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A STUDY OF STICKING OF AEROSOLS DUE TO COAGULATION AND CONDENSATION IN THE ATMOSPHERE

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Abstract: Under the effects of electric field and couple stress parameter on sticking of aerosols as a mixture of deformable agglomeration and coalescence of aerosols in the atmosphere is investigated.Analytical solutions are obtained using regular perturbation technique.The solutions are computed and the results are depicted graphically through figures. The graphs for velocity, and skin friction coefficient for both solid phase and fluid phase of atmospheric aerosols through horizontal channel are presented for different values of electric number and couple stress parameter.

Keywords: couple stress fluid, electrodes, skin friction coefficient, mixture theory.

1. INTRODUCTION

Air pollution in cities and towns has changed in composition and character during last twenty years. Earlier air pollution was cause mainly by domestic heating and industry. However, car traffic has gradually gained importance and is now the dominant source of air pollution.The national capital,New Delhi is one of the most polluted cities in the world and has very high levels of air pollution. This trend is not limited to New Delhi but plagues a majority of India's urban areas. Particles suspended in air act as siteson which water can condense and thus play a principal role in the watercycle and the formation of rain. High concentration of some particles are extremely explosive and low concentration of other particles are extremely toxic. Whether we realize it or not,we are at all times surrounded by literally thousands of small particles called aerosols and their importance to the natural functioning of the earth is incalculable[14].At high aerosol concentration, individual aerosol particles combine together to form larger aerosol particles. This process is called coagulation mainly caused by the random movements and subsequent collision and coalescence of aerosol particles. Coagulation is mainly controlled by the diffusion coefficient of particles which in turn is related to the particle mobility, which is the average drift velocity of the particle per unit driving force. Condensation of water vapour takes place on the aerosols with increasing relative humidity (ratio of the actual water vapour pressure of air to that when saturated) and evaporation of water present on the aerosols takes place with decreasing relative humidity. This means that physical parameters such as mean density, refractive index and hence the radiative properties of aerosols depend on relative humidity.

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Carlo method for particle coagulation in spatially homogeneous systems. Jianzhong Lin et al., [9] have analyzed nanoparticle migration in a fully developed turbulent pipe flow considering the particle coagulation. Anand and Mayya [2] have studied a simplified approach for solving coagulation-diffusion equation to estimate atmospheric background particle number leading factors contributed by emissions from localized sources. Chan et al.,[6]have investigated simultaneous numerical simulation of nano and fine particle coagulation and dispersion in a round jet. Schmidt [17] revealed experimental study of electro-Coalescence in a unit reactor. Anand and Mayya [1] have considered coagulation in a diffusing Gaussian aerosol puff and compared analytical approximations with numerical solution. Sun et al.,[22,23] have derived Monte Carlo simulation of multicomponent aerosols undergoing simultaneous coagulation and condensation. Margaritis Kostoglou and Athanasios Konstandopoulos[11] have studied evolution of aggregate size and fractal dimension during Brownian coagulation. Richard and Fangqun Yu [15] have analysed particle size distributions in an expanding plume undergoing simultaneous coagulation and condensation. Turco and Yu [24] have derived an analytical solution of the equations describing aerosol coagulation in an expanding plume.

Usually, when the earth's local weather is fine, the electric field is about 180Vm-1-280Vm-1, depending on the concentration of aerosols [3]. When the aerosols are continuously deforming with relative motion resulting in particles colliding and coalescing to form larger particles where an electric field induces a dipole in uncharged aerosol particles the charges induced on the closest sides of the neighbouring particles are of opposite sign.These particlesexperience an attractive force which can eventually lead to the particles colliding causing agglomeration or coalescence. Jayaratne and Verma [8] investigated the environmental aerosols and their effect on the earth's local fair weather electric field. The couple stress fluid theory developed by Stokes [18,19,20,21] represents the simplest generalization of the classical theory which allows for polar effects such as the presence of couple stresses and body couples. Rudraiah and Devaraju [16] revealed the effects of reaction rate and large size deformable aerosols on dispersion in atmospheric flow regarded as the turbulent fluid saturated porous media. Meenapriya and Nirmala Ratchagar [12,13] studied the dispersion of atmospheric aerosols through channel with temperature distributions in the presence of electric field. In the literature [4,5,10] mixture theory has been used to derive the basic equations assuming the mixture of aerosol and atmospheric fluid as deformable porous media.

From the above references, we note that it is of great importance to study the effects of coagulation and condensation of aerosols by considering the mechanism of mixture of aerosols with couple stress fluid. The simultaneous effect of electric field and couple stress on the coagulation and condensation of aerosols through a channel is investigated. By using the mechanism of mixture of aerosols and couple stress fluid taken into account the combined effect of electric field, deformable aerosols and settling of larger particles using the Reynolds averaging procedure supplemented ,the mathematical model is formulated.

2. MATHEMATICAL FORMULATION

We consider a two dimensional geometry as shown in Fig. 1. It consists of flow through a symmetrical channel extended to infinity on both directions of the x-axis. The channel is filled with poorly conducting fluid regarded as a mixture of aerosol and couple stress fluid with embedded electrodes of different potentials at y=0 and y=h. The applied pressure gradient $\frac{\partial p}{\partial x}=G(t)$ produces an axially directed flow. Then the required momentum equations are given by

$$\overline{\rho^{\beta}}\left(\frac{\overline{\partial q_i^{\beta}}}{\partial t}+\overline{q_j^{\beta}}\frac{\overline{\partial q_i^{\beta}}}{\partial x_j}\right)=-\phi^{\beta}\frac{\partial \overline{p}}{\partial x_i}+\left[X_p(\mu-\mu_a)+\mu_a+\mu_e\right]\frac{\overline{\partial^2 q_i^{\beta}}}{\partial x_i \partial x_j}\mp K\left(\overline{q_i^s}-\overline{q_i^f}\right)+\rho_e\overline{E_i} \tag{1}$$

where

($\mp$) denotes negative sign for aerosols phase and positive sign for fluid phase, ϕ^{β} the volume fraction, (i=1,2,3) the velocity of corresponding phases, β=s for solid phase, β=f for fluid phase of aerosols, Eithe electric field, $\mu_a$the apparent viscosity, μ_e the eddy viscosity, μ the lame constant, ρ_e the density of charges.

For the mixture of aerosol and couple stress fluid with the assumptions stated above becomes

$$\rho^s\frac{\partial u^s}{\partial t}=\mu_s\frac{\partial^2 u^s}{\partial y^2}-\phi^s G-K\left(u^s-u^f\right)+\rho_e E_x-\lambda^s\frac{\partial^4 u^s}{\partial y^4} \tag{2}$$

$$\rho^f\frac{\partial u^f}{\partial t}=\mu_f\frac{\partial^{2f} u^f}{\partial y^2}-\phi^f G-K\left(u^s-u^f\right)+\rho_e E_x-\lambda^f\frac{\partial^4 u^f}{\partial y^4} \tag{3}$$

where $\mu_s=\mu+\mu_e$is one of the turbulent lame constants, $\mu_f=\mu_a+\mu_e$the effective visocity of poorly conducting couple stress fluid, k thelinear drag coefficient that is the Darcy resistance offered by solid to fluid, G the pressure gradient andλ the couple stress parameter. We make these equations dimensionless using

$$y^* = \frac{y}{h}, t^* = \frac{t}{t_0}, u^{s^*} = \frac{\mu_s}{h^2 G_0} u^s, u^{f^*} = \frac{\mu_f}{h^2 G_0} u^f, G^* = \frac{G}{G_0},$$

$$E_x^* = \frac{E_x}{\frac{V}{h}}, \rho_e^* = \frac{\rho_e}{\frac{\varepsilon_0 V}{h^2}}, \phi^* = \frac{\phi}{V}, \nu^f = \frac{\mu_f}{\rho^f}, \nu^s = \frac{\mu_s}{\rho^s}, l^2 = \frac{\lambda}{\mu} \tag{4}$$

where h, G_0,V and t_0 are the characteristic length, pressure gradient, electric potential and time respectively and the asterisks (*) denote the dimensionless quantities.

