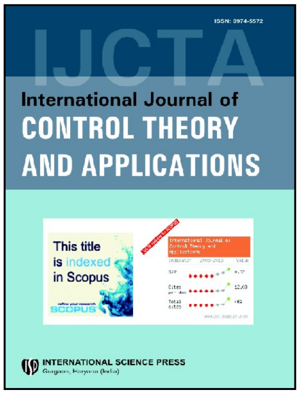

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Application of UKF and MGAEKF for Bearings and Elevation Angles-only Target Tracking

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Abstract: The objective of this research work is to track the target even though range measurements are not available. Bearing and elevation measurements are used to find out the target path. Modified gain angles-only extended Kalman filter (MGAEKF) and Unscented Kalman filter (UKF) are used to processing the noise-corrupted measurements and to analyse the target motion. It is observed that the results are more accurate with UKF than that of MGAEKF.***Keywords:*** Stochastic theory, Statistical signal processing, Applied statistics, Estimation theory.

1. INTRODUCTION

In underwater, passive target tracking is generallyfollowed to track a submarine target [1].The observer submarine is assumed to be moving at low speeds to reduce self-noise for tracking of the targets. In conventional submarines, bearings-only measurements are available. These days, submarines with sonar are coming up having the facility to get target elevation measurements also. In this paper, research is towards submarine (observer) tracking another submarine using elevation and bearing measurements. As angle measurements are only available, the process is highly nonlinear and hence unscented angles-only Kalman filter (UAKF)and modified gain angles-only extended Kalman filter(MGAEKF), nonlinear filters are explored for this application, as shown in the Figure.1 [1-3].Ownship is S-maneuvered for early observability of the process and to obtain the solution fast as shown in Figure 2 [4-6].The estimated target range, course, bearing and speed (RCBS) are utilizedin weapon guidance algorithm (which is not discussed here).

Section 2 deals with modeling of measurements, state vector, UAKF and MGAEKF.Section 3 describes generalised simulator. Section 4 deals with results obtained for different scenarios in simulation. Finally, the paper is concluded in section 5.

2. MATHEMATICAL MODELING

2.1. Measurements and state vector

Let $X_s(k)$ be state vector with $x(k), y(k), ¿(k), R_x(k), R_y(k)$ and $R_z(k)$ are velocity and range components in x, y and z directionsrespectively. The state equation is

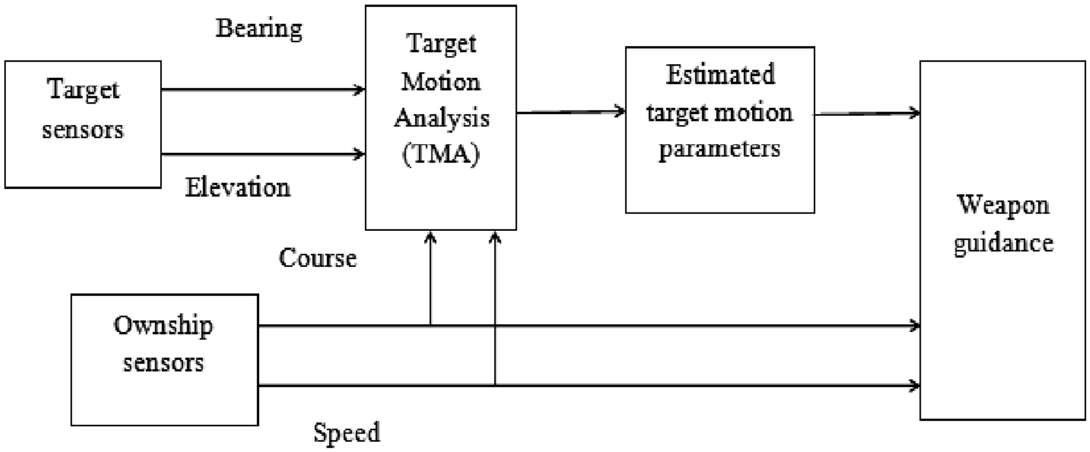

Figure 1: Block diagram of passive target tracking using bearing and elevation measurements

$$X_S(k+1) = Ø(k+1/k)\, X_S(k) + b(k+1)\, \omega(k) \tag{1}$$

where, $\omega(k)$ is noise having zero mean white Gaussian power spectral density. $Ø(k+1/k)$ istransient matrix. $b(k+1)$ is deterministic vector.

