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State Observer Design of Chaotic Systems and Application to Secure Communication

Yeong-Jeu Sun*

ABSTRACT

In this paper, the horizontal platform system is considered and the state observation problem of such a system is investigated. A simple observer for the horizontal platform system is developed to guarantee the global exponential stability of the resulting error system. Applications of proposed state observer scheme to secure communication as well as some numerical simulations are given to demonstrate the feasibility and effectiveness of the obtained results. Besides, the guaranteed exponential convergence rate of the proposed state observer and that of the proposed secure communication can be arbitrarily pre-specified.

Keywords: Observer, secure communication, chaos.

I. INTRODUCTION

From practical considerations, it is either impossible or inappropriate to measure all the elements of the state vector. In particular, states observation is more intricate when system is nonlinear, chaotic, or stochastic in model or parameters. Observing system states has come to take its pride of place in control design, system identification, and filter theory, which has taken up engineers' attention from early 1960s. In recent years, a wide variety of methodologies have been proposed for the observer design of systems, such as Chebyshev neural network (CNN), sliding-mode observer (SMO), passivation of error dynamics, separation principle, and frequency domain analysis. For more detailed knowledge, one can refer to [1]-[13] and the references therein. Obviously, most of the observer designs focus on the fast response and high tracking accuracy. For the fast response, the use of a sigmoid function in a boundary layer is particularly popular. However, the observer error cannot be guaranteed to converge to zero within the boundary layer [1].

Since chaotic system is highly sensitive to initial conditions and the output behaves like a random signal, several kinds of chaotic systems have been widely applied in various applications such as secure communication, ecological systems, system identification, master-slave chaotic systems, chemical reactions, and biological systems; see, for instance, [13]-[25] and the references therein.

In this paper, the state estimator of the horizontal platform systems is studied. A simple observer for such systems is provided to guarantee the global exponential stability of the resulting error system. Furthermore, the guaranteed exponential convergence rate can be assigned arbitrarily. Applications of proposed state observer scheme to secure communication as well as numerical simulations are offered to evidence the utility and effectiveness of the main results.

This paper is organized as follows. The problem formulation and main result are presented in Section II. Several numerical simulations are given in Section III to illustrate the main result. Finally, conclusions are made in Section IV.

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II. PROBLEM FORMULATION AND MAIN RESULT

In this paper, we consider the following horizontal platform systems:

$$\dot{x}_1(t) = x_2(t), \tag{1a}$$

$$\dot{x}_2(t) = -ax_2(t) - b\sin(x_1(t)) + l\cdot\sin(x_1(t))\cos(x_1(t)) + d\cos(wt), \ \forall \ t \geq 0, \tag{1b}$$

$$y(t) = cx_1(t), \ \forall \ t \geq 0, \tag{1c}$$

where $x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T \in \Re^{2\times 1}$ is the state vector, $y(t) \in \Re$ is the system output, and a, b, c, d, l, w are system parameters, with $c \neq 0$. It is noted that the system (1) with $a = \frac{4}{3}, \ b = 3.776, \ l = 4.6\times 10^{-6}, \ d = \frac{43}{3}, \ w = 1.8$, displays chaotic behavior [22]. It is a well-known fact that since states are not always available for direct measurement, states must be estimated. The objective of this paper is to search an observer for the system (1) such that the global exponential stability of the resulting error systems can be guaranteed. In what follows, A^T is used to denote the transpose for a matrix A, $\|x\| := \sqrt{x^T \cdot x}$ denotes the Euclidean norm of the column vector x, and $|a|$ denotes the absolute value of a real number a.

Before presenting the main result, let us introduce a definition which will be used in the main theorem.

Definition 1: The system (1) is exponentially state reconstructible if there exist an observer $E\dot{z}(t) = f(z(t), y(t))$ and positive numbers k and α such that

$$\|e(t)\| := \|x(t) - z(t)\| \leq k\exp(-\alpha \ t), \ \forall \ t \geq 0,$$

where $z(t)$ expresses the reconstructed state of the system (1). In this case, the positive number α is called the exponential convergence rate.

Now we present the main result for the state observer of system (1).

Theorem 1: The system (1) is exponentially state reconstructible. Besides, a suitable observer is given by

$$\begin{cases} z_1(t) = \frac{1}{c}y(t), \\ \dot{z}_2(t) = -\alpha z_2(t) + \frac{\alpha - a}{c}\dot{y}(t) + N(y(t)) + d\cos(wt), \end{cases} \tag{2}$$

with $N(y) := -b\sin\left(\frac{y}{c}\right) + \frac{l}{2}\sin\left(\frac{2y}{c}\right)$ and $\alpha > 0$. In this case, the guaranted exponential convergence rate is given by α.