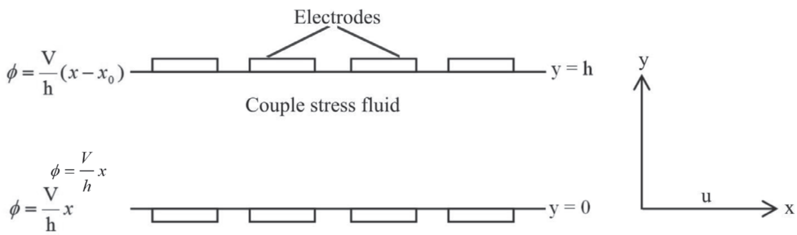

Fig.1 Physical configuration

Substituting equation (4) into equation (2) and (3), simplifying and for simplicity neglecting the asterisks, we get

$$\frac{\partial u^s}{\partial t} = R_1 \frac{\partial^2 u^s}{\partial y^2} - R_1 \phi^s G - R_2 \left(u^s - R_3 u^f\right) + w_e \rho_e E_x - \frac{R_1}{a^2} \frac{\partial^4 u^s}{\partial y^4} \tag{5}$$

$$\frac{\partial u^f}{\partial t} = R_4 \frac{\partial^2 u^f}{\partial y^2} - R_4 \phi^f G + R_2 \lambda_1 \left(u^s - R_3 u^f\right) + w_e \rho_e E_x - \frac{R_4}{a^2} \frac{\partial^4 u^f}{\partial y^4} \tag{6}$$

whereR_i(i= 1 to 4) are dimensionless numbers defined by

$$R_1 = \frac{\mu_s t_0}{\rho^s h^2}, R_2 = \frac{K t_0}{\rho^s}, R_3 = \frac{\mu_s}{\mu_f}, R_4 = \frac{\mu_f t_0}{\rho^f h^2}, \lambda_1 = \frac{\nu^f}{\nu^s},$$

$$w_e = \frac{\varepsilon_0 V^2}{\rho^s h^2 \left(\overline{u}^s\right)^2}, w_e = \frac{\varepsilon_0 V^2}{\rho^f h^2 \left(\overline{u}^f\right)^2}, a = \frac{h}{l} \tag{7}$$

where we the electric number, ε_0 a di electric constant, V applied electric potential, h the thickness of the medium, ρ the density of medium.

The boundary conditions are $u_0^s = 0$ at y=0,1 and $\frac{d^2 u_0^s}{dy^2} = 0$ at y=0,1 (8)

The equation of continuity of charges

$$\frac{\partial \rho_e}{\partial t} + \frac{\partial(\rho_e q_i)}{\partial x_i} + \frac{\partial J_i}{\partial x_i} = 0 \tag{9}$$

and using Ohm's law for a poorly conducting fluid $(\sigma << 1)$ and neglecting convection current $\rho_e q_i$ compared to conduction current, we denote $D_i=\varepsilon_0 E_i$, $J_i=\sigma E_i$. From these, while using the assumption $\sigma<<1$, equation (9) becomes

$$\frac{\partial \rho_e}{\partial t} + \sigma \frac{\partial E_i}{\partial x_i} + E_i \frac{\partial \sigma}{\partial x_i} = 0 \left(\because \frac{\partial q_i}{\partial x_i} = 0 \right) \tag{10}$$

Using Maxwell's equation and taking $\sigma = \sigma$ (y), (y), we get

$$\frac{\partial^2 \phi}{\partial y^2} + \frac{1}{\sigma} \frac{\partial \phi}{\partial y} \frac{\partial \sigma}{\partial y} = 0 \tag{11}$$

with the boundary conditions due to potentials $\phi = \frac{Vx}{h}$ at y=0 and $\phi = \frac{V(x - x_0)}{h}$ due to embedded electrodes as shown in Fig. 1. Making ϕ dimensionless using V, the boundary conditions are

$$\left.\begin{array}{lll} \phi = x & at & y = 0 \\ \phi = x - x_0 & at & y = 1 \end{array}\right\} \tag{12}$$

Since $\sigma<<1$, perturbation on it is negligible and hence it depends on the conduction temperature T_b namely $\frac{d^2 T_b}{dy^2} = 0$ with the boundary conditions (13)

$$\left.\begin{array}{lll} T_b = T_0 & at & y = 0 \\ T_b = T_1 & at & y = 1 \end{array}\right\} \tag{14}$$

is $T_b - T_0 = \Delta T_y$ (15)

Then σ is given by $\sigma = (1+\alpha y) \approx e^{\alpha y}$ (since $\alpha<<1$) (16)

where $\alpha = \alpha_h \Delta T$

equation (11) using (16) becomes

$$\frac{d^2 \phi}{dy^2} + \alpha \frac{d\phi}{dy} = 0 \tag{17}$$

The solution of equation (17) using the boundary condition given in equation (12) is

$$\phi = x - \frac{x_0}{1-e^{-\alpha}}\left(1-e^{-\alpha y}\right) \tag{18}$$

Substituting equation (18) into E_x and ρ_e, we get

$$E_x = -\frac{\partial \phi}{\partial x} = -1 \tag{19}$$

$$\rho_e = -\frac{\partial^2 \phi}{\partial y^2} = \frac{x_0 \alpha^2 e^{-\alpha y}}{1-e^{-\alpha}} \tag{20}$$

3. ANALYTICAL SOLUTIONS

The required basic equations and the corresponding boundary conditions are given by (5) to (20). These equations are coupled linear partial differential equations. Although it is possible to decouple these equations using a suitable operator, the resulting (PDE) are of higher order and cumbersome hence the required boundary conditions have to be extrapolated. Therefore, to avoid this process we use a regular perturbation technique choosing $\in = R_2R_3 (= K\ \mu_s\ t_0/\rho^s \mu_f)$ to be a small perturbation parameter. The assumption $K\ \mu_s/\rho^s << \mu_f/t_0$ is valid in our study of quasi-steady flow where t0 is very large and μ_f, the viscosity of couple stress fluid is small. Using this technique, we find the solutions of equations (5) and (6) in the form

$$u^s = u_0^s + \in u_1^s + \ldots, \qquad u^f = u_0^f + \in u_1^f + \ldots \tag{21}$$

Substituting equation (21) and (19) into equations (5) and (6) and assuming the normal mode solutions of the form

$$\left\{u_i^f(y,t), u_i^s(y,t), G(t)\right\} = \left\{u_i^f(y), u_i^s(y), G_0\right\} e^{-nt} \tag{22}$$

where i=0,1….

On simplifying them, we get

$$\frac{d^4 u_0^s}{dy^4} - a^2 \frac{d^2 u_0^s}{dy^2} + a_0^2 a^2 u_0^s = -a^2 \phi^s G_0 + a^2 a_1 \frac{w_e}{R_1} e^{-\alpha y} \tag{23}$$

$$\frac{d^4 u_0^f}{dy^4} - a^2 \frac{d^2 u_0^f}{dy^2} - a^2 b_0^{\,2} u_0^f = -a^2 \phi^f G_0 + a^2 \frac{R_2}{R_4} \lambda_1 u_0^s + a^2 \frac{w_e\, a_1}{R_4} e^{-\alpha y} \tag{24}$$

$$\frac{d^4u_1^s}{dy^4}-a^2\frac{d^2u_1^s}{dy^2}+a_0^2a^2u_1^s=a^2\frac{u_0^f}{R_1} \tag{25}$$

$$\frac{d^4u_1^f}{dy^4}-a^2\frac{d^2u_1^f}{dy^2}-a^2b_0^2u_1^f=-a^2\frac{\lambda_1}{R_4}u_0^f+a^2\frac{R_2}{R_4}\lambda_1u_1^s \tag{26}$$

where $a_0^2=\left(\frac{R_2-n}{R_1}\right), b_0^2=\frac{n}{R_4}$ and $a_1=\frac{x_0\alpha^2}{1-e^{-\alpha}}e^{nt}$.

