IJCTA, 8(3), 2015, pp. 1235-1242

Design of Hybrid Model Predictive Controller for a Hybrid Three Tank System

Cyril Joseph*, Dr. V.I. George, Neethu Narayanan and Saranya T.S.

Abstract: The hybrid systems contain two different types: subsystems with continuous dynamic behavior and subsystems with discrete dynamic behavior that not only coexist but also interact with one another. In recent years study of hybrid systems has caught the attention of many researchers. Motivation for this is due to the reason that almost all the systems in laboratory and industries exhibits hybrid dynamic behavior. And designing a controller for such a system must take into account the challenges of interacting continuous and discrete dynamics within the system. Model Predictive Controller is a widely appreciated controller however in this paper a hybrid MPC controller is designed for a three-tank benchmark system modelled as a hybrid system. The controller so designed was found to be stable displaying no oscillations even under load and servo disturbances applied.

***Keywords**:* Hybrid Systems, Three Tank System, Model Predictive Control (MPC)

1. INTRODUCTION

Many dynamical systems incorporate behaviors typical of continuous-time systems with behaviors of discrete-time systems. For example, in switched electrical circuit, currents and voltages that change continuously according to classical network laws also change discontinuously due to the opening or closing of switches. Also some biological system behaves similarly, with continuous change during normal operations and discontinuous change due to an impulsive stimulus. Finally, modern control algorithms often lead to both kind of behavior, either digital components used in implementations or logic and decision making encoded in the control algorithm. These examples fit into the class of hybrid dynamical systems [1], or simply hybrid systems. Hybrid systems [2] feature difference equations for discrete dynamics and differential equations for continuous dynamics. Hybrid systems arise in embedded control when digital computers, controllers and subsystems modeled as finite-state machines are coupled with plants and controllers modeled by partial or ordinary differential or difference equations. Thus, such system arises whenever one mix logical decision making with generation of continuous-valued control laws. Generally speaking, hybrid systems [3] are mixtures of real time dynamics and discrete events. These continuous and discrete dynamics not only coexist, but interact and changes occur both in responses to discrete as well as continuous events.

There are five subclasses [4] for hybrid systems. They are Mixed logical dynamical (MLD) systems, piecewise affine (PWA) systems, linear complementarity (LC) systems, max min-plus-scaling (MMPS) systems and extended linear complementarity (ELC) systems. The five subclasses of hybrid systems are equivalent [5]. These equivalences are based on assumptions related to the well-posedness (which means existence and uniqueness of solution trajectories) and boundedness of system variables.

Manipal Institute of Technology, Manipal, Udupi, 576104
* e-mail: cyril.cholenkery@gmail.com

Hybrid models are important for number of problems in the analysis of systems, for example, the computations of trajectory, stability, control, and safety analysis. Several good control approaches were proposed in various literature. However, model predictive control approach for hybrid systems are most promising ones at the moment, and are thoroughly investigated in recent years. In this paper the focus is on structured controller design for an industrially relevant hybrid system which is considered to be a benchmark i.e. a three tank flow system .The controller designed here is an extension of the model predictive control (MPC) [6] framework for continuous systems, so as to adapt for hybrid systems. The MPC is a very popular control scheme employed in industries such as oil refining and process industry and has satisfactorily proved its usefulness. MPC offers many attractive features that make this control scheme interesting and relevant for extension to hybrid systems.

The paper is organized as follows. In Sec. 2 the modeling of three tank system as a hybrid system is discussed. The problems of model predictive control of hybrid systems are addressed in Sec. 3. The proposed algorithm, applied to the model of a three tank system is discussed in Sec. 4. The results are given in Sec. 5. Sec. 6 deals with the conclusion and future scope.

