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PID, I-PD and PD-PI Controller Design for the Ball and Beam System: A Comparative Study

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Abstract : This article addresses the design of PID, I-PD and PD-PI controller for the ball and beam system. The ball and beam system, a non linear one, has been linearised around the equilibrium point to obtain the linearised model transfer function. The PID controller has been tuned using ITAE criterion and the same set of parameters has been utilized for designing both the I-PD and PD-PI controller. The performance of the system with the above mentioned controllers has been compared. The result of comparison reveals that PD-PI controller outperforms both the PID and I-PD controller. In future, for improving the performance of a system, PD-PI controller can be designed for other class of plants.

Keywords : Ball and beam system; PID; I-PD; PD-PI.

1. INTRODUCTION

The ball and beam system (BBS) is considered as one of the benchmark problems for studying control system. The task of controller design becomes very difficult because of its non linear and unstable behaviour. A variety of classical and modern control techniques have been adopted to obtain satisfactory response from the BBS. In [1], nonlinear PD regulation for the BBS has been explained. Synchronization of BBS with neural compensation is discussed in [2]. Oh *et al.* described the design of a fuzzy cascade controller for BBS. Sliding mode control of BBS has been illustrated in [4]. In [5], interpolating sliding mode observer for BBS is discussed. Jerome *et al.* in their paper [6] discussed the non linear controller design for the BBS using state dependent Riccati equation. H-infinity PID controller design has been discussed in [7] for BBS. In [8, 9], fractional order controller design for BBS is explained. Keshmeri *et al.* explained modeling and control of BBS using model based and non model based approach [10].

PID controller has always been a choice of control engineers for a long time because of its simple structure and easy implementation. But the performance of a system with PID controller can be significantly improved by slightly modifying the control structure.

In this paper, apart from designing PID controller, I-PD and PD-PI controller has also been designed for the BBS. The non linear model of BBS has been linearised around the equilibrium point to obtain linearised model transfer function. The PID controller parameters have been identified by minimizing the ITAE criterion. The same controller parameters have been utilized for the design of I-PD and PD-PI controller. It has been found that, with the same set of controller parameters, I-PD controller has the potential to provide better response in terms of overshoot, gain margin, phase margin and complementary sensitivity as compare to the PID controller. It has also been observed that PD-PI controller outperforms both PID and I-PD controller.

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This paper consists of five sections. Section 1 is about the introduction of the paper. In section 2, dynamics of the BBS is discussed and linearised model transfer function has been obtained. Section 3 deals with the design of PID, I-PD and PD-PI controller. Along with discussions, the response of the system with different controllers has been provided in Section 4. The conclusion part and the future scope of research are highlighted in section 5.

2. BALL AND BEAM SYSTEM

In BBS, a ball is placed on a beam where it is allowed to move along the length of the beam. A lever arm is attached to a servo gear at one end and the beam at the other. The lever changes the angle of the beam by α when the servo gears turns by an angle of θ. Gravity causes the ball to roll along the beam when there is any change of beam angle from horizontal position. In this study, it has been assumed that ball rolls without slipping and friction between ball and beam is very small (negligible).

The schematic diagram of BBS is depicted in Fig. 1

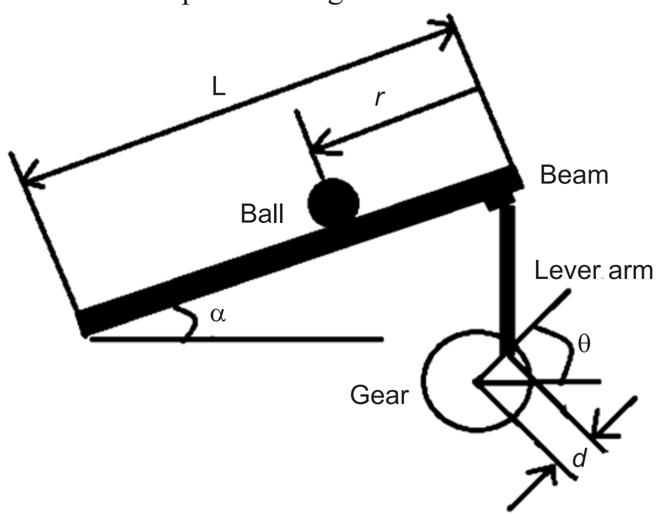

Figure 1: Schematic diagram of BBS

Parameters and their values associated with the BBS are summarized in Table. 1 [11]