The boundary conditions as given in equation (8),

$$u_i^s=0 \quad at \quad y=0,1 \qquad u_i^f=0 \quad at \quad y=0,1$$
$$\frac{d^2u_i^s}{dy^2}=0\,at \quad y=0,1 \qquad \frac{d^2u_i^f}{dy^2}=0\,at \quad y=0,1 \tag{27}$$

Similarly, we can get equations for i >1, but we restrict only to i=0 and 1. The solutions of equation (23) to (26) satisfying the condition given by equation (27), are

$$u_0^s=c_1\cos a_0a_y+c_2\sin a_0ay+c_3e^{a\sqrt{1-a_0^2}y}+c_4e^{-a\sqrt{1-a_0^2}y}$$
$$-\frac{\phi^sG_0}{a_0^2}+\frac{1}{\alpha^2\left(\alpha^2-a^2\right)+a_0^2a^2}a^2a_1\frac{We}{R_1}e^{-\alpha y} \tag{28}$$

$$u_0^f=d_1e^{ab0y}+d_2e^{-ab0y}+d_3e^{a\sqrt{b_0^2+1y}}+d_4e^{-a\sqrt{b_0^2+1y}}+f_1+f_2\cos a_0ay$$

$$+f_3\sin a_0ay+f_4e^{a\sqrt{1-a_0^2}y}+f_5e^{-a\sqrt{1-a_0^2}y}+f_6e^{-\alpha y} \tag{29}$$

$$u_1^s=g_1\cos a_0a_y+g_2\sin a_0ay+g_3e^{a\sqrt{1-a_0^2}y}+g_4e^{-a\sqrt{1-a_0^2}y}+h_1e^{ab0y}$$

$$+h_2e^{-ab0y}+h_3e^{a\sqrt{b_0^2+1y}}+h_4e^{-a\sqrt{b_0^2+1y}}+h_5+h_6\cos a_0ay+h_7\sin a_0ay+h_8e^{a\sqrt{1-a_0^2}y}$$
$$+h_9e^{-a\sqrt{1-a_0^2}y}+h_{10}e^{-\alpha y} \tag{30}$$

$$u_1^f = j_1 e^{ab0y} + j_2 e^{-ab0y} + j_3 e^{a\sqrt{b_0^2+1}y} + j_4 e^{-a\sqrt{b_0^2+1}y} - m_1 e^{ab0y} - m_2 e^{-ab0y}$$
$$- m_3 e^{a\sqrt{b_0^2+1}y} - m_4 e^{-a\sqrt{b_0^2+1}y} + m_5 - m_6 \cos a_0 ay - m_7 \sin a_0 ay - m_8 e^{a\sqrt{1-a_0^2}y} - m_9 e^{-a\sqrt{1-a_0^2}y}$$
$$- m_{10} e^{-\alpha y} + m_{11} \cos a_0 ay + m_{12} \sin a_0 ay + m_{13} e^{a\sqrt{1-a_0^2}y} + m_{14} e^{-a\sqrt{1-a_0^2}y} + m_{15} e^{ab_0 y}$$
$$+ m_{16} e^{-ab0y} + m_{17} e^{a\sqrt{b_0^2+1}y} + m_{18} e^{-a\sqrt{b_0^2+1}y} + m_{19} \cos a_0 ay + m_{20} \sin a_0 ay + m_{21} e^{a\sqrt{1-a_0^2}y}$$
$$+ m_{22} e^{-a\sqrt{1-a_0^2}y} + m_{23} e^{-\alpha y} \qquad (31)$$

where the coefficients c_i(i=1,2,3,4), d_i(i=1,4), g_i(i=1,…4) j_i(i=1,2,3,4), f_i(i=1…6), h_i(i=1…10), m_i(i=1,2,3…23) are given in the appendix .

From equation (21), using equations (28) to (31) we get

$$u^s = u_0^s + \in u_i^s = s_1 \cos a_0 ay + s_2 \sin a_0 ay + s_3 e^{a\sqrt{1-a_0^2}y} + s_4 e^{-a\sqrt{1-a_0^2}y}$$
$$- s_5 + s_6 e^{-\alpha y} + s_7 e^{ab_0 y} + s_8 e^{-ab_0 y} + s_9 e^{a\sqrt{b_0^2+1}y} + s_{10} e^{-a\sqrt{b_0^2+1}y}$$
$$+ s_{11} e^{-\alpha y} \qquad (32)$$

$$u^f = u_0^f + \in u_1^f = p_1 e^{ab0y} + p_2 e^{-ab0y} + p_3 e^{a\sqrt{b_0^2+1}y} + p_4 e^{-a\sqrt{b_0^2+1}y} + p_5 + p_6 \cos a_0 ay$$
$$+ p_7 \sin a_0 ay + p_8 e^{a\sqrt{1-a_0^2}y} + p_9 e^{-a\sqrt{1-a_0^2}y} + p_{10} e^{-\alpha y} \qquad (33)$$

where the coefficients s_i(i=1…, 11) and p_i (i=1,…10) are given in the appendix

To find skin friction,

In many environmental pollution problems it is advantageous to know the skin friction at the boundaries. These can be determined once we know the velocity. The skin friction at the walls is defined as

$$\tau = \mu\left(\frac{du}{dy}\right)_{y=0 \text{ and } h}$$

making this dimensionless using the scale for u used earlier we get,

$$\tau = \left(\frac{du}{dy}\right)_{y=0 \text{ and } 1}$$

where $\frac{du}{dy}$ can be obtained using (32) and (33).

The analytical results for both cases (for solid phase and fluid phase) are represented through figures.

4. RESULTS AND DISCUSSIONS

To understand the behaviour of aerosols formed by sticking due to coagulation and condensation requires the study of velocity profiles and skin friction coefficient for both solid phase and fluid phase aerosols in the atmosphere.In this paper, an attempt has been made to study the simultaneous effect of electric field and couple stress parameter on atmospheric aerosols consisting of mixture of solid phase and liquid phase of aerosols. The velocity and the skin friction coefficient is obtained analytically and they are numerically computed and the results are depicted graphically. Figures 2 and 3 represent the effect of electric number on the velocity profiles of aerosols and couple stress fluid. It is seen that us increases with an increase we but uf decreases with an increase in we. In figure 4 and 5 the skin friction coefficient of solid phase is plotted against y for different values of electric number and couple stress parameter. It is observed that the enhancement of electric number and couple stress parameter increases the skin friction coefficient for both solid phase and fluid phase of aerosols. Figure 6 and 7 represent the skin friction coefficient of fluid phase is plotted against y for different values of electric number and couple stress parameter. It is noted that the skin friction coefficient decreases with increase in electric number but it increases with increase in couple stress parameter.