2. MODELING OF THE THREE-TANK SYSTEM

The system consists of three tanks [7] with same cross-sectional area and are supported with two independent identical pumps on Tanks, q_1 and q_2. Each tank in the system is interconnected to each other by the switching valves V_{23} and V_{13}, which are normally open and close depending on the conditions. The liquid levels in each tank are defined as h_1, h_2 and h_3 and these are measured with continuous valued level sensors. The output flow of system is through the three valves on the three tanks V_1, V_2 and V_3 respectively.

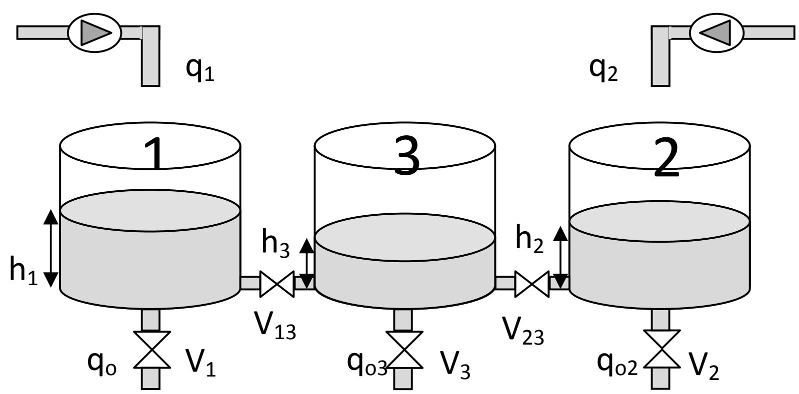

Figure 1: The three tank benchmark system model

The material balance equation for the model shown above is given as:

$$\frac{dh_1}{dt} = \frac{q_1}{S_1} - sign(h_1 - h_3)V_{13}\frac{S_{13}}{S_1}\sqrt{|2g(h_1 - h_3)|} - V_1\frac{S_{01}}{S_1}\sqrt{2gh_1} \tag{1}$$

$$\frac{dh_2}{dt} = \frac{q_2}{S_2} - sign(h_2 - h_3)V_{23}\frac{S_{23}}{S_2}\sqrt{|2g(h_2 - h_3)|} - V_2\frac{S_{02}}{S_2}\sqrt{2gh_1} \tag{2}$$

$$\frac{dh_3}{dt} = sign(h_1 - h_3)V_{13}\frac{S_{13}}{S_3}\sqrt{|2g(h_1 - h_3)|} + sign(h_2 - h_3)V_{23}\frac{S_{23}}{S_3}\sqrt{|2g(h_2 - h_3)|} - V_3\frac{S_{o3}}{S_3}\sqrt{2gh_3} \tag{3}$$

Where:

h_1, h_2, h_3 – levels in respective tanks

S_1, S_2, S_3 – cross sections of tanks (tanks dimensions are equal)

S_{13}, S_{23} – cross section of digital valves between tanks

S_{o1}, $S_{o2,}$ S_{o3} – cross section of output valves

q_1, q_2 – inflow through pumps

$V_{12,}$ V_{23} – status of digital valves between tanks (0-closed, 1-opened)

$V_{1,}$ V_2, V3 – status of output valves (0-closed, 1-opened)

g –gravitational acceleration

Given that for the system at hand S=S_1, S2, S3, S_{i3}=S_{13}= S_{23}, S_{oi}=S_{o1}=S_{o2}, i=1, 2 and for different problems, the equations become simpler. These equations are linearized for further needs.

