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A Robust Control Chart for Variability with Modified Trimmed Mean and Standard Deviation

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Abstract : Control Chart is useful to identify the process is under control or not. Usually, Mean and Standard deviation of the samples plays a vital role for the construction of control charts. This paper mainly focuses on the dispersion measures like, Range, Quartile deviation and Standard deviation as the quality characteristics. Classical method of estimating the parameters of the sampling distribution may be affected by the presence of outliers. In order to overcome such situation, robust estimators, which are the functions of the all sample observations and to estimate the trimmed measures, are introduced in Engineering machine learning applications. Authors are estimate the trimmed measures, which are constructed with the data between 25% to 75% sample observations and construct the control chart for variability. Finally, this robust control chart is compared with *s*-chart in terms of its efficiency to detect outliers or assignable causes of variation.

Keywords : Robust control chart, Control limits, Trimmed Mean etc.,

1. INTRODUCTION

Shewhart control chart for sample standard deviation is a widely used process control technique. Performance of control chart is depending on designing its limits, as narrowing or widening limits influence probability of type 1 and type 2 errors respectively . Its performance can also affected by the presence of outliers. As sample standard deviation is defined on all data points in a sample, it is influenced by the extreme values present in the subgroup. If subgroup taken in phase 1 contain outliers, 3-sigma limits of *s*-chart may inflate and outliers remain unnoticed. This chart is based on the assumption that the underlying distribution of quality characteristics is approximately normally distributed, even though the actual process dispersion of *s* has long right tail. If robust measures are used to estimate process dispersion and to construct the limits, It can overcome such problem.

A robust estimator is an estimator that is insensitive to changes in the underlying distribution and also resistant against the presence of outliers. There are many robust measures of location and scale available in literature. Wilcox (2012) says by substituting robust measures of location and scale for the usual mean and standard variance, it should be possible to obtain test statistics which are insensitive to the combined effects of varianceheterogeneiety and non-normality. When robust measures are used to estimate parameters of quality characteristics constructing limits, control chart procedures become robust. Rocke (1989) suggested that in order to identify outliers easily, limits of a control chart should frame on robust measures while non-robust measures should plotted on it.

2. REVIEW OF LITERATURE

Iglewicz andLangenberg (1986) proposed mean range chart with control limits determined by Trimmed mean of the subgroup means and the trimmed mean of the range. Rocke (1989) proposed standard deviation control charts based on the mean or the trimmed mean of the subgroup ranges or subgroup Inter Quartile Ranges. Several

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authors have developed robust control chart based on various measures of scale namely Median Absolute Deviation Tukey (1948), Trimmed mean and its standard error are more appealing because of its computational simplicity. Apart from that, these measures are less affected by departures from normality than the usual mean and standard deviation.

Standard error of trimmed mean is not sufficient to estimate process dispersion because of trimming subgroup (**Dixon** and **Yuen**, 1974). Of trimmed Standard estimator of variance of trimmed is obtained through Winsorization (**Wilcox**, 2012). **Caperra**and **Rivest** , (1985) derived an exact formula for variance of the trimmed mean as a function of order statistics, when trimmed percentage is small.

Melan Terek and MatusTibensky(2014) deals with an analysis of how to use certain measures of location in analysis of wages, some of the situations , the mean is located in tail of the distribution and gives very biased idea about the location of the distribution . then by using trimmed mean and M-estimators are characterised and explained about the nature of the data.

As trimmed makes reduction in dispersion , the effect of loss due to trimming so that its robust qualities are not much distributed. A robust control chart for controlling process dispersion is developed based on this modified measure.

3. METHODS AND DISCUSSIONS

Usually, the control charts are useful to determine whether the process is under control or not. Actually themachinary valuesare X_1, X_2, …………., X_n an order statistics.