Proof. From (1), (2) with

$$e_i(t) := x_i(t) - z_i(t), \ \forall \ i \in \{1, \ 2\},$$

it can be readily obtained that

$$
\begin{aligned}
e_1(t) &= x_1(t) - z_1(t) \\
&= x_1(t) - \frac{z_1(t)}{c} \\
&= x_1(t) - x_1(t) = 0, \quad \forall\, t \ge 0;
\end{aligned}
$$

$$
\begin{aligned}
\dot{e}_2(t) &= \dot{x}_2(t) - \dot{z}_2(t) \\
&= -ax_2(t) + \alpha z_2(t) - \frac{\alpha - a}{c}\dot{y}(t) - b\sin(x_1(t)) + l\sin(x_1(t))\cos(x_1(t)) - N(y(t)) \\
&= -ax_2(t) + \alpha z_2(t) - (\alpha - a)\dot{x}_1(t) - b\sin\left(\frac{y_1(t)}{c}\right) + l\sin\left(\frac{y_1(t)}{c}\right)\cos\left(\frac{y_1(t)}{c}\right) \\
&\quad - N(y(t)) \\
&= -ax_2(t) + \alpha z_2(t) - (\alpha - a)x_2(t) - b\sin\left(\frac{y_1(t)}{c}\right) + \frac{l}{2}\sin\left(\frac{2y_1(t)}{c}\right) - N(y(t)) \\
&= -\alpha x_2(t) + \alpha z_2(t) + N(y(t)) - N(y(t)) \\
&= -\alpha\, e_2(t), \quad \forall\, t \ge 0.
\end{aligned}
$$

This implies that

$$
\begin{aligned}
&\frac{d\left[e_2(t)\exp(\alpha\ \ t)\right]}{dt} = 0 \\
&\Rightarrow e_2(t)\exp(\alpha\ \ t) = e_2(0)\exp(\alpha 0) \\
&\Rightarrow e_2(t) = \exp(-\alpha\ \ t)e_2(0) \\
&\Rightarrow |e_2(t)| = |e_2(0)| \cdot \exp(-\alpha\ \ t), \ \forall\ \ t \ge 0.
\end{aligned}
$$

Consquently, we conclude that

$$
\|e(t)\| = \sqrt{e_1^2(t) + e_2^2(t)} \le |e_2(0)| \cdot \exp(-\alpha\ \ t), \ \forall\ \ t \ge 0\,. \tag{3}
$$

This completes the proof.

Remark 1: In Theorem 1, we have shown that the state observer of the horizontal platform systems can always be achieved with any pre-specified converence rate.

III. APPLICATION WITH NUMERICAL SIMULATIONS

For any information vector $h(t)$ in the transmitter system, the objective of secure communication system is to recover the message $h(t)$ in the receiver system. Let us consider the following secure communication system and the proposed scheme is illustrated in Figure 1.

Transmitter

$$
\dot{x}_1 = x_2\,, \tag{4a}
$$

$$
\dot{x}_2 = -ax_2 - b\sin(x_1) + l \cdot \sin(x_1)\cos(x_1) + d\cos(wt), \ \forall\ \ t \ge 0\,, \tag{4b}
$$

