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Curvelet Transformation Based Image Denoising

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ABSTRACT

Image denoising is an important task in image processing. Denoising is a preprocessing method which provides a quality image without any noise. In this paper curvelet transformation method is used to do image denoising. The limitations of the existing approaches are: one-dimensional transformations, transformations applied on geometricaledges of the images. But in this paper real two dimensional transform based capturing the intrinsic geometrical structure as the visual information for denoising. To do this a multi-objective curvelet wavelet transform method is used. The experiment is carried out in MATLAB software and the performance is compared with the filter based image denoising. From the experimental results it is concluded that the proposed denoising technique performs better in terms of PSNR.

Keywords: Image Processing, Image Denoising, Curvelet Transform, Image Transformations, Filter.

1. BACKGROUND STUDY

The secure ofovercoming some of the intrinsic resolution restrictions of flow-cost imaging sensors (e.g. cell phone or surveillancecameras) is offered by Super-resolution (SR) image reconstruction which is at present avery active area of research and it offers allowing better utilization of the increasing capabilityof high-resolution displays (e.g. high-definition LCDs). In medical imaging and satellite imaging suchresolution-enhancing technology may also prove to be necessarywhere diagnosisor scrutiny from low-quality images can be extremely difficult.Conventional approaches to generating a super-resolution imagetypically require as input *multiple* low-resolution imagesof the same scene, which are aligned with sub-pixel precision.By fusing the low-resolutionimages, in terms of on reasonable assumptions or prior knowledgeabout the observation model, the SR task is cast as the opposite problem of recovering theoriginal high-resolution image which maps the high-resolution imageto the low-resolution ones.

One of the most important and serious tasks in any kind of image processing areas such as medical image processing, satellite image processing, real time image processing, robot vision and space exploration etc., is image denoising. The probabilistic behavior and statistical behavior are determined by the pattern. Speckle noise poison noise, tiny noise, frequency noise, device based noise, Gaussian noise are the types of the noises focused here to remove from satellite images. There are various algorithms were proposed to remove linear and nonlinear noises from the images in order to increase the quality of the images and performance of the image processing results. The quality of the image is spoiled by the noises. During the image capturing, the noise may be created according to the problem available in the electronic camera circuit. One of the noise is distributed evenly on the image is white Gaussian noise.

Dealing with additive noise is linear and efficient technique while nonlinear filters are resourceful to contractwith the multiplicative and function based noise. Curvelet and Ridgelet techniques remove the noises and artifacts on the image linearly. For denoising curvelet transform based noise removal is considered

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as a best method till now. The best method is selected among various methods is by comparing the PSNR value of the image, where it determines the visual quality of an image.

The primary reconstruction restricted for SR is that the recovered image, after applying thesame production model, should replicate the observed lowresolutionimages. However, SR image reconstruction is usuallya severely ill-posed problem because of the insufficientnumber of low resolution images, ill-conditioned registrationand unknown blurring operators and the solution from thereconstruction constraint is not unique. Various regularizationmethods have been proposed to further stabilize the inversionof these ill-posed problems. Redundant representations and sparsity have been usedin the past decade successfully for the denoising problem.

Wavelet coefficients of an image is taken to calculate the sparsity value using shrinkage algorithms and these are leading algorithms applied for image denoising[1, 2, 3, 4, 5, 6].One of the main reasons for focusing on redundant representation is to use the shift invariance property of the images [7]. Separating one dimensional wavelets is not suitable for image denoising. Since various multi-scaled and multi-directional, redundant transformation techniques were introduced. They are together with the Curvelet[8], Contourlet [9], Wedgelet [10], Bandlet [11], the steerablewavelet [12], and more.Beginningof the matching pursuit [13, 14] in parallel, the and the basis pursuit denoising[15], gave rise to the ability to address the imagedenoising problem as a direct sparse decomposition techniqueover redundant dictionaries. Some of the best available image denoisingmethods – see [16, 18, and 19] for few representativeworks will include all these to lead further. One of the existing worksto consider is training the dictionaryusing patches from the corrupted image itself. Thisidea of learning a dictionary that yields sparse representationsfor a set of training image-patches has been studied in a sequence of works [22, 23, 24, 25, and 26].

