AN INVENTORY MODEL WITH POWER DEMAND PATTERN, WEIBULL DISTRIBUTION DETERIORATION WITH PARTIAL BACKLOGGING

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ABSTRACT: Inventory models in which the demand rate depends on the inventory level are based on the common real-life observations that greater product availability tends to stimulate more sales. Mathematical methods help in deriving certain rule and which may suggest how to minimize the total inventory cost in case of deterministic demand. Here an attempt has been made for obtaining a deterministic inventory model for power demand pattern incorporating two-parameter Weibull distribution deterioration with partial backlogging.

Keywords: EOQ, Deterioration, Power demand pattern.

1. INTRODUCTION

The observation of markets reveals that the demand rate is usually influenced by the amount of stock level. During the stock out period, either all the demand is back ordered, in which all customers wait until their demand is satisfied or all the demand is lost. However in many real inventory systems, demand can be captive partially, for customers whose needs are not crucial at that time can wait for the item to be replenished while others who can not wait will fill their demands from some other sources.

A number of researchers have worked on inventory with constant demand rate, time varying demand patterns. A few of the researchers have considered the demand of the items as power demand pattern.

Datta and Pal [3] have developed an order level inventory system with power demand pattern, assuming the deterioration of items governed by a special form of Weibull density function

$$\theta(t) = \theta_0 t; \quad 0 \langle \theta_0 \langle 1, t \rangle \rangle 0.$$

They used special form of Weibull density function to sidetrack the mathematical complications in deriving a compact EOQ model. Gupta and Jauhari [4] has developed an EOQ model for deteriorating items with power demand pattern with an additional feature of permissible delay in payments. They also used special form of Weibull density function for deterioration of items.

A step forward to special form of Weibull density function, here we shall develop an EOQ model with power demand pattern and using actual form of Weibull density function $Z(t) = \alpha\beta t^{\beta-1}$, where $\alpha(0\langle \alpha \leq 1), \beta\rangle 0$ for deterioration of items. Despite of all mathematical intricacies, expressions for various inventory parameters are obtained.

Here we shall develop the same problem with power demand pattern with partial backlogging.

2. ASSUMPTIONS AND NOTATIONS

Inventory model is developed under the following assumptions and notations.

Assumptions:

(i) Replenishment rate is infinite.

(ii) The lead-time is zero.

(iii) Shortages in inventory are allowed and are partially backlogged.

(iv) The demand is given by the power demand pattern for which, demand upto time t is assumed to be

$$d\left(\frac{t}{T}\right)^{\frac{1}{n}}$$

Where d is the demand size during the fixed cycle time T, $(0\langle n \langle \infty)$ is the pattern index and $\left(\frac{dt^{\frac{(1-n)}{n}}}{nT^{\frac{1}{n}}}\right)$ is the demand rate at time t.

(v) The rate of deterioration at any time $t \rangle 0$ follows the two-parameter Weibull distribution $Z(t) = \alpha\beta t^{\beta-1}$, where $\alpha\,(0\,(\alpha \ll 1)$, is the scale parameter and $\beta\,(\rangle\, 0)$ is the shape parameter.

(vi) It is assumed that only a fraction of demand is back logged. The longer the waiting time is, the smaller is the backlogging. $\beta(t)$ denote this backlogging rate, where t is the waiting time upto the next replenishment. $\beta(t) = \frac{1}{(1+\delta t)}$, where δ is the backlogging parameter.

Notations:

T – The fixed length of each ordering/production cycle.

C_1 – The holding cost, per unit time.

C_3 – The cost of each deteriorated unit.

C_2 – The shortage cost per backlogged items.

C_4 – The unit cost of lost sales

3. DEVELOPMENT OF THE MODEL

Let S be the number of items produced or purchased at the beginning of the cycle. It will be the initial inventory at time $t = 0$ and d be the demand during period T. Now, the inventory level S gradually falls during time period $(0, t_1)$, due to demand and deterioration. At time $t = t_1$, the inventory level becomes zero and shortages are allowed. During the shortage, demand is partially backlogged.