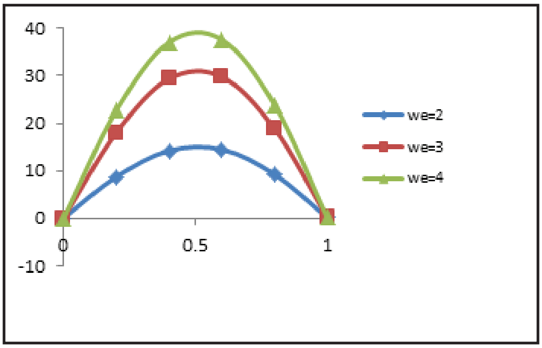

Figure 2. Velocity profiles of aerosols for different values of electric number

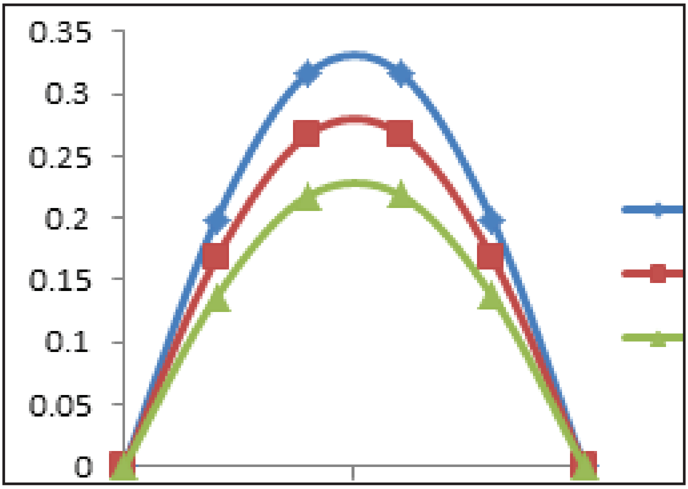

Figure 3. Velocity profiles for couple stress fluid for different values of electric number

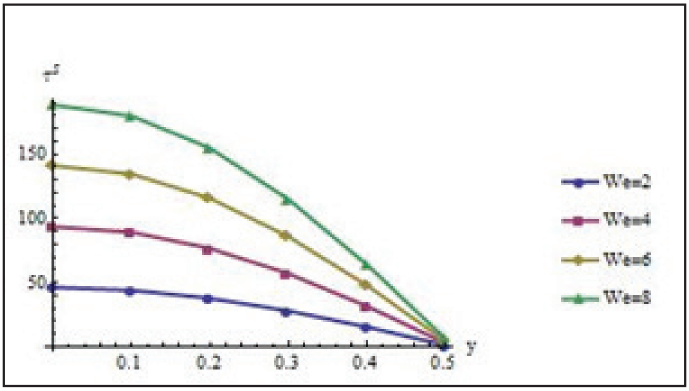

Figure 4. Skin friction coefficient of aerosols for different values of electric number

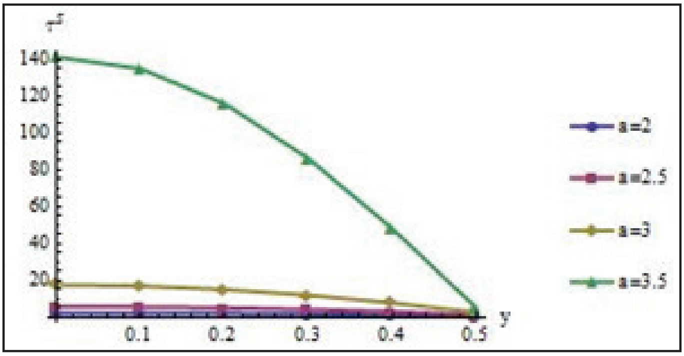

Figure 5. Skin friction coefficient of aerosols for different values of couple stress parameter

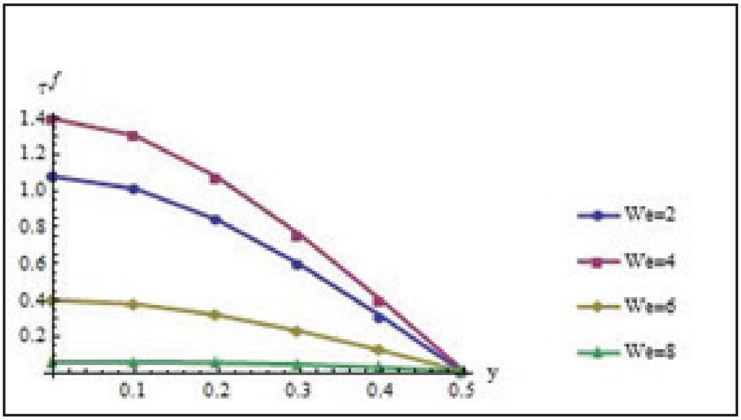

Figure 6. Skin friction coefficient of couple stress fluid for different values of electric number

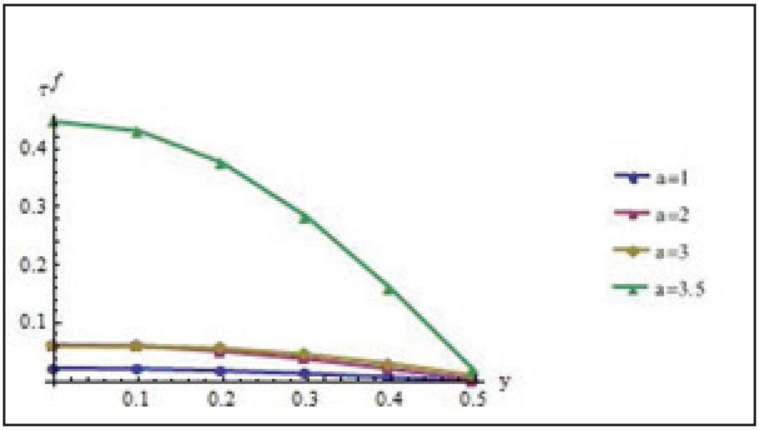

Figure 7. Skin friction coefficient of couple stress fluid for different values of couple stress parameter

5. CONCLUSION

We have investigated the effects of electric field and couple stress on the velocity profiles and skin friction coefficient of coagulation and condensation of aerosols. It is concluded that the parameters that influence velocity and skin friction coefficient of solid and fluid phase of aerosols are couple stress parameter and electric number. Here the behaviour of velocity and skin friction of solid phase aerosols increaseswith increase in electric number and couple stress parameter but influid phase of aerosols it decreases with increase in electric number and increases with increases in couple stress parameter. The proposed model and anlaysis presented here also suggest that the velocity profiles and skin friction of pollutants would depend upon the rate of introduction of the parameters in the atmosphere.

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APPENDIX

$$a_2 = \left(\frac{G_0}{a_0^2}\right); \qquad a_3 = \frac{a^2 a_1 w_e}{R_1\left(\alpha^2\left(\alpha^2 - a^2\right) + a^2 a_0^2\right)}; \quad b_0 = \frac{t}{R_4};$$

$$f_1 = \frac{G_0}{b_0^2} + \frac{R_2\lambda}{R_4}\frac{G_0}{a_0^2 b_0^2}; \quad f_2 = \left(\frac{a^2 R_2\lambda}{R_4}\left(\frac{c_1}{a_0^2 a^4\left(a_0^2+1\right) - a^2 b_0^2}\right)\right);$$

$$f_3 = \left(\frac{a^2 R_2\lambda}{R_4}\left(\frac{c_2}{a_0^2 a^4\left(a_0^2+1\right) - a^2 b_0^2}\right)\right); \quad f_4 = \left(\frac{a^2 R_2\lambda}{R_4}\left(\frac{c_3}{a_0^2 a^4\left(a_0^2+1\right) - a^2 b_0^2}\right)\right);$$

$$f_5 = \left(\frac{a^2 R_2\lambda}{R_4}\left(\frac{c_4}{a_0^2 a^4\left(a_0^2+1\right) - a^2 b_0^2}\right)\right);$$

$$f_6 = \left(\frac{a^2 a_1\, we}{\alpha^2\left(\alpha^2 - a^2\right) - a^2 b_0^2}\right)\left(\frac{1}{R_4} + \frac{\lambda R_2 a_1 a^2}{R_1 R_4\left(\alpha^2\left(\alpha^2 - a^2\right) - a^2 a_0^2\right)}\right);$$

$$c_1 = \left(a_0^2\left(a_3 - a_2\right) + \frac{\alpha^2}{a^2} a_3 + a_2 - a_3\right);$$

$$c_2 = \frac{1}{(\sin[a_0 a])}\left(\left(\left(\cos\left[a_0 a\right]\right) - e^{a\sqrt{1-a_0^2}}\right) a_0^2 a_3 - a_0^2 a_2 + \frac{\alpha^2}{a^2} a^3\right) -$$

$$\left(\cos\left[a_0 a\right]\right)\left(a_3 - a_2\right) - a_2 + a_3 e^{-\alpha} - a_0^2 a_3\left(-1 + a^2 e^{a\sqrt{1-a_0^2}}\right) -$$