2. PROBLEM STATEMENT

Communication system improves its quality of service through visual perceptions. It is essential to differentiate the original data transformed than the noise affected data also it ensured that the transmitted data is not affected by any noise. Noise created from improper sources used for production or capturing the images whereas the noisy signals are deviated from real signals expected by the user. After denoising the performance of the image processing task become more accurate. Since all image processing tasks do image denoising to preserve the image features. In this paper a curvelet transformation based image denoising process is applied. The curvelet transformation result obtained after adding additive Gaussian noise. Then curvelet transformation approach is implemented. First the input image is feed into a filter, and thenthe

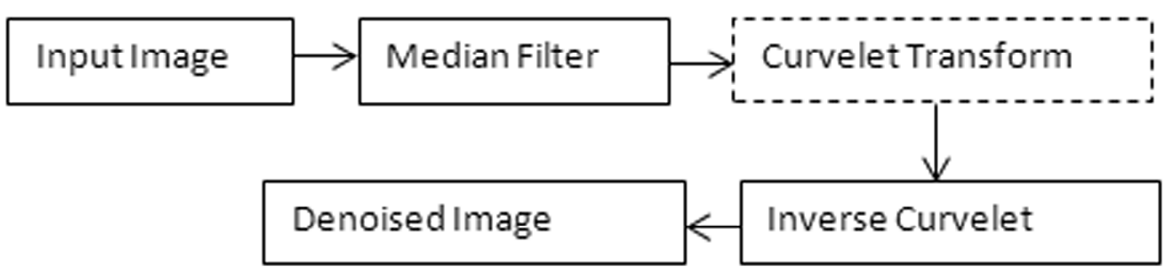

Figure-1: Proposed System Model

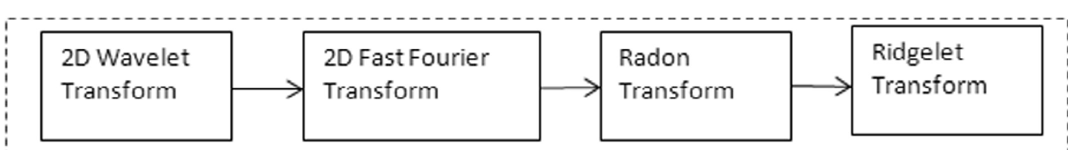

Figure-2: Curvelet Transform

filter output is applied to curvelet transform. The resultant image is denoised image where its quality can be compared with the input image quality in terms of PSNR.

3. WIENER FILTERING

Due to simplicity and speed Wiener filter is used by more research works. Wiener filter is behaves as simple because it uses linear equations to calculate a set of optimal filter weights which decreases the noise level of a received signal. Correlation is estimated using the covariance of the noise which leads to calculate the Gaussian noise occurrence in the signal. By utilizing the noise values the optimal filter weights are determined. By comparing with the input signal the noise behavior is estimated and find out the optimal noise distributed in the signal. Once the Gaussian noise is removed the image processing steps become very speed.

De-convolution is also a method applied for inverse filtering. Inverse filters are used to recover the images from the blurred data using low pass filters. These filters are mainly used to remove the additive noises which are more sensitive. In order to save restoration time, instead of using multiple degradation algorithms, it can be used a single degradation algorithm by combining all the features together. For smoothen the image from various noises an optimum noise-removal method is given by Wiener Filter (WF). Blurring and additive noises are combinedly taken out by the wiener filters. Also WF minimizes the MSE of the images. The Wienerfiltering is a linear estimation of the original image. The approach is based on a stochastic framework. The orthogonallyprinciple implies that the Wiener filter in Fourier domain can be expressed as follows:

$$W\left(f_1,f_2\right)=\frac{H^*\left(f_1,f_2\right)S_{xx}\left(f_1,f_1\right)}{\left|H\left(f_1,f_2\right)\right|^2 S_{xx}\left(f_1,f_2\right)+S_{\eta\eta}\left(f_1,f_2\right)} \tag{1}$$

