IJCTA, 9(7), 2016, pp. 585-616
ISSN: 0974-5572

Mining Trapezoidal Intuitionistic Fuzzy Correlation Rules for Eigen Valued Magdm Problems

John Robinson P.* and Jeeva S.**

ABSTRACT

In this article, we propose a new framework for mining correlation rules from trapezoidal intuitionistic fuzzy information. The data from a Multiple Attribute Group Decision Making (MAGDM) problem with trapezoidal intuitionistic fuzzy data are first pre-processed using some arithmetic aggregation operators. The aggregated data in turn are processed for efficient data selection through fuzzy correlation rule mining where the unwanted or less important decision variables are pruned from the decision making system. Normalization of eigen value matrix is utilised for determining the decision makers weights by which a decision-maker can overcome the drawbacks in the conventional methods of Decision Support Systems (DSS) especially when dealing with large data-set. The algorithm is also presented, in which the technique of Fuzzy Correlation Rule Mining (FCRM) is fused into the MAGDM problem, in order to enhance the efficiency and accuracy in decision making environment. A numerical illustration is presented to show the effectiveness and accuracy of the developed algorithm.

Keywords: MAGDM, Data mining, Decision support systems, Fuzzy Correlation rule mining, Correlation coefficient of Trapezoidal intuitionistic fuzzy sets, Aggregation operators, Eigen Matrix.

1. INTRODUCTION

The progress of Information Technology has determined companies to record a large quantity of information in huge databases. Due to the fact that a lot of useful knowledge is hidden in these databases, the companies need to extract this knowledge in order to help the decision-making an effective process. Data mining, also known as knowledge discovery in databases, provides efficient automated techniques for discovering potentially useful, hidden knowledge or relations among data from large databases. Data mining functions include classification, clustering, prediction, regression, and link analysis (associations), etc.Data analysts are primarily concerned with discerning trends in the data and thus a system that provides approximate answers in a timely fashion would suit their requirements better. Mining association rules represent an unsupervised data mining method that allows identifying interesting associations, correlations between items, and frequent patterns from large transactional databases and this problem was first introduced by Agrawal et al.,(1993). Most association rule mining algorithms employ a support-confidence framework. Often, many interesting rules can be found using low support thresholds. Although minimum support and confidence thresholds help weed out or exclude the exploration of a good number of uninteresting rules, many rules so generated are still not interesting to the users. Unfortunately, this is especially true when mining at low support thresholds or mining for long patterns. This has been one of the major bottlenecks for successful application of association rule mining. It is important to see that strong rules are not always interesting. Whether or not a rule is interesting can be assessed either subjectively or objectively (Han & Kamber, 2006). Ultimately, only the user can judge if a given rule is interesting, and this judgement, being subjective, may differ from one user to another. However, objective interestingness measures, based on the statistics "behind" the data, can be used as one step toward the goal of weeding out interesting rules from presentation to the user. The tools and

*,** PG and Research Department of Mathematics, Bishop Heber College,Tiruchirappalli-620017, Tamilnadu, India, *Email: robijohnsharon@gmail.com*

technologies that have been developed in fuzzy Set Theory have the potential to support all of the steps that comprise a process of model induction or knowledge discovery (Bandemer & Nather, 1992). In particular, fuzzy set theory can already be used in the data selection and preparation phase, e.g., for modelling vague data in terms of fuzzy sets, to "condense" several crisp observations into a single fuzzy one, or to create fuzzy summaries of the data (Laurent, 2003). As the data to be analysed thus becomes fuzzy, one subsequently faces a problem of fuzzy data analysis.

In the real world there are vaguely specified data values in many applications and fuzzy set theory has been proposed to handle such vagueness by generalizing the notion of membership in a set. Essentially, in a Fuzzy Set (FS) each element is associated with a point-value selected from the unit interval [0,1], which is termed the grade of membership in the set. An Intuitionistic Fuzzy Set (IFS), is a further generalization of a FS. Atanassov, (1986; 1989) introduced the concept of intuitionistic fuzzy sets (IFSs), which is the generalization of the concept of fuzzy sets and has been found to be compatible to deal with vagueness.Instead ofusing point-based membership as in FSs,interval-based membership is used in IFS. The interval-based membership in IFSs is more expressive in capturing vagueness of data. Fuzzy set theory has long been introduced to handle inexact and imprecise data, since in the real world there is vague information about different applications. In fuzzy set theory, each object $u \in U$ is assigned a single real value, called the grade of membership,between zero and one. (Here U is a classical set of objects, called the universeof discourse). Gau & Buehrer, (1994) point out that the drawback of using the single membership value in fuzzy set theory is that the evidence for $u \in U$ and the evidence against $u \in U$ are in fact mixed together. In order to tackle thisproblem, they proposed the notion of Vague Sets (VSs), which allow using interval-based membership instead of using point-based membership as inFSs. The interval-based membership generalization in VSs is more expressive in capturing vagueness of data. However, VSs are shown to be equivalent to that of Intuitionistic Fuzzy Sets (IFSs). In the present days, different higher order Fuzzy Sets is available in the literature, and one among them is the Trapezoidal Intuitionistic Fuzzy Sets (TzIFS).Trapezoidal IFS is a special case of IFSs, with two characterizations, namely the trapezoidal fuzzy characterization and the intuitionistic fuzzy characterization (Robinson&Amirtharaj,2011b; 2012b).

In an age of extensive competition among organizations, managers search for efficiency and excellence to solve decision problems. An effective group decision mechanism will enhance the quality of the group decision making process, and thereby improve the organization's performance. Due to the development of e-democracy and information technology, decision makers are now able to conclude a group decision without face-to-face meetings. These results the problem of aggregation of preferences being solved and the managerial operations of the organization being enhanced without much time consumption.Decision making is the study of identifying and choosing alternatives based on the values and preferences of the decision maker. Making a decision implies that there are alternative choices to be considered, and in such cases many of these alternatives as possible should be identified and the one that best fits with our goals, objectives, desires, values, should be chosen. Decision making should start with the identification of the decision maker(s) and stakeholder(s) in the decision, reducing possible disagreement about problem definition, requirements, goals and criteria. It is very important to make a distinction between the cases where we have a single or multiple criteria. When a decision problem has a single criterion or a single aggregate measure,then the decision can be made implicitly by determining the alternative with the best value of the single criterion or aggregate measure. When a decision problem has a finite number of criteria or multiple criteria,and the number of the feasible alternatives (the ones meetingrequirements) is infinite,and then the decision problem belongs to the field of multiple criteria optimization. Also, techniques of multiple criteria optimization can be used when thereare a finite number of feasible alternatives, but are given only in implicit form. This research work focuses on decision making problems when the number of the criteria (attribute) and alternatives is finite, and the alternatives are explicitlygiven. Problems of this type are called Multi Attribute Decision Making (MADM) problems.

Consider a multi-attribute decision making problem with *m* criteria and *n* alternatives. Let $C_1, ..., C_m$ and $A_1, ..., A_n$ denote the criteria and alternatives, respectively. A standard feature of multi-attribute decision making methodology is the decision tableas shown in the following. In this table each row belongs to a criterion and each

$$
\begin{array}{cc|cccc|}
 & & x_1 & \cdots & & x_n \\
 & & A_1 & \cdots & & A_n \\
\hline
w_1 & C_1 & a_{11} & . & . & a_{m1} \\
. & . & . & . & . & . \\
. & . & . & . & . & . \\
w_m & C_m & a_{m1} & . & . & a_{mn} \\
\hline
\end{array}
$$

column describes the performance of an alternative. The score a_{ij} describes the performance of the alternative A_j against the criterion C_i. For the sake of simplicity it is assumed that a higher score value means a better performance,as any goal of minimization can easily be transformed into a goal of maximization.

Here the weights w_i reflects the relative importance of criteria C_i to the decision, and is assumed to be positive. These weights are usually determined on subjective basis andthey represent the opinion of a single decision maker or synthesize the opinions of a group ofexperts using a group decision technique. The values $x_1, ..., x_n$ are associated with the alternatives in the decision table, and are the final ranking values of the alternatives A_j. Usually, a higher ranking value means a better performance of the alternative;hence the alternative with the highest ranking value is the best of the alternatives.Group decision is usually understood as aggregating different individual preferences on a given set of alternatives to a single collective preference. It is assumed that the individuals participating in making a group decision face the same common problem and are all interested in finding a solution. A group decision situation involves multiple actors (decision makers), each with different skills, experience and knowledge related to different aspects (criteria) of the problem.In a correct method for synthesizing group decisions, the competence of different actors to different professional fields has also to be considered. It is assumed that each actor considers the same sets of alternatives and criteria. It is also assumed that there is a special actor with authority to establish consensus rules and determine voting powers to group members on different criteria. Many researchers call this entity the Supra Decision Maker (SDM). The final decision is derived by aggregating (synthesizing) the opinions of group members according to rules and priorities defined by the SDM. There are several approaches to extend the basic multi attribute decision making techniques for group decision. Consider a decision problem with l group members (decision makers) $D_1, .., D_l$, n alternatives $A_1, ..., A_n$ and m criteria $C_1, .., C_m$. In case of a factual criterion, the evaluation scores must be identical for any alternative and any decision maker, while subjective (judgmental) criteria can be evaluated differently by each decision maker. In this work,Multiple Attribute Group Decision Making (MAGDM) problems with applications of different classes of aggregation operators,and correlation coefficient in trapezoidalintuitionistic fuzzy environment as the ranking method have been concentrated upon.

Knowledge management can be defined as the uncovering and managing of various levels of knowledge within individuals, teams, decision makers or within an organization. The aim of Decision Support Systems as Knowledge management is to improve organizational performance.Multiple Attribute decision support systems are provided to assist decision makers with an explicit and comprehensive tool and techniques in order to evaluate alternatives in terms of different factors and importance of their weights. Some of the common Decision Support System (DSS) techniques for Multi-Attribute Decision-Making (MADM) are (Cheng, 2000; Power, 2013):

- Simple Additive Weighted (SAW)
- Weighted Product Method (WPM)
- Cooperative Game Theory (CGT)

- Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)
- Elimination Et Choice Translating Reality with complementary analysis (ELECTRE)
- Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE)
- Analytical Hierarchy Process (AHP)

Adewumi et al., (2013), Hayez et al., (2012) and Power et al., (2011) have presented novel approaches of DSS techniques in the recent times. The MAGDM model proposed in this paper is a framework that can consider huge data of diverse measures, seeking to identify solutions close to an ideal and far from a nadir solution. In addition to the above mentioned Decision Support System (DSS) techniques already available in the literature, in this paper we propose a new DSS technique called the MAGDM-Miner, where data mining techniques like Association rules and Correlation rules are fused into the MAGDM algorithm to put the decision making situation at ease. The merit of the proposed method is that it can deal with both quantitative and qualitative assessment in the process of evaluation with less computation load. The MAGDM-Miner in this paper utilizes the mining of correlation rules for trapezoidal intuitionistic fuzzy data, in order to prune and eliminate some unwanted decision variables from the decision making environment. The Apriori algorithm (Han & Kamber, 2006) is adopted into the MAGDM to present a new MAGDM-Miner algorithm with mining trapezoidalintuitionistic fuzzy correlation rules for discovering frequent itemsets so that the unwanted decision alternatives can be dropped from the final decision making scenario. A numerical example is presented to explain the developed decision making model.

2. LITERATURE REVIEW

Many researchers have applied the IFS theory to the field of decision making. Chen & Tan, (1994) presented some products for dealing with multi attribute decision making (MADM) problems based on vague sets. Szmidt & Kacprzyk,(2000; 2002; 2003)introduced several distance functions and similarity measures for IFSs which were later used in various MAGDM problems. Herrera et al., (1999) developed an aggregation process for combining numerical, interval valued and linguistic information, and then proposed different extensions of this process to deal with contexts in which can appear information such as IFSs or multi-granular linguistic information. Xu & Yager, (2006) developed some geometric aggregation operators for MADM problems. Li & Nan, (2011) extended the TOPSIS method under intuitionistic fuzzy environment for multiple attribute decision making problems. Wu &David, (2012) applied TOPSIS in data mining techniques. Li,(1999; 2005; 2008) presented new methods for handling multiple attribute fuzzy decision making problems, where the characteristics of the alternatives are represented by intuitionistic fuzzy sets. Liu& Wang, (2007) developed an evaluation function for the decision making problem to measure the degrees to which alternatives satisfy/do not satisfy the decision maker's requirement. Also Hong & Choi, (2000), Liu, (2004) and Liu & Guan, (2008; 2009) provided some new techniques for handling multiple attribute fuzzy decision making problems based on vague set theory. Liu, (2009), Liu et al., (2012),Wei, (2008; 2010), Wei et al., (2011; 2012) and Wei & Zhao, (2012) contributed novel approaches to the field of fuzzy decision making. Nayagam et al., (2011) proposed a novel accuracy function for MCDM problems for interval valued intuitionistic fuzzy sets.

Bustince & Burillo, (1995) and Hong, (1998) focused on the correlation degree of interval valued intuitionistic fuzzy sets. Kao & Liu, (2002) introduced fuzzy measures for the correlation coefficient of fuzzy numbers lying in the interval [-1,1]. Buckley et al., (2004), using the approach of fuzzy probabilities, provided a tool which could be utilized for the fuzzy correlation coefficient applications. Zeng & Li, (2007) focused on probability spaces to define a new kind of correlation for intuitionistic fuzzy sets. Gerstenkon & Manko, (1991) defined the correlation of intuitionistic fuzzy sets, where Hong & Hwang, (1995) defined it on probability spaces. Chaudhuri & Bhattachary, (2001) defined a correlation coefficient between two fuzzy membership functions. Hung & Wu, (2002) introduced the concepts of positively and negatively correlated results based on the concept of centroid for intuitionistic fuzzy sets lying in the interval [-1,1]. Hong, (1998) studied the correlation coefficient of interval-valued intuitionistic fuzzy sets in probability spaces. Mitchell, (2004) adopted a statistical view point to interpret intuitionistic fuzzy sets as an ensemble of ordinary fuzzy set, and defined correlation coefficient of intuitionistic fuzzy sets by using the correlation coefficient of two ordinary fuzzy sets and a mean aggregation function.Wei et al.,(2011) appliedinterval valued intuitionistic fuzzy correlation in decision making analysis. Park et al., (2009) also worked on the correlation coefficient of interval valued intuitionistic fuzzy sets and applied in MAGDM problems. Robinson & Amirtharaj, (2011a; 2011b; 2012a; 2012b; 2014a; 2014b; 2014c) defined correlation coefficient for different higher order intuitionistic fuzzy sets and utilized in MAGDM problems. Robinson et al., (2015) have proposed new methods for mining environmental data using some bio-statistical rules.