Basic parameters of the system are:

A	0.150	diameter of tanks
V	0.015m	diameter of digital valves
V_3	0.030m	diameter of output valves
L	0.28m	maximum water level in tank
T_s	0.01	variable sampling time

Problem:

Level of water in each tank should be constant and set at following values, h1=60mm, h_2=40mm, h_3=20mm. Both pumps are working, so there are two inputs, but only one output valve V_3 is (always) opened. (There is an assumption that h_1>h_3 and h_2>h_3)

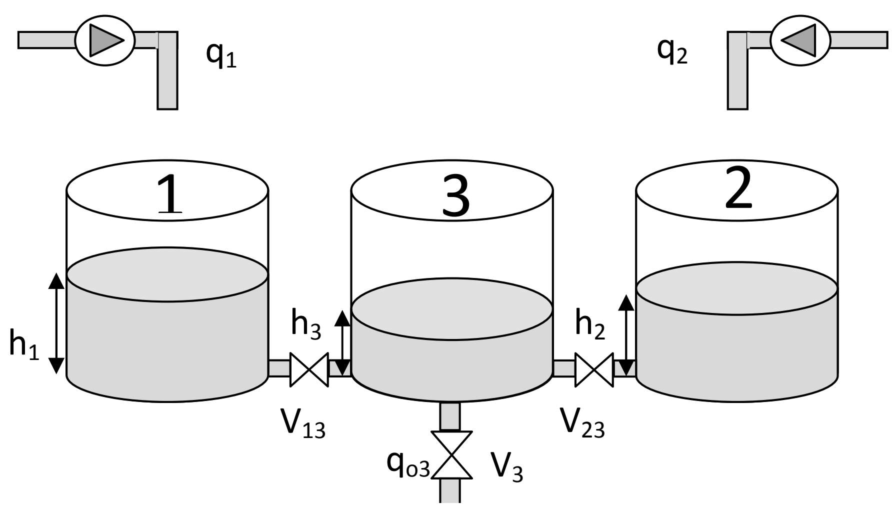

Figure 2: The three tank system model

Linearized equations take the form:

$$\frac{d\Delta h_1}{dt} = \frac{\Delta q_1}{S} - \left(V_{13} \frac{S_{i3}}{S} \frac{g}{\sqrt{2g(h_{10} - h_{30})}} \Delta h_1 - V_{13} \frac{S_{i3}}{S} \frac{g}{\sqrt{2g(h_{10} - h_{30})}} \Delta h_3 \right) \tag{4}$$

$$\frac{d\Delta h_2}{dt} = \frac{\Delta q_2}{S} - \left(V_{23} \frac{S_{i3}}{S} \frac{g}{\sqrt{2g(h_{20} - h_{30})}} \Delta h_2 - V_{23} \frac{S_{i3}}{S} \frac{g}{\sqrt{2g(h_{10} - h_{30})}} \Delta h_3 \right) \tag{5}$$

$$\begin{aligned} \frac{d\Delta h_3}{dt} = {} & V_{13} \frac{S_{i3}}{S} \frac{g}{\sqrt{2g(h_{10} - h_{30})}} \Delta h_1 - V_{13} \frac{S_{i3}}{S} \frac{g}{\sqrt{2g(h_{10} - h_{30})}} \Delta h_3 \\ & + V_{23} \frac{S_{i3}}{S} \frac{g}{\sqrt{2g(h_{20} - h_{30})}} \Delta h_2 - V_{23} \frac{S_{i3}}{S} \frac{g}{\sqrt{2g(h_{10} - h_{30})}} \Delta h_3 \\ & + - \frac{S_{oi}}{S} \frac{g}{\sqrt{2gh_{30}}} \Delta h_3 \end{aligned} \tag{6}$$

Hence the state-space model matrices are:

$$A = \begin{bmatrix} -C_{13} & 0 & C_{13} \\ 0 & -C_{23} & C_{23} \\ C_{13} & C_{23} & -C_{13} - C_{23} - C_3 \end{bmatrix}, B = \begin{bmatrix} \frac{1}{S} & 0 \\ 0 & \frac{1}{S} \\ 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{7}$$

where:

$$C_{13} = V_{13} \frac{S_{i3}}{S} \frac{g}{\sqrt{2g(h_{10} - h_{30})}} \tag{8}$$

$$C_{23} = V_{23} \frac{S_{i3}}{S} \frac{g}{\sqrt{2g(h_{20} - h_{30})}} \tag{9}$$