4. CURVELETS TRANSFORMATION

In this paper, we report initial efforts at image denoising based popular transformation method proposed and utilized in recent research works is taken here instead of wavelet representation of image data. Below explained transformation method is new and still it is in under-development. Software for computing these new transforms is still in a formative stage, as various trades-offs and choices are still being puzzled through. Using scaling law $width \approx length^2$ the accurate width and length can be scaled. It concludes that if the anisotropy increases then the scale is decreases like power law. The thresholding of discrete curvelet coefficients provide near optimal N-terms representations. In order to get more precise understanding of curvelet it is necessary to gather the knowledge about ridgelet and radon transform, which are described below.

Let us consider a function $f(x, y)$ is a two-dimensional function providing spare representation of the smooth function and the straight edges using splendid locus of ridgelet function.

$$\forall L^2\left(R^2\right)$$

obtaining the ridgelet coefficients is

$$R_f\left(a,b,\theta\right)$$

from the inner product of ridgelet coefficient with the frame like function as:

$$\psi_{a,b,\theta}\left(x\right)$$

is the wavelet in crosswise coordination constant comprising the line

$$x_1\cos\theta+x_2\sin\theta$$

is a constant. $\forall a>0$, *for each* $b \in R$ and $\forall\theta\in\left[0,2\pi\right]$, the bivariate ridgelet $\Psi_{a,b,\theta}$ is obtained from

$$\frac{1}{\sqrt{a}}\varphi\left(\frac{x\cos\theta + y\sin\theta - b}{a}\right) - (yy)$$
$$R$$

The (yy) is constant with the lines

$$x_1 cos\theta + x_2 sin\theta = const$$

Transverse to these ridges it is a wavelet. Consider the integrable bivariate function f(x), the ridgelet coefficients by

$$R_f(a,b,\theta) = \int \varphi_{a,b,\theta}\theta(x) f(x)\,dx$$

The reconstruction is obtained using the following formulae as:

$$f(x) = \int_0^{2\pi}\int_{-\infty}^{\infty}\int_0^{\infty} R_f(a,b,\vartheta)\varphi_{a,b,\theta}(x)\frac{da}{a^3}db\frac{d\theta}{4\pi}\quad(ss)$$

Equation (ss) is valid for both, square integrable and integrable. From this an arbitrary function using wavelet as a continuous superposition of ridgelets can be written as:

$$\int |f(x)|^2\,dx = \int_0^{2\pi}\int_{-\theta}^{\theta}\int_0^{\theta} |R_f(a,b,\theta)|^2 \frac{da}{a^3}db\frac{d\theta}{4\pi}$$

4.1. Radon Transform

Ridgelet coefficients are calculated by a basic fundamental tool used to view the ridgelet analysis in the form of wavelet analysis over radon domain. The radon transform of an object f is the collection of line integrals indexed by $(\theta, t) \in [0, 2\pi] \in R$.

$$Rf(\theta, t) = \int f(x_1, x_2)\delta(x_1 cos\theta + x_2 sin\theta - t)dx_1 dx_2 \tag{6}$$

Where, δ is the Dirac distribution and the ridgelet coefficients $R_f(a, b, \theta)$ of an object f are given by analysis of the radon transform via

$$R_f(a,b,\theta) = \int Rf(\theta,t)\,a^{-\frac{1}{2}}\varphi((t-b)/a)\,dt$$

In the above equation, it is clear that the ridgelet transformis one dimensional wavelet transform to the slices of the radon transform, whereas θ is variable constant and t is changing one.