Haleh et al., (2012) have applied data mining techniques in MCDM problems involving educational databases to evaluate question weights in scientific examinations. Kweku-Muata & Osei-Bryson, (2004) worked on the evaluation of decision trees through MCDM approaches. Kaplan, (2006) proposed asolid waste management system model and optimization using MCDM applications and data mining techniques.The case study provided by Peng et al., (2011) demonstrated that combining data mining and MCDM methods provided objective and comprehensive assessments of huge data sets. Khan et al., (2008) provide various means where data mining techniques enhances the Decision Support Systems.In this work we have proposed a new framework combining Decision Making methods and Data Mining techniques with the application of correlation coefficient of trapezoidal IFS.The method to find the correlation coefficient of trapezoidal IFS proposed by Robinson & Amirtharaj, (2012b) is used for the intuitionistic fuzzy correlation rule mining.

3. MINING FUZZY CORRELATION RULES

Frequent patterns are patterns that appear in a data set frequently. Finding such frequent patterns plays an essential role in mining fuzzy associations, fuzzy correlations and many other interesting relationships among data. Frequent pattern mining searches for recurringrelationships in a dataset which is the work of the Apriori algorithm. If a fuzzy item set almost occurs in all records, then it may frequently occur with other fuzzy item-sets also (Agrawal et al., 1993). In order to find out useful relationships between the fuzzy item-sets based on fuzzy statistics, fuzzy correlation rules (Lin et al., 2007) are generated.The discovery of interesting correlation relationships in huge amounts of business transaction records can help in many decision making processes.

Let $I = \{I_1, I_2, ..., I_m\}$ be a set of fuzzy items. Let *D*, the task-relevant fuzzy data, be a set of database transactions where each transaction *T* is a set of fuzzy items such that $T \subseteq I$. Let *A* be a set of fuzzy items. A transaction *T* is said to contain *A* if and only if $A \subseteq T$. A fuzzy association rule is an implication of the form $A \Rightarrow B$, where $A \subset I$, $B \subset I$ and $A \cap B = \phi$. The rule $A \Rightarrow B$ holds in the transaction set *D* with fuzzy support *s*, where *s* is the percentage of transactions in *D* that contain $A \cup B$ which is taken to be the probability $P(A \cup B)$. (The notation $P(A \cup B)$ indicates that a transaction contains every item in A and in B (Han & Kamber, 2006)). The rule $A \Rightarrow B$ has fuzzy confidence *c* in the transaction set *D*, where *c* is the percentage of transactions in *D* containing *A* that also contain *B*. This is taken to be the conditional probability $P(B / A)$.

$$\begin{aligned} &\textit{fuzzysupport } (A \Rightarrow B) = P(A \cup B) \\ &\textit{fuzzyconfidence } (A \Rightarrow B) = P(B / A) \end{aligned} \tag{1}$$

A set of items is referred to as an item-set. An item-set that contains k items is called a k-item-set. The occurrence frequency of an item-set is the number of transactions that contain the item-set. The fuzzy item-set support defined in equation (1) is called relative support, whereas the occurrence frequency is called the absolute support. If the relative support of an item-set *I*, satisfies a pre-specified minimum support threshold, then *I* is a frequent fuzzy item-set. From (1) we have:

$$\textit{fuzzy confidence } (A \Rightarrow B) = P(B / A)$$

$$= \frac{\text{support}(A \cup B)}{\text{support}(A)}$$

$$= \frac{\text{support}\_count(A \cup B)}{\text{support}\_count(A)} \quad (2)$$

Equation (2) shows that the fuzzy confidence of rule $A \Rightarrow B$ can be easily derived from the support counts of A and $A \cup B$. Once the support counts of A, B and $A \cup B$ are found, it is straightforward to derive the corresponding association rules $A \Rightarrow B$ and $B \Rightarrow A$, and check whether they are strong. Thus the problem of mining fuzzy association rules can be reduced to that of mining frequent fuzzy item-sets. A fuzzy item-set X is closed in a fuzzy data set S if there exists no proper super-item set Y such that Y has the same support count as X in S. An item-set X is a closed frequent fuzzy item-set in set S if X is both closed and frequent in S.

The fuzzy item-sets which frequently occur together in large databases are found using fuzzy association rules (Lin et al., 2007; Dubois & Prade, 2003). Zhang et al.,(2006) proposed an effectiveframework for Association Rule Miningin XML Data. All the methods used for mining fuzzy association rules are based upon a support-confidence framework where fuzzy support and fuzzy confidence are used to identify the fuzzy association rules. Let $F = \{\tilde{r}_1, \tilde{r}_2, ..., \tilde{r}_m\}$ be a set of fuzzy items, $T = \{t_1, t_2, ..., t_n\}$ be a set of fuzzy records, and each fuzzy record t_i is represented as a vector with m values, $(\tilde{r}_1(t_i), \tilde{r}_2(t_i), ..., \tilde{r}_m(t_i))$, where $\tilde{r}_j(t_i)$ is the degree that $\tilde{r}_j$ appears in record t_i, $\tilde{r}_j(t_i) \in [0,1]$. Then a fuzzy association rule is defined as an implication form such as $F_X \Rightarrow F_Y$, where $F_X \subset F$, $F_Y \subset F$ are two fuzzy item-sets. The fuzzy association rule $F_X \Rightarrow F_Y$ holds in T with the fuzzy support (fsupp $(\{F_X, F_Y\})$) and the fuzzy confidence (fconf $(F_X \Rightarrow F_Y)$). The fuzzy support and fuzzy confidence are given as follows (Lin et al., 2007):

$$\text{fsupp}\left(\{F_X, F_Y\}\right) = \frac{\sum_{i=1}^{n}\left(\min f_j(t_i)\right)}{n}, \text{ where } f_j \in \{F_X, F_Y\} \quad (3)$$

$$\text{fconf}\left(F_X \Rightarrow F_Y\right) = \frac{\text{fsupp}\left(\{F_X, F_Y\}\right)}{\text{fsupp}\left(\{F_X\}\right)} \quad (4)$$

If the fsupp $(\{F_X, F_Y\})$ is greater than or equal to a predefined threshold, minimal fuzzy support, and the fconf $(F_X \Rightarrow F_Y)$ is also greater than or equal to a predefined threshold, minimum fuzzy confidence, then $F_X \Rightarrow F_Y$ is considered as an interesting fuzzy association rule, and it means that the presence of the fuzzy item-set F_x in a record can imply the presence of the fuzzy item set F_Y in the same record. Fuzzy correlation analysis is used to determine the linear relationship between any two fuzzy item-sets. As it is seen, the fuzzy support and fuzzy confidence measures are insufficient at filtering out uninteresting association rules. To tackle this weakness, a fuzzy correlation measure can be used to augment the fuzzy support-confidence framework for fuzzy association rules. Hence the correlation between the fuzzy item-sets A and B becomes necessary. There are many different fuzzy correlation measures (Hong & Hwang, 1995; Bustince & Burillo, 1995; Robinson & Amirtharaj, 2011a; 2012b; Robinson &Amirtharaj, 2014a; Park et al., 2009; Zeng & Li, 2007) from which a suitable method is chosen.

3.1. The General Apriori Algorithm

Mining fuzzy association rules is better done by finding frequent fuzzy item-sets using candidate generation method. Apriori is a seminal algorithm proposed for mining frequent fuzzy item-sets. The algorithm uses prior knowledge of frequent fuzzy item-set properties. Apriori employs an iterative approach known as level-wise search, where k-itemsets are used to explore (k+1)–itemsets.The general Apriori algorithm consists of two steps, namely (i) the join step and (ii) the prune step, for candidate generation.

Step 1: The set of 1-itemsets is found by scanning the fuzzy database to accumulate the count for each item, and collecting those items that satisfy minimum support. The resulting set is denoted by L_1.

Step 2: The set of frequent fuzzy 2-itemset is found by scanning the fuzzy database, and collecting those items that satisfy minimum support and highest correlation coefficient.

L_1 is used to find L_2.

Step 3: The set of frequent fuzzy 3-itemset is also found by the same method. L_2 is used to find L_3, and so on, until no more frequent fuzzy k-item-sets can be found.

The finding of each L_k requires one full scan of the database.

Pseudo-code for Apriori Algorithm:

C_k: Candidate itemset of size k

L_k : frequent itemset of size k

L_1 = {frequent items};

For (k = 1; L_k!=∅; k++) **do begin**

C_{k+1} = candidates generated from L_k;

//that iscartesian product L_{k-1} x L_{k-1} and eliminating

any k-1 size item-set that isnot frequent // **For each**transactiontin database **do** increment the count of all

candidates in C_{k+1}that are contained int

L_{k+1}= candidates in$C_{k+1}$with min_support

End

Return *”$_k L_k$;

4. BASIC CONCEPTS OF INTUITIONISTIC FUZZY SETS(IFSS)

Vagueness and uncertainty are the two important aspects of imprecision. IFS is an intuitively straight forward extension of Zadeh’s fuzzy sets (1965), which is composed of membership and non-membership grades.

Definition: Intuitionistic Fuzzy set (Atanassov, 1986; 1989)

Let a set X be the universe of discourse. An IFS A in X is an object having the form $A=\{\langle x, \mu_A(x),\ \nu_A(x)\rangle\ /\ x \in X\}$, where $\mu_A: X\to[0, 1]$, $\nu_A: X\to[0, 1]$ define the degree of membership and the degree of non-membership respectively, of the element $x\in X$ to the set A, which is a subset of X, and for every element of $x\in X$, $0 \le \mu_A(x) + \nu_A(x) \le 1$.

Definition: (Trapezoidal fuzzy number, TzFN)

It is a fuzzy number represented with three points as follows: $A = (a_1, a_2, a_3, a_4)$ and its membership function is given as

$$\mu_A(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1} & \text{for } a_1\le x<a_2\\ 1 & \text{for } a_2\le x<a_3\\ \dfrac{a_3-x}{a_4-a_3} & \text{for } a_3\le x<a_4\\ 0 & \text{otherwise}\end{cases} \tag{5}$$

Definition: Trapezoidal intuitionistic fuzzy number TzIFN (Wei, 2010).

A TzIFN is an IFS in R with the following membership function $\mu_A(x)$ and non-membership function $v_A(x)$:

$$\mu_A(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1} & \text{for } a_1\le x<a_2\\ 1 & \text{for } a_2\le x<a_3\\ \dfrac{a_3-x}{a_4-a_3} & \text{for } a_3\le x<a_4\\ 0 & \text{otherwise}\end{cases}\qquad \nu_A(x)=\begin{cases}\dfrac{a_2-x}{a_2-a_1'} & \text{for } a_1'\le x<a_2\\ 0 & \text{for } a_2\le x<a_3\\ \dfrac{x-a_3}{a_4'-a_3} & \text{for } a_3\le x<a_4'\\ 1 & \text{otherwise}\end{cases} \tag{6}$$

Where $a_1'\le a_1\le a_2\le a_3\le a_4\le a_4'$ and $\mu_A(x)$, $v_A(x)\le 0.5$ for $\mu_A(x)=v_A(x)$. This TzIFN is denoted by $(a_1,a_2,a_3,a_4;a_1',a_2,a_3,a_4')$.

This TzIFN is also denoted as: $A=\langle([a_1,a_2,a_3,a_4];\mu_A),([a_1',a_2,a_3,a_4'];\nu_A)\rangle$.

5. CORRELATION COEFFICIENT OF TRAPEZOIDAL INTUITIONISTIC FUZZY SETS

Since a trapezoidal intuitionistic fuzzy number(TzIFN) is characterized by the trapezoidal entries and the membership, non-membership values, no conventional methods of finding the correlation coefficient can be used. Hence in this workwe usethe method of finding the correlation coefficient fortrapezoidal IFNs proposed by Robinson &Amirtharaj, (2012b) which has concentrated on both the above characterizations.

Definition: Given a trapezoidal fuzzy number $A=(a,b,c,d)$, the Graded Mean Integration Representation (GMIR) of A is defined as (Chen & Hsieh, 1999):

$$P(\tilde{A})=\frac{a+2b+2c+d}{6}. \tag{7}$$

A TzIFS is represented as $A=([a,b,c,d];\mu,\gamma)$, where $[a,b,c,d]$ represent the trapezoidal fuzzy entries and μ,γ represent the membership, non-membership grades respectively. Utilizing the GMIR for a trapezoidal fuzzy entry together with the membership, non-membership grades of the TzIFS, we define the correlation of TzIFS as follows:

Let $A=([a_1,b_1,c_1,d_1];\mu_A,\gamma_A)$, $B=([a_2,b_2,c_2,d_2];\mu_B,\gamma_B)$ be two trapezoidal IFSs (TzIFS). Then for each $A, B\in$TzIFS(X), the informational trapezoidal intuitionistic energy of A is defined as follows(Robinson & Amirtharaj, 2012b):

$$E_{TzIFS}(A) = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{a_1 + 2b_1 + 2c_1 + d_1}{6}\right]^2 \left(\mu_A{}^2(x_i) + \gamma_A{}^2(x_i) + \pi_A{}^2(x_i)\right) \tag{8}$$

and

$$E_{TzIFS}(B) = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{a_2 + 2b_2 + 2c_2 + d_2}{6}\right]^2 \left(\mu_B{}^2(x_i) + \gamma_B{}^2(x_i) + \pi_B{}^2(x_i)\right). \tag{9}$$

Now the correlation of A and B is defined as:

$$C_{TzIFS}(A,B) = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{a_1 + 2b_1 + 2c_1 + d_1}{6}\right]\left[\frac{a_2 + 2b_2 + 2c_2 + d_2}{6}\right]\left(\mu_A(x_i)\mu_B(x_i) + \gamma_A(x_i)\gamma_B(x_i) + \pi_A(x_i)\pi_B(x_i)\right) \tag{10}$$

Then the correlation coefficient between A and B is defined as:

$$K_{TzIFS}(A,B) = \frac{C_{TzIFS}(A,B)}{\sqrt{E_{TzIFS}(A).E_{TzIFS}(B)}}. \tag{11}$$

Proposition: For A, $B \in$ TzIFS(X), we have:

i) $0 \le K_{TzIFS}(A,B) \le 1$,

ii) $C_{TzIFS}(A,B) = C_{TzIFS}(B,A)$,

iii) $K_{TzIFS}(A,B) = K_{TzIFS}(B,A)$,

iv) $K_{TzIFS}(A,B) = 1$, iff A = B.

The following theorems are true for the correlation coefficient of trapezoidal intuitionistic fuzzy sets (Robinson & Amirtharaj, 2012b).

Theorem 1: For A, B $\in$TzIFS(X), then $0 \le K_{TzIFS}(A,B) \le 1$.

Theorem 2: $K_{TzIFS}(A,B) = 1 \Leftrightarrow A = B$.

Theorem 3: $C_{TzIFS}(A,B) = 0 \Leftrightarrow$ A and B are non-fuzzy sets and satisfy the condition $\mu_A(x_i) + \mu_B(x_i) = 1$ or $\gamma_A(x_i) + \gamma_B(x_i) = 1$ or $\pi_A(x_i) + \pi_B(x_i) = 1, \quad \forall x_i \in$ X.