For clarity of concepts, the observer and target encounter in horizontal plane is shown in Figure 2. Z(k) is measurement vector and it is

$$Z(k) = \begin{bmatrix} B_m(k) \\ \theta_m(k) \end{bmatrix} \tag{2}$$

where, $B_m(k)$and $\theta_m(k)$are bearing and elevationmeasurements respectively.

Actual bearing and elevation angles are $B(k)$ and

$\theta(k)$ respectively and these are

$$B(k) = \tan^{-1}\left(\frac{R_x(k)}{R_y(k)}\right) \tag{3}$$

$$\theta(k) = \tan^{-1}\left(\frac{R_{xy}(k)}{R_z(k)}\right) \tag{4}$$

The noises $\eta(k)$ and $\gamma(k)$are uncorrelated Gaussian noises. The measurement equation is written as

$$Z(k) = H(k)\, X_S(k) + \xi(k) \tag{5}$$

where, *H(k) is relation between state and measurements.*

2.2. UAKF &MGAEKF algorithms

UAKF algorithm is presented in detail in [4-6] and MGAEKF algorithm is presented in Table 1.

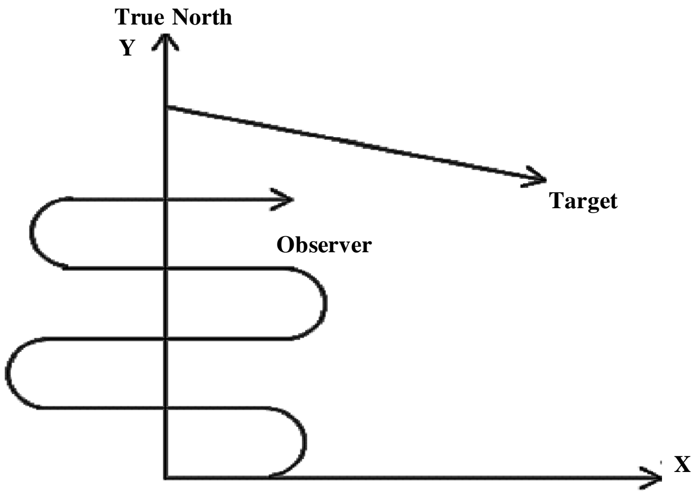

Figure 2: Target and observer Encounter

Table 1
MGAEKF & UKF algorithm

1. To start with X(0/0)and P(0/0), initial state vector and its covariance matrix respectively are chosen.
2. Kalman gain is given as

$$K(k+1/k) = P(k+1/k)h^T(k+1/k)(h((k+1)/k)P((k+1/k)h^T((k+1)/k)+r(k))^{-1} \quad (6)$$

here, *r(k)* is measurement covariance matrix.

3. Updated state vector is

$$X(k+1/k+1) = X(k+1/k) + K(k+1)(z(k+1) - h(k+1)X(k+1/k) \quad (7)$$

4. Updated state covariance matrix is

$$P(k+1/k+1) = (I - K(k+1)g(z(k1), X(k+1/k).P(k+1/k)(I-(k+1)g\left(z(k+1), X\left(k+{}^{1}\!/\!_{k}\right)\right)^T + K(k+1)\, r(k)G(k+1)^T \quad (8)$$

$$G(k)=\begin{bmatrix} 0 & 0 & 0 & \frac{\cos(\hat{B}(k))}{R_{xy}} & \frac{-\sin(\hat{B}(k))}{R_{xy}} & 0 \\ 0 & 0 & 0 & \frac{\cos(\phi_m(k))*\sin\left(\frac{\hat{B}(k)+B_m(k)}{2}\right)}{\hat{R}*\cos\left(\frac{B_m(k)-\hat{B}(k)}{2}\right)} & \frac{\cos\left(\frac{\hat{B}(k)+B_m(k)}{2}\right)*\cos(\phi_m(k))}{\hat{R}*\cos\left(\frac{B_m(k)-\hat{B}(k)}{2}\right)} & \frac{-\sin(\phi_m(k))}{\hat{R}} \end{bmatrix} \quad (9)$$