4.2. Discrete Curvelet Transform of Continuous Function

One of the continuous functions is $f(x_1, x_2)$ called as discrete curvelet transform which utilizes a dyadic sequence of measures and a bank of filters $[(\Delta_\theta f, \Delta_1 f, \Delta_2 f, \ldots\ldots)]$ and it has a property that the pass band filter Δ_s is concentrated near the frequencies $[2^{2s}, 2^{2s+1}]$. For example,

$$\Delta_s = \psi_{2s} * f, \hat{\psi}_{2s}(\xi) = \dot{\psi}(2^{-2s}\xi)$$

In wavelet theory, if one uses decomposition into dyadic sub bands $[2^{2s}, 2^{2s+1}]$. Opposite to that, subbands utilize DCT well worth remembering.

Initially the input image divided into subbands, and then spatial representation and finally applied with ridgelet transform over each block which is shown in Figure-3. The entire process is mathematically represented in the following steps as:

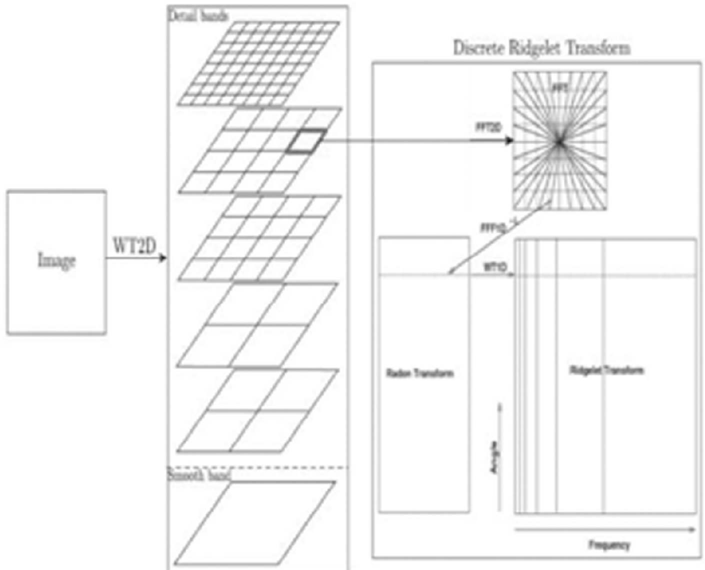

Figure 3: First Generation Discrete curvelet Transform (DCTG1) flowchart.

1. Subbands decomposition:

$$f \to \left(P_0 f, \Delta_1 f, \Delta_2 f, ...\right)$$

2. Smooth partitioning:

$$\Delta_s f \to \left(wQ\Delta_s f\right) Q \in Q_s$$

3. Renormalization:

$$gQ = 2^{-s}\left(T_Q\right)^{-1}\left(wQ\Delta_s f\right), Q \in Q_s$$

4. Ridgelet Analysis:

$$\alpha_\mu = \left|\left\langle gQ, \rho\lambda \right\rangle, \mu = \left(Q, \lambda\right)\right|$$

After applying the ridgelet transform both the dyadic subbands [2^{2s}, 2^{2s+1}] and [2^{2s}, 2^{2s+2}]. The entire functionality of the paper is given in the form of Algorithm, where it can be coded in any computer programming language and the performance is verified.

Algorithm Curvelet_Based_Denoising ( )

{

1. Read the input image
2. Median filter based noise removal
3. Apply 2D wavelet transform method
4. Apply 2D Fast Fourier Transform method
5. Apply Radon transform method
6. Apply Reidgelet Transform Method
7. Reconstruct the image
8. Compute PSNR for reconstructed image and input image compare.

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5. EXPERIMENTAL ANALYSIS

This proposed approach is programmed in MATLAB software and experimented using various kinds of images like Medical images, Satellite Images, Still Images and Benchmark images taken from matlab software itself. The sample input images from various categories are given in the following Table-1.

Table 1
Various Kinds of Input Images Taken for Experiment

Medical Images	Natural Images	Satellite Images	Matlab Images

Table 2
Input Images vs. Gray scale Image

Medical Images	Natural Images	Satellite Images	Matlab Images

All the images taken as color image with various resolution and obtained from various sources. Since these images are different with each other in terms of their property values, taken for experimenting and evaluating the performance of the proposed approach. Before going to apply all the transformation function all the images are converted into gray scale image whereas the noise level computation will be an easy process. The corresponding gray scale images given in Table-1 are given in Table-2.