Table 1
Parameters and their values associated with BBS

Symbol	*Description*	*Value*
m	Mass of the ball	0.11 kg
R	Radius of the ball	0.015 m
d	Lever arm offset	0.04 m
g	Gravitational Acceleration	9.8 m/s^2
L	Length of the beam	0.4 m
J	Ball's moment of inertia	$\frac{2}{5}mR^2$ kgm^2
r	Ball position coordinate	–
α	Beam angle coordinate	–
θ	Servo gear angle	–

The Lagrangian equation of motion for the ball can be expressed by the following differential equation:

$$\left(\frac{J}{R^2}+m\right)\ddot{r} + mg\sin(\alpha) - mr\left(\dot{\alpha}\right)^2 = 0 \tag{1}$$

The linear approximation of the system can be found by Linearizing the above equation around the beam angle, $\alpha = 0$, and is given by

$$\left(\frac{J}{R^2} + m\right)\ddot{r} = -mg\alpha \tag{2}$$

The linear relation between beam and gear angle can be approximated as

$$\alpha = \frac{d}{L}\theta \tag{3}$$

Substituting the above value of in equation (2) and taking Laplace transform, the following equation can be obtained

$$\left(\frac{J}{R^2} + m\right)R(s)\, s^2 = -mg\,\frac{d}{L}\,\theta(s) \tag{4}$$

Rearranging the equation (4), the transfer function from gear angle $\theta(s)$ to ball position is given by

$$G_P(s) = \frac{R(s)}{\theta(s)} = -\frac{mgd}{L\left(\frac{J}{R^2} + m\right)}\frac{1}{s^2} \tag{5}$$

Substituting the values of system parameters in equation (5), the transfer function becomes

$$G_P(s) = \frac{R(s)}{\theta(s)} = \frac{0.7}{s^2} \tag{6}$$

3. DESIGN OF CONTROLLERS

This section of the paper deals with the design of PID, I-PD and PD-PI controller for the ball beam system.

3.1. Design of PID Controller

The general structure of PID controller is

$$G_{PID}(s) = K_P + \frac{K_I}{s} + K_D s \tag{7}$$

where, K_P, K_I and K_D represent proportional, integral and derivative gain respectively.

Characteristic equation of the system with PID controller for unity feedback is given

$$1 + G_P(s)G_{PID}(s) = 0 \tag{8}$$

i.e.

$$1 + \frac{0.7}{s^2}\left(K_P + \frac{K_I}{s} + K_D s\right) = 0 \tag{9}$$

To obtain PID controller parameter (K_P, K_I and K_D) values, minimization of the integral of time-weighted absolute error (ITAE) is usually referred to as a good tuning criterion in the literatures [12]. There are mainly three steps involved for finding PID controller parameter values using ITAE index. In the very first step closed loop Simulink model of the system with controller is developed. In the second step a MATLAB *m*-file with an objective function is created which calculates the ITAE index. In the final step minimization of ITAE index is done using a MATLAB function (the function *'fmincon'* has been used in this study) from optimization tool box.

As the range of unknowns affect the optimality of the solution, in the beginning wider solution space is considered. After getting the initial solution, in the subsequent steps the solution space has been shorten. The range and initial assumption of parameters for writing the MATLAB code is decided after a number of trial runs and provided in Table 2.

Table 2

Initial guess and range of controller parameters

Parameter	K_P	K_I	K_D
Initial value	34	17	12
Lower range	33	15	10
Upper range	35	20	13

The values of PID controller parameters are found after optimization and given in Table 3.

Table 3

PID controller parameter values after optimizing the objective function

Parameter	K_P	K_I	K_D
Value	33	15	13

3.2. Design of I-PD Controller

The control structure of the I-PD controller with plant and unity feedback is shown in the Fig. 2

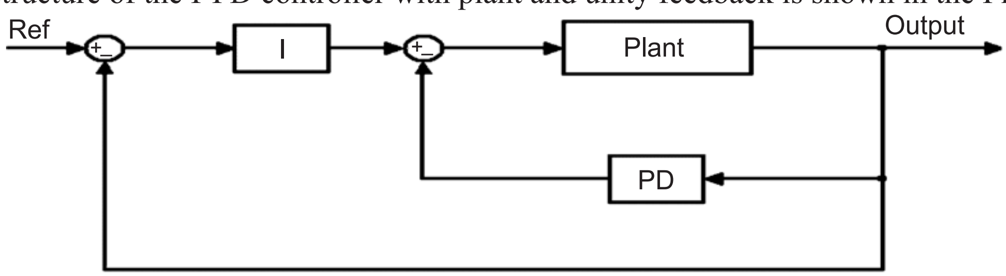

Figure 2: Control Structure of an I-PD controller

As different signal paths are present for set-point and process outputs, it has got more flexibility to satisfy the design specifications accurately.