$$
y = cx_1 \tag{4c}
$$

$$
\phi_h(t) = C_h x(t) + h(t) \tag{4d}
$$

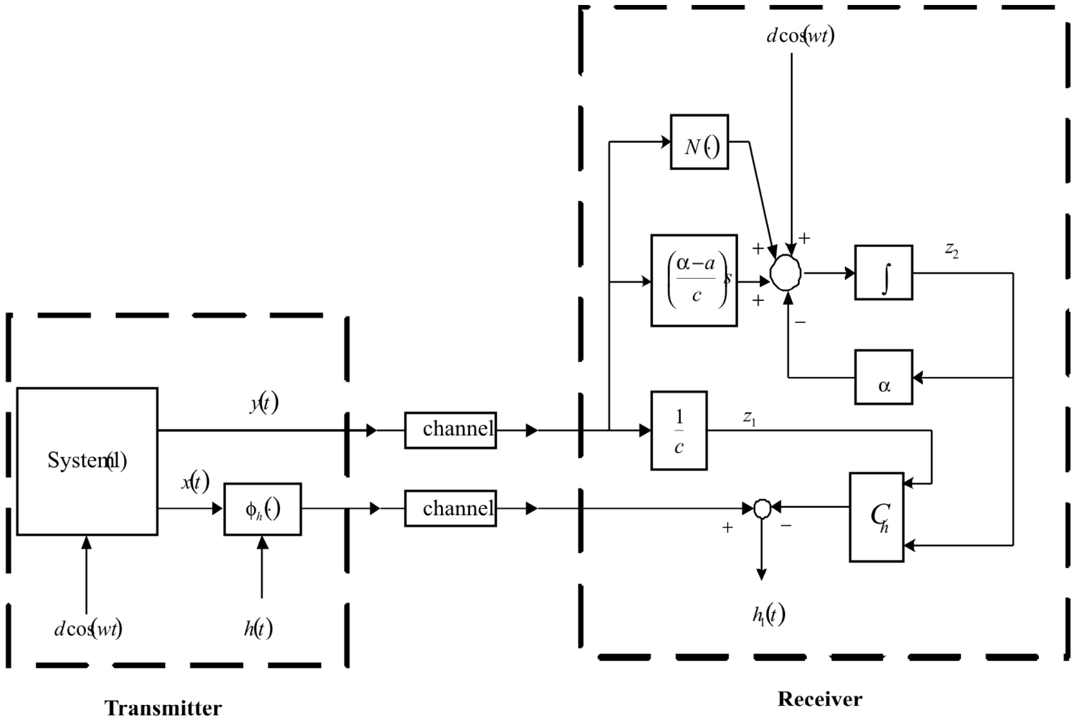

Figure 1: Secure Communication System ($h(t)$ is the Information Vector and $h_1(t)$ is the Recovered Vector

Receiver:

$$z_1 = \frac{1}{c} y \tag{5a}$$

$$\dot{z}_2 = -\alpha z_2 + \frac{\alpha - a}{c} \dot{y} + N(y) + d\cos(wt), \tag{5b}$$

$$h_1(t) = \phi_h(t) - C_h z(t) \tag{5c}$$

where $x(t) := \begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^T \in \Re^{2\times 1}$, $z(t) := \begin{bmatrix} z_1(t) & z_2(t) \end{bmatrix}^T \in \Re^{2\times 1}$, $h(t) \in \Re^{q\times 1}$ is the information vector, $C_h \in \Re^{q\times 2}$, $h_1(t) \in \Re^{q\times 1}$ is the signal recovered from $h(t)$, with $c \neq 0$, $\alpha > 0$, and $q \in N$. By Theorem 1 with (3)-(5), one can see that

$$\begin{aligned} \|h_1(t) - h(t)\| &= \|\phi_h(t) - C_h z(t) - \phi_h(t) + C_h x(t)\| \\ &\le \|C_h\| \cdot \|e(t)\| \\ &\le |e_2(0)| \cdot \|C_h\| e^{-\alpha t}, \ \forall \ t \ge 0. \end{aligned}$$

This implies that one can recover the message $h(t)$ in the receiver system, with the guaranteed exponential convergence rate α. In other words, the synchronization of signals $h(t)$ and $h_1(t)$ for the proposed secure communication (4) and (5) can always be achieved with any pre-specified converence rate α.

With, e.g., $\alpha = 10,\ C_h = \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 1 & 0.5 \end{bmatrix}$, the real message $h(t)$, the recoverd message $h_1(t)$, and the error signal are depicted in Figure 2-Figure 4, respectively, which clearly indicateds that the real message $h(t)$ is recoved after 0.6 seconds.

