing of best model with regard to efficiency value is used as the criterion of the choosing. Efficiency value is used since it compares the estimation of best model of each polynominal ordo, the result of efficiency value can be seen as follows.

Table 4
The estimation of the best function of Truncated Spline

No	*GCV Minimum (Linier)*	*GCV Minimum (Quadratic)*	*ER*
1	2,445	0,424	5,767

Table 4 shows that the value of ER (5.767) > 1, which means that quadratic nonparametric regression model is more efficient up to six times compared to linier nonparametric regression model. The Graphic of Best Model Estimation of the growth of green bean plants *(Vigna Radiata L.)*

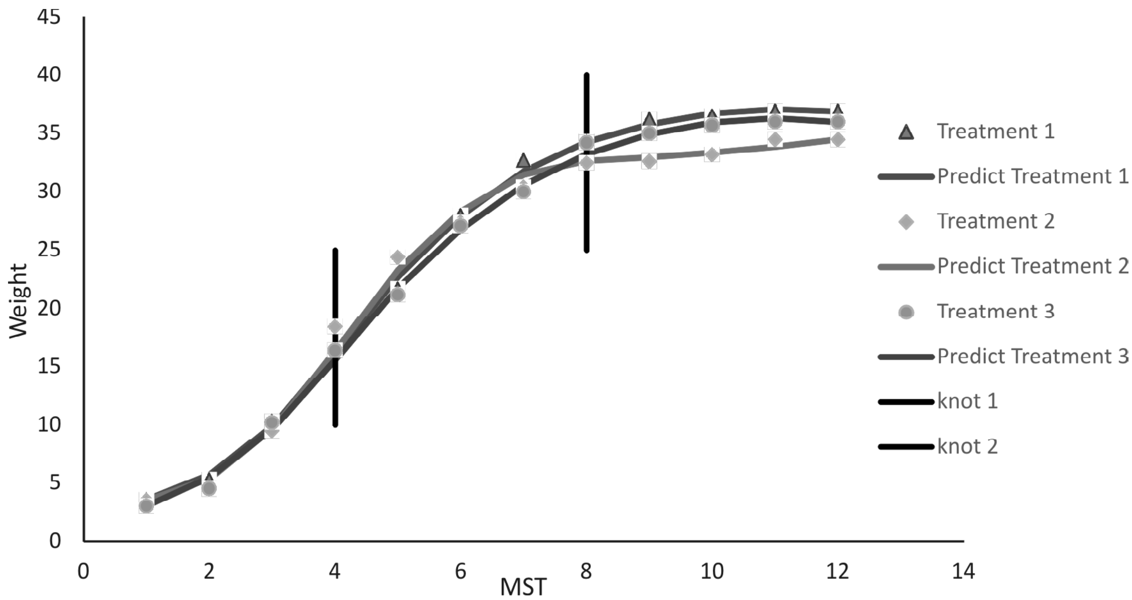

Figure 3: The graphic of the weight of green bean plants *(Vigna Radiata L.)* towards age

Figure 3 shows the changes of the pattern of the curve at knot number-4 which means that it is changed at age 4 and weight 8 of green bean plants (*Vigna Radiata L.*) so that pattern changed occurred and the curve is divided into three regime. The estimation of truncated spline polynominal orde non-parametric regression model is presented in the following:

$$\hat{f}_{1t} = 3.414 - 0.882x_t + 1.017x_t^2 - 1.700(x_t - 4)_+^2 + 0.398(x_t - 8)_+^2$$
$$\hat{f}_{2t} = 4.082 - 1.797x_t + 1.215x_t^2 - 2.176(x_t - 4)_+^2 + 1.015(x_t - 8)_+^2$$
$$\hat{f}_{3t} = 2.279 - 0.148x_t + 0.868x_t^2 - 1.465(x_t - 4)_+^2 + 0.262(x_t - 8)_+^2$$

The interpretation of the estimation of best function of non-parametric truncated spline. Here is the interpretation of the estimation of best function for treatment 1.

$$\hat{f}_{1t} = 3.414 - 0.882x_t + 1.017x_t^2 - 1.700(x_t - 4)_+^2 + 0.398(x_t - 8)_+^2$$
$$= \begin{cases} 3.414 - 0.882x_t + 1.017x_t^2 & ; \quad x_t < 4 \\ -23.786 + 12.718x_t - 0.603x_t^2 & ; \quad 4 < x_t < 8 \\ 28.886 - 7.248x_t + 1.415x_t^2 & ; \quad x_t > 8 \end{cases}$$

Determine the optimal point, for second time to:

1. When $x_t < 4$, obtained:

$$\frac{d\hat{f}_{2t}}{dx_t} = 3.414 - 0.882x_t + 1.017x_t^2$$
$$= 0.882 + 2.034x_t = 0$$
$$x_t = 0.433$$

$$\frac{d^2\hat{f}_{2t}}{dx_t^2} = 0,882 + 2.034x_t$$
$$= 2.034$$

The interpretation of first area curve, if seen from the first and second lowering, valued > 0, then at the age of 0.433 (MST) is the optimal growth of green bean plants (*Vigna Radiata L.*) suffered from weight lowering.

2. When $4 < x_t < 8$, obtained:

$$\frac{d\hat{f}_{2t}}{dx_t} = -23.786 + 12.718x_t - 0.603x_t^2$$
$$= 12.718 - 1.206x_t = 0$$
$$x_t = 10.546$$

$$\frac{d^2\hat{f}_{2t}}{dx_t^2} = 12.718 - 1.206x_t$$
$$= -1.206$$

Interpretation of second area curve, if seen from first and second lowering, valued < 0, then at the age of 10.5 (MST) is the optimal growth of green bean plants (*Vigna Radiata L.*) suffered from weight increasing.

3. When $x_t < 8$, obtained:

$$\frac{d\hat{f}_{2t}}{dx_t} = 28.886 - 7.248x_t + 1.415x_t^2$$
$$= -7.248 + 2.830x_t = 0$$
$$x_t = 2.561$$

$$\frac{d^2\hat{f}_{2t}}{dx_t^2} = -7.248 + 2.830x_t$$
$$= 2.830$$

The interpretation of third area curve, if seen from first and second lowering, valued > 0, then at the age of 2.5 (MST) is the optimal point outside the curve which means that the growth of green bean plants is always on the range of 8 to 12 (MST).

4. CONCLUSION

Based on the analysis result and discussion, the conclusion of this study are: (1) Obtained optimal knots on the growth of green beans (*Vigna Radiata L.*) with a quadratic polynomial order of 2 knots in point 4 and 8 with minimum GCV value of 0.424. (2) Based on the chart the best model estimation. There were changes in the pattern of growth curve of green beans (*Vigna Radiata L.*) with 3 treatment at the age of 4 and 8 (MST). The suggestions of the research are: (1) this research is limited to 2 knot, the succeeding research could be developed to exceed the limit of this research, (2) the data used in this research is limited to balanced longitudinal data, the succeeding research could be developed to employ truncated spline-based simulation data, and (3) the succeeding research could explain the relation between the value of $R^2_{adjusted}$ and autocorrelation rate.

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