Theorem 4: $E_{TzIFS}(A) = 1 \Leftrightarrow A$ is a non-fuzzy set.

The above described correlation coefficient uniquely combines the trapezoidal fuzzy characterization and the intuitionistic fuzzy characterization of TzIFNs. In the following, some arithmetic aggregation operators in decision making for TzIFNs will be presented.

6. ARITHMETIC AGGREGATION OPERATORS WITH TZIFNS

Let $\tilde{R} = (\tilde{r}_{ij})_{m\times n} = \left(\left[a_{ij}, b_{ij}, c_{ij}, d_{ij}\right]; u_{ij}, v_{ij}\right)_{m\times n}$ be a normalized trapezoidal intuitionistic fuzzy decision making matrix, where $0 \le a_{ij} \le b_{ij} \le c_{ij} \le d_{ij} \le 1,\ 0 \le u_{ij} + v_{ij} \le 1$. For a normalized trapezoidal intuitionistic fuzzy decision making matrix, the trapezoidal intuitionistic fuzzy positive ideal solution and trapezoidal intuitionistic fuzzy negative ideal solution are defined as follows:

$$\tilde{r}^+ = ([a^+, b^+, c^+, d^+]; u^+, v^+,) = ([\ 1, 1, 1, 1]; 1, 0), = ([a^-, b^-, c^-, d^-]; u^-, v^-) = ([\ 0, 0, 0, 0]; 0, 1).$$

Definition: Let $a_j,\ j=1,\ 2,\ \ldots,\ n$ be a collection oftrapezoidal intuitionistic fuzzy numbers. Let the collection of all TzIFNsbe denoted by *Q*. The Trapezoidal Intuitionistic Fuzzy Weighted Arithmetic Averaging (*TzIFWAA*) operator is defined as:

$$TzIFWAA: Q^n \to Q$$

$$TzIFWAA\left(\left(a_1, a_2, \ldots, a_n\right)\right) = \sum_{j=1}^{n} a_j \omega_j$$

$$= \left(\left[\sum_{j=1}^{n} a_j \omega_j, \sum_{j=1}^{n} b_j \omega_j, \sum_{j=1}^{n} c_j \omega_j, \sum_{j=1}^{n} d_j \omega_j \right]; 1-\prod_{j=1}^{n} (1-\mu_{a_j})^{\omega_j}, \prod_{j=1}^{n} (\gamma_{a_j})^{\omega_j} \right) \tag{12}$$

where, $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight vector of $a_j,\ j=1,\ 2, \ldots,\ n$ and for $\omega_j > 0,\ \sum_{j=1}^{n} \omega_j = 1.$

Definition: Let $a_j,\ j=1,\ 2, \ldots,\ n$ be a collection of TzIFNs.Then a Trapezoidal Intuitionistic Fuzzy Ordered Weighted Averaging *(TzIFOWA)* operator of dimension *n* is amapping*TzIFOWA: $Q^n \to Q$*,that has an associated vector $w = (w_1, w_2, \ldots, w_n)^T$ such that $w_j > 0$, and $\sum_{j=1}^{n} w_j = 1.$

Furthermore, $$TzIFOWA_w\left(a_1, a_2, \ldots, a_n\right) = \sum_{j=1}^{n} \tilde{a}_{\sigma(j)} w_j$$

$$= \left(\left[\sum_{j=1}^{n} a_{\sigma(j)} w_j, \sum_{j=1}^{n} b_{\sigma(j)} w_j, \sum_{j=1}^{n} c_{\sigma(j)} w_j, \sum_{j=1}^{n} d_{\sigma(j)} w_j \right]; 1-\prod_{j=1}^{n} (1-\mu_{\tilde{a}_{\sigma(j)}})^{w_j}, \prod_{j=1}^{n} (\gamma_{\tilde{a}_{\sigma(j)}})^{w_j} \right) \tag{13}$$

Where $\left(\sigma(1), \sigma(2), \ldots, \sigma(n)\right)$ is a permutation of (1, 2, ..., n) such that $\tilde{a}_{\sigma(j-1)} \geq \tilde{a}_{\sigma(j)}$ for all *j = 2, ..., n.*

Definition: A Trapezoidal Intuitionistic Fuzzy Hybrid Aggregation*(TzIFHA)* operator of dimension n is the mapping *TzIFHA: $Q^n \to Q$* that has an associated vector $w = (w_1, w_2, \ldots, w_n)^T$ such that $w_j > 0$, and $\sum_{j=1}^{n} w_j = 1.$

Furthermore, $$TzIFHA_{\omega,w}\left(a_1, a_2, \ldots, a_n\right) = \sum_{j=1}^{n} \dot{\tilde{a}}_{\sigma(j)} w_j$$

$$= \left(\left[\sum_{j=1}^{n} \dot{a}_{\sigma(j)} w_j, \sum_{j=1}^{n} \dot{b}_{\sigma(j)} w_j, \sum_{j=1}^{n} \dot{c}_{\sigma(j)} w_j, \sum_{j=1}^{n} \dot{d}_{\sigma(j)} w_j \right]; 1-\prod_{j=1}^{n} (1-\dot{\mu}_{\tilde{a}_{\sigma(j)}})^{w_j}, \prod_{j=1}^{n} (\dot{\gamma}_{\tilde{a}_{\sigma(j)}})^{w_j} \right) \tag{14}$$

Where $\dot{\tilde{a}}_{\sigma(j)}$ is the j^{th} largest of the weighted *TzIFNs* $\dot{\tilde{a}}_j$, where $\dot{\tilde{a}}_j = \tilde{a}_j^{n\omega_j},\ j=1,2\ldots\ldots n$, $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ be the weight vector of $a_j,\ j=1,2,\ldots,n$ and for $\omega_j > 0,\ \sum_{j=1}^{n} \omega_j = 1,$ *n* is the balancing co-efficient.

7. DETERMINING DECISION MAKER WEIGHTS BY NORMALIZED EIGEN VALUE MATRIX

In many of the real-life decision making problems, the information about attributes (weights) provided by the decision-makers is usually incompletely known because of time pressure, lack of knowledge or data and expert's limited expertise about the problem domain. So an interesting and important issue is how to utilize the collective trapezoidal intuitionistic fuzzy decision matrix and the unknown weight information to find the most desirable alternative(s). Let us suppose that the decision makers provide their weights about the attributes in the form of a square matrix of order three. Then we shall employ the method of finding the eigen values of the matrix and finally normalize to obtain the unknown weights of the decision makers. Let the matrix be as follows:

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ -1 & 2 & 1 \end{bmatrix}$$

Find all the eigen values of the matrix using the Jacobi method. Iterate till the off-diagonal elements in magnitude are less than 0.0005. The largest off-diagonal element in magnitude in a_{12}. Therefore, $\tan\theta = \dfrac{2a_{12}}{a_{11} - a_{22}} = \dfrac{4}{0} = \infty$,

or $\theta = \dfrac{\pi}{4}$

$$S_1 = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The first rotation gives

$$B_1 = S_1^{-1} A S_1 = \begin{bmatrix} 3 & 0 & 1/\sqrt{2} \\ 0 & -1 & 3/\sqrt{2} \\ 1/\sqrt{2} & 3/\sqrt{2} & 1 \end{bmatrix} \; ie., B_1 = S_1^{-1} A S_1 = \begin{bmatrix} 3 & 0 & 0.707107 \\ 0 & -1 & 2.121320 \\ 0.707107 & 2.121320 & 1 \end{bmatrix}$$

The largest off-diagonal elements in magnitude in B_1 *is* a_{23}. Therefore,

$$\tan 2\theta = \frac{2a_{23}}{a_{22} - a_{33}} = -2.121320, \text{ or } \theta = -0.565143,$$

$$\cos\theta = 0.844512, \sin\theta = -0.535537$$

$$S_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

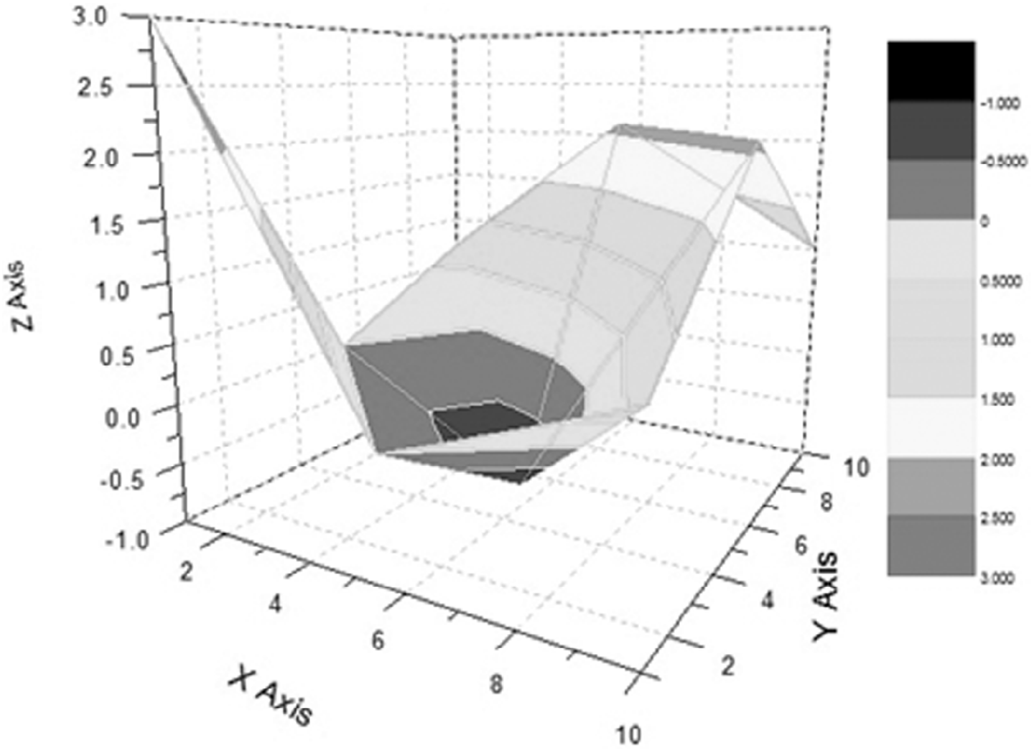

Figure 1: 3-D Representation of B1

The second rotation gives

$$B_2 = S_2^{-1} B_1 S_2 = \begin{bmatrix} 3 & -0.378682 & 0.597160 \\ -0.378682 & -2.345209 & -0.000003 \\ 0.597160 & -0.000003 & 2.345209 \end{bmatrix}$$

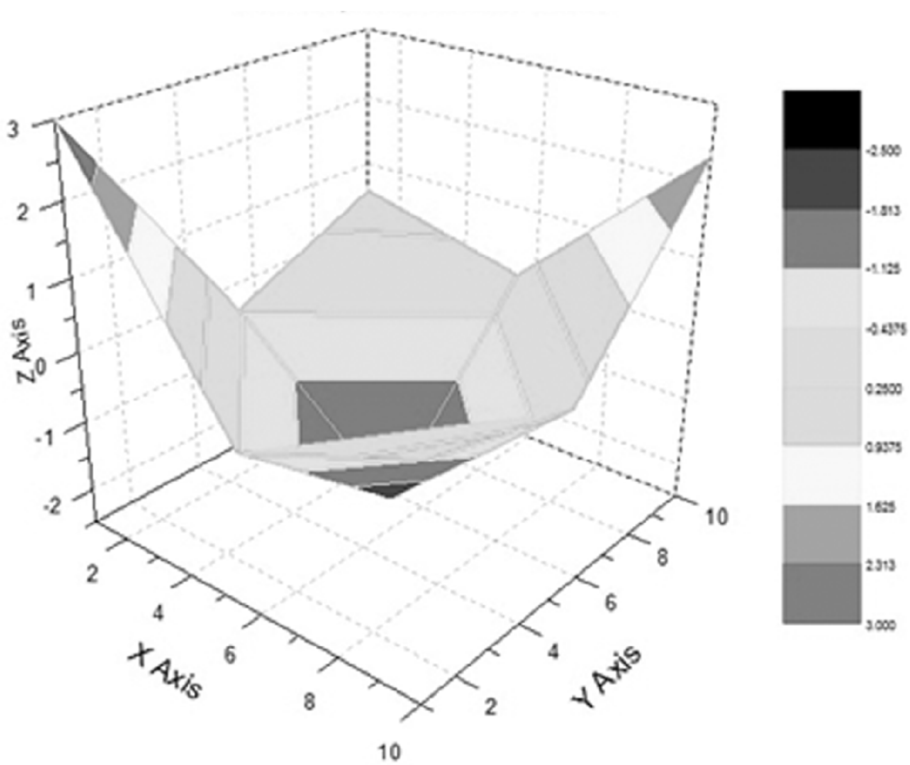

Figure 2: 3-D Representation of B2

The largest off-diagonal elements in magnitude in B_2 *is* a_{13}. Therefore,

$$\tan 2\theta = \frac{2a_{13}}{a_{11} - a_{33}} = 1.823969, \text{ or } \theta = 0.534647,$$

$$\cos\theta = 0.860449, \sin\theta = -0.509537$$

$$S_3 = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

The third rotation gives

$$B_3 = S_3^{-1} B_2 S_3 = \begin{bmatrix} 3.353625 & -0.325837 & -0.000001 \\ -0.325837 & -2.345208 & 0.192952 \\ 0.000001 & 0.192952 & 1.991585 \end{bmatrix}$$

The largest off-diagonal elements in magnitude in B_3 *is* a_{12}. Therefore,

$$\tan 2\theta = \frac{2a_{12}}{a_{11} - a_{22}} = -0.114352, \text{ or } \theta = -0.056929,$$

$$\cos\theta = 0.998380, \sin\theta = -0.056898$$

$$S_4 = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

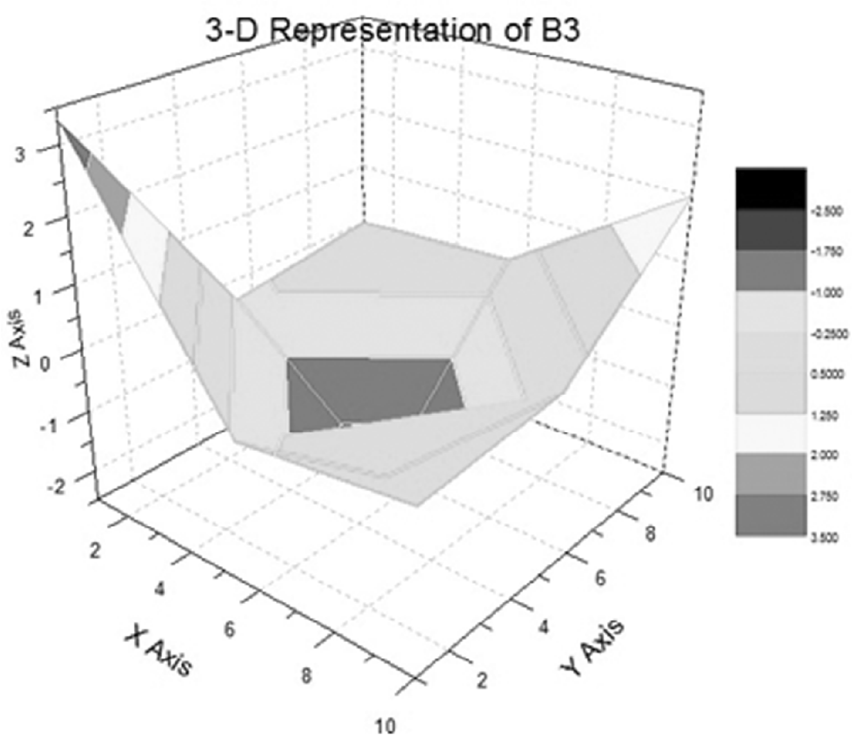

Figure 3: 3-D Representation of B3

The fourth rotation gives

$$B_4 = S_4^{-1} B_3 S_4 = \begin{bmatrix} 3.372195 & 0 & -0.010979 \\ 0 & -2.363717 & 0.192639 \\ -0.010979 & 0.192639 & 1.991585 \end{bmatrix}$$