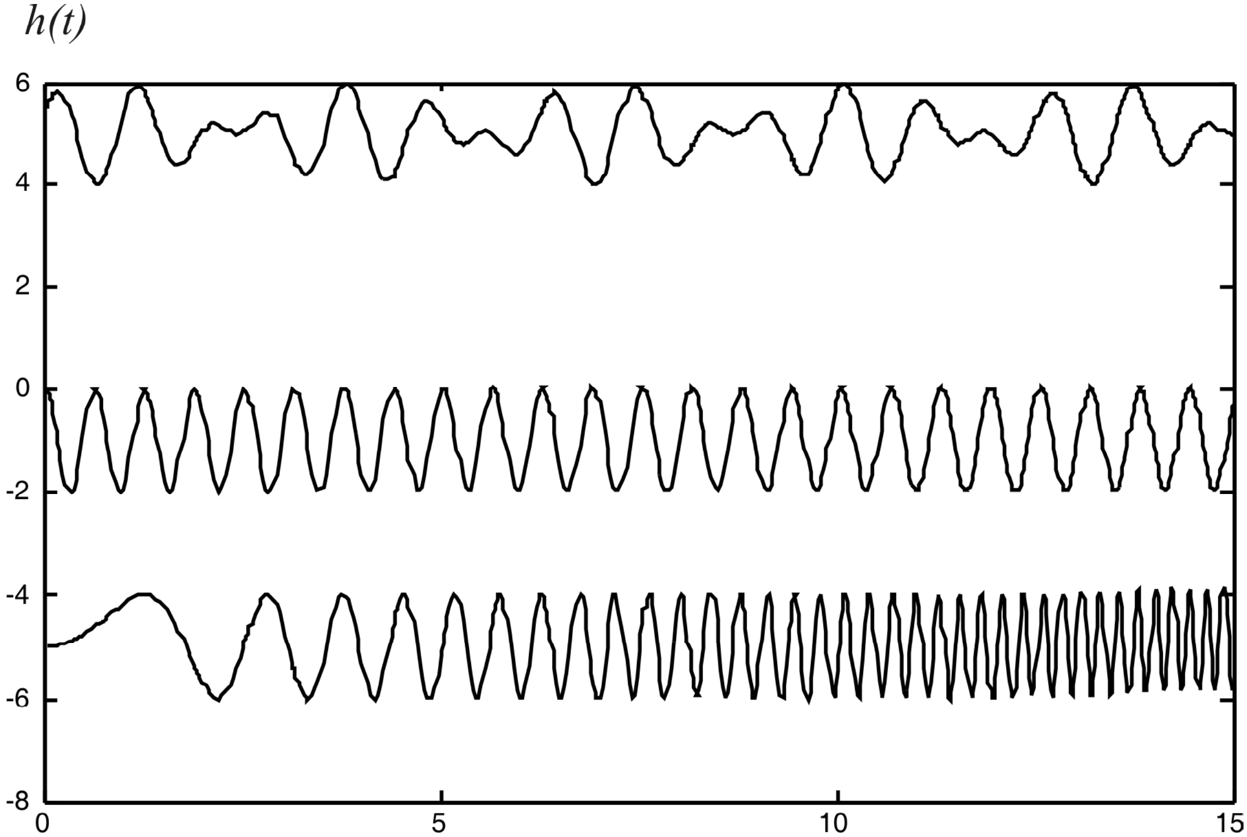

Figure 2: Real Message of $h(t) \in \Re^{3\times1}$ Described in the Transmitter of (4)

