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BLOOD FLOW THROUGH AN ARTERY WITH STENOSIS: AN APPLICATION OF POWER LAW FLUID

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Abstract: The presence of a stenosis in the artery is always affects the nature of blood flow from its usual state to a distributed flow; therefore the effect of size of stenosis is very important on blood flow through an artery. Here in this model, assume that the blood behaves like a power law fluid in a uniform circular tube with an axially non-symmetric but radially symmetric stenosis. Further the governing equations for laminar, incompressible and power-law fluid flow, subject to the boundary conditions is then solved numerically. The analytical expressions for pressure drop, flux (flow rate), and resistance to flow and wall shear stress have been obtained, and the numerical values extracted from these analytical expressions are presented graphically. Here in this work shear stress is taken less than 20 sec^{-1} and the diameter of the tube is less than 0.2 mm. Through this model we observed that, if the height of stenosis increased the pressure drop and flux also increased the resistance to flow, wall shear stress decreases as stenosis size increases. This information of blood could be useful in the development of new diagnosis tools for many diseases.

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1. INTRODUCTION

It has been suggested that the deposition of cholesterol on the arterial wall and proliferation of connective tissue may be responsible for the abnormal growth in the lumen of the artery. Many cardiovascular diseases such as due to the hemodynamics behaviour of the blood flow is influenced by the presence of the arterial stenosis. If the stenosis is present in an artery, normal blood flow is disturbed. The intimal thickening of stenotic artery was understood as an early process in the beginning of atherosclerosis, while is the leading cause of death in many countries. There is considerable evidence that vascular fluid dynamics plays an important role in the development and progression of arterial stenosis, which is one of the most widespread diseases in human beings. The fluid mechanical study, especially of power law fluid of blood flow in artery bears some important aspects due to the engineering interest as well as the feasible medical applications. Stenosis is one of the most wide-spread arterial diseases and its effect on the cardiovascular system has been determined by the studying the flow characteristics of blood in stenotic region in artery.

During the past few decay several studies were conducted in this direction of power law fluid and blood flow problems. Initially Young (1979) discussed about some problems in fluid mechanics of arterial stenosis, while Shukla *et al*., (1980) studied the effects of stenosis on non-Newtonian flow of blood in an artery. It has been suggested that the deposits of cholesterol on the arterial wall and proliferation of connective tissue are responsible for the abnormal growth in the lumen of an artery. The actual causes of stenosis are not well known but its effect on the cardiovascular system can be understood by studying the blood flow in its vicinity. Ahmed and Giddens (1983) studied about the velocity measurements in steady flow through axisymmetric stenosis at moderate Reynolds number, while Back *et al*., (1986) realizing the fact that the pulsatile nature of the flow cannot be neglected. Haldar (1987) deal with the problem of oscillatory blood flow through a rigid tube with mild constriction under a simple harmonic pressure gradient and has examined the effects of stenosis on the flow field.

It has been reported by Tu and Deville (1992) that the assumption of Newtonian behaviour of blood is acceptable for high shear rate flow. It has also been pointed out that in some diseased conditions (e.g. patients with severe myocardial infarction) cerebro-vascular diseases and hypertension, blood exhibits remarkable non-Newtonian properties. Again Tu and Deville (1996) investigated the pulsatile flow of blood in stenosed arteries. Zendehbudi and Moayer (1999) have studied the comparison of physiological and simple pulsatile flows through stenosed arteries. It is true that the Casson fluid model can be used for moderate shear rates $\gamma < 10\, s^{-1}$ in smaller diameter tubes whereas, the Herschel-Bulkley fluid model can be used at still lower shear rate of flow in very narrow arteries. Leuprecht and Perktold (2001) have studied the computer simulation of non-Newtonian effects on blood flows in large arteries. Neofytou and Drikakis (2003) reported that a non-Newtonian flow instability in a channel with a sudden expansion. Chakravarty *et al*., (2004) presented a theoretical investigation to examine the significant characteristics of the two-layered non-Newtonian rheology of blood flowing through a tapered flexible artery in the presence of stenosis under a pulsatile pressure gradient. A mathematical model for blood flow in magnetic field is studied by Tzirtzilakis (2005) while Misra and Shit (2006, 2007) studied in two different situations on the blood flow through arterial stenosis by treating blood as a non-Newtonian (Herschel-Bulkley fluid) fluid model. It is generally well known that blood, being a suspension of red cells in plasma, behaves like a non-Newtonian fluid at low shear rates. Misra *et al*., (2008) also conducted a theoretical study for the effects of multiple stenosis, while fluid flow analysis of blood flow through multistenosis arteries in the presence of magnetic field is investigated by Verma and Parihar (2010), where the effect of magnetic field and shape of stenosis on the flow rate is studied. Singh and Rathee (2010) studied the analytical solution of two-dimensional model of blood flow with variable viscosity through an indented artery due to LDL effect in the presence of magnetic field.