Characteristic equation of the system with I-PD controller for unity feedback is given by

$$1 + G_P(s)(G_I(s) + G_{PD}(s)) = 0 \tag{10}$$

i.e.

$$1 + \frac{0.7}{s^2}\left(K_P + \frac{K_I}{s} + K_D s\right) = 0 \tag{11}$$

Considering unity feedback, it is interesting to note that the characteristic equation of the system with PID controller is identical with that of the I-PD controller as long as the controller parameter values are identical.

Instead of designing the I-PD controller from the scratch, PID controller parameter values will be used to design the I-PD controller. It will help to analyze that by keeping same set of parameter values how it is possible to obtain smoother response just by changing the control structure from PID to I-PD.

3.3. Design of PD-PI Controller

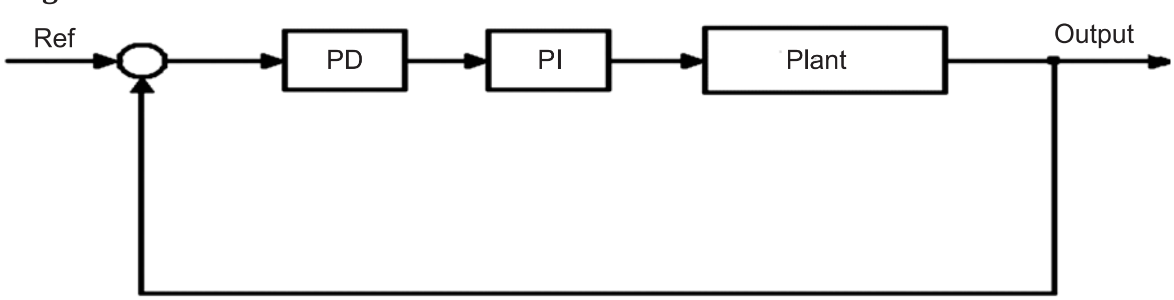

Figure 3: Control Structure of PD-PI controller

The design of PD-PI controller is discussed in [13] where two individual PD and PI block is placed in series with the plant and proportional gain of the PD block is considered as unity. The control structure of PD-PI controller with plant and unity feedback is provided in Fig. 3.

The characteristic equation of the system with PD-PI controller for unity feedback is given by

$$1 + G_P(s)(G_{PD}(s)G_{PI}(s)) = 0 \tag{12}$$

i.e.

$$1+\frac{0.7}{s^2}\left((1+K_D s)(K_P+\frac{K_I}{s})\right) = 0 \tag{13}$$

As explained earlier, PID controller parameters are again utilized for designing the PD-PI controller.

4. RESULTS AND DISCUSSION

As the input signal, unit step signal has been considered for this study. The simulation is carried out for 50 seconds and the response of simulation with PID, I-PD and PD-PI controller has been provided in Fig. 4

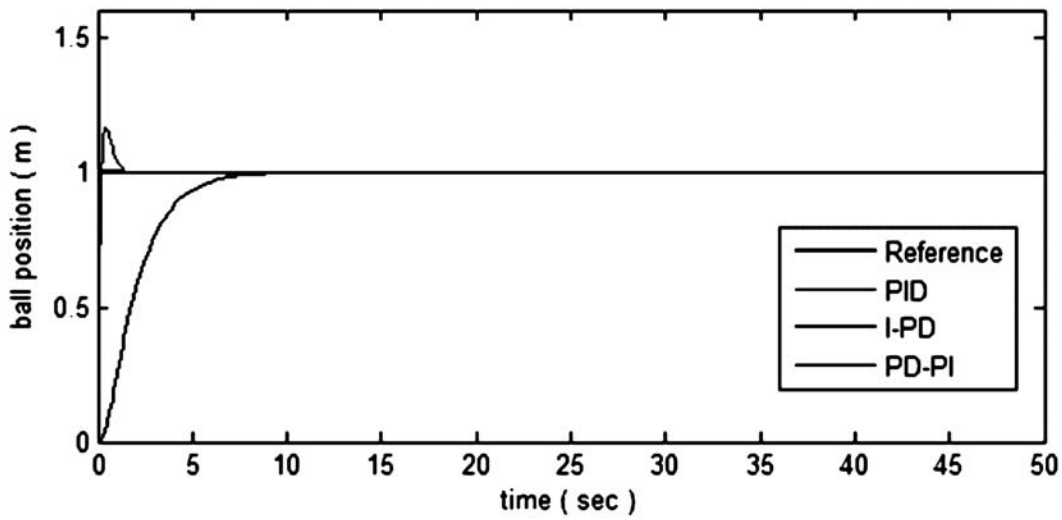

Figure 4: Step response of BBS with different controllers

The performance of the BBS with PID, I-PD and PD-PI controller is summarized in Table 4