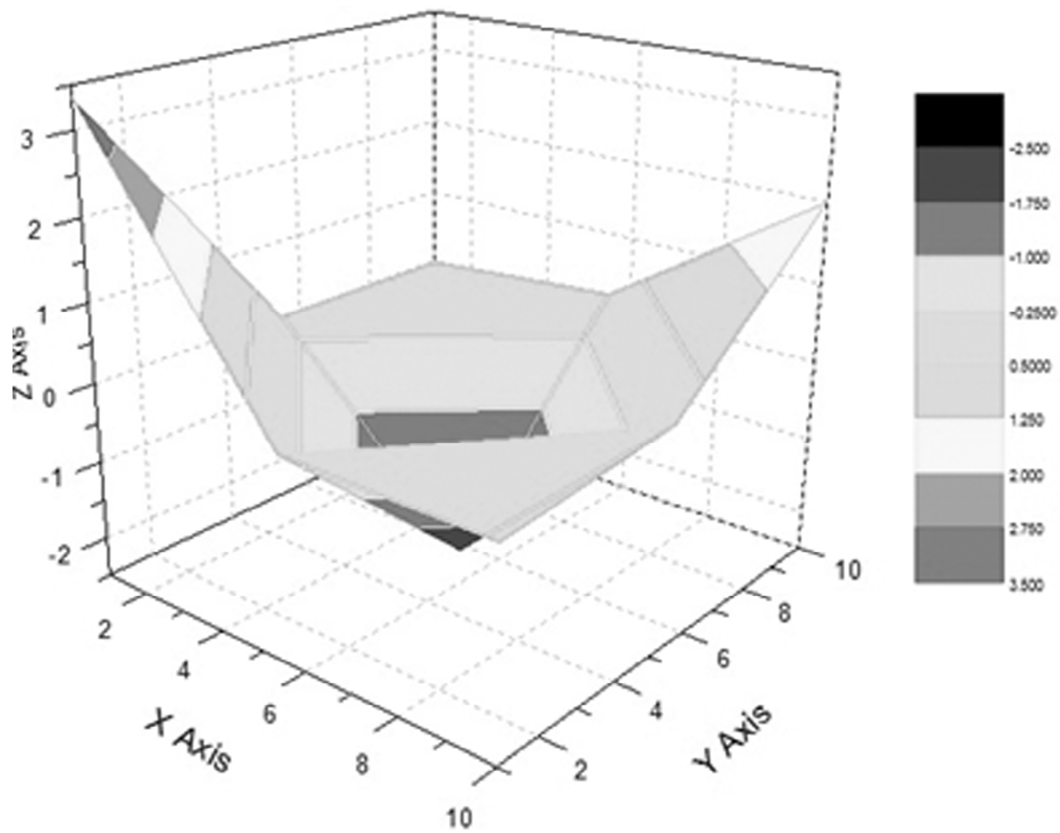

Figure 4: 3-D Representation of B4

The largest off-diagonal elements in magnitude in B_4 *is* a_{23}. Therefore,

$$\tan 2\theta = \frac{2a_{23}}{a_{22} - a_{33}} = -0.088462, \text{ or } \theta = -0.044116,$$

$$\cos\theta = 0.999027, \sin\theta = -0.044102$$

$$S_5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

The fifth rotation gives

$$B_5 = S_5^{-1} B_4 S_5 = \begin{bmatrix} 3.372195 & 0.000484 & -0.010968 \\ 0.000484 & -2.372281 & 0.000003 \\ -0.010968 & 0.000003 & 2.000089 \end{bmatrix}$$

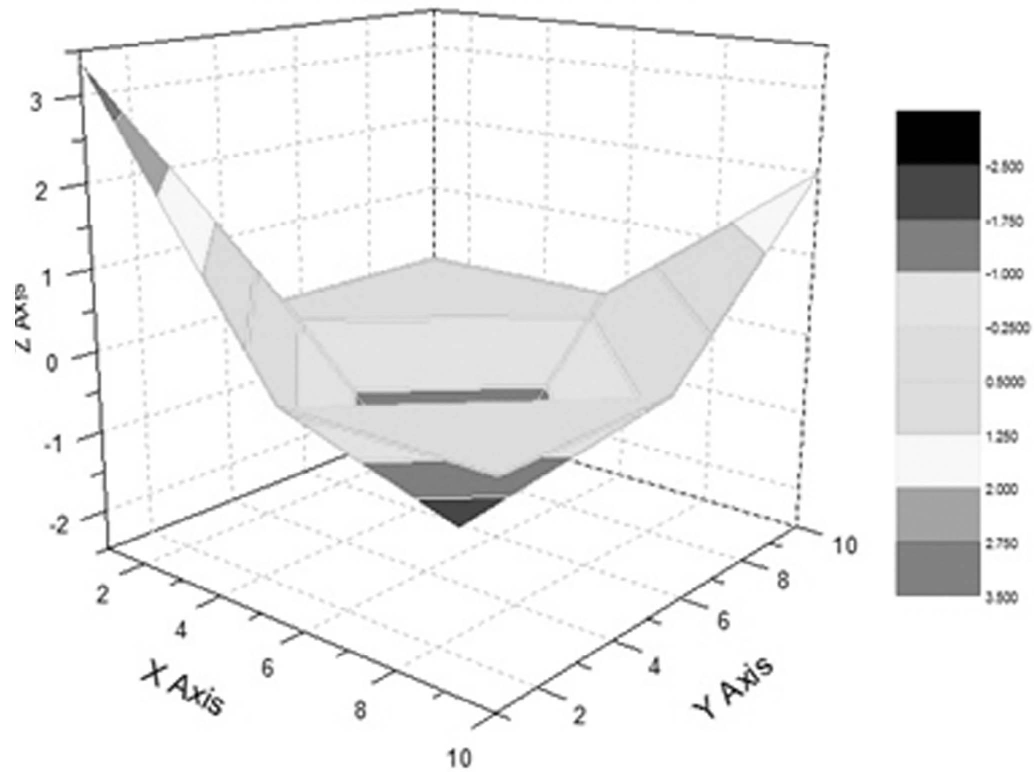

Figure 5: 3-D Representation of B5

The largest off-diagonal elements in magnitude in B_5 *is* a_{13}. Therefore,

$$\tan 2\theta = \frac{2a_{13}}{a_{11} - a_{33}} = -0.015987, \text{ or } \theta = -0.007992,$$

$$\cos\theta = 0.999968, \sin\theta = -0.007992$$

$$S_6 = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

The sixth rotation gives

$$B_6 = S_6^{-1} B_5 S_6 = \begin{bmatrix} 3.372283 & 0.000484 & 0 \\ 0.000484 & -2.372281 & 0 \\ 0 & 0 & 1.999998 \end{bmatrix}$$

Hence, the eigenvalues are 3.372283, -2.372281 and 1.999998 (approximately 3.4, -2.4, 2).

Consider the Spectrum (collection of positive eigen values) of the above problem (3.4, 2.4, 2) and three decision makers giving three possible optimal solutions to the weighting vector for any decision making problem.

Formulation of the decision problem by first Decision Maker (DM):

D.M (i):

$$Max\, z = 3.4w_1 + 2.4w_2 + 2w_3$$

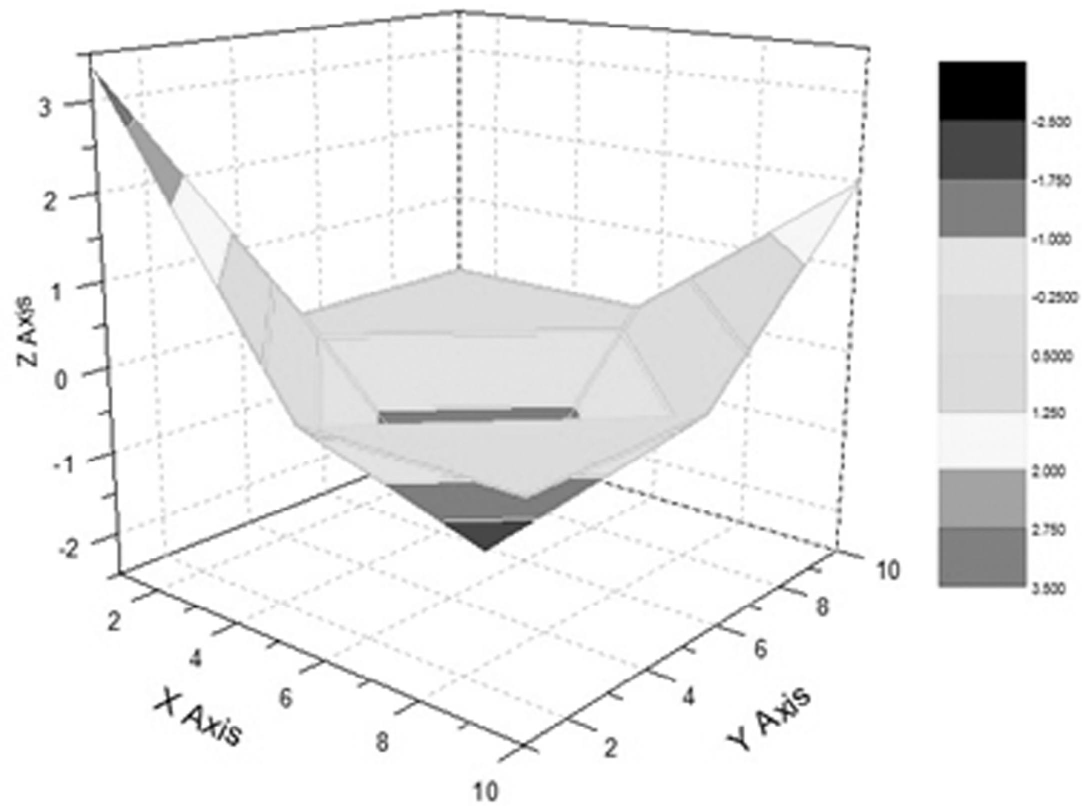

Figure 6: 3-D Representation of B6

Subject to constrains,

$w_1 + w_2 \le 0.6;\ \ w_1 + w_3 \le 0.8;\ \ w_2 + w_3 \le 0.7$
$w_1, w_2, w_3 \ge 0.$

Solution:

$$Max\, z = 3.4w_1 + 2.4w_2 + 2w_3 + 0s_1 + 0s_2 + 0s_3$$

Subject to constrains,

$w_1 + w_2 + s_1 + 0s_2 + 0s_3 = 0.6$
$w_1 + w_3 + 0s_1 + s_2 + 0s_3 = 0.8$
$w_2 + w_3 + 0s_1 + 0s_2 + s_3 = 0.7$
$w_1, w_2, w_3, s_1, s_2, s_3 \ge 0$

Table 1
Optimal table of DM (1)

C_B	y_B	C_j / x_B	3.4 / w_1	2.4 / w_2	2 / w_3	0 / s_1	0 / s_2	0 / s_3
3.4	w_1	0.35	1	0	0	1/2	1/2	–1/2
2	w_3	0.45	0	0	1	–1/2	1/2	1/2
2.4	w_2	0.25	0	1	0	1/2	–1/2	1/2
$z_j - c_j$		2.69	3.4	0	0	1.9	1.5	0.5

Hence the weight vector from DM (i) is

$w_1 = 0.35 \quad w_2 = 0.25 \quad w_3 = 0.45$

Formulation of the decision problem by second Decision Maker (DM):

D.M (ii):

$$Max\, z = 3.4w_1 + 2.4w_2 + 2w_3$$

Subject to constrains,

$$w_1 + w_2 \le 0.5; \; w_1 + w_3 \le 0.7; \; w_2 + w_3 \le 0.6$$
$$w_1, w_2, w_3 \ge 0$$

Solution:

$$Max\, z = 3.4w_1 + 2.4w_2 + 2w_3 + 0s_1 + 0s_2 + 0s_3$$

Subject to constrains,

$$w_1 + w_2 + s_1 + 0s_2 + 0s_3 = 0.5$$
$$w_1 + w_3 + 0s_1 + s_2 + 0s_3 = 0.7$$
$$w_2 + w_3 + 0s_1 + 0s_2 + s_3 = 0.6$$
$$w_1, w_2, w_3, s_1, s_2, s_3 \ge 0$$

Table 2
Optimal table of DM (2)

C_B	y_B	C_j x_B	3.4 w_1	2.4 w_2	2 w_3	0 s_1	0 s_2	0 s_3
3.4	w_1	0.3	1	0	0	1/2	1/2	–1/2
2	w_3	0.4	1	0	1	–1/2	1/2	1/2
2.4	w_2	0.2	0	1	0	1/2	–1/2	1/2
$z_j - c_j$		2.3	2	0	0	1.9	1.5	0.5

Hence the weight vector from DM (ii) is

$w_1 = 0.3 \quad w_2 = 0.2 \quad w_3 = 0.4$

Formulation of the decision problem by third Decision Maker (DM):

D.M (iii):

$$Max\, z = 3.4w_1 + 2.4w_2 + 2w_3$$

Subject to constrains,

$$w_1 + w_2 \le 0.6; \; w_1 + w_3 \le 0.9; \; w_2 + w_3 \le 0.5$$
$$w_1, w_2, w_3 \ge 0$$

Solution:

$$Max\, z = 3.4w_1 + 2.4w_2 + 2w_3 + 0s_1 + 0s_2 + 0s_3$$

Subject to constrains,

$$w_1 + w_2 + s_1 + 0s_2 + 0s_3 = 0.6$$
$$w_1 + w_3 + 0s_1 + s_2 + 0s_3 = 0.9$$
$$w_2 + w_3 + 0s_1 + 0s_2 + s_3 = 0.5$$
$$w_1, w_2, w_3, s_1, s_2, s_3 \geq 0$$