A mathematical model of blood flow in porous vessel having double stenosis in the presence of an external magnetic field has been investigated by Sinha *et al*., (2011) while Shit and Roy (2012) put forwarded a mathematical analysis for the unsteady flow of blood through arteries having stenosis, in which blood was treated as a Newtonian, viscous, incompressible fluid. Recently Kumar et al (2012) worked on a biomagentic fluid dynamic model for the MHD couette flow between two infinite horizontal parallel porous plates and gives that the main flow component decreases with the increase of Hartmann number and the velocity decreases with the increases of the injection suction parameter, they also investigate the effects of Hartmann number and suction parameter along with the velocity and temperature distribution. Recently Linge *et al*., (2013) studied the pulsatile spiral blood flow through arterial stenosis. Pulsatile spiral blood flow in a modeled three-dimensional arterial stenosis, with a 75% cross-sectional area reduction, is investigated by using numerical techniques. Two-equation k-ω model is used for the simulation of the transitional flow with Reynolds numbers 500 and 1000. They found the spiral component increases the static pressure in the vessel during the deceleration phase of the flow pulse. Therefore on the basis of above information the power law fluid flow through a stenosis in artery with axisymmetric blood flow, is considered here.

2. MATHEMATICAL FORMULATION

Consider a circular cylinder as arterial segment having axisymmetric blood flow with this axially symmetric stenosed artery. Here the blood is assumed as a power law fluid and flow of blood is considered to be steady and laminar. If w is the axial velocity, τ is the shear stress, τ_0 is the yield stress and μ is the viscosity of the blood then the constitutive equations may be expressed as:

$$e = f(\tau) = -\frac{dw}{dr} = \left\{ \begin{array}{ll} \left(\dfrac{\tau}{\mu}\right)^{1/n} & \text{for} \quad \tau \geq \tau_0 \\ 0 & \text{for} \quad \tau \leq \tau_0 \end{array} \right\} \tag{1}$$

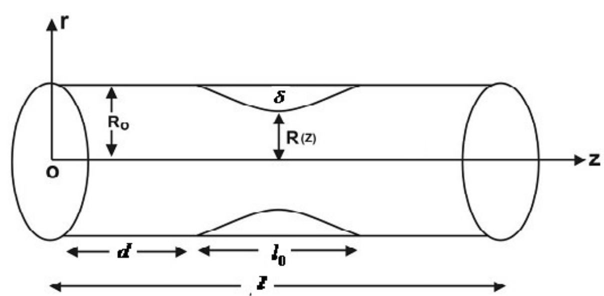

Figure 1(a): Schematic Diagram of Artery with Stenosis

Further if R_0 is the radius of artery, δ is the height of the stenosis and R is the radius of abnormal artery then the geometrical description of the figure 1(a) is given by the following equation:

$$\frac{R}{R_0} = \begin{cases} 1 - \frac{\delta}{2R_0}\left[1 + \cos\frac{2\pi}{l_0}\left(z - d - {l_0}/{2}\right)\right] & d \le z \le d + l_0 \\ 1 & \text{Otherwise} \end{cases} \tag{2}$$

Now if the velocity is parallel to the axis then it will be a function of r only and the velocity will be maximum on the axis while zero on the surface. So the non-zero component of strain rate is given by the Naveir-Stokes equation are:

$$e = -\left(\frac{dw}{dr}\right). \tag{3}$$

For the non-Newtonian fluid, the constitutive equation may be expressed as:

$$\tau = f(e) = f\left(-\frac{dw}{dr}\right), \tag{4}$$

and the expression for τ is,

$$\tau = \frac{1}{2}\,\mathrm{Pr}. \tag{5}$$

From equation (4) and (5) we conclude the following:

$$-\frac{1}{2}\,\mathrm{Pr} = f\left(-\frac{dw}{dr}\right), \tag{6}$$

while for the power law fluid, above may be expressed as:

$$-\left(\frac{1}{2}\frac{\mathrm{Pr}}{\mu}\right)^{1/n} = \left(\frac{dw}{dr}\right). \tag{7}$$

The boundary conditions are:

$$\left\{\begin{array}{lll} {\partial w}/{\partial r} = 0 & \text{at} & r = 0 \\ w = 0 & \text{at} & r = R_0 \end{array}\right\}. \tag{8}$$