Table 4
Performance comparison of the BBS with different controllers

Method / Performance	*PID*	*I-PD*	*PD-PI*
Maximum Overshoot (%)	15.5	0	0.174
Peak time (sec)	0.367	NA	0.043
Rise time (sec)	0.155	3.93	0.00728
Settling time (sec)	1.18	7.28	0.0128
Gain margin (dB)	-26	26	-73.2
Phase margin (degree)	74.6	79.8	89.9
Sensitivity	1	1.1407	1
Complementary sensitivity	1.2007	1	1.0003
IAE	0.1879	2.2	0.007188
ISE	0.05833	1.307	0.002165
ITAE	0.1143	3.973	0.01286
ITSE	0.007881	1.274	1.073×10^{-6}

From the data available in Table 4, it is observed that as the control structure changes from PID to I-PD and PD-PI controller, the response of the system gets improved. In I-PD controller the system response has become smooth as compare to the PID controller by compromising the settling time and rise time, whereas, in PD-PI controller, the smooth response has been achieved without compromising performance of the system.

5. CONCLUSION

In this article, the design of PID, I-PD and PD-PI controller has been discussed for the BBS. The PID controller parameters have been found by minimizing the ITAE criterion and the same set of values are further used for designing the I-PD and PD-PI controller. The performance of these controllers has been compared. The result of comparison reveals that I-PD and PD-PI controller has the potential to make the system response smooth as compare to the PID controller. The best response of the BBS has been found in case of PD-PI controller. This particular approach of using PD-PI controller can be extended to other class of plants for improving the performance of the system.

6. REFERENCES

1. W. Yu, "Nonlinear PD Regulation for Ball and Beam System," *International Journal of Electrical Engineering Education*, Vol. 46, pp. 37–59, 2009.
2. X. Li and W. Yu, "Synchronization of Ball and Beam Systems with Neural Compensation," *International Journal of Control, Automation and Systems*, Vol. 8, No. 3, pp.491-496, 2010.
3. S.K Oh, H.J. Jang and W. Pedrycz, "The Design of a Fuzzy Cascade Controller for Ball and Beam System: A Study in Optimization with the Use of Parallel Genetic Algorithms," *Engineering Applications of Artificial Intelligence*, Vol. 22, pp. 261–271, 2009.
4. N.B. Almutairi and M. Zribi, "On the Sliding Mode Control of a Ball on a Beam system," Nonlinear Dynamics, Vol. 59, pp. 221-238, 2009.
5. M. L. Hammadih, K. A. Hosani and I. Boiko, "Interpolating sliding mode observer for a ball and beam system," *International Journal of Control*, Vol. 89, Iss. 9, pp. 1879-1889, 2016.
6. E. V. Kumar, J. Jerome and G. Raaja, "State Dependent Riccati Equation based Nonlinear Controller Design for Ball and Beam System" *Procedia Engineering,* Vol. 97, 1896 – 1905, 2014.
7. A. Jose, K. K. Avinashe, M. Dhanoj and E.S. Yadav, "H-infinity PID controller for a Ball and Beam System," *International Journal of Innovative Research In Electrical, Electronics, Instrumentation And Control Engineering,* Vol. 3, Iss. 1, pp. 91-95, 2015.
8. S. K. Choudhary, "Fractional Order Feedback Control of a Ball and Beam System," *International Journal of Computer, Information, Systems and Control Engineering,* Vol. 8, No. 7, pp. 1120-1126, 2014.
9. M. Korkma and O. Aydogdu, "Fractional Order Controller Design for Ball and Beam System," *Applied Mechanics and Materials*, Vols. 313-314, pp. 544-548, 2013.
10. M. Keshmiri, A. F. Jahromi, A. Mohebbi, M. H. Amoozgar and W. F. Xie, "Modeling And Control Of Ball And Beam System Using Model Based And Non-Model Based Control Approaches," *International Journal On Smart Sensing And Intelligent Systems,* Vol. 5, No. 1, pp. 14-35, 2012.
11. S. Sathiyavathi and K. Krishnamurthy, "PID Control of Ball and Beam System- A Real Time Experimentation," *Journal of Scientific and Industrial Research*, Vol. 72, pp. 481 - 484, 2013.
12. F. G. Martins, "Tuning PID controllers using the ITAE criterion," *International Journal of Engineering Education*, Vol. 21, No. 5,pp. 867-873, 2005.
13. T. Jain and M. J. Nigam, "Optimization of PD-PI Controller using Swarm Intelligence," *Journal Of Theoretical And Applied Information Technology*, Vol. 4, No. 11, pp. 1013-1018, 2008.