Table 3
Optimal table of DM (3)

C_B	y_B	C_j / x_B	3.4 / w_1	2.4 / w_2	2 / w_3	0 / s_1	0 / s_2	0 / s_3
3.4	w_1	0.5	1	0	0	1/2	1/2	–1/2
2	w_3	0.4	0	0	1	–1/2	1/2	1/2
2.4	w_2	0.1	0	1	0	1/2	–1/2	1/2
$z_j - c_j$		2.74	0	0	0	2.1	1.3	0.5

Hence the weight vector from DM (iii) is

$$w_1 = 0.5 \quad w_2 = 0.1 \quad w_3 = 0.4$$

Construct a matrix with the vector obtained from the three decision makers as follows:

$$w = \begin{bmatrix} 0.35 & 0.25 & 0.45 \\ 0.3 & 0.2 & 0.4 \\ 0.5 & 0.1 & 0.4 \end{bmatrix}$$

Let S represent the eigen value matrix:

$$S = \begin{bmatrix} 3.4 & 0.000484 & 0 \\ 0.000484 & -2.372281 & 0 \\ 0 & 0 & 1.999998 \end{bmatrix}$$

$$S^T = \begin{bmatrix} 3.4 & 0.000484 & 0 \\ 0.000484 & -2.372281 & 0 \\ 0 & 0 & 1.999998 \end{bmatrix}$$

$$S^T w = \begin{bmatrix} 3.4 & 0.000484 & 0 \\ 0.000484 & -2.372281 & 0 \\ 0 & 0 & 1.999998 \end{bmatrix} \times \begin{bmatrix} 0.35 & 0.25 & 0.45 \\ 0.3 & 0.2 & 0.4 \\ 0.5 & 0.1 & 0.4 \end{bmatrix}$$

$$S^T w = \begin{bmatrix} 1.1901452 & 0.850968 & 1.5301936 \\ -2.0721116 & -0.4743351 & -0.9486946 \\ 0.999999 & 0.1999998 & 0.7999992 \end{bmatrix}$$

$$\left(S^T w\right)^T = \begin{bmatrix} 1.1901452 & -2.0721116 & 0.999999 \\ 0.850968 & -0.4743351 & 0.1999998 \\ 1.5301936 & -0.9486946 & 0.7999992 \end{bmatrix}$$

$$\left(S^T w\right)^T \left(S^T w\right) = \begin{bmatrix} 1.1901452 & -2.0721116 & 0.999999 \\ 0.850968 & -0.4743351 & 0.1999998 \\ 1.5301936 & -0.9486946 & 0.7999992 \end{bmatrix} \times \begin{bmatrix} 1.1901452 & 0.850968 & 1.5301936 \\ -2.0721116 & -0.4743351 & -0.9486946 \\ 0.999999 & 0.1999998 & 0.7999992 \end{bmatrix}$$

$$\left(S^T w\right)^T \left(S^T w\right) = \begin{bmatrix} 6.613688 & 2.1956246 & 4.5869524 \\ 2.1956496 & 0.9891409 & 1.9121447 \\ 4.5869524 & 1.9121447 & 3.8815117 \end{bmatrix}$$

Taking column average and dividing each entry of that column, we get:

$$\omega = \begin{bmatrix} 0.4936955 & 0.4307756 & 0.4418770 \\ 0.1638998 & 0.1940668 & 0.18420352 \\ 0.3424047 & 0.3751576 & 0.3739195 \end{bmatrix}$$

Taking row average of the above matrix, we get:

$$\omega = \begin{bmatrix} 0.455449366 \\ 0.180723373 \\ 0.363827266 \end{bmatrix}$$

To find the weight vector: $v = w \times \omega$

$$v = \begin{bmatrix} 0.35 & 0.25 & 0.45 \\ 0.3 & 0.2 & 0.4 \\ 0.5 & 0.1 & 0.4 \end{bmatrix} \times \begin{bmatrix} 0.45544 \\ 0.18072 \\ 0.36383 \end{bmatrix}$$

$$v = \begin{bmatrix} 0.36831 \\ 0.31831 \\ 0.39132 \end{bmatrix}$$

Which will represent the weight vector of the decision makers obtained from normalizing the eigen value matrix.

The MAGDM-Miner Algorithm and its pseudocode are presented in the following.

8. THE MAGDM-MINER ALGORITHM FOR TRAPEZOIDAL IFS

Let $A = \{A_{1,} A_{2,...,} A_m\}$ be a discrete set of alternatives, and $G = \{G_1, G_2,...,G_n\}$ be the set of attributes, $w = (w_1, w_2, ..., w_n)$ is the weighting vector of the attribute G_j, $(j = 1, 2,...., n)$, where $w_j \in [0,1]$, $\sum_{j=1}^{n} w_j = 1$. Let $D = \{D_1, D_2,...., D_t\}$ be the set of decision makers, $v = (v_1, v_2, ..., v_n)$ be the weighting vector of decision makers, with $v_k \in [0,1]$, $\sum_{k=1}^{t} v_k = 1$; Let us suppose that, $\tilde{R}_k = \left(\tilde{r}_{i,j}^{(k)}\right)_{m\times n} = ([a_{ij}^{(k)}, b_{ij}^{(k)}, c_{ij}^{(k)}, d_{ij}^{(k)}]; u_{ij}^{(k)}, v_{ij}^{(k)})_{mxn}$ is the trapezoidal intuitionistic fuzzy decision matrix, with $u_{ij}^{(k)} \in [0,1], v_{ij}^{(k)} \in [0,1], u_{ij}^{(k)} + v_{ij}^{(k)} \le 1$, where $i = 1, 2,..., m;\ j = 1, 2,..., n;\ k = 1, 2, ..., t$. In the following, we apply the *TzIFWAA* and *TzIFHA* operator to multiple attribute group decision making based on trapezoidal intuitionistic fuzzy information. The method involves the following steps.

Step-1: Utilize the decision information given in thetrapezoidal intuitionistic fuzzy decision matrix $\tilde{R}_k$, and the *TzIFWAA* operator to derive the individual overall preferencetrapezoidal intuitionistic fuzzy values $\tilde{r}_\iota(k)$ of the alternative A_i.

Step-2: Utilize the *TzIFHA* operator to derive the collective overall preferencetrapezoidal intuitionistic fuzzy values $\tilde{r}_\iota, (i = 1, 2, ..., m)$ of the alternative $A_{i,}$ $v = (v_1, v_2, ..., v_n)$ being the weighting vector of decision makers, with $v_k \in [0,1]$, $\sum_{k=1}^{t} v_k = 1$; and $w = (w_1, w_2, ..., w_n)$ being the associated weighting vector of the *TzIFHA* operator with $w_j > 0$, and $\sum_{j=1}^{n} w_j = 1$.

Step-3: Utilize the Apriori algorithm and mining trapezoidal intuitionistic fuzzy correlation analysis to identify the closely related itemsets of the collective overall preference trapezoidal intuitionistic fuzzy values of step 2, to eliminate some of the non-interesting or less important decision variables. Using equations (9), (10) and (11), the trapezoidal intuitionistic fuzzy correlation coefficient is given by:

$$K_{TzIFS}(A,B) = \frac{C_{TzIFS}(A,B)}{\sqrt{E_{TzIFS}(A).E_{TzIFS}(B)}}$$

Assume that $F = \{\tilde{r}_1, \tilde{r}_2, ..., \tilde{r}_m\}$ be a set of fuzzy items; $T = \{t_1, t_2, ..., t_n\}$ be a random sample with n fuzzy data records, and each sample record t_i is represented as a vector with m values, $\left(\tilde{r}_1(t_i), \tilde{r}_2(t_i), ..., \tilde{r}_m(t_i)\right)$, where $\tilde{r}_j(t_i)$ is the degree that the fuzzy item $\tilde{r}_j$ occurs in the record t_i, where $\tilde{r}_j(t_i) \in [0,1]$. We need three predefined thresholds to be defined, namely, the minimal fuzzy support, the minimal fuzzy confidence and the minimal fuzzy correlation coefficient, and in our decision making situation we require only two of the thresholds, the minimal fuzzy support and the minimal fuzzy correlation coefficient.The join step and prune step for mining fuzzy correlation rules is as follows:

(3.1) The fuzzy support $\text{fsupp}\ (\tilde{r}_i)$, of each trapezoidal IFS item is computed.

(3.2) Let $L_1 = \left\{\tilde{r}_p / \tilde{r}_p \in F,\ \text{fsupp}(\tilde{r}_p) \ge s_{\tilde{r}}\right\}$ be the set of frequent fuzzy itemsets whose size is equal to 1.

(3.3) Let $C_2 = \{(F_A, F_B)\}$ be the set of all combinations of two elements belonging to L_1, where $F_A, F_B \in L_1, F_A \neq F_B$. That is, $C_2 = \{(F_A, F_B)\}$ is generated by L_1 joint with L_1. Since F_A, F_B are the elements of L_1, the size of each element of C_2 is 2.

(3.4) For each element of C_2, (F_A, F_B), the fuzzy support, fsuppis $\{(F_A, F_B)\}$ computed by using the comparison of the hesitation degree of each intuitionistic fuzzy information, and then the trapezoidal intuitionistic fuzzy correlation coefficient between F_A, F_B, $K(F_A, F_B)$ is computed fromequation (11). Calculate the Median value $K_{MED}(F_A, F_B)$ of all the correlation coefficientand $K(F_A, F_B)$ consider all the $K(F_A, F_B) > K_{MED}(F_A, F_B)$ for the next level.

(3.5) For each element, whose fuzzy support is greater than or equal to $s_{\tilde{r}}$ and the maximum fuzzy correlation coefficients with their correlation coefficient $K(F_A, F_B) > K_{MED}(F_A, F_B)$ of C_2, will be an element of L_2. Hence, L_2 is the set of the frequent combinations of two fuzzy itemsets, and the size of each element being 2.

(3.6) Next, each C_k, $k \geq 3$, is generated by L_{k-1} joint with L_{k-1}. Assume that (F_W, F_X) and (F_Y, F_Z) are two elements of L_{k-1}, where $F_X = F_Y$. If the size of the combination $(F_X, \{F_W, F_Z\})$ is k, and (F_W, F_Z) is also a frequent combination of 2-fuzzy itemsets, then the combination $(F_X, \{F_W, F_Z\})$ is an element with size k of C_k. For each element ofC_k, its fuzzy support and fuzzy correlation coefficient are still used to find the elements of L_k.

(3.7) When each L_k, $k \geq 2$, is obtained, for each element of L_k, (F_G, F_H), the 2-candidate fuzzy correlation rules can be generated. At this level, the itemsets with the highest correlation coefficient can be selected or ranked, which can be considered as interesting fuzzy correlation rules.

The loop will stop when next C_{k+1} cannot be generated.

Step 4: Using the trapezoidal intuitionistic fuzzy correlation rule mining from step-3 we can eliminate the unwanted or less-important decision variables. Find the relationship of the remaining decision variables with the positive ideal solutions for trapezoidal intuitionistic setsusing the same correlation coefficient formula (11):

$$\left\{ K^{+}_{TzIFS}(\tilde{r}_i, \tilde{r}^{+}) = \frac{C_{TzIFS}(\tilde{r}_i, \tilde{r}^{+})}{\sqrt{E_{TzIFS}(\tilde{r}_i).E_{TzIFS}(\tilde{r}^{+})}} \right. \quad \text{, with positive ideal solution } \tilde{r}^{+} = ([1,1,1,1];1,0)$$

Step 5: Rank all the alternatives $A_i (i = 1, 2, \ldots, m)$ and select the best one in accordance with the highest correlation coefficient obtained.

Pseudo-code for MAGDM-Miner Algorithm:

C_k: Candidate itemset of size k

L_k : frequent itemset of size k

Input {*Trapezoidal Intuitionistic Fuzzy Decision Matrices*}

Compute {*TzIFWAA&TzIFHA*}

Generate {*Individual Preference Decision Matrix*}

Generate {*Collective Overall Preference Decision Matrix*}

L_1 = {frequent items};

For (k = 1; L_k! = ∅; k++)**do begin**

While $f\text{supp}(item\text{-}set) \geq Threshold$ **do**

C_{k+1}= candidates generated fromL_k;

//that iscartesian product $L_{k-1} \times L_{k-1}$ and eliminating

any k-1 size item-set that isnot frequent //

Compute {*Correlation coefficient between item-sets* $K(F_A, F_B)$ }

Compute {*Median of all* $K_{MED}(F_A, F_B)$ }

While $K(F_A, F_B) > K_{MED}(F_A, F_B)$ **do**

For each transaction database **do** increment the count of all candidates in C_{k+1} that are contained in

L_{k+1}= candidates in$C_{k+1}$with $f\text{supp}(item\text{-}set) \geq Threshold$ and $K(F_A, F_B) > K_{MED}(F_A, F_B)$

End

Return $\cup_k L_k$;

9. NUMERICALILLUSTRATION

A company needs to select, over a set of potential suppliers, a fixed number of them in order to satisfy its demands. At the first step, the company finds five strategic suppliers and wishes to choose three suppliers among the five suppliers. The most significant attributes of the company for this selection are:

G_1 is the ***Price***

G_2 is the ***Quality***

G_3 is the ***Flexibility***

G_4 is the ***Service***

The behaviours of the suppliers are fuzzy in nature and doesnot have much information to describe these behaviours. However it has information given in decision matrices by three experts in the form of trapezoidal intuitionistic fuzzy numbers. The company decided to use a voting system (experts group have to give a mark for each attribute, for each supplier) for the decision making purpose. The five possible alternatives A_i (suppliers) are to be evaluated using the trapezoidal intuitionistic fuzzy numbers by the three decision makers whose weighting vector is $v = (v_1, v_2, ..., v_n) = (0.36831, 0.31831, 0.39132)$ obtained from the normalized eigen value matrix, and under the four attributes whose weighting vector is ω = (0.2, 0.1, 0.3, 0.4). The decision matrices (5 × 4) are respectively:

$$R_1 = \begin{bmatrix} ([0.5,0.6,0.7,0.8];0.5,0.4) & ([0.1,0.2,0.3,0.4];0.6,0.3) & ([0.5,0.6,0.8,0.9];0.3,0.6) & ([0.4,0.5,0.6,0.7];0.2,0.7) \\ ([0.6,0.7,0.8,0.9];0.7,0.3) & ([0.5,0.6,0.7,0.8];0.7,0.2) & ([0.4,0.5,0.7,0.8];0.7,0.2) & ([0.5,0.6,0.7,0.9];0.4,0.5) \\ ([0.1,0.2,0.4,0.5];0.6,0.4) & ([0.2,0.3,0.5,0.6];0.5,0.4) & ([0.5,0.6,0.7,0.8];0.5,0.3) & ([0.3,0.5,0.7,0.9];0.2,0.3) \\ ([0.3,0.4,0.5,0.6];0.8,0.1) & ([0.1,0.3,0.4,0.5];0.6,0.3) & ([0.1,0.3,0.5,0.7];0.3,0.4) & ([0.6,0.7,0.8,0.9];0.2,0.6) \\ ([0.2,0.3,0.4,0.5];0.6,0.2) & ([0.3,0.4,0.5,0.6];0.4,0.3) & ([0.2,0.3,0.4,0.5];0.7,0.1) & ([0.5,0.6,0.7,0.8];0.1,0.3) \end{bmatrix}$$

$$R_2 = \begin{bmatrix} ([0.4,0.5,0.6,0.7];0.4,0.3) & ([0.1,0.2,0.3,0.4];0.5,0.2) & ([0.4,0.5,0.7,0.8];0.2,0.5) & ([0.3,0.4,0.5,0.6];0.1,0.6) \\ ([0.5,0.6,0.7,0.8];0.6,0.2) & ([0.4,0.5,0.6,0.7];0.6,0.1) & ([0.3,0.4,0.6,0.7];0.6,0.1) & ([0.4,0.5,0.6,0.8];0.3,0.4) \\ ([0.1,0.2,0.3,0.4];0.5,0.3) & ([0.1,0.2,0.4,0.5];0.4,0.3) & ([0.4,0.5,0.6,0.7];0.4,0.2) & ([0.2,0.4,0.6,0.8];0.5,0.2) \\ ([0.2,0.3,0.4,0.5];0.7,0.1) & ([0.1,0.2,0.3,0.5];0.5,0.2) & ([0.1,0.2,0.4,0.6];0.2,0.3) & ([0.5,0.6,0.7,0.8];0.1,0.5) \\ ([0.1,0.2,0.3,0.4];0.5,0.1) & ([0.2,0.3,0.4,0.5];0.3,0.2) & ([0.1,0.2,0.3,0.4];0.6,0.2) & ([0.4,0.5,0.6,0.7];0.4,0.2) \end{bmatrix}$$

$$R_3 = \begin{bmatrix} ([0.6,0.7,0.8,0.9];0.4,0.5) & ([0.2,0.3,0.4,0.5];0.5,0.4) & ([0.6,0.7,0.9,1.0];0.2,0.7) & ([0.5,0.6,0.7,0.8];0.1,0.8) \\ ([0.7,0.8,0.9,1.0];0.6,0.4) & ([0.6,0.7,0.8,0.9];0.6,0.3) & ([0.5,0.6,0.8,0.9];0.6,0.3) & ([0.6,0.7,0.8,1.0];0.3,0.6) \\ ([0.2,0.3,0.5,0.6];0.5,0.5) & ([0.3,0.4,0.6,0.7];0.4,0.5) & ([0.6,0.7,0.8,0.9];0.4,0.4) & ([0.4,0.6,0.8,1.0];0.5,0.4) \\ ([0.4,0.5,0.6,0.7];0.7,0.2) & ([0.2,0.4,0.5,0.6];0.5,0.4) & ([0.2,0.4,0.6,0.8];0.2,0.5) & ([0.7,0.8,0.9,1.0];0.1,0.7) \\ ([0.3,0.4,0.5,0.6];0.5,0.3) & ([0.4,0.5,0.6,0.7];0.3,0.4) & ([0.3,0.4,0.5,0.6];0.6,0.2) & ([0.6,0.7,0.8,0.9];0.4,0.4) \end{bmatrix}$$

Step-1: Utilize the decision information given in the trapezoidal intuitionistic fuzzy decision matrix R_k, and the *TzIFWAA* operator to derive the individual overall preference intuitionistic triangular fuzzy values $\tilde{r}_i$(k) of the alternative A_i.

$$\begin{aligned} \tilde{r}_1^{(1)} &= ([0.42,0.52,0.65,0.75];0.3472,0.5490) & \tilde{r}_1^{(2)} &= ([0.33,0.43,0.56,0.66];0.2446,0.4431) \\ \tilde{r}_2^{(1)} &= ([0.49,0.59,0.72,0.86];0.6041,0.3129) & \tilde{r}_2^{(2)} &= ([0.39,0.49,0.62,0.76];0.4996,0.2000) \\ \tilde{r}_3^{(1)} &= ([0.31,0.45,0.62,0.76];0.4229,0.3270) & \tilde{r}_3^{(2)} &= ([0.23,0.37,0.52,0.63];0.4622,0.2259) \\ \tilde{r}_4^{(1)} &= ([0.34,0.48,0.61,0.74];0.4565,0.3464) & \tilde{r}_4^{(2)} &= ([0.28,0.38,0.51,0.65];0.3424,0.2837) \\ \tilde{r}_5^{(1)} &= ([0.33,0.43,0.53,0.63];0.4715,0.1990) & \tilde{r}_5^{(2)} &= ([0.23,0.33,0.43,0.53];0.4798,0.1741) \end{aligned}$$

and

$$\begin{aligned} \tilde{r}_1^{(3)} &= ([0.52,0.62,0.75,0.85];0.2468,0.5895) \\ \tilde{r}_2^{(3)} &= ([0.59,0.69,0.82,0.96];0.4996,0.4913) \\ \tilde{r}_3^{(3)} &= ([0.41,0.55,0.72,0.86];0.4622,0.4277) \\ \tilde{r}_4^{(3)} &= ([0.44,0.58,0.71,0.84];0.3424,0.4657) \\ \tilde{r}_5^{(3)} &= ([0.43,0.53,0.63,0.73];0.4875,0.3067) \end{aligned}$$

Step-2: Utilize the *TzIFHA* operator to derive the collective overall preference trapezoidal intuitionistic fuzzy values $\tilde{r}_i$ of the alternative A_i.

Then we have:

$$\begin{aligned} \tilde{r}_1^{(1)} &= ([0.42,0.52,0.65,0.75];0.3472,0.5490) \\ \tilde{r}_1^{(2)} &= ([0.33,0.43,0.56,0.66];0.2446,0.4431) \\ \tilde{r}_1^{(3)} &= ([0.52,0.62,0.75,0.85];0.2468,0.5895) \end{aligned}$$

Where $v = (v_1, v_2, ..., v_n)$ = (0.35, 0.40, 0.25), w = (0.20, 0.50, 0.30)

$$\dot{\tilde{a}}_1 = a_1^{(n\times v_1)} = 0.4022, \dot{\tilde{a}}_2 = a_2^{(n\times v_2)} = 0.2644, \dot{\tilde{a}}_3 = a_3^{(n\times v_3)} = 0.6124,$$

$$\dot{\tilde{b}}_1 = b_1^{(n\times v_1)} = 0.5033, \dot{\tilde{b}}_2 = b_2^{(n\times v_2)} = 0.3632, \dot{\tilde{b}}_3 = b_3^{(n\times v_3)} = 0.6987,$$

$$\dot{\tilde{c}}_1 = c_1^{(n\times v_1)} = 0.6361, \dot{\tilde{c}}_2 = c_2^{(n\times v_2)} = 0.4987, \dot{\tilde{c}}_3 = c_3^{(n\times v_3)} = 0.8059,$$

$$\dot{d}_1 = d_1^{(n\times v_1)} = 0.7393, \dot{d}_2 = d_2^{(n\times v_2)} = 0.6074, \dot{d}_3 = d_3^{(n\times v_3)} = 0.8852,$$

$$\dot{\tilde{\mu}}_1 = \mu_1^{(n\times v_1)} = 0.3293, \dot{\tilde{\mu}}_2 = \mu_2^{(n\times v_2)} = 0.1846, \dot{\tilde{\mu}}_3 = \mu_3^{(n\times v_3)} = 0.4523,$$

$$\dot{\tilde{\gamma}}_1 = \gamma_1^{(n\times v_1)} = 0.5328, \dot{\tilde{\gamma}}_1 = \gamma_2^{(n\times v_2)} = 0.3765, \dot{\tilde{\gamma}}_1 = \gamma_3^{(n\times v_3)} = 0.6378.$$

Utilizing TzIFHA operator we get:

$$\tilde{r}_1 = \left(\left[0.4029, 0.5004, 0.6288, 0.7289\right];\ 0.2933, 0.5030\right)$$

$$\tilde{r}_2 = \left(\left[0.4680, 0.5662, 0.6955, 0.8365\right];\ 0.5490, 0.2736\right)$$

$$\tilde{r}_3 = \left(\left[0.3001, 0.4349, 0.5959, 0.7356\right];\ 0.4376, 0.2866\right)$$

$$\tilde{r}_4 = \left(\left[0.3342, 0.4582, 0.5860, 0.7188\right];\ 0.3963, 0.3247\right)$$

$$\tilde{r}_5 = \left(\left[0.3137, 0.4097, 0.5071, 0.6058\right];\ 0.4718, 0.1912\right)$$

Step-3: The fuzzy support $f\,\mathrm{supp}(\tilde{r}_i)$, of each trapezoidal IFS item is computed. Let $L_1 = \left\{\tilde{r}_i / \tilde{r}_i \in F, \mathrm{fsupp}(\tilde{r}_i) \le s_{\tilde{r}}\right\}$ be the set of frequent fuzzy itemsets whose size is equal to 1, where $s_{\tilde{r}}$ is fixed to be 0.5. The GMIR and $E(\tilde{r}_i)$ are calculated using (7) and (9) respectively, and are recorded in Table-4 and the comparison of the above five decision variables can be seen in Figure 7.

Let C_2 be the set of all combinations of two elements belonging to L_1. That is, C_2 is generated by L_1 joint with L_1, and the size of each element of C_2 is 2. For each element of C_2, the fuzzy support is taken to be the hesitation degree of each intuitionistic fuzzy information satisfying the threshold condition fsupp $(\tilde{r}_i) \le s_{\tilde{r}}$, and

Table 4
GMIR values, fsupport and Informational intuitionistic energy of the decision variables.

L_1	*GMIR values*	*Hesitation Degree* πi *fsupport*	$E(\tilde{r}_i)$
$\tilde{r}_1$	0.5650	0.2037	0.1214
$\tilde{r}_2$	0.6379	0.1774	0.1658
$\tilde{r}_3$	0.5162	0.2758	0.0931
$\tilde{r}_4$	0.5235	0.2817	0.0936
$\tilde{r}_5$	0.4588	0.3370	0.0784

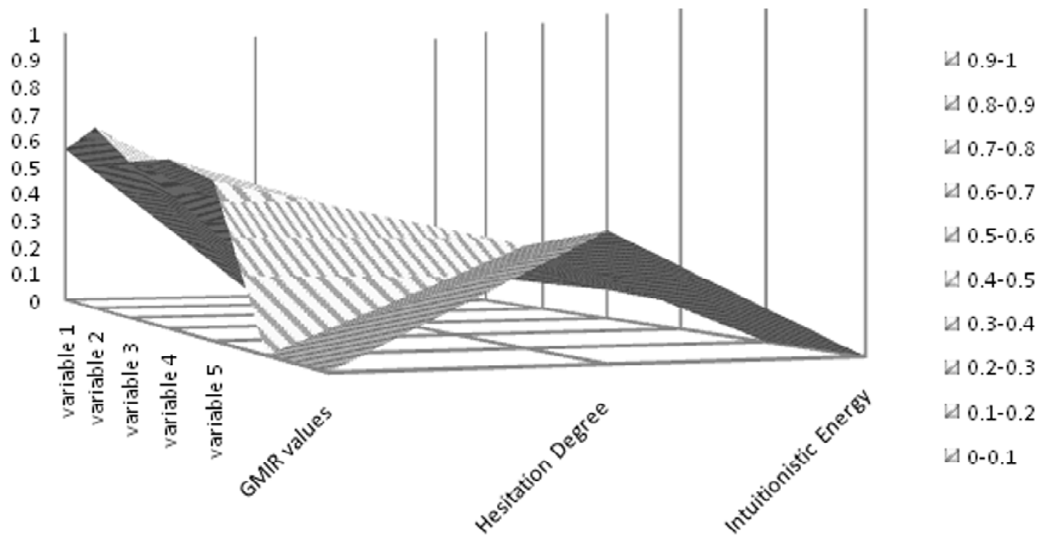

Figure 7: Comparison of the 5 decision variables

Table 5
Correlation coefficient of 2-itemsets.

C_2	Fuzzy support (fsupp)	Correlation $C(\tilde{r}_i, \tilde{r}_j)$	Correlation Coefficient $K(\tilde{r}_i, \tilde{r}_j)$	$K(A,B) > K_{MED}(A,B)$
$(\{\tilde{r}_1, \tilde{r}_2\})_{r_{12}}$	0.1774.4722	0.1206	0.8504	$0.8504 < K_{MED}(A,B)$
$(\{\tilde{r}_1, \tilde{r}_3\})_{r_{13}}$	0.2037	0.0958	0.9012	$0.9012 < K_{MED}(A,B)$
$(\{\tilde{r}_1, \tilde{r}_4\})_{r_{14}}$	0.2037	0.0996	0.9287	$0.9287 < K_{MED}(A,B)$
$(\{\tilde{r}_1, \tilde{r}_5\})_{r_{15}}$	0.2037	0.0786	0.8061	$0.8061 < K_{MED}(A,B)$
$(\{\tilde{r}_2, \tilde{r}_3\})_{r_{23}}$	0.1774	0.1207	0.9718	$0.9718 > K_{MED}(A,B)$
$(\{\tilde{r}_2, \tilde{r}_4\})_{r_{24}}$	0.1774	0.1190	0.9558	$0.9558 > K_{MED}(A,B)$
$(\{\tilde{r}_2, \tilde{r}_5\})_{r_{25}}$	0.1774	0.1086	0.9526	$0.9526 = K_{MED}(A,B)$
$(\{\tilde{r}_3, \tilde{r}_4\})_{r_{34}}$	0.2758	0.0930	0.9967	$0.9967 > K_{MED}(A,B)$
$(\{\tilde{r}_3, \tilde{r}_5\})_{r_{35}}$	0.2758	0.0839	0.9824	$0.9824 > K_{MED}(A,B)$
$(\{\tilde{r}_4, \tilde{r}_5\})_{r_{45}}$	0.2817	0.0826	0.9649	$0.9649 > K_{MED}(A,B)$

then the trapezoidal intuitionistic fuzzy correlation coefficient, $K(A,B)$ is computed by (11). All the necessary information of C_2 are recorded in Table-5 and Figure 8.

Collect all the $K(A,B)$ values from Table-5 that satisfy the threshold condition $K(A,B) > K_{MED}(A,B)$. Hence from Table-5 we can observe six interesting 2-candidate sets which will proceed to the next stage.