$$C_3 = \frac{S_{oi}}{S} \frac{g}{\sqrt{2gh_{30}}} \Delta h_3 \tag{10}$$

3. MODEL PREDICTIVE CONTROL

Model predictive control (MPC) is a leading control technique used for difficult multivariable control problems. The basic theory can be summed up as. If we wish to control a MIMO process, satisfying inequality constraints on the output and input variables. If a definite dynamic model of the process is achievable, current measurements can be used to predict the future values of outputs. Then the relevant changes in the input variables can be determined based on both the predictions and the measurements. The changes in the particular input variables are coordinated after seeing the input-output relationships described by the process model.

MPC can be also used to control hybrid systems [8]-[9]. This can be realized by computing the cost function for different configurations of discrete values and choosing one assuring the smallest result. The algorithm used in this report is presented below:

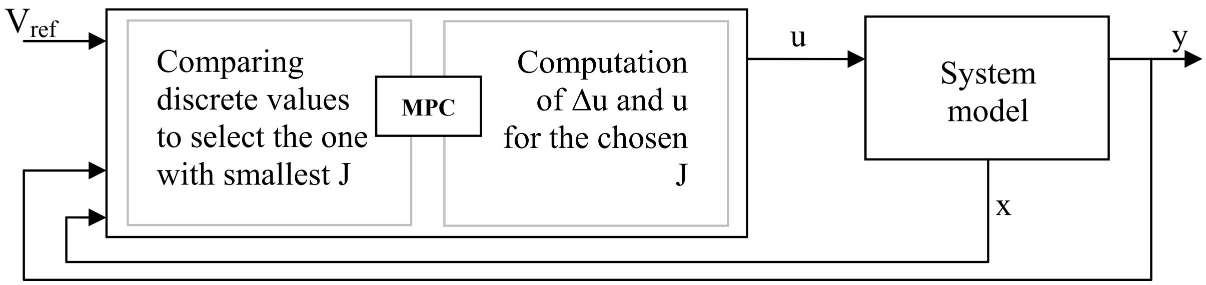

Figure 3: MPC block diagram

4. MATLAB IMPLEMENTATION

The model implemented in this paper is a modified version of the one proposed in [10], so as to be compatible for hybrid systems.

A general continuous system in state space is defined by:

$$\dot{x}\,(t) = Ax(t) + Bu(t) \qquad (11)$$
$$y\,(t) = Cx(t)$$

however, the augmented discrete time model for use in predictive control is given by

$$\begin{bmatrix} \Delta x(k+1) \\ y(k+1) \end{bmatrix} = \begin{bmatrix} A & o^T \\ CA & I_{q\times q} \end{bmatrix} \begin{bmatrix} \Delta x(k) \\ y(k) \end{bmatrix} + \begin{bmatrix} B \\ CB \end{bmatrix} \Delta u(k) \qquad (12)$$

where the new state variable vector $x(k) = [\Delta x(k)^T y(k)^T]^T$ and $I_{q\times q}$ is the identity matrix with dimensions $q \times q$ and o is a zero matrix with dimensions $q \times n_1$.

The cost function and control signal computed according to the receding horizon principle are

$$\Delta U = (\varphi^T\varphi + \overline{R})^{-1}(\varphi^T\overline{R_s}r(k_i) - \varphi^T Fx(k_i)) \qquad (13)$$

$$J_{min} = (R_s - Fx(k_i))^T(R_s - Fx(k_i) - (R_s - Fx(k_i))^T\varphi(\varphi^T\varphi + \overline{R})^{-1}\varphi^T(R_s - Fx(k_i) \qquad (14)$$

The hybrid MPC controller design algorithm is shown in the flowchart below:

5. RESULTS

The MPC controller designed is found to perfectly stabilize the system as shown in Fig. 5(a) which in it can be observed that the level in tanks h_1, h_2, and h_3 holds their set points at 60, 40 and 20cm respectively. The input flow control signal is shown in Fig. 5(b)