$$\frac{a_3 e^{-\alpha}}{a^2}\left(a_0^2 a^2 + 2\alpha^2\right) + a_3 e^{a\sqrt{1-a_0^2}}\left(a_0^2 a^2 - \alpha^2\right)\Bigg);$$

$$c_3 = \left(a_0^2(a_3 - a_2) + \frac{\alpha^2}{a^2} a_3 - \left(\frac{1}{a^2\left(e^{a\sqrt{1-a_0^2}} - e^{-a\sqrt{1-a_0^2}}\right)} \right. \right.$$

$$\left. \left. \left(a^2 a_0^2 a_3 \left(-1 + a^2 e^{a\sqrt{1-a_0^2}} \right) + a_3 e^{-\alpha}\left(a_0^2 a^2 + 2\alpha^2\right) - a^2 a^3 e^{a\sqrt{1-a_0^2}} \left(a_0^2 a^2 - \alpha^2\right) \right) \right) \right);$$

$$c_4 = \left(\left(\left(\frac{1}{a^2\left(e^{a\sqrt{1-a_0^2}} - e^{-a\sqrt{1-a_0^2}}\right)} \right. \right. \right.$$

$$\left. \left. \left(a^2 a_0^2 a_3 \left(-1 + a^2 e^{a\sqrt{1-a_0^2}} \right) + a_3 e^{-\alpha}\left(a_0^2 a^2 + 2\alpha^2\right) - a^2 a^3 e^{a\sqrt{1-a_0^2}} \left(a_0^2 a^2 - \alpha^2\right) \right) \right) \right);$$

$$d_1 = \left(- f_1\left(b_0^2 + 1\right)\left(\frac{\left(1 - e - ab_0\right)}{e^{ab0} - e^{-ab_0}} \right) - f_2\left(b_0^2 + 1\right) \frac{\left(1 - e^{-ab_0} + \cos\left[a_0 a\right]\right)}{\left(e^{ab_0} - e^{-ab_0}\right)} - \right.$$

$$f_2 a_0^2 \frac{\left(-e^{-ab_0} + \cos\left[a_0 a\right] + \cos\left[t_2 a\right]\right)}{\left(e^{ab_0} - e^{-ab_0}\right)} - f_4 b_0^2 \left(\frac{e^{-ab_0}}{e^{ab0} - e^{-ab_0}} \right) +$$

$$f_4 a_0^2 \frac{\left(e^{-ab_0}\right)}{\left(e^{ab_0} - e^{-ab_0}\right)} + f_5 b_0^2 \left(\frac{e^{-ab_0}}{\left(e^{ab_0} - e^{-ab_0}\right)} \right) + f_5 a_0^2 \left(\frac{e^{-ab_0}}{\left(e^{ab_0} - e^{-ab_0}\right)} \right) +$$

$$\left(\frac{f_6}{\left(e^{ab_0} - e^{-ab_0}\right)} \right) \left(-\frac{\alpha^2}{a^2} e^{-ab_0} + b_0^2 e^{-ab_0} + e^{-ab_0} - b_0^2 e^{-\alpha} + \frac{\alpha^2}{a^2} e^{-\alpha} \right) -$$

$$\left. \left(\frac{f_3\left(\sin\left[a_0 a\right]\right)}{\left(e^{ab_0} - e^{-ab_0}\right)} \right) \left(a_0^2 + b_0^2 + 1\right) - \frac{f_4 e^{a\sqrt{1-a_0^2}}\left(a_0^2 + b_0^2\right)}{e^{ab_0} - e^{-ab_0}} - \frac{f_5 e^{a\sqrt{1-a_0^2}}\left(a_0^2 + b_0^2\right)}{e^{ab_0} - e^{-ab_0}} \right);$$

$$d_2 = \left(\frac{1}{\left(e^{ab_0} - e^{-ab_0}\right)}\right)\left(f_1\left(b_0^2+1\right)\left(1-e^{ab_0}\right) + f_2\left(b_0^2+1\right)\left(\cos\left[a_0 a\right] - e^{ab_0}\right) + \right.$$

$$f_2 a_0^2\left(\cos\left[a_0 a\right]\right)\left(1-e^{ab_0}\right) + f_3\left(\sin\left[a_0 a\right]\right)\left(a_0^2+b_0^2+1\right) + f_4 e^{ab_0}\left(b_0^2-a_0^2\right) +$$

$$f_5 e^{ab_0}\left(b_0^2-a_0^2\right) + f_4 e^{a\sqrt{1-a_0^2}}\left(a_0^2 b_0^2\right) + f_5 e^{-a\sqrt{1-a_0^2}}\left(a_0^2+b_0^2\right) +$$

$$\left. f_6\left(-b_0^2 e^{ab_0} + \frac{\alpha^2}{a^2}e^{ab_0} - e^{ab_0} + b_0^2 e^{-\alpha} + e^{-\alpha}\frac{e^{-\alpha}\alpha^2}{a^2}\right)\right);$$

$$d_3 = \left(f_1 b_0^2\left(\frac{-e^{-a\sqrt{b_0^2+1}}+1}{e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{b_0^2+1}}}\right) + f_2 b_0^2\left(\frac{-e^{-a\sqrt{b_0^2+1}} + \cos\left[a_0 a\right]}{e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{b_0^2+1}}}\right)\right.;+$$

$$f_2 a_0^2\frac{\left(-\left(\cos\left[a_0 a\right]\right)e^{-a\sqrt{b_0^2+1}} + \left(\cos\left[a_0 a\right]\right)\right)}{e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{b_0^2+1}}} + \frac{f_3\left(\sin\left[a_0 a\right]\right)\left(a_0^2+b_0^2\right)}{e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{b_0^2+1}}}$$

$$f_4\frac{\left(-e^{-a\sqrt{b_0^2+1}} - b_0^2 e^{-a\sqrt{b_0^2+1}} + a_0^{2-a\sqrt{b_0^2+1}}\right)}{e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{b_0^2+1}}} + \frac{f_4 e^{a\sqrt{1-a_0^2}\left(b_0^2-1+a_0^2\right)}}{e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{b_0^2+1}}}$$

$$-f_5\frac{\left(-e^{-a\sqrt{b_0^2+1}} + a_0^2 e^{-a\sqrt{b_0^2+1}} - b_0^{2-a\sqrt{b_0^2+1}}\right)}{e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{b_0^2+1}}} + f_5\frac{\left(e^{-a\sqrt{1-a_0^2}}\left(b_0^2-1+a_0^2\right)\right)}{e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{b_0^2+1}}} +$$

$$\left. b_0^2 f_6 - \frac{\alpha^2}{a^2}f_6 + \frac{f_6}{a^2}\left(\frac{\alpha^2 e^{a\sqrt{b_0^2+1}} - b_0^2 a^2 e^{a\sqrt{b_0^2+1}} + e^{-\alpha}a^2 b_0^2 - \alpha^2 e^{-\alpha}}{e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{b_0^2+1}}}\right)\right);$$

$$d_4 = \left(\left(\left(\frac{1}{\left(e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{b_0^2+1}}\right)}\right)\left(f_1 b_0^2\right)\left(e^{a\sqrt{b_0^2+1}} - 1\right) + f_2 b_0^2\left(e^{a\sqrt{b_0^2+1}} - \cos(a_0 a) -\right.\right.\right.$$

$$f_2 a_0^2\left(-e^{a\sqrt{b_0^2+1}}\cos(a_0 a) + \cos(a_0 a)\right) - f_3(\sin a_0 a)\left(a_0^2 + b_0^2\right) +$$

$$f_4 e^{a\sqrt{b_0^2+1}}\left(-1 + a_0^2 - b_0^2\right) - f_4 e^{a\sqrt{1-a_0^2}}\left(b_0^2 - 1 + a_0^2\right) + f_5 e^{a\sqrt{b_0^2+1}}\left(-1 + a_0^2 - b_0^2\right) -$$