The find mean of all sample observations say $\overline{X}1, \overline{X}2............., \overline{Xn}$. Then we can obtain mean of means of the different samples taken from the machinary data available in VIT.After that obtain standard deviation of the samples means.Then construct control limits UCL, CL and LCL.

The control limits $\overline{\overline{x}} \pm 3\sigma\overline{x}$ *;* CL = $\overline{\overline{X}}$

Upper control limit UCL = $\overline{\overline{x}} + 3\sigma\overline{x}$

Lower control limit LCL = $\overline{\overline{x}} - 3\sigma\overline{x}$

The given all sample means are within the control limits, then process is under control otherwise it is not under control. If it is not under the control the process then we suggest to the machine expert to the standard machinary part using statistics process control technique.

Let X_1, X_2, …………., X_n are sample merged values then the number of values not finite. Then if we apply the usual procedures it will take more time and expenses.

In order to overcome thus situations, we are neglecting 50% of the data by using some standard statistical procedures.

Let $\overline{X}1, \overline{X}2, ..., \overline{Xn}$ are the sample means. Then write the data as a order statistics. Then find the 50% of data scattered in between the Box –plot by using Intra Quartile Range (Q3-Q1).

We can take the data from Q1 to Q3 as a order statistics like $\overline{X}1 <, \overline{X}2 <, ..., < \overline{Xn}$ and (neglecting 50% data).

Starting from 0 to 25% and 75% to 100%. Then find mean and standard deviation for the above said data. Then construct control limits for the given order statistics like $\overline{X}1 <, \overline{X}2 <, ..., < \overline{Xn}$

$$\text{CL} = \overline{\overline{X}}$$

$$\text{LCL} = \overline{\overline{x}} - 3\sigma\overline{x}$$

$$\text{UCL} = \overline{\overline{x}} + 3\sigma\overline{x}$$

Let the data $X_1 < X_2 < X_3 < X_4 < \ldots\ldots\ldots < X_n$ be a set of n observations collected from the population and Let the sample size n may be a finite or infinite .In this situation, the estimation of the mean and standard deviation is difficult.The construction of control charts are obviously depends on the standard characteristics like Mean and Standard deviation. The given data is set of observations with different samples and finding mean for each sample is of course difficult. But the control charts for variability, we can find the standard measures and fix the following limits for construction of control chart.

Lower control chart $= \overline{X} - 3\sigma_{\overline{X}}$; Central Line = $\overline{X}$ and Upper control Limit = $\overline{X} - 3\sigma_{\overline{X}}$;

The mean of the sample observations $\overline{X}$ and the standard deviation $\sigma_{\overline{X}}$.

Now, The data can bearranged in ascending order with respect to the different sample means and neglecting the 50% of the data by using some standard measures for variability. The measure of variability play a vital role for the construction of control charts. To draw the BOX-PLOT to the arranged data for the means of the different samples and it covers the data from first quartile to third quartile of the arranged one. For the construction of plotting the data on graph used the R programming. In Robust data analysis, Wilcox(2003) fixing the proportion of observations to be trimmed ,without looking at the data ,avoid certain theoretical problems when one is testing hypothesis.

Figure (1) graphically illustrates one of the practicaladvantage ofa 25% to 75% data can be used for finding trimmed mean and trimmed standard deviation. With the help of 50% data, obtained mean and standard deviation for construction of the control chart for variability.

The Trimmed Mean with the range of the data already given is as follows:

$$\widetilde{X} = \frac{1}{n - n_1 - n_2} \sum_{n=n_1+1}^{n-n_2} X_n$$

and

$$\text{Var}(X) = \frac{1}{n - n_1 - n_2} \sum_{n=n_1+1}^{n-n_2} (X_n - \widetilde{X});$$

By using the above measures we can again find the Lower control Limit, Central Line and Upper Control Limit.

We draw the control chart for variability with the help of the trimmed mean and trimmed standard deviation and it in figure (2). The calculated all means of the samples are scatted within the control limits ,then the process is under control .Otherwise the process is out of control.