Let $I(t)$ be the on-hand inventory, then the various states of the system are governed by the following differential equations:

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1} I(t) = -\frac{dt^{\frac{(1-n)}{n}}}{nT^{\frac{1}{n}}} \tag{1}$$

where

$$Z(t) = \alpha\beta t^{\beta-1} \tag{2}$$

$$\frac{dI(t)}{dt} = -\frac{\dfrac{dt^{\frac{(1-n)}{n}}}{nT^{\frac{1}{n}}}}{1+\delta(T-t)}, \qquad t_1 \le t \le T, \tag{3}$$

Using (2) in (1), the Solution $I(t)$ is,

$$I(t)\, e^{\alpha t^\beta} = -\frac{dt^{\frac{(1-n)}{n}}}{nT^{\frac{1}{n}}} e^{\alpha t^\beta} dt$$

$$I(t) = -\frac{de^{-\alpha t^\beta}}{nT^{\frac{1}{n}}} \int_0^{t_1} t^{\frac{(1-n)}{n}} (1+\alpha t^\beta)\, dt$$

$$= -\frac{de^{-\alpha t^\beta}}{nT^{\frac{1}{n}}} \left[\frac{t^{\frac{(1-n)}{n}+1}}{\frac{(1-n)}{n}+1} \right] - \frac{d\alpha e^{-\alpha t^\beta}}{nT^{\frac{1}{n}}} \int_0^{t_1} t^{\frac{(1-n)}{n}} e^{-\alpha t^\beta} dt$$

Solving further, on expanding $e^{\alpha t^\beta} = 1 + \alpha t^\beta$, gives

$$I(t) = Se^{-\alpha t^\beta} - \frac{d}{T^{\frac{1}{n}}} e^{-\alpha t^\beta} t^{\frac{1}{n}} - \frac{d\alpha}{T^{\frac{1}{n}}} e^{-\alpha t^\beta} \frac{t^{\frac{(1+n\beta)}{n}}}{(1+n\beta)}. \tag{4}$$

Using $I(t_1) = 0$, in (4) gives,

$$S = \frac{dt_1^{\frac{1}{n}}}{T^{\frac{1}{n}}} + \frac{d\alpha}{(1+n\beta)T^{\frac{1}{n}}} t_1^{\frac{(1+n\beta)}{n}} \tag{5}$$

Solution of equation (3) gives, with the boundary condition $I(t_1) = 0$ at $t = t_1$.

$$I(t) = \frac{d}{T^{\frac{1}{n}}}\left((1-\delta T)\,[T^{\frac{1}{n}} - t_1^{\frac{1}{n}}] + \frac{\delta}{1+n}[T^{\frac{1+n}{n}} - t_1^{\frac{1+n}{n}}]\right) \tag{6}$$

The total amount of deteriorated units,

$$= S - \int_0^{t_1} \frac{dt^{\frac{(1-n)}{n}}}{nT^{\frac{1}{n}}}\, dt$$

Using (5) in the above, the total amount of deteriorated units in $[0, t_1]$

$$= \frac{dt_1^{\frac{1}{n}}}{T^{\frac{1}{n}}} + \frac{d\alpha}{(1+n\beta)T^{\frac{1}{n}}} t_1^{\frac{(1+n\beta)}{n}} - \int_0^{t_1} \frac{dt^{\frac{(1-n)}{n}}}{nT^{\frac{1}{n}}}\, dt$$

$$= \frac{d\alpha}{(1+n\beta)T^{\frac{1}{n}}} t_1^{\frac{(1+n\beta)}{n}}$$

Average total cost per unit is given by

$$C(S,T) = C_3 C_3 \frac{d\alpha}{(1+n\beta)T^{\frac{1}{n}}} t_1^{\frac{(1+n\beta)}{n}} + \frac{C_1}{T}\int_0^{t_1} I(t)\,dt + \frac{C_2}{T}\int_{t_1}^{T} I(t)\,dt$$
$$+ \frac{C_4}{T}\int_{t_1}^{T}\left[1 - \frac{1}{1+\delta(T-t)}\right]\left\{\frac{dt^{\frac{1-n}{n}}}{nT^{\frac{1}{n}}}\right\} dt$$

Substituting the values of $I(t)$ and eliminating S using (4) and integrating, we get,

$$C(S,T) = C_3 \frac{d\alpha}{(1+n\beta)T^{\frac{1}{n}}} t_1^{\frac{(1+n\beta)}{n}} + \frac{C_1}{T}\int_0^{t_1}\left(Se^{-\alpha t^{\beta}} - \frac{d}{T^{\frac{1}{n}}} e^{-\alpha t^{\beta}} t^{\frac{1}{n}} - \frac{d\alpha}{T^{\frac{1}{n}}} e^{-\alpha t^{\beta}} \frac{t^{\frac{(1+n\beta)}{n}}}{(1+n\beta)}\right)$$
$$+ \frac{C_2}{T}\int_{t_1}^{T} \frac{d}{T^{\frac{1}{n}}}\left((1-\delta T)\left[T^{\frac{1}{n}} - t_1^{\frac{1}{n}}\right] + \frac{\delta}{1+n}\left[T^{\frac{1+n}{n}} - t_1^{\frac{1+n}{n}}\right]\right)$$
$$+ \frac{C_4}{T}\int_{t_1}^{T}\left[1 - \frac{1}{1+\delta(T-t)}\right]\left\{\frac{dt^{\frac{1-n}{n}}}{nT^{\frac{1}{n}}}\right\} dt$$