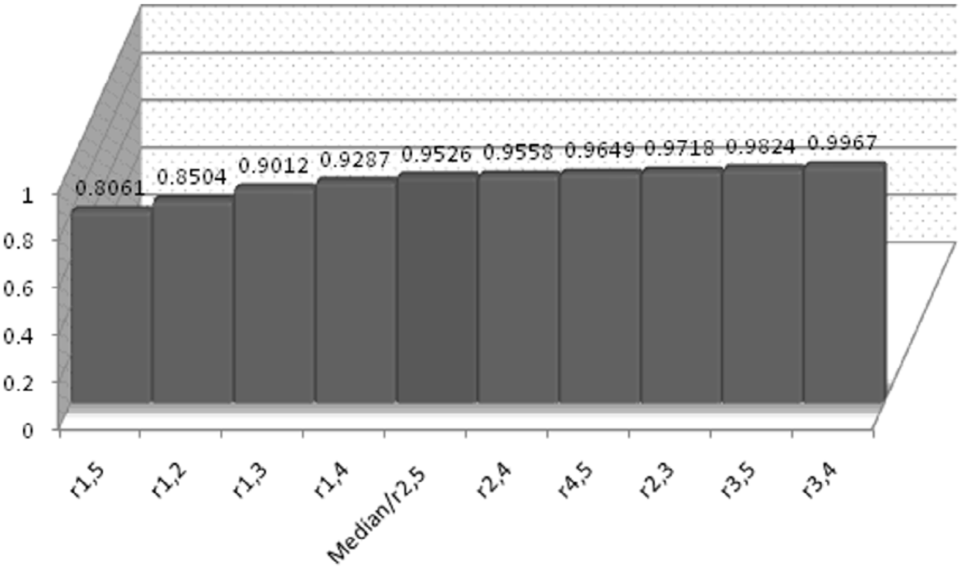

Figure 8: Correlation coefficient of 2-itemsets in ascending order

(r_{ij} refers $(\{\tilde{r}_i\},\{\tilde{r}_j\})$).

Table 6

Correlation coefficients satisfying threshold condition $K(A,B) > K_{MED}(A,B)$

C_2	*Correlation Coefficient* $K(\tilde{r}_i,\tilde{r}_j)$
$(\{\tilde{r}_2,\tilde{r}_3\})$	0.9718
$(\{\tilde{r}_2,\tilde{r}_4\})$	0.9558
$(\{\tilde{r}_3,\tilde{r}_4\})$	0.9967
$(\{\tilde{r}_3,\tilde{r}_5\})$	0.9824
$(\{\tilde{r}_4,\tilde{r}_5\})$	0.9649

Thus $L_2 = \left\{(\{\tilde{r}_2,\tilde{r}_3\}),(\{\tilde{r}_2,\tilde{r}_4\}),(\{\tilde{r}_3,\tilde{r}_4\}),(\{\tilde{r}_3,\tilde{r}_5\}),(\{\tilde{r}_4,\tilde{r}_5\})\right\}$

When L_2 is obtained, C_3 is generated by L_2 joint with L_2.

$$C_3 = \begin{Bmatrix} (\{\tilde{r}_4\},\{\tilde{r}_2,\tilde{r}_3\}),(\{\tilde{r}_5\},\{\tilde{r}_2,\tilde{r}_3\}),(\{\tilde{r}_3\},\{\tilde{r}_2,\tilde{r}_4\}), \\ (\{\tilde{r}_5\},\{\tilde{r}_2,\tilde{r}_4\}),(\{\tilde{r}_2\},\{\tilde{r}_3,\tilde{r}_4\}),(\{\tilde{r}_5\},\{\tilde{r}_3,\tilde{r}_4\}), \\ (\{\tilde{r}_2\},\{\tilde{r}_3,\tilde{r}_5\}),(\{\tilde{r}_4\},\{\tilde{r}_3,\tilde{r}_5\}),(\{\tilde{r}_2\},\{\tilde{r}_4,\tilde{r}_5\}), \\ (\{\tilde{r}_3\},\{\tilde{r}_4,\tilde{r}_5\}) \end{Bmatrix}$$

In order to find the fuzzy correlation coefficient between the itemsets of C_3, it is necessary to compute the trapezoidal intuitionistic fuzzy information of the itemset $\{\tilde{r}_i,\tilde{r}_j\}$, hesitation degree $\pi_{i,j}$ of $\{\tilde{r}_i,\tilde{r}_j\}$ and $E(\tilde{r}_{i,j})$, which is recorded in Table-7.

$$\{\tilde{r}_i,\tilde{r}_j\} = \left(\left[\max(a_i,a_j), \max(b_i,b_j), \max(c_i,c_j), \max(d_i,d_j)\right];\ \max(u_i,u_j),\ \min(v_i,v_j)\right)$$

where $\tilde{r}_i = \left(\left[a_i,b_i,c_i,d_i\right]; u_i, v_i\right)$ and $\tilde{r}_j = \left(\left[a_j,b_j,c_j,d_j\right]; u_j, v_j\right)$.

As the computations were done for C_2, a similar procedure follows for C_3, and the information are recorded in Table-8 and Figure 9.

Table 7
GMIR values, fsupport and Informational intuitionistic energy of 2-itemsets.

L_2	GMIR values	Hesitation Degree $\pi_{i,j}$ fsupport	$E(\tilde{r}_{i,j})$
$(\{\tilde{r}_2,\tilde{r}_3\})$	0.6379	0.1774	0.1658
$(\{\tilde{r}_2,\tilde{r}_4\})$	0.6379	0.1774	0.1658
$(\{\tilde{r}_3,\tilde{r}_4\})$	0.5296	0.2758	0.0981
$(\{\tilde{r}_3,\tilde{r}_5\})$	0.5184	0.3370	0.1001
$(\{\tilde{r}_4,\tilde{r}_5\})$	0.5235	0.3370	0.1021

Table 8
Correlation coefficients of 3-itemsets.

C_3	Fuzzy support (fsupp)	Correlation $C(\tilde{r}_p,\tilde{r}_{i,j})$	Correlation Coefficient $K(\tilde{r}_p,\tilde{r}_{i,j})$	Threshold $K(A,B) > K_{MED}(A,B)$
$(\{\tilde{r}_4\},\{\tilde{r}_2,\tilde{r}_3\})_{r_{4,23}}$	0.1774	0.1190	0.9558	$0.9558 = K_{MED}(\tilde{r}_p,\tilde{r}_{i,j})$
$(\{\tilde{r}_5\},\{\tilde{r}_2,\tilde{r}_3\})_{r_{5,23}}$	0.1774	0.1086	0.9526	$0.9526 < K_{MED}(\tilde{r}_p,\tilde{r}_{i,j})$
$(\{\tilde{r}_3\},\{\tilde{r}_2,\tilde{r}_4\})_{r_{3,24}}$	0.1774	0.1207	0.9718	$0.9718 > K_{MED}(\tilde{r}_p,\tilde{r}_{i,j})$
$(\{\tilde{r}_5\},\{\tilde{r}_2,\tilde{r}_4\})_{r_{5,24}}$	0.1774	0.1086	0.9526	$0.9526 < K_{MED}(\tilde{r}_p,\tilde{r}_{i,j})$
$(\{\tilde{r}_2\},\{\tilde{r}_3,\tilde{r}_4\})_{r_{2,34}}$	0.1774	0.1242	0.9738	$0.9738 > K_{MED}(\tilde{r}_p,\tilde{r}_{i,j})$
$(\{\tilde{r}_5\},\{\tilde{r}_3,\tilde{r}_4\})_{r_{5,34}}$	0.2758	0.0860	0.9806	$0.9806 > K_{MED}(\tilde{r}_p,\tilde{r}_{i,j})$
$(\{\tilde{r}_2\},\{\tilde{r}_3,\tilde{r}_5\})_{r_{2,35}}$	0.1774	0.1227	0.9524	$0.9524 < K_{MED}(\tilde{r}_p,\tilde{r}_{i,j})$
$(\{\tilde{r}_4\},\{\tilde{r}_3,\tilde{r}_5\})_{r_{4,35}}$	0.2817	0.0933	0.9638	$0.9638 > K_{MED}(\tilde{r}_p,\tilde{r}_{i,j})$
$(\{\tilde{r}_2\},\{\tilde{r}_4,\tilde{r}_5\})_{r_{2,45}}$	0.1774	0.1239	0.9523	$0.9523 < K_{MED}(\tilde{r}_p,\tilde{r}_{i,j})$
$(\{\tilde{r}_3\},\{\tilde{r}_4,\tilde{r}_5\})_{r_{3,45}}$	0.2758	0.0957	0.9816	$0.9816 > K_{MED}(\tilde{r}_p,\tilde{r}_{i,j})$

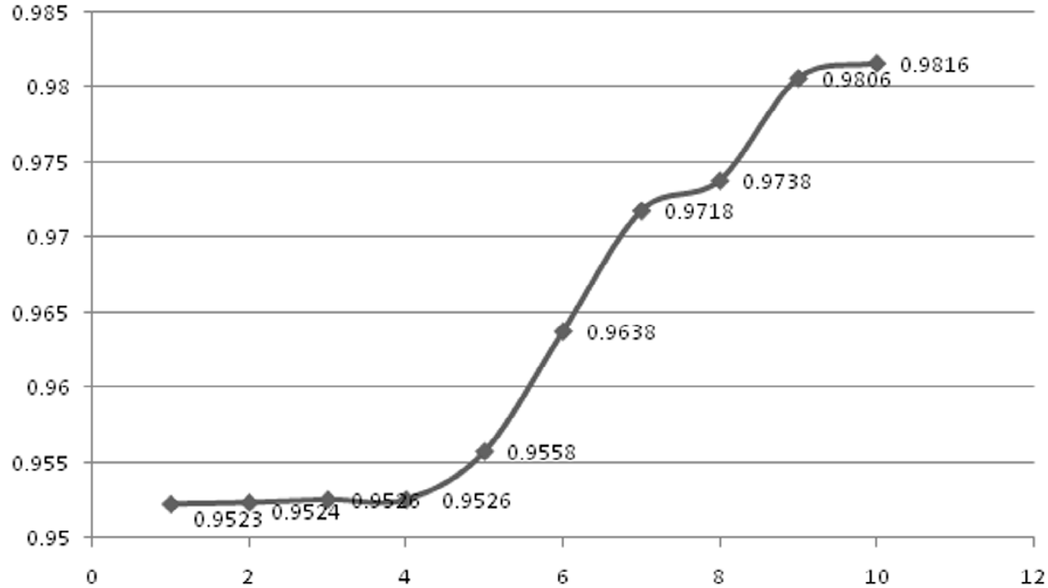

Figure 9: Correlation coefficient of 3-itemsets in ascending order ($r_{i,jk}$ refers $(\{\tilde{r}_i\},\{\tilde{r}_j,\tilde{r}_k\})$).

From Table-8, the itemsets of C_3 with highest correlation coefficient that satisfy the threshold condition $K(\tilde{r}_p,\tilde{r}_{i,j}) > K_{MED}(\tilde{r}_p,\tilde{r}_{i,j})$ can be considered for L_3.

Thus
$$L_3 = \left\{ \begin{array}{c} (\{\tilde{r}_3\},\{\tilde{r}_2,\tilde{r}_4\}),(\{\tilde{r}_2\},\{\tilde{r}_3,\tilde{r}_4\}), \\ (\{\tilde{r}_5\},\{\tilde{r}_3,\tilde{r}_4\}),(\{\tilde{r}_4\},\{\tilde{r}_3,\tilde{r}_5\}),(\{\tilde{r}_3\},\{\tilde{r}_4,\tilde{r}_5\}) \end{array} \right\}$$

From L_3 it can be observed that the combination $(\{\tilde{r}_3\},\{\tilde{r}_4\},\{\tilde{r}_5\})$ occurs more frequently than the combination $(\{\tilde{r}_2\},\{\tilde{r}_3\},\{\tilde{r}_4\})$. We can elicit three most interesting relationships

$$\left\{(\{\tilde{r}_5\},\{\tilde{r}_3,\tilde{r}_4\}),(\{\tilde{r}_4\},\{\tilde{r}_3,\tilde{r}_5\}),(\{\tilde{r}_3\},\{\tilde{r}_4,\tilde{r}_5\})\right\}$$

And in this combination of three interesting relationships, the one with the highest correlation coefficient is $\left(\{\tilde{r}_5\},\{\tilde{r}_3,\tilde{r}_4\}\right)$. Hence we can stop at this stage and derive some important trapezoidal intuitionistic fuzzy correlation rules as follows:

$$\left\{\{\tilde{r}_3\}\to\{\tilde{r}_4\},\{\tilde{r}_4\}\to\{\tilde{r}_3\},\{\tilde{r}_3\}\to\{\tilde{r}_5\},\{\tilde{r}_5\}\to\{\tilde{r}_3\},\{\tilde{r}_4\}\to\{\tilde{r}_5\},\{\tilde{r}_5\}\to\{\tilde{r}_4\}\right\} \quad \text{and}$$
$$\left\{\{\tilde{r}_5\}\to\{\tilde{r}_3,\tilde{r}_4\},\{\tilde{r}_3,\tilde{r}_4\}\to\{\tilde{r}_5\},\{\tilde{r}_4\}\to\{\tilde{r}_3,\tilde{r}_5\},\{\tilde{r}_3,\tilde{r}_5\}\to\{\tilde{r}_4\},\{\tilde{r}_3\}\to\{\tilde{r}_4,\tilde{r}_5\},\{\tilde{r}_4,\tilde{r}_5\}\to\{\tilde{r}_3\}\right\}$$

Step-4: To calculate the correlation coefficient between the selected decision variables, $\tilde{r}_i = ([a_i, b_i, c_i, d_i]; u_{i}, _{i})$, $i = 2, 3, 4$ and the trapezoidal intuitionistic fuzzy positive ideal solution $\tilde{r}^+ = ([1, 1, 1, 1]; 1, 0)$.

$$K_{TzIFS}(\tilde{r}_i,\tilde{r}^+) = \frac{C_{TzIFS}(\tilde{r}_i,\tilde{r}^+)}{\sqrt{E_{TzIFS}(\tilde{r}_i).E_{TzIFS}(\tilde{r}^+)}}.$$

Hence the calculated values are given as follows:

$K_{TzIFS}(\tilde{r}_3, \tilde{r}^+) = 0.7403$, $K_{TzIFS}(\tilde{r}_4, \tilde{r}^+) = 0.6782$, $K_{TzIFS}(\tilde{r}_5, \tilde{r}^+) = 0.7732$

Step-5: Rank all the alternatives A_i(i = 1, 2, 3, 4, 5) based on the values of $K_{TzIFS}(\tilde{r}_i, \tilde{r}^+)$.

$$A_5 > A_3 > A_4 .$$

Hence, the best supplier is A_5 among the chosen three suppliers. It can be observed that initially there were five suppliers and using the MAGDM-Miner, the final ranking is reduced to only three potential suppliers.