5. For next iteration

$X\,(k/k) = X(k+1/k+1))$ (10)

$P\,(k/k) = P(k+1/k+1))$ (11)

6. Predicted state vector is

$X\,(k + 1/k) = \phi\,(k+1/k)\,X\,(k/k)$ (12)

7. Predicted state covariance matrix is

$P\,(k + 1/k) = \phi\,(k + 1/k)\,\phi^T\,(k + 1/k) + Q\,(k + 1/k)$ (13)

Here *Q(k)* is plant noise covariance matrix

Algorithm flow is shown in Figure 3[4-13].

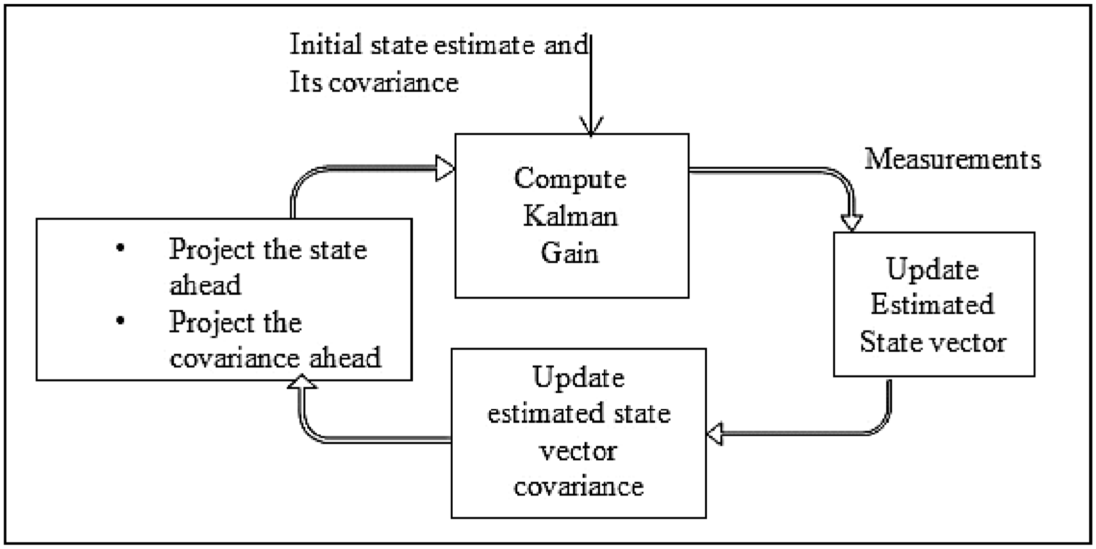

Figure 3: The flow of the algorithms

3. GENERALISED SIMULATOR

Let initial position of the target be (x_t, y_t, z_t) andthe target moves with velocity v_t.After time t seconds,observer position changes. Change in the observer position is given by

$$dx_0 = v_0 * \sin(ocr) * \sin(oph) * t \tag{14}$$

$$dy_0 = v_0 * \cos(ocr) * \sin(oph) * t \tag{15}$$

$$dz_0 = v_0 * \cos(oph) * t \tag{16}$$

where ocr and oph are observer course and pitchrespectively. Now the new observer position becomes