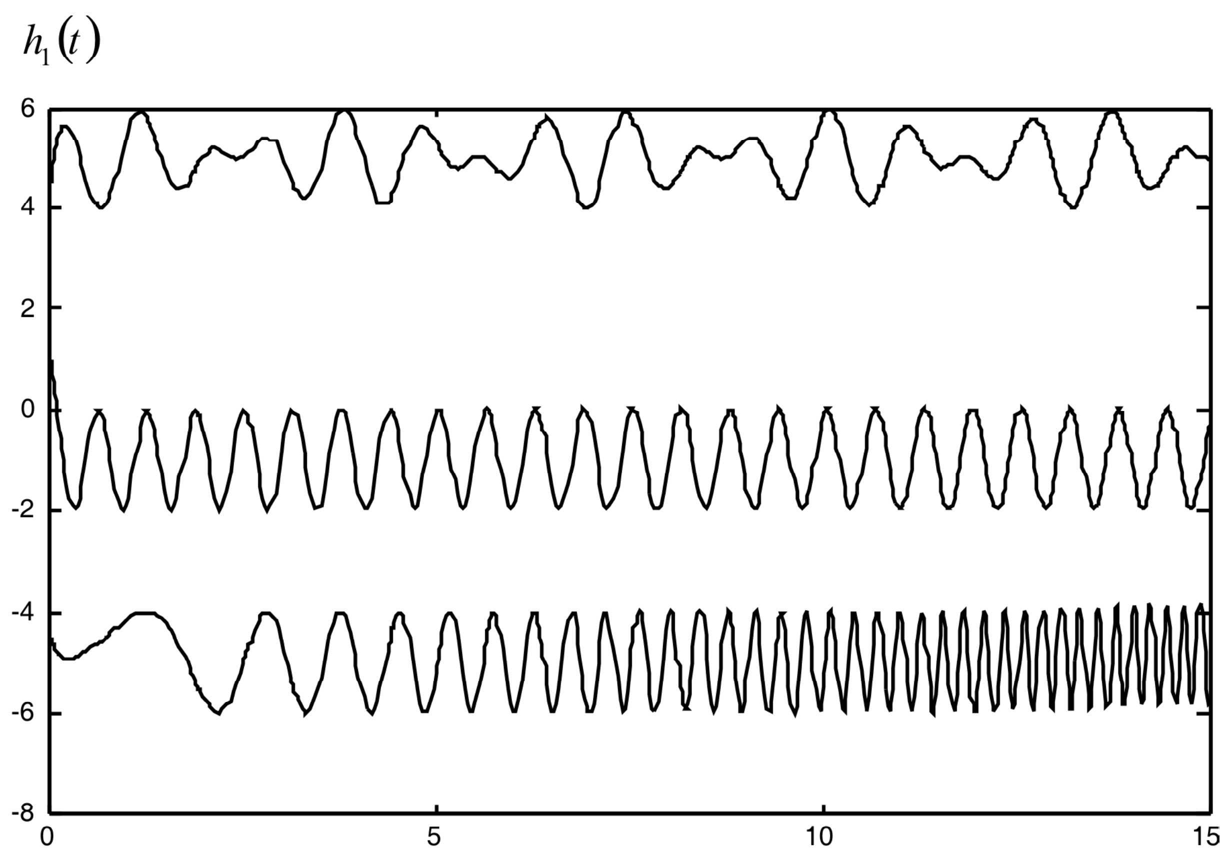

Figure 3: Recoverd Message of $h_1(t) \in \Re^{3\times1}$ Described in in the Receiver of (5)

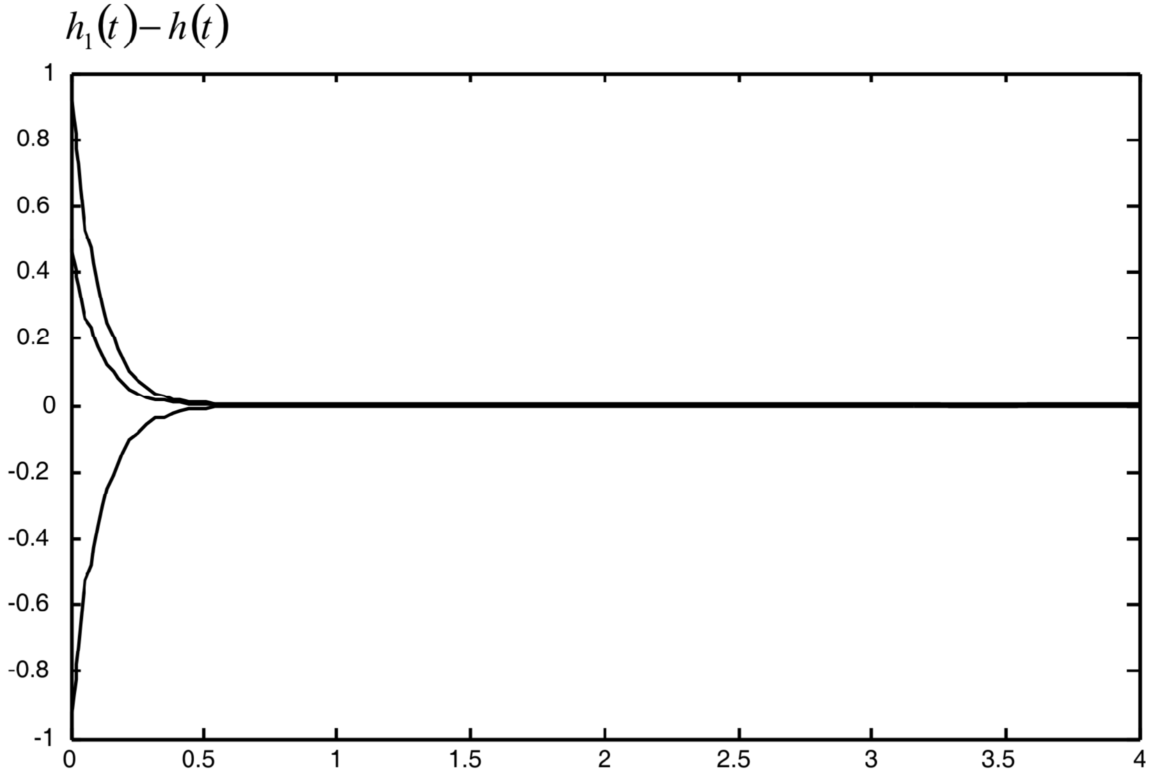

Figure 4: Error Signal of $h_1(t) - h(t)$

IV. CONCLUSIONS

In this paper, the horizontal platform chaotic system has been considered and the state observation problem of such a system has been investigated. A simple state observer for the horizontal platform system has been developed to guarantee the global exponential stability of the resulting error system. Applications of proposed state observer scheme to secure communication as well as some numerical simulations have also been presented to illustrate the practicability and effectiveness of the main results. Meanwhile, we have shown that the guaranteed exponential convergence rate of the proposed state observer and that of the proposed secure communication can be arbitrarily pre-specified.

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