On integrating the equation (7) and using the above boundary conditions (8), we may have:

$$w = \int_0^R \left(\frac{1}{2}\frac{\mathrm{Pr}}{\mu}\right)^{1/n} dr = \left(\frac{1}{2}\frac{P}{\mu}\right)^{1/n} \int_0^R r^{1/n}\, dr$$

or
$$w = \left(\frac{1}{2}\frac{P}{\mu}\right)^{1/n} \frac{n}{n+1}\left[R^{\frac{1}{n}+1}\right]. \tag{9}$$

Now the flux Q through the artery can be written in the following form:

$$Q = \int_0^R 2\pi r\, w\, dr\,,$$

which may be expressed, along with equation (9):

$$Q = \frac{n\pi}{(3n+1)}\left(\frac{1}{2}\frac{\mathrm{P}}{\mu}\right)^{1/n}\left[R^{\frac{1}{n}+1}\right]^{\frac{1}{n}+3} \tag{10}$$

and
$$P = -\frac{dp}{dz} = 2\mu\left[\frac{(3n+1)Q}{n\pi}\right]^n \frac{1}{R^{3n+1}}. \tag{3.11}$$

Integrating equation (11) along the length of artery and using the following conditions:

$$\begin{aligned} p &= p_1 \quad \text{at} \quad z = 0 \\ p &= p_2 \quad \text{at} \quad z = l \end{aligned} \tag{12}$$

Now the pressure drop along with no-slip conditions may be written as:

$$\Delta p = p_2 - p_1 = 2\mu\left[\frac{(3n+1)Q}{n\pi}\right]^n \int_0^l \frac{1}{R^{3n+1}}\, dz \tag{13}$$

which may also be written as:

$$\Delta p = \frac{2\mu l_0}{\pi R_0^{\,3n+1}}\left[\frac{(3n+1)Q}{n\pi}\right]^n \int_0^l \frac{1}{(a - b\cos w)^{3n+1}}\, dw\,. \tag{14}$$

Now if there is no stenosis i.e. $\delta = 0$ and $f\left(\frac{\delta}{R_0}\right) = 1$ then the pressure drop across the stenosis length is given by:

$$(\Delta p)_{\delta=0} = \frac{2\mu l_0}{\pi R_0^{3n+1}}\left[\frac{(3n+1)Q}{n\pi}\right]^n. \tag{15}$$

Dividing the equation (14) by equation (15) then we have,

$$\frac{\Delta p}{(\Delta p)_{\delta=0}} = \int_0^l \frac{1}{(a - b\cos w)^{3n+1}}\, dw \tag{16}$$

Now solve the equation (16) for the different value of n,

First we take $n = \frac{1}{3}$ then we have,

$$\frac{\Delta p}{(\Delta p)_{\delta=0}} = \left(1 - \frac{\delta}{2R_0}\right)\left(1 - \frac{\delta}{R_0}\right)^{-3/2} \tag{17}$$

Now for $n = \frac{2}{3}$,

$$\frac{\Delta p}{(\Delta p)_{\delta=0}} = \left(1 - \frac{\delta}{R_0} + \frac{3}{8}\frac{\delta}{R_0}\right)\left(1 - \frac{\delta}{R_0}\right)^{-5/2} \tag{18}$$

Now for $n = 1$

$$\frac{\Delta p}{(\Delta p)_{\delta=0}} = \left(1 - \frac{\delta}{2R_0}\right)\left(1 - \frac{\delta}{R_0} + \frac{3}{8}\frac{\delta^2}{R^2{}_0}\right)\left(1 - \frac{\delta}{R_0}\right)^{-7/2} \tag{19}$$

Similarly for the other different value of n, the definite integral (16) has to be evaluated numerically. Again the resistance to flow or the resistive impedance is denoted by λ and is defined as:

$$\lambda = \frac{\Delta p}{Q}.$$

From equations (10) and (12), we expressed as:

$$\lambda_0 = \frac{2\mu Q^{n-1}}{R_0^{\,3n+1}}\left[\frac{3n+1}{n\pi}\right]^n \int_0^l 1\big/ R_0^{\,3n+1} dz\,. \tag{20}$$

If there is no stenosis in artery ($\delta = 0$), then the resistance to flow may be expressed as:

$$\lambda_N = \frac{2\mu l Q^{n-1}}{R_0^{\,3n+1}}\left[\frac{3n+1}{n\pi}\right]^n. \tag{21}$$