After a sequence of image processing processes applied on the input images, the final result obtained is noiseless image. In order to verify the noise level the quality of the images are calculated using PSNR and MSE value of each image. But in this paper the features are extracted after transformation through their coefficients. The following Table-2 shows the transformation features extracted using CWT method.

The feature values are extracted from the test images using CWT method is shown in Table-2. The transformation method has an inbuilt formulation for computing and extracting the selective features from the input segmented image.

In order to obtain the accuracy the input image is converted into gray scale image. The relevant gray scaled images for the input images are given in Table-2. The input images are given in Table-1 and the relevant gray scale images are given in Table-2. Then the images are applied in to curvelet transform whereas curvelet transform method extracts the wavelet coefficients as features. The feature coefficients are extracted from ridgelet and radon transforms method. Both the methods are concentrating on the internal structure of the images and the curve structure of the images respectively. During the transformations and reconstructions the additional noise data occurred in the image or the signal are removed automatically. The main difference between the input image and the noise removed images are measured and compared using PSNR and MSE values calculated on the images.

6. Performance Calculation

This paper presents a method which removes the noise and provides a denoised image using various stages of the curvelet transform method integrated with median filter. In order to evaluate the image quality PSNR value of each image is calculated. The formula used for calculating PSNR is given below.

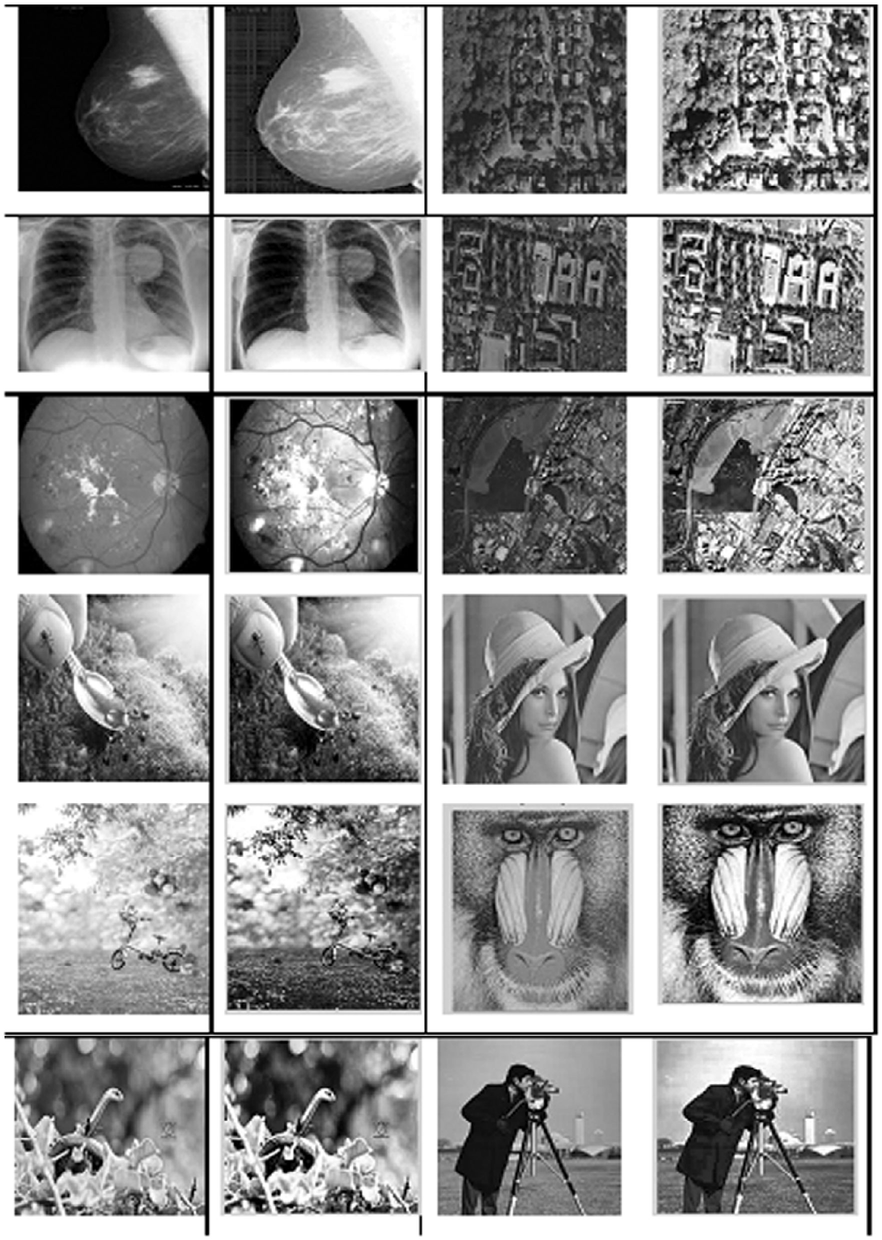
$$PSNR = 10 \cdot \log_{10}\left(\frac{MAX_I^2}{MSE}\right)$$