$$= C_3 \frac{d\alpha}{(1+n\beta)T^{\frac{1}{n}}} t_1^{\frac{(1+n\beta)}{n}} + \frac{C_1}{T}\int_0^{t_1}\left[\left(\left\{\frac{dt_1^{\frac{1}{n}}}{T^{\frac{1}{n}}} + \frac{d\alpha}{(1+n\beta)T^{\frac{1}{n}}} t_1^{\frac{(1+n\beta)}{n}}\right\} e^{-\alpha t^{\beta}}\right.\right.$$

$$\left.\left. - \frac{d}{T^{\frac{1}{n}}}(1-\alpha t^{\beta})t^{\frac{1}{n}} - \frac{d\alpha}{(1+n\beta)T^{\frac{1}{n}}}(1-\alpha t^{\beta})t^{\frac{(1+n\beta)}{n}}\right)\right] dt$$

$$+ \frac{C_2}{T}\int_{t_1}^{T} \frac{d}{T^{\frac{1}{n}}}\left((1-\delta T)\left[T^{\frac{1}{n}} - t_1^{\frac{1}{n}}\right] + \frac{\delta}{1+n}\left[T^{\frac{1+n}{n}} - t_1^{\frac{1+n}{n}}\right]\right)$$

$$+ \frac{C_4}{T}\int_{t_1}^{T}\left[1 - \frac{1}{1+\delta(T-t)}\right]\left\{\frac{dt^{\frac{1-n}{n}}}{nT^{\frac{1}{n}}}\right\} dt$$

$$= C_3 \frac{d\alpha}{(1+n\beta)T^{\frac{1}{n}}} t_1^{\frac{(1+n\beta)}{n}} + \frac{C_1 d}{T^{\frac{1+n}{n}}}\int_0^{t_1}\left[\frac{t_1^{\frac{1+n}{n}}}{1+n} - \frac{\alpha t_1^{(\beta+1)+\frac{1}{n}}}{(\beta+1)} + \frac{\alpha t_1^{\frac{1+n(\beta+1)}{n}}}{1+n\beta}\left\{1 + \frac{n^2\beta}{1+n(\beta+1)}\right\}\right.$$

$$+ \frac{C_2 d}{T^{\frac{1}{n}+1}}\left[(1-\delta T)\left(T^{\frac{1}{n}} - t_1^{\frac{1}{n}}\right) + \frac{\delta}{1+n}\left(T^{\frac{1+n}{n}} - t_1^{\frac{1+n}{n}}\right)\right] \tag{7}$$

$$+ \frac{C_4 d}{T^{\frac{1}{n}+1}}\left[\frac{nT^{\frac{1+n}{n}}}{1+n} - Tt_1^{\frac{1}{n}} + \frac{t_1^{\frac{1+n}{n}}}{1+n}\right]$$

(neglecting higher powers of α^2, α^3, ...)

For the minimization of cost, we set

$$\frac{dC(T)}{dT} = 0$$

$$= C_1 d\alpha t_1^{\beta+1}\left[\frac{\beta - n\beta - n^2\beta}{(\beta+1)(1+n\beta)}\right] + C_3 d\alpha T t_1^{\beta} + [C_4\delta - C_2\delta + C_1]\, t_1^{\frac{1}{n}+n}$$

$$+ d\delta T[C_2 - C_4] - C_2 d = 0$$

On solving it, we obtain value of t_1 and let this value of t_1 be the optimum value t_1^* (say), Subtituting the optimum value t_1^*. in (5), we get the optimum value of S is,

$$S^* = \frac{dt_1^{*\frac{1}{n}}}{T^{\frac{1}{n}}} + \frac{d\alpha}{(1+n\beta)T^{\frac{1}{n}}} t_1^{*\frac{(1+n\beta)}{n}}$$

The obvious relation between S and Q is

$$Q = S + d - \frac{dt_1^{\frac{1}{n}}}{T^{\frac{1}{n}}}$$

Using optimum values S^* and t_1^*, the optimum value of Q^* is,

$$Q^* = d + \frac{d\alpha}{(1+n\beta)T^{\frac{1}{n}}} t_1^{*\frac{(1+n\beta)}{n}}$$

The minimum value of C is $C(t_1^*)$ and obtained by using (7).

4. CONCLUSION

In this paper we have considered an inventory model for the items having stock dependent demand. From the above work, an inventory model is studied using Weibull distribution deterioration with partial backlogging. It is a general observation that an increase in the selling price of the commodity will deter its customers from opting that item in future. We can also extend this problem using reserve inventory cases.

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