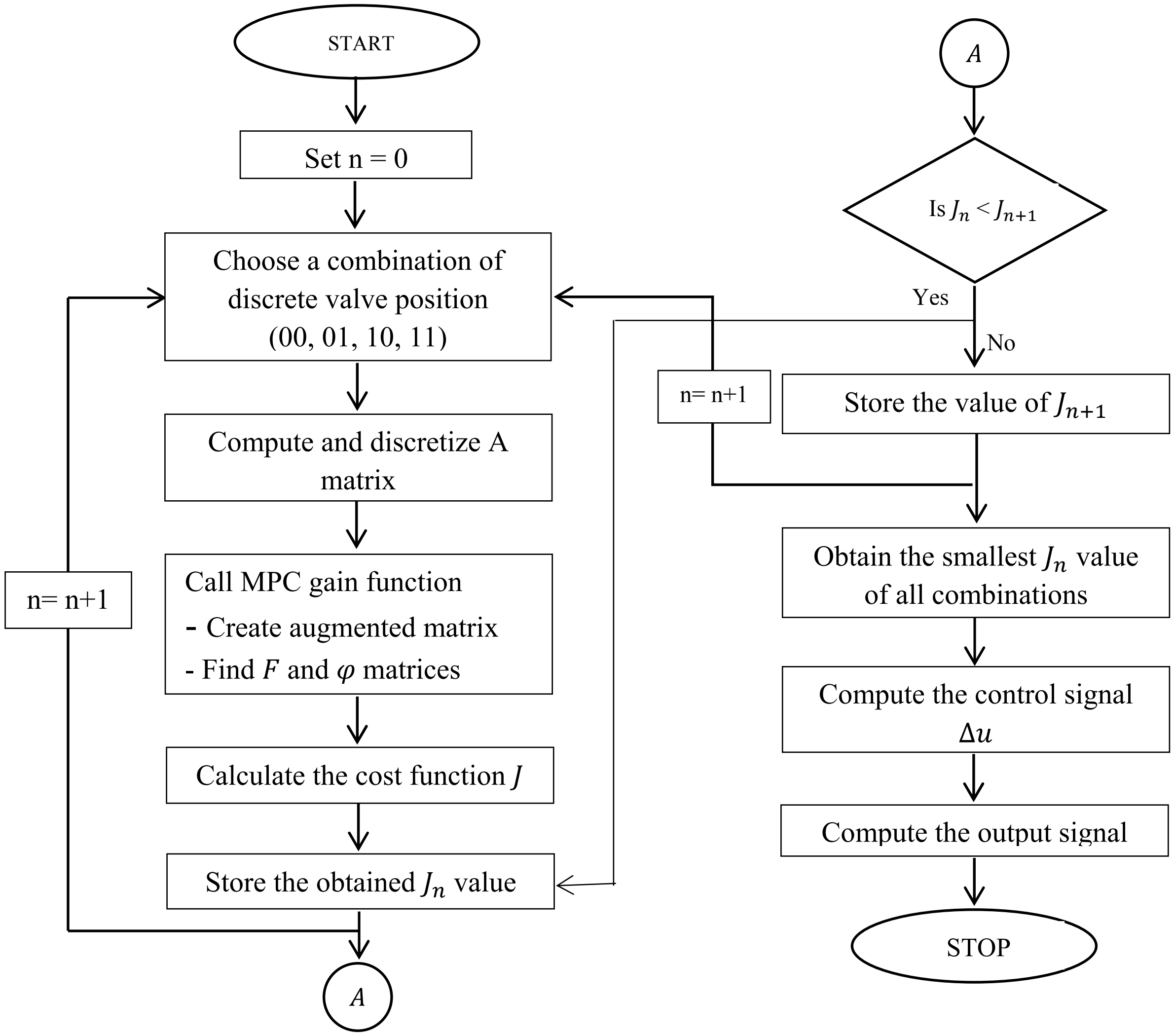

Figure 4: Flow chart for hybrid MPC controller design

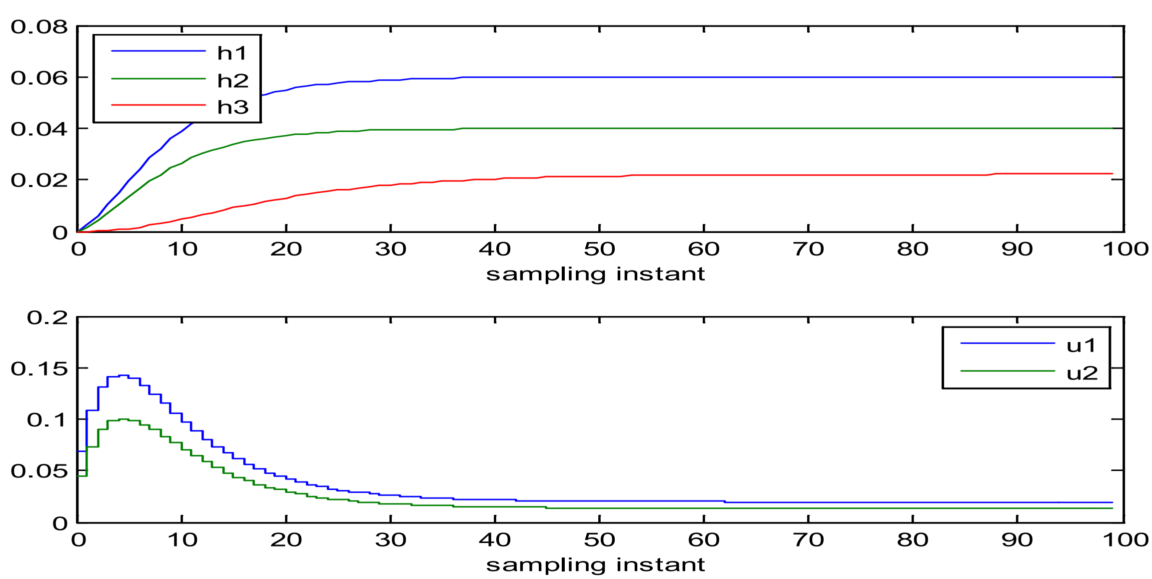

Fig. 5: (a) Output: Level of the tanks (b) Control: Input flow

In order to verify the stability of the controller, a servo disturbance was introduced at the 80th sampling instant, however the system was able to track the new set points of 70, 50 and 25cm for tanks h_1, h_2, and h_3 within an interval of 20 sampling instants. This can be observed in Fig. 6(a) & (b)

For further verification a load disturbance was applied, again at the 80th sampling time, and the controller designed proved to be capable of withstanding this too by tracking back the original set point within matter of seconds.