$$f_5 e^{-a\sqrt{1-a_0^2}}\left(b_0^2 - 1 + a_0^2\right) - \frac{f_6}{a^2}\left(\alpha^2 e^{a\sqrt{b_0^2+1}} + e^{-\alpha} a^2 b_0^2 - \alpha^2 e^{-\alpha} - b_0^2 a^2 e^{a\sqrt{b_0^2+1}}\right)\Bigg)\Bigg)$$

$$g_1 = h_1\left(a_0^2 + b_0^2 - 1\right) + h_2\left(a_0^2 + b_0^2 - 1\right) + h_3\left(a_0^2 + b_0^2\right) + h_4\left(a_0^2 + b_0^2\right) + h_5\left(a_0^2 - 1\right)$$

$$- h_6 + h_{10}\left(a_0^2 + \frac{\alpha^2}{a^2} - 1\right);$$

$$g_2 = \frac{1}{\sin(a_0 a)}\Bigg(- h_1\left(a_0^2 + b_0^2 - 1\right)\cos(a_0 a) - h_2\left(a_0^2 + b_0^2\right) - 1\Big)\cos(a_0 a) - h_3\left(a_0^2 + b_0^2\right)\cos(a_0 a)$$

$$- h_4\left(a_0^2 + b_0^2\right)\cos(a_0 a) - h_5\left(a_0^2 - 1\right)\cos(a_0 a) + h_6\cos(a_0 a) - h_{10}\left(a_0^2 + \frac{\alpha^2}{a^2} - 1\right)\cos(a_0 a)$$

$$- \frac{\left(1 - a_0^2\right)}{a_0^2} h_1 e^{a b_0}\left(a_0^2 + b_0^2\right) - \frac{\left(1 - a_0^2\right)}{a_0^2} h_2 e^{-a b_0}\left(a_0^2 + b_0^2\right) -$$

$$- \frac{\left(1 - a_0^2\right)}{a_0^2} h_3 e^{a\sqrt{b_0^2+1}}\left(a_0^2 + b_0^2 + 1\right) - \frac{\left(1 - a_0^2\right)}{a_0^2} h_4 e^{-a\sqrt{b_0^2+1}}\left(a_0^2 + b_0^2 + 1\right) - \left(1 - a_0^2\right) h_5$$

$$- \frac{\left(1 - a_0^2\right)}{a_0^2} h_8 e^{a\sqrt{1-a_0^2}} - \frac{\left(1 - a_0^2\right)}{a_0^2} h_9 e^{-a\sqrt{1-a_0^2}} - \frac{\left(1 - a_0^2\right)}{a_0^2} h_{10} e^{-\alpha}\left(a_0^2 + \frac{\alpha^2}{a^2}\right) +$$

$$\frac{b_0^2}{a_0^2} h_1 e^{a b_0} + \frac{b_0^2}{a_0^2} h_2 e^{-a b_0} + \frac{\left(b_0^2 + 1\right)}{a_0^2} h_3 e^{a\sqrt{b_0^2+1}} + \frac{\left(b_0^2 + 1\right)}{a_0^2} h_4 e^{-a\sqrt{b_0^2+1}} -$$

$$h_6\cos(a_0 a) - h_7\sin(a_0 a) + \frac{\left(1 - a_0^2\right)}{a_0^2} h_8 e^{a\sqrt{1-a_0^2}} + \frac{\left(1 - a_0^2\right)}{a_0^2} h_9 e^{-a\sqrt{1-a_0^2}} + \frac{\alpha^2}{a^2 a_0^2} h_{10} e^{-\alpha}\Bigg);$$

$$g_3 = \frac{1}{e^{a\sqrt{1-a_0^2}} - e^{-a\sqrt{1-a_0^2}}}\left(h_1\left(a_0^2+b_0^2\right)\left(-e^{ab_0}+e^{-a\sqrt{1-a_0^2}}\right)+h_2\left(a_0^2+b_0^2\right)\left(-e^{-ab_0}+e^{-a\sqrt{1-a_0^2}}\right)+\right.$$

$$h_3\left(a_0^2+b_0^2\right)\left(-e^{a\sqrt{b_0^2+1}}+e^{-a\sqrt{1-a_0^2}}\right)-h_8\left(e^{a\sqrt{1-a_0^2}}-e^{-a\sqrt{1-a_0^2}}\right)+h_4\left(a_0^2+b_0^2\right)\left(-e^{-a\sqrt{b_0^2+1}}+e^{-a\sqrt{1-a_0^2}}\right)+$$

$$h_5a_0^2\left(-1+e^{-a\sqrt{1-a_0^2}}\right)+h_{10}\left(a_0^2+\frac{\alpha^2}{a^2}\right)\left(-e^{-\alpha}+e^{-a\sqrt{1-a_0^2}}\right)+h_3\left(e^{-a\sqrt{1-a_0^2}}-e^{a\sqrt{b_0^2+1}}\right)+h_4\left(e^{-a\sqrt{1-a_0^2}}-e^{-a\sqrt{b_0^2+1}}\right)$$

$$g_4 = \frac{1}{e^{a\sqrt{1-a_0^2}} - e^{-a\sqrt{1-a_0^2}}}\left(\left(a_0^2+b_0^2\right)\left(-e^{a\sqrt{1-a_0^2}}\right)(h_1+h_2+h_3+h_4)-h_5a_0^2\left(e^{a\sqrt{1-a_0^2}}-1\right)+h_{10}\left(a_0^2+\frac{\alpha^2}{a^2}\right)\right.$$

$$\left(-e^{a\sqrt{1-a_0^2}}+e^{-\alpha}\right)+\left(a_0^2+b_0^2\right)\left(h_1e^{ab_0}+h_2e^{-ab_0}+h_3e^{a\sqrt{b_0^2+1}}+h_4e^{-a\sqrt{b_0^2+1}}\right)+(h_3+h_4)\left(-e^{a\sqrt{1-a_0^2}}\right)+$$

$$\left.h_3e^{a\sqrt{b_0^2+1}}+h_4e^{-a\sqrt{b_0^2+1}}-h_9\left(e^{a\sqrt{1-a_0^2}}-e^{-a\sqrt{1-a_0^2}}\right)\right);$$

$$j_1 = \left(\frac{1}{e^{ab_0}-e^{-ab_0}}\left(m_1\left(e^{ab_0}-e^{-ab_0}\right)-m_5\left(b0^2+1\right)\left(-e^{-ab_0}+1\right)\right)+m_6\left(b_0^2+a_0^2+1\right)\left(-e^{-ab_0}+\cos(a_0a)\right)+\right.$$

$$m_7\sin(a_0a)\left(b_0^2+a_0^2+1\right)+m_8\left(b_0^2+a_0^2\right)\left(-e^{-ab_0}+e^{a\sqrt{1-a_0^2}}\right)+m_9\left(\left(b_0^2+a_0^2\right)\left(-e^{-ab_0}+e^{-a\sqrt{1-a_0^2}}\right)\right)+m_{10}\left(b_0^2-\frac{\alpha^2}{a^2}+1\right)$$

$$\left(-e^{-ab_0}+e^{-\alpha}\right)-m_{11}\left(b_0^2+a_0^2+1\right)\left(-e^{-ab_0}+\cos(a_0a)\right)-m_{12}\sin(a_0a)\left(b_0^2+a_0^2+1\right)-m_{13}\left(b_0^2+a_0^2\right)\left(-e^{-ab_0}+e^{a\sqrt{1-a_0^2}}\right).$$

$$-m_{14}\left(b_0^2+a_0^2\right)\left(-e^{-ab_0}+e^{-a\sqrt{1-a_0^2}}\right)-m_{15}\left(e^{ab_0}-e^{-ab_0}\right)-m_{19}\left(b_0^2+a_0^2+1\right)\left(-e^{-ab_0}+\cos(a_0a)\right)-$$