10. DISCUSSION AND FUTURE RESEARCH

This paper proposed an effective group decision mechanismwhich enhances the quality of the group decision making process, and thereby improving theperformance of any organization. This group decision model, whichdiffers from the traditional ones, based on extended MAGDM, and considers three aspects namelyattribute weights, alternative priorities, and group ideal solutionsto be taken into the construction. Therefore, the proposed model would result in adecision which is more realistic and acceptable for decision makers, because it utilizes correlation coefficient which is used for mining data sets and removing the unwanted variables from the decision making system.From the comparison, it is observed that the final ranking of the best alternative remains unchanged in all three methods. The advantage of the proposed MAGDM-Miner algorithm when compared with our earlier methods (Robinson & Amirtharaj, 2012b; 2014a; 2014b; 2014c) is that the less important or the uninteresting decision alternatives can be removed or neglected from the decision situation. Since the data is in the form of TzIFNs, our proposed method is advantageous than the other methods because it includes correlation rule mining in the decision making process through a newly proposed algorithm. The paper discussed how the conventional association rule mining could be supplemented with additional interesting measure based on statistical significance and correlation analysis together with the decision maker's weight determining methods. In future, various other statistical techniques like multi-variable correlation analysis can be utilized to develop new frameworks for mining huge dataset efficiently.

11. CONCLUSION

In this paper, the MAGDM problems under trapezoidal intuitionistic fuzzy environment was investigated, and a new method for decision making is proposed based on data mining techniques. The method of finding the correlation coefficient of trapezoidal intuitionistic fuzzy numbers (Robinson&Amirtharaj,2012b) was used for efficiently mining correlation rules for trapezoidal IFS. In the context of data-modeling and decision making, the new algorithm combines the mining of trapezoidal intuitionistic fuzzy correlation rules and MAGDM techniques for removing some of the uninteresting, unwanted and/or less-important decision variables from the decision making environment, especially when huge data is involved. Also the unknown weights of the decision makers were determined using eigen value matrix and normalized eigen values using Jacobi method. Finally, an illustration was presented to demonstrate and validate the effectiveness of the proposed method. This method of decision making together with the MAGDM Mining for TzIFNs proves to be a better technique because of its exclusiveness in dealing with imprecise data. The proposed method could be applied in complicated domains where large measures of vagueness could be found.

REFERENCES

[1] Adewumi,J.R., Ilemobade,A.A., van Zyl, J.E., (2013). Application of a Multi-Criteria Decision Support Tool in Assessing the Feasibility of Implementing Treated Wastewater Reuse , *International Journal Decision Support Systems Technology*. 5(1), 1-23.

[2] Agrawal.R, Imielinski.T and Swami.A.(1993).Mining Association Rules between Sets of Items in Large Databases.*Proceedings of the ACM SIGMOD International Conference on Management of Data*, Washington D.C., 207-216.

[3] Atanassov, K. (1986). Intuitionistic fuzzy sets.*Fuzzy Sets and Systems*, 20, 87- 96.

[4] Atanassov, K. (1989).More on intuitionistic fuzzy sets.*Fuzzy Sets and Systems*, 33, 37- 46.

[5] Bandemer, H &Nather, W, (1992).Fuzzy Data Analysis.Kluwer AcademicPublishers, Dordrecht.

[6] Bustince, H.,&Burillo, P. (1995).Correlation of interval-valued intuitionistic fuzzy sets.*Fuzzy Sets and Systems*, 74, 237-244.

[7] Buckley, J.J., Reilly, K., Zhang, X., (2004), Fuzzy probabilities for web planning, *Soft Computing*, 8, 464-476.

[8] Chaudhuri, B. B. and Bhattachary, P., (2001), On correlation between fuzzy sets. *Fuzzy Sets and Systems*, 118, 447-456.

[9] Chen, S.M.,& Tan,J.M. (1994). Handling multi-criteria fuzzy decision making problems based on vague sets. *Fuzzy Sets and Systems*,67, 163-172.

[10] Chen, S. H., & Hsieh, C. H., (1999).Graded Mean Integration Representation of

[11] Generalized Fuzzy Number, *Journal of the Chinese Fuzzy System Association* **5**(2),1-7.

[12] Cheng, S. K. (2000). Development of a Fuzzy Multi-Criteria Decision Support System for Municipal Solid Waste Management.*A master thesis of applied science in Advanced Manufacturing and Production Systems,* University of Regina, Saskatchewan.

[13] Dubois, D.E.&Prade.H. (2003).A Note on Quality Measures for Fuzzy Association Rules.*Proceedings of the 10thInternational Fuzzy Systems Association World Congress (IFSA-03), Lecture Notes in Artificial Intelligence*- 2715, *Springer-Verlag*, 346-353.

[14] Gau, W.L., Buehrer, D.J., (1994), Vague sets, *IEEE Transactions on Systems, Man and Cybernetics, Part A*, 23, 610 - 614.

[15] Gerstenkorn, T., Manko, J., (1991), Correlation of intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 44, 39-43.

[16] Haleh, H., Ghaffari, A., and Meshki, A.K. (2012). A Combined Model of MCDM and Data Mining for Determining Question Weights in Scientific Exams.*Applied Mathematical Sciences*, 6(4), 173 – 196.

[17] Han, J, &Kamber, M. (2006). Data Mining: Concepts and Techniques. *Elsevier*.

[18] Hayez, Q., De Smet, Y.,&Bonney, J. (2012).D-Sight: A New Decision Making Software to Address Multi-Criteria Problems. *International Journal Decision Support Systems Technology*. 4(4), 1-23.

[19] Herrera, F., Martinez, L.,&Sanchez,P.J. (1999).Managing non-homogenous information in group decision making.*European Journal of Operational Research*, 116, 115-132.

[20] Hong, D. H., (1998), A note on correlation of interval- valued intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 95, 113-117.

[21] Hong, D.H , Choi D.H., (2000). Multicriteria fuzzy decision making problems based on vague set theory, *Fuzzy Sets and Systems*, 114 , 103-113.

[22] Hong, D. H. and Hwang, S. W., (1995), Correlation of intuitionistic fuzzy sets in probability spaces.*Fuzzy Sets and Systems*, 75, 77-81.

[23] Hung, W. L. and Wu, J. W., (2002), Correlation of intuitionistic fuzzy sets by centroid method. *Information Sciences*, 144, 219-225.

[24] Kao,C.,& Liu, S.T., (2002), Fuzzy measures for correlation coefficient of fuzzy numbers, *Fuzzy Sets and Systems*, 128, 267-275.

[25] Kaplan, P,O,. (2006). A New Multiple Criteria Decision Making Methodology for Environmental Decision Support.*A dissertation to the Graduate Faculty of North Carolina State University for the Degree of Doctor of Philosophy*.Raleigh, North Carolina.

[26] Khan, S., Ganguly, A.R., & Gupta, A., (2008). Data Mining and Data Fusion for enhanced Decision Support. *Hand-Book on Decision Support Systems*. 1(4), 581-608.

[27] Kweku-Muata&Osei-Bryson, (2004).Evaluation of decision trees: a multi-criteria approach. *Computers & Operations Research*. 31, 1933 – 1945.

[28] Laurent, A. (2003).Generating fuzzy summaries: a new approach based on fuzzyMulti-dimensional databases. Intelligent Data Analysis Journal, 7(2), 155-177.

[29] Li, D.F. (1999). Fuzzy Multi attribute decision making models and methods with incomplete information. *Fuzzy Sets and Systems*, 106(2), 113-119.

[30] Li, D.F. (2005). Multi attribute decision making models and methods using intuitionistic fuzzy sets. *Journal of Computer and System Sciences*, 70, 73-85.

[31] Li, D.F. (2008). Extension of the LINMAP for multi attribute decision making under Atanassov's intuitionistic fuzzy environment. *Fuzzy Optimization and Decision Making,* 7(1), 17-34.

[32] Li, D.F.,& Nan, J.X. (2011).Extension of TOPSIS for Multi-attribute group decision making under Atanassov IFS environments.*International Journal of Fuzzy System Applications*, 1(4), 47-61.

[33] Lin, N.P. Chen, H-J. Chueh, H-E. Hao, W-H. & Chang, C-I. (2007). A Fuzzy Statistics based method for mining fuzzy correlation rules, *WSEAS Transactions on Mathematics*, 6(11),852-858.

[34] Liu, H.W. (2004). Vague set methods of multi-criteria fuzzy decision making, *Systems Engineering-theory & Practice,* 5(5): 214–220.

[35] Liu, P. D., (2009). A novel method for hybrid multiple attribute decision making, *Knowledge-based Systems*, 22(5), 388-391.

[36] Liu, P. D., & Guan, Z. L.(2008). Research on group decision making based on the vague set and hybrid aggregation operators, *Journal of Wuhan University of Technology*, 30(10): 152–155.

[37] Liu, P. D., & Guan, Z. L. (2009). An approach for multiple attribute decision-making based on Vague sets, *Journal of Harbin Engineering University*, 30(1), 106–110.

[38] Liu, J., Li, Y., Deng, X., Wei, D., & Deng, Y. (2012). Multi-Criteria Decision Making based on Interval-valued Intuitionistic Fuzzy sets. *Journal of Information and Computational Science*. 9 (4), 1107-1114.

[39] Liu, P. D., & Wang, T. J. (2007). A method for multiple attribute decision making with triangular fuzzyNumber and partial attribute weight information, *Journal of Information and Computational Science*, 4(3): 1017–1022.

[40] Mitchell, H.B, (2004), A correlation coefficient for intuitionistic fuzzy sets, *International Journal of Intelligent Systems*, 19, 483-490.

[41] Nayagam, L. G., Muralikrishnan, S., & Sivaraman, G. (2011). Multi-Criteria Decision Making based on Interval-Valued Intuitionistic Fuzzy Sets. *Expert Systems with Applications*, 38, 1464-1467.

[42] Park, D.G., Kwun, Y. C., Park, J.H., & Park, I.Y. (2009).Correlation coefficient of interval-valued intuitionistic fuzzy sets and its application to multiple attribute group decision making problems. *Mathematical and Computer Modeling*, 50, 1279-1293.

[43] Peng, Y., Zhang, Y., Tang, Y., & Shiming, L., (2011). An Incident Information Management framework based on Data Integration, Data mining, and Multi Criteria Decision Making, *Decision Support Systems,* 51(2), 316-327.

[44] Power, D.J., (2013). Engineering effective decision support technologies: New models and applications. Hershey: IGI Global.

[45] Power, D.J., Roth, R.M., & Karsten, R. (2011).Decision support for Crisis Management. *International Journal of Decision Support System Technology*, 3(2), 44-56.

[46] Robinson, J.P., Amirtharaj, E.C.H, (2011a). A short primer on the Correlation coefficient of Vague sets, *International Journal of Fuzzy System Applications*, 1(2), 55-69 .

[47] Robinson, J.P., Amirtharaj, E.C.H, (2011b). Extended TOPSIS with correlation coefficient of Triangular Intuitionistic fuzzy sets for Multiple Attribute Group Decision Making, *International Journal of Decision Support System Technology*, 3(3), 15-40.

[48] Robinson, J.P., Amirtharaj, E.C.H, (2012a). Vague Correlation coefficient of Interval Vague sets, *International Journal of Fuzzy System Applications*, 2(1), 18-34.

[49] Robinson, J.P., Amirtharaj, E.C.H, (2012b). A Search for the Correlation coefficient of Triangular and Trapezoidal intuitionistic Fuzzy sets for Multiple Attribute Group Decision Making,*Communications in Computer and Information Science - 283, Springer-Verlag*, 333-342.

[50] Robinson, J.P., Amirtharaj, E.C.H, (2014a). Efficient Multiple Attribute Group Decision Making models with Correlation coefficient of Vague sets, *International Journal of OperationsResearch and Information Systems*. 5 (3), 27-51.

[51] Robinson, J.P., Amirtharaj, E.C.H, (2014b). MAGDM Problems with Correlation coefficient of Triangular Fuzzy IFS, *International Journal of Fuzzy System Applications*, 4 (1), 1-32.

[52] Robinson, J.P., Amirtharaj, E.C.H, (2014c). MADM Problems with Correlation Coefficient of Trapezoidal Fuzzy Intuitionistic Fuzzy Sets, *Advances in Decision Sciences*, Volume 2014, Article ID 159126, 1-10. http://dx.doi.org/10.1155/2014/159126.

[53] Robinson, J.P., Sonia, M.D., & Rajasekaran, S. (2015). Mining Efficient Fuzzy Bio-Statistical Rules for Association of Sandalwood in Pachaimalai Hills, *International Journal of Agricultural and Environmental Information Systems.* 6 (2), 40-76.

[54] Szmidt, E. &Kacprzyk, J., (2000), Distances between intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 114, 505-518.

[55] Szmidt, E.,&Kacprzyk,J. (2002).Using intuitionistic fuzzy sets in group decision making.*Control and Cybernetics*, 31, 1037-1053.

[56] Szmidt, E.,&Kacprzyk,J. (2003).A consensus-reaching process under intuitionistic fuzzy preference relations.*International Journal of Intelligent Systems*, 18, 837-852.

[57] Wei, G., (2008). Induced intuitionistic fuzzy ordered weighted averaging operator and its application to MAGDM, *LNAI-5009*, *Springer-Verlag Berlin Heidelberg*.

[58] Wei, G., (2010). Some arithmetic aggregation operators with intuitionistic trapezoidal fuzzy numbers and their application to group decision making. *Journal of Computers*, 5 (3), 345-351.

[59] Wei, G., Wang H.J.,&Lin,R., (2011). Application of correlation coefficient to interval-valued intuitionistic fuzzy multiple attribute decision-making with incomplete weight information. *Knowledge and Information Systems*.26, 337-349.

[60] Wei, G., Zhao,X., (2012). Some induced correlated aggregating operators with intuitionistic fuzzy information and their application to multiple attribute group decision making. *Expert Systems with Applications*, 39, 2026–2034.

[61] Wei, G., Zhao,X., Lin,R., Wang H.J. (2012). Generalized triangular fuzzy correlated averaging operator and their application to multiple attribute decision making.*Applied Mathematical Modelling.* 36, 2975–2982.

[62] Wu, D., &David L. Olson. (2006). A TOPSIS Data MiningDemonstration andApplicationto Credit Scoring. *International Journal of Data Warehousing and Mining,* 2(3), 16-26.

[63] Xu, Z.S., &Yager,R.R. (2006).Some geometric aggregation operators based on Intuitionistic Fuzzy sets.*International Journal of General Systems*, 35, 417-433.

[64] Zadeh, L. A., (1965), Fuzzy sets. *Information Control*, 8, 338-353.

[65] Zeng, W., & Li, H., (2007). Correlation Coefficient of Intuitionistic Fuzzy sets.*Journal of Industrial Engineering International,*3, 33-40.

[66] Zhang, J., Liu, H., Ling, T.W., Bruckner, R.M., Tjoa, A.M. (2006). A Framework for EfficientAssociation Rule Miningin XML Data, *Journal of Data Base Management,* 17(3), 19-40.