Table 3
Features extracted using Curvelet Wavelet Transform Method

Images	*STD*
Image1	68.96
Image2	71.12
Image3	75.31
Image4	76.32
Image5	73.89
Image6	71.87
Image7	69.44
Image8	74.63
Image9	71.25
Iamge10	76.42
Image11	79.23
Image12	65.89

Table 4
Input Image vs. Denoised Image

Input Images	Denoised Images	Input Images	Denoised Images



$$= 10 \cdot \log_{10}\left(\frac{MAX_I}{MSE}\right)$$

$$= 20 \cdot \log_{10}\left(MAX_I\right) - 10 \cdot \log_{10}\left(MSE\right)$$

$$MSE = \frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}\left[I(i,j) - K(i,j)\right]^2$$

The term m and n denotes the number of rows and columns of the input image. The input image is represented as I and the noisy image is represented as K.

Table-5
dB vs. PSNR for Proposed Approach

Proposed algorithm in (dB)	Curvelet Transform (dB)	Noisy Image (dB)
34.98	28.11	27.989
30.324	22.879	22.0
27.03	19.11	18.32
24.654	17.01	16.01
19.90	14.32	14.98
23.12	19.34	18.21

Table-6: dB vs. PSNR for Existing Approach

		PSNR	
a(db)	*Proposed algorithm in (dB)*	*Curvelet transform (dB)*	*Nosiy image (dB)*
10	36.4913	28.2318	28.1118
20	31.3377	22.3253	22.0901
30	28.1687	18.9076	18.5945
40	25.9377	16.4319	16.0804
50	24.0712	14.5199	14.1539
60	22.5495	12.9343	12.5663
70	21.3423	11.6289	11.2541

The quantitative comparison among the images is using the PSNR and MSE values of each pixels represented by. In order to test the proposed algorithm the test images are corrupted with Gaussian noise with the STD () as 10.The following table shows the PSNR value of the images. Table-6 shows the dB versus PSNR value of the images experimented. Comparing with the existing approach proposed approach obtained good quality in terms of PSNR.

7. CONCLUSION

The main objective of this paper is to denoise the images under various categories. Medical images, natural images, satellite images and still images are considered and taken for experiment. In order to obtain best quality the images a curvelet wavelet transformation method is utilized. This curvelet wavelet method includes various additional functionalities with the curvelet transformation. 2D transformation, reconstruction and curve based and surface based image features are separately transformed and noises are removed. There are three different ways are used for noise removal here. One is by wiener filter, another is by radon transformation and the final one is by wavelet transformation. Curvelet transformation preserves the curvature structure and maintains the originality of the image while reconstruction. These functionalities make the noisy image as denoised images and quality one.

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