$$m_{20}\sin(a_0a)\left(b_0^2+a_0^2+1\right)-m_{21}\left(b_0^2+a_0^2\right)\left(-e^{-ab_0}+e^{a\sqrt{1-a_0^2}}\right)-m_{22}\left(\left(b_0^2+a_0^2\right)\left(-e^{-ab_0}+e^{-a\sqrt{1-a_0^2}}\right)\right)-$$

$$\left.m_{23}\left(b_0^2-\frac{\alpha^2}{a^2}+1\right)\left(-e^{-ab_0}+e^{-\alpha}\right)\right);$$

$$j_2 = \left(\frac{1}{e^{ab_0}-e^{-ab_0}}\left(m_2\left(e^{ab_0}-e^{-ab_0}\right)-m_5\left(b0^2+1\right)\left(-e^{ab_0}+1\right)\right)+m_6\left(b_0^2+a_0^2+1\right)\left(e^{ab_0}+\cos(a_0a)\right)+\right.$$

$$m_7\sin(a_0a)\left(b_0^2+a_0^2+1\right)+m_8\left(b_0^2+a_0^2\right)\left(e^{ab_0}-e^{a\sqrt{1-a_0^2}}\right)+m_9\left(\left(b_0^2+a_0^2\right)\left(e^{ab_0}-e^{a\sqrt{1-a_0^2}}\right)\right)+m_{10}\left(b_0^2-\frac{\alpha^2}{a^2}+1\right)$$

$$\left(e^{ab_0}-e^{-\alpha}\right)-m_{11}\left(b_0^2+a_0^2+1\right)\left(e^{ab_0}-\cos(a_0a)\right)+m_{12}\sin(a_0a)\left(b_0^2+a_0^2+1\right)-m_{13}\left(b_0^2+a_0^2\right)\left(e^{ab_0}-e^{a\sqrt{1-a_0^2}}\right).$$

$$-m_{14}\left(b_0^2+a_0^2\right)\left(e^{ab_0}-e^{-a\sqrt{1-a_0^2}}\right)-m_{16}\left(e^{ab_0}-e^{-ab_0}\right)m_{19}\left(b_0^2+a_0^2+1\right)\left(e^{ab_0}-\cos(a_0a)\right)+$$

$$m_{20}\sin(a_0a)\left(b_0^2+a_0^2+1\right)-m_{21}\left(b_0^2+a_0^2\right)\left(e^{ab_0}-e^{a\sqrt{1-a_0^2}}\right)-m_{22}\left(\left(b_0^2+a_0^2\right)\left(e^{ab_0}-e^{-a\sqrt{1-a_0^2}}\right)\right)-$$

$$\left.m_{23}\left(b_0^2-\frac{\alpha^2}{a^2}+1\right)\left(e^{ab_0}-e^{-\alpha}\right)\right);$$

$$j_3 = \Bigg(m_3 + b_0^2 m_5 \left(\frac{-e^{-a\sqrt{b_0^2+1}} + 1}{e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{b_0^2+1}}} \right) - m_6 \left(b_0^2 + a_0^2\right) \left(\frac{-e^{-a\sqrt{b_0^2+1}} + \cos(a_0 a)}{e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{b_0^2+1}}} \right) -$$

$$m_7 \frac{\sin(a_0 a)(b_0^2 + a_0^2)}{e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{b_0^2+1}}} - m_8 \left(b_0^2 + a_0^2 - 1\right) \left(\frac{-e^{-a\sqrt{b_0^2+1}} + e^{a\sqrt{1-a_0^2}}}{e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{b_0^2+1}}} \right) - m_9 \left(b_0^2 + a_0^2 - 1\right) \left(\frac{-e^{-a\sqrt{b_0^2+1}} + e^{-a\sqrt{1-a_0^2}}}{e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{b_0^2+1}}} \right)$$

$$-m_{10} \left(b_0^2 - \frac{\alpha^2}{a^2} \right) \left(\frac{-e^{-a\sqrt{b_0^2+1}} + e^{-\alpha}}{e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{b_0^2+1}}} \right) + m_{11} \left(b_0^2 + a_0^2\right) \left(\frac{-e^{-a\sqrt{b_0^2+1}} + \cos(a_0 a)}{e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{b_0^2+1}}} \right) + m_{12} \frac{\sin(a_0 a)(b_0^2 + a_0^2)}{e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{b_0^2+1}}}$$

$$m_{13} \left(b_0^2 + a_0^2 - 1\right) \left(\frac{-e^{-a\sqrt{b_0^2+1}} + e^{a\sqrt{1-a_0^2}}}{e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{b_0^2+1}}} \right) + m_{14} \left(b_0^2 + a_0^2 - 1\right) \left(\frac{-e^{-a\sqrt{b_0^2+1}} + e^{-a\sqrt{1-a_0^2}}}{e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{b_0^2+1}}} \right) -$$

$$m_{17} + m_{19} \left(b_0^2 + a_0^2\right) \left(\frac{-e^{-a\sqrt{b_0^2+1}} + \cos(a_0 a)}{e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{b_0^2+1}}} \right) + m_{20} \frac{\sin(a_0 a)(b_0^2 + a_0^2)}{e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{b_0^2+1}}} +$$

$$m_{21} \left(b_0^2 + a_0^2 - 1\right) \left(\frac{-e^{-a\sqrt{b_0^2+1}} + e^{a\sqrt{1-a_0^2}}}{e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{b_0^2+1}}} \right) + m_{22} \left(b_0^2 + a_0^2 - 1\right) \left(\frac{-e^{-a\sqrt{b_0^2+1}} + e^{-a\sqrt{1-a_0^2}}}{e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{b_0^2+1}}} \right) +$$

$$m_{23} \left(b_0^2 - \frac{\alpha^2}{a^2} \right) \left(\frac{-e^{-a\sqrt{b_0^2+1}} + e^{-\alpha}}{e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{b_0^2+1}}} \right) \Bigg);$$

$$j_4 = \Bigg(\frac{1}{e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{b_0^2+1}}} \Big\{ m_4 \left(e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{b_0^2+1}} \right) + b_0^2 m_5 \left(e^{a\sqrt{b_0^2+1}} - 1 \right) - m_6 \left(b_0^2 + a_0^2\right)$$

$$\left(e^{a\sqrt{b_0^2+1}} - \cos(a_0 a) \right) + m_7 \sin(a_0 a)(b_0^2 + a_0^2) - m_8 \left(b_0^2 + a_0^2 - 1\right) \left(e^{a\sqrt{b_0^2+1}} - e^{a\sqrt{-1a_0^2}} \right) -$$

$$m_9 \left(b_0^2 + a_0^2 - 1\right) \left(e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{-1a_0^2}} \right) - m_{10} \left(b_0^2 - \frac{\alpha^2}{a^2} \right) \left(e^{a\sqrt{b_0^2+1}} - e^{-\alpha} \right) +$$

$$m_{11} \left(b_0^2 + a_0^2\right) \left(e^{a\sqrt{b_0^2+1}} - \cos(a_0 a) \right) - m_{12} \sin(a_0 a)(b_0^2 + a_0^2) + m_{13} \left(b_0^2 + a_0^2 - 1\right) \left(e^{a\sqrt{b_0^2+1}} - e^{a\sqrt{-1a_0^2}} \right) +$$

$$m_{14} \left(b_0^2 + a_0^2 - 1\right) \left(e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{-1a_0^2}} \right) - m_{18} \left(e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{b_0^2+1}} \right) + m_{19} \left(b_0^2 + a_0^2\right)$$

$$\left(e^{a\sqrt{b_0^2+1}} - \cos(a_0 a) \right) - m_{20} \sin(a_0 a)(b_0^2 + a_0^2) + m_{21} \left(b_0^2 + a_0^2 - 1\right) \left(e^{a\sqrt{b_0^2+1}} - e^{a\sqrt{-1a_0^2}} \right) +$$