4. NUMERICAL ILLUSTRATIONS:

A food company puts mango juice into cans advertised as containing 10 ounces of the juice. The weight of the juice drain from cans immediately after filling for 20 samples are taken by random method.

The samples means of the given values :

9.5 10.0 10.0 10.5 11.5 11.5 12.0 12.5 12.5 13.0 13.0 13.5 13.5 13.5 14.0 15.0 15.0 16.5 17.5 18.5

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
9.50	11.50	13.00	13.15	14.25	18.50

Grand Mean = 13.15 ;

Standard Deviation = 2.455391;

Upper Control Limit = 20.51617;

Control Limit = 13.15;

Lower control limit = 5.783826;

New method applied for calculations is as follows:

Median (Mo) = 13.0 #By grouping procedure

N+1/4 = 9+1/4 = 5

Q1 = 12, Q3 = 15,

Mode = 13.0, $\overline{X}$ = 13.5; σx = 9.5

LCL = 10.45,

CL = 13.5 ;

UCL = 16.15

Since all the points are fall in within control limits then the process is under control and hence there is nothing to worry.

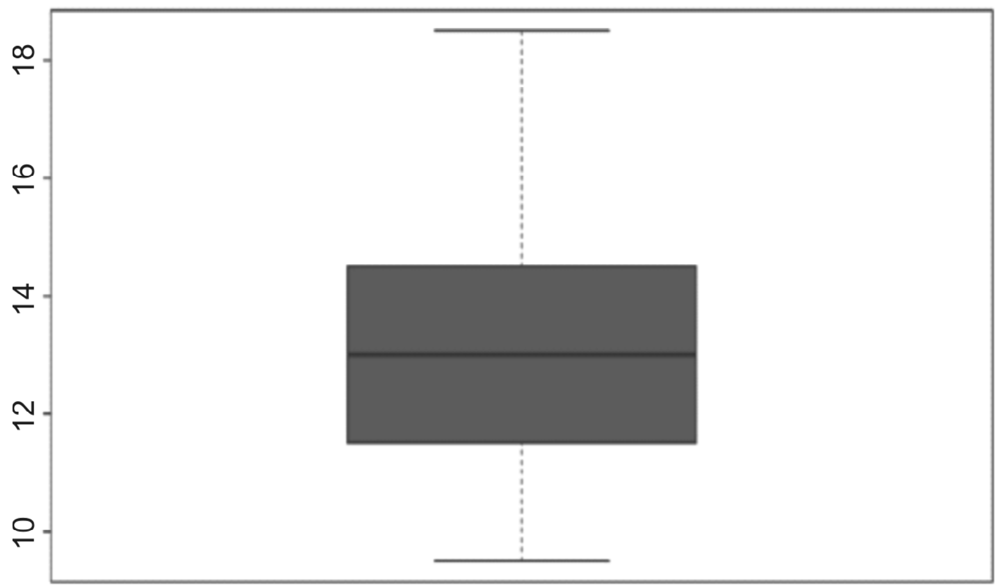

Fig. 1. Boxplot diagram

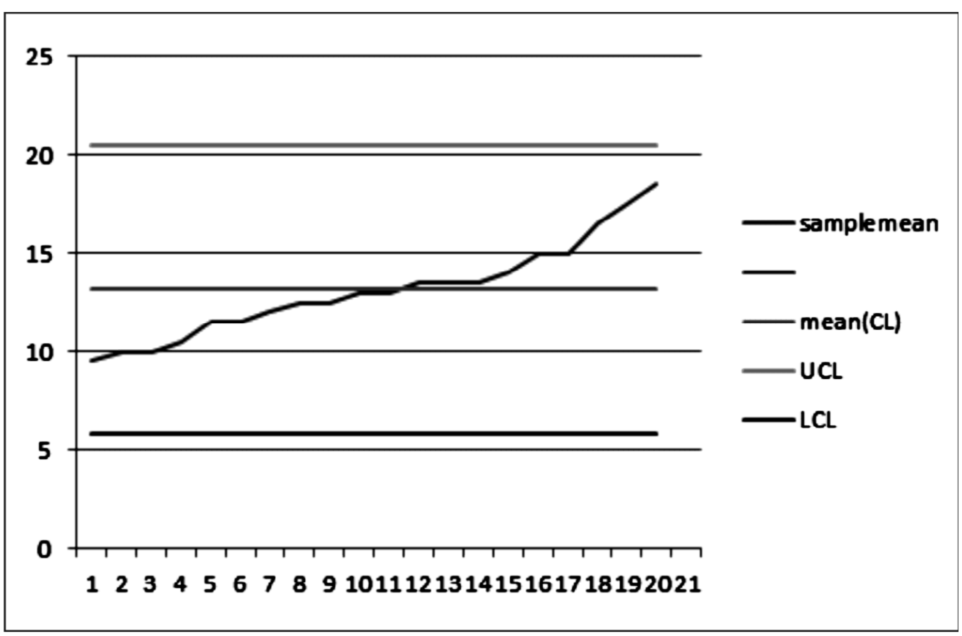

Fig. 2. Mean Chart for the original data.

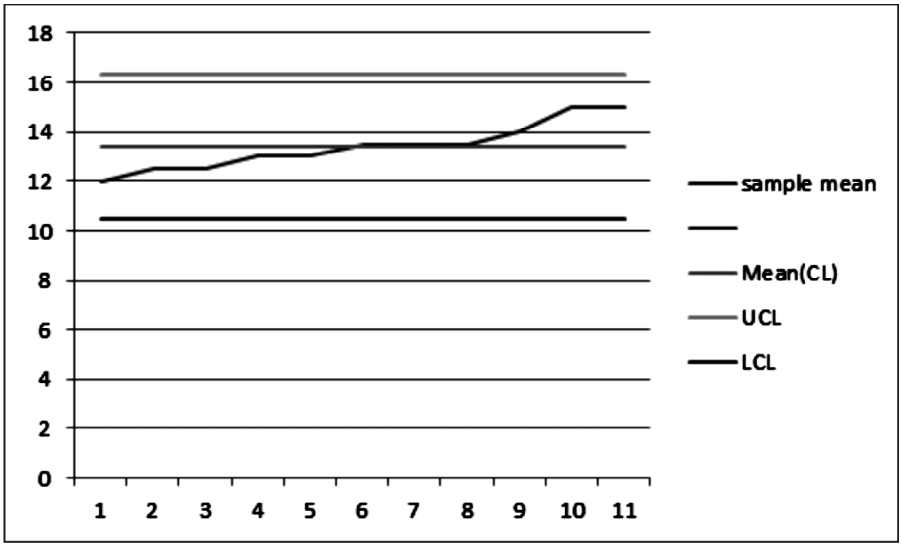

Fig. 3. Control Chart with Trimmed mean.

From figure –II, we observe that the process or product is under control by using all observations. But the Figure-III, is constructed by using the trimmed mean and Trimmed Standard Deviation. Finally , we conclude that the both methods are applicable for the incoming lots are under control or not.

The data by using both the methods.Then the taken process is under control. Here proposed technique gives the conclusions with 50The% of the data indexed with Quartile Range which we call as Robust modified control chart for variability with trimmed mean and trimmed standard deviation.

5. CONCLUSION

We observe that the robust technique is easily given the process is under control or not with 50% the data only. But the usual procedure takes maximum time and money. Finally, we proposed method is good for process control in statistical Quality control. For the convenience of the control chart , we can use and apply for process and product control with the values between the above 25% and below 75% of the incoming lots. It is convenient to construct control charts with minimum number of observations is possible .This method is also the feasible for the process and product control techniques.

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