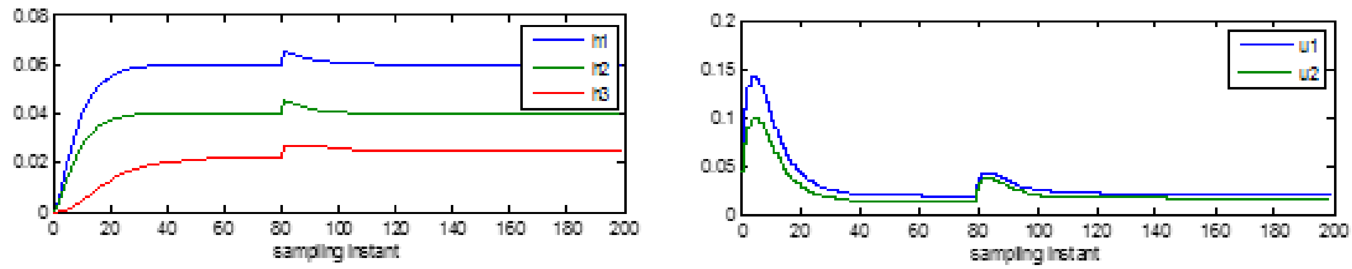

Figure 6: Servo disturbance (a) Tank levels (b) Control signal variations

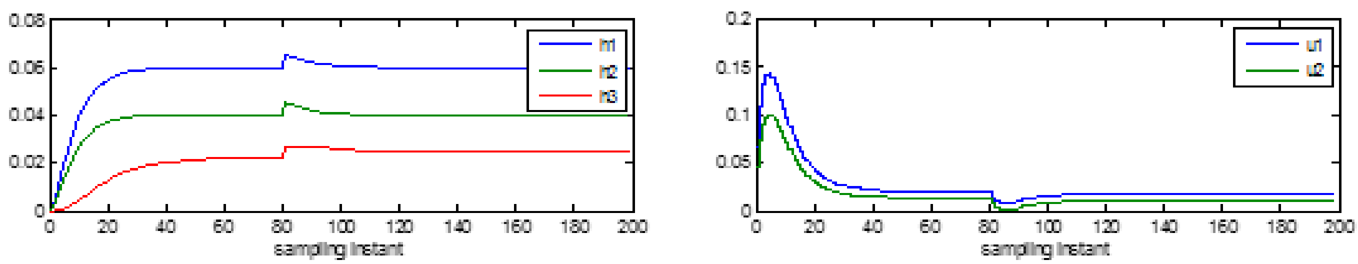

Figure 7: Load disturbance (a) Tank levels (b) Control signal variations

6. CONCLUSION

Although model predictive control theories are developed for linear systems, it is evident from this paper that they work justifiably for linearized hybrid systems as well with modifications as depicted in the above algorithm. The response obtained from the hybrid MPC controlled system designed was found to be smooth without any oscillations or overshoots. The system remained stable even under application of servo changes by varying input as well as the applied load disturbances. The response achieved the required set points in minimum time and even under duress caused by the load and servo changes.

References

1. Rafal Goebel, Ricardo G. Sanfelice, and Andrew R. Teel. Hybrid Dynamical Systems. *IEEE control systems magazine*, April 2009.
2. Branicky, M.S. *Studies in Hybrid Systems: Modeling, Analysis, and Control*. ScD thesis, Massachusetts Institute of Technology, Cambridge, MA, 1995.
3. M.S. Branicky, V.S. Borkar, and S.K. Mitter. A unified framework for hybrid control: Model and optimal control theory. *IEEE Transactions on Automatic Control*, 43(1):31–45, 1998.
4. W.P.M.H. Heemels. J.M. Schumacher. and S. Weiiahd. Linear complementarity systems. *SIAM J. Appl. Math*. G0(4): 1234-1269, 2000.

5. W.P.M.H. Heemels, B. De Schutter, and A. Bemporad Equivalence of hybrid dynamical models. *Automotica*, 37(7), July 2001.

6. Eduardo F. Camacho, Carlos Bordons, *Model Predictive Control*, Springer, London 1999.

7. J.L. Villa, M. Duque, A. Gauthier, N. Rakoto-Ravalontsalama, *MLD Control of Hybrid Systems: Application to the Three-Tank Benchmark Problem,* 2003.

8. M. Lazar, W. P. M. H. Heemels, S. Weiland, A. Bemporad, Stabilizing Model Predictive Control of Hybrid Systems*, IEEE Transactions on automatic control*, Vol. 51, No 11, November 2006.

9. A. Bemporad, W.P.M.H. Heemels, and B. De Schutter, On hybrid systems and closed-loop MPC systems, *IEEE Transactions on Automatic Control*, vol. 47, no. 5, pp. 863–869, May 2002.

10. Linping Wang, *Model Predictive Control System Design and Implementation Using MATLAB,* Springer, London 2009.