$$m_{22} \left(b_0^2 + a_0^2 - 1\right) \left(e^{a\sqrt{b_0^2+1}} - e^{-a\sqrt{-1a_0^2}} \right) + m_{23} \left(b_0^2 - \frac{\alpha^2}{a^2} \right) \left(e^{a\sqrt{b_0^2+1}} - e^{-\alpha} \right) \Big\} \Bigg);$$

$$h_1 = \frac{d_1}{\left(a^2 b_0^2 \left(b_0^2 - 1\right) + a_0^2\right) R_1}; \quad h_2 = \frac{d_2}{\left(a^2 b_0^2 \left(b_0^2 - 1\right) + a_0^2\right) R_1}; \quad h_3 = \frac{d_3}{\left(a^2 b_0^2 \left(b_0^2 - 1\right) + a_0^2\right) R_1};$$

$$h_4 = \frac{d_4}{\left(a^2 b_0^2 \left(b_0^2 - 1\right) + a_0^2\right) R_1}; \quad h_5 = \frac{f_1}{R_1 a_0^2}; \quad h_6 = \frac{f_2}{a^2 a_0^2 \left(a_0^2 + 1\right) + a_0^2) R_1}; \quad h_7 = \frac{f_3}{a^2 a_0^2 \left(a_0^2 + 1\right) + a_0^2) R_1};$$

$$h_8 = \frac{f_4}{a^2 a_0^2 \left(a_0^2 + 1\right) + a_0^2) R_1}; \quad h_9 = \frac{f_5}{a^2 a_0^2 \left(a_0^2 + 1\right) + a_0^2) R_1}; \quad h_{10} = \frac{a^2 f_6}{\alpha^2 \left(\alpha^2 - a^2\right) + a^2 a_0^2) R_1};$$

$$m_1 = \frac{\lambda}{R_4} \frac{d_1}{a^2 b_0^2 \left(b_0^2 - 1\right) - b_0^2}; \quad m_2 = \frac{\lambda}{R_4} \frac{d_2}{a^2 b_0^2 \left(b_0^2 - 1\right) - b_0^2}; \quad m_3 = \frac{\lambda}{R_4} \frac{d_3}{a^2 b_0^2 \left(b_0^2 - 1\right) - b_0^2};$$

$$m_4 = \frac{\lambda}{R_4} \frac{d_4}{a^2 b_0^2 \left(b_0^2 - 1\right) - b_0^2}; \quad m_5 = \frac{\lambda f_1}{R_4 b_0^2} - \frac{R_2 \lambda h_5}{R_4 b_0^2}; \quad m_6 = \frac{\lambda f_2}{R_4 \left(a^2 a_0^2 (a_0^2 + 1\right) - b_0^2};$$

$$m_7 = \frac{\lambda f_3}{R_4 \left(a^2 a_0^2 (a_0^2 + 1\right) - b_0^2}; \quad m_8 = \frac{\lambda f_4}{R_4 \left(a^2 a_0^2 (a_0^2 + 1\right) - b_0^2}; \quad m_9 = \frac{\lambda f_5}{R_4 \left(a^2 a_0^2 (a_0^2 + 1\right) - b_0^2};$$

$$m_{10} = \frac{a^2 \lambda}{R_4} \frac{f_6}{a^2 \left(\alpha^2 - a^2\right) - a^2 b_0^2}; \quad m_{11} = \frac{R_2 \lambda}{R_4} \frac{g_1}{a^2 a_0^2 \left(a_0^2 + 1\right) - b_0^2}; \quad m_{12} = \frac{R_2 \lambda}{R_4} \frac{g_2}{a^2 a_0^2 \left(a_0^2 + 1\right) - b_0^2};$$

$$m_{13} = \frac{R_2 \lambda}{R_4} \frac{g_3}{a^2 a_0^2 \left(a_0^2 + 1\right) - b_0^2}; \quad m_{14} = \frac{R_2 \lambda}{R_4} \frac{g_4}{a^2 a_0^2 \left(a_0^2 + 1\right) - b_0^2}; \quad m_{15} = \frac{R_2 \lambda}{R_4} \frac{g_4}{a^2 b_0^2 \left(b_0^2 - 1\right) - b_0^2};$$

$$m_{16} = \frac{R_2 \lambda}{R_4} \frac{h_2}{a^2 b_0^2 \left(b_0^2 - 1\right) - b_0^2}; \quad m_{17} = \frac{R_2 \lambda}{R_4} \frac{h_3}{a^2 b_0^2 \left(b_0^2 + 1\right) - b_0^2}; \quad m_{18} = \frac{R_2 \lambda}{R_4} \frac{h_4}{a^2 b_0^2 \left(b_0^2 + 1\right) - b_0^2};$$

$$m_{19} = \frac{R_2 \lambda}{R_4} \frac{h_6}{a^2 a_0^2 \left(a_0^2 + 1\right) - b_0^2}; \quad m_{20} = \frac{R_2 \lambda}{R_4} \frac{h_7}{a^2 a_0^2 \left(a_0^2 + 1\right) - b_0^2}; \quad m_{21} = \frac{R_2 \lambda}{R_4} \frac{h_8}{a^2 a_0^2 \left(a_0^2 + 1\right) - b_0^2};$$

$$m_{22} = \frac{R_2 \lambda}{R_4} \frac{h_9}{a^2 a_0^2 \left(a_0^2 + 1\right) - b_0^2}; \quad m_{23} = \frac{R_2 \lambda}{R_4} \frac{h_{10}}{\alpha^2 \left(\alpha^2 - a^2\right) - a^2 b_0^2}; \quad q_1 = \frac{\sin[a_0 a]}{2 a_0 a};$$

$$q_2 = \frac{\cos[a_0 a] - 1}{2 a_0 a}; \quad q_3 = \frac{e^{a\sqrt{1-a_0^2}} - 1}{2^{a\sqrt{1-a_0^2}}}; \quad q_4 = \frac{e^{a\sqrt{1-a_0^2}} - 1}{2^{a\sqrt{1-a_0^2}}}; \quad q_5 = \frac{e^{-a} - 1}{2\alpha};$$

$$q_6 = \frac{e^{a b_0} - 1}{2 a b_0}; \quad q_7 = \frac{e^{-a b_0} - 1}{2 a b_0}; \quad q_8 = \frac{e^{a\sqrt{b_0^2+1}} - 1}{2a\sqrt{b_0^2 + 1}}; \quad q_9 = \frac{e^{-a\sqrt{b_0^2+1}} - 1}{2a\sqrt{b_0^2 + 1}};$$

$$q_{10} = \frac{e^{-\alpha} - 1}{2\alpha};$$

$s_1=c_1+\in g_1+\in h_6$; $s_2=c_2+\in g_2+\in h_7$; $s_3=c_3+\in g_3+\in h_8$; $s_4=c_4+\in g_4+\in h_9$;

$$s_5 = \frac{G_0}{a^2} - \in h_5; \quad s_6 = \frac{1}{\alpha^2\left(\alpha^2 - a^2\right) + a^2 a_0^2} a^2 a_1 \frac{w_e}{R_1};$$

$s_7=\in h_1$;$s_8=\in h_2$;$s_9=\in h_3$;$s_{10}=\in h_4$;$s_{11}=\in h_{10}$;

$p_1=d_1+\in j_1-\in m_1+\in m_{15}$; $p_2=d_2+\in j_2-\in m_2+\in m_{16}$; $p_3=d_3+\in j_3-\in m_3+\in m_{17}$;
$p_4=d_4+\in j_4-\in m_4+\in m_{18}$; $p_5=f_1+\in m_5$; $p_6=f_2-\in m_6+\in m_{11}+\in m_{19}$;
$p_7=f_3-\in m_7+\in m_{12}+\in m_{20}$; $p_8=f_4-\in m_8+\in m_{13}+\in m_{21}$; $p_9=f_5-\in m_9+\in m_{14}+\in m_{22}$;
$p_{10}=f_6-\in m_{23}-\in m_{10}$;

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