The non-dimensional form of resistance to flow, denoted by $\overline{\lambda}$, is given as

$$\overline{\lambda} = \frac{\lambda_0}{\lambda_N}$$

$$\overline{\lambda} = 1 - \frac{l_0}{l} + \frac{1}{l}\int_d^{d+l_0} \frac{dz}{\left[R\big/ R_0\right]^{3n+1}} \tag{22}$$

The expression (22) may reduce to:

$$\overline{\lambda} = 1 - \frac{l_0}{l} + \frac{l_0}{2\pi l}\int_0^{2\pi l} \frac{dz}{\left[a + b\cos\theta\right]^{3n+1}} \tag{23}$$

Also the ratio of shearing stress on with and without stenosis can be written as;

$$\overline{\tau} = \frac{\tau_0}{\tau_N} = \left[\frac{R}{R_0}\right]^{3n}, \tag{24}$$

where n is power law index.

3. NUMERICAL RESULTS AND DISCUSSION

Blood flow through an artery is mainly depends on the pressure gradient and resistance to flow. In this work, we have worked on the effects of the stenosis in an artery by considering the blood as power-law fluid, and our results are based on the mathematical analysis indicates that the pressure drop and flux varying markedly across the stenotic lesion. Here we see

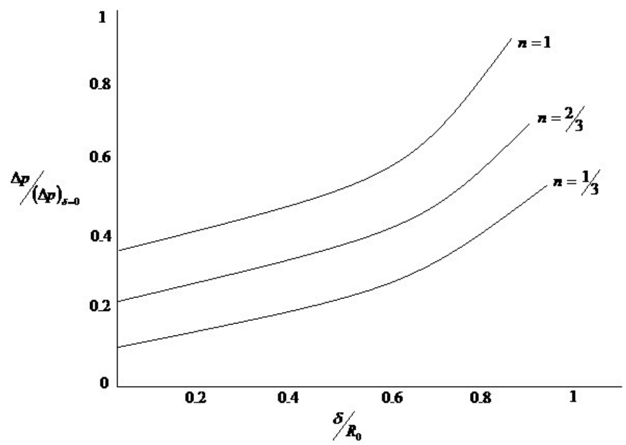

Figure 1(b): Pressure Drop Across Stenosis Size for Power Law Fluid

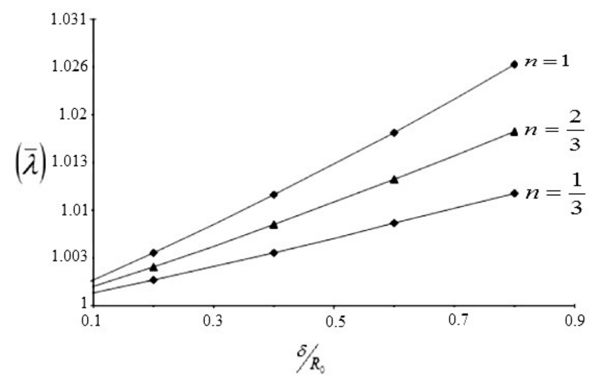

Figure 1(c): Resistance to Flow with Stenosis Size for Power Law Fluid

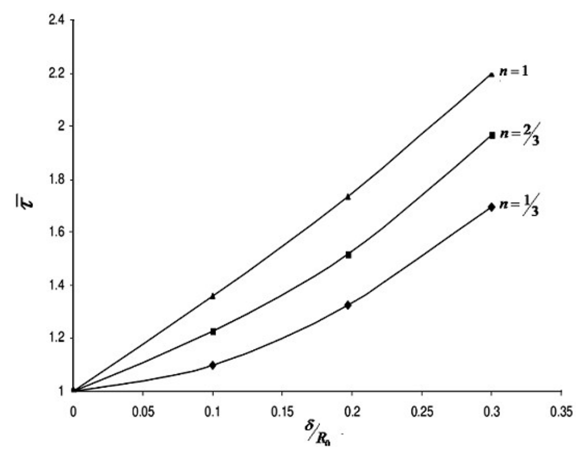

Figure 1(d): Wall Shear Stress with Respect to Stenosis Size

that if the size of stenosis increases the pressure drop and flux also increases which has been shown by the figure 1(b), while figures 1(c) and 1(d) shows that the resistance to flow and wall shear stress decreases as stenosis size increases. We use the numerical technique to solve the analytical results of this model with considering the temperature 25.5 °C and for this result the shear stress is taken less than 20 sec^{-1} and the diameter of the tube is less than 0.2 mm. It appears that the non-Newtonian behaviour of the blood is helpful in the functioning of diseased arterial circulation.

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