$$x_0 = x_0 + dx_0 \tag{17}$$

$$y_0 = y_0 + dy_0 \tag{18}$$

$$z_0 + z_0 + dz_0 \tag{19}$$

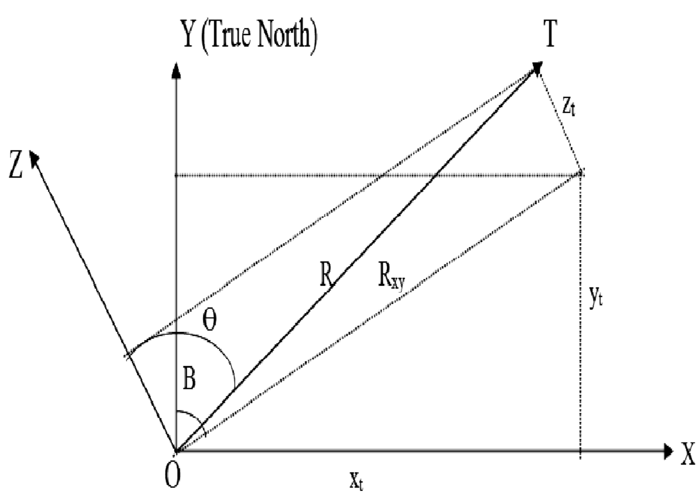

Figure 4: Target and observer positions

From Figure 4

$$x_t = R_{xy} * \sin(B) \tag{20}$$

$$y_t = R_{xy} * \cos(B) \tag{21}$$

$$\sin(\theta) = R_{xy} / R \tag{22}$$

Substituting equations (38) in (36) and (37)

$$x_t = R * \sin(\theta) * \sin(B) \tag{23}$$

$$y_t = R * \sin(\theta) * \cos(B) \tag{24}$$

$$z_t = R * \cos(\theta) \tag{25}$$

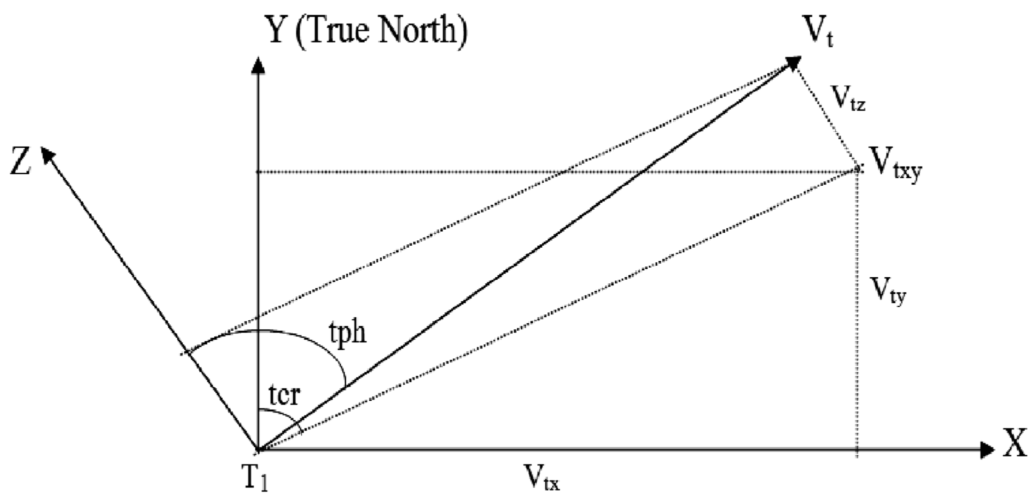

Figure 5: Target and observer velocities

When the target is in motion with velocity v_t, change in target position after t seconds, from Figure 5.

$$dx_t = v_t * \sin(tcr) * \sin(tph) * t \tag{26}$$

$$dy_t = v_t * \sin(tcr) * \sin(tph) * t \tag{27}$$

$$dz_t = v_t * cos(tph) * t \tag{28}$$

where tcr and tph are target course and pitch respectively.

Now the new target position is

$$x_t = x_t + dx_t \tag{29}$$

$$y_t = y_t + dy_t \tag{30}$$

$$z_t = z_t + dz_t \tag{31}$$

Target true bearing,range and elevation are

$$true\,bearing = \tan^{-1}\left(\frac{x_t - x_0}{y_t - y_0}\right) \tag{32}$$

$$true\,range = \sqrt{(x_t - x_0)^2 + (y_y - y_0)^2 + (z_t - z_0)^2} \tag{33}$$

$$true\,elevation = \tan^{-1}\left(\frac{R_{xy}}{z_t - z_0}\right) \tag{34}$$

Since the measurements are affected by noise in real situations, noise is added to these measurements.

Measured bearing = true bearing+ sigma b

Measured range = true range + sigma r

Measured elevation = true elevation+ sigma e

where sigma b, sigma r and sigma e are 1ó values of white Gaussian process.

The details are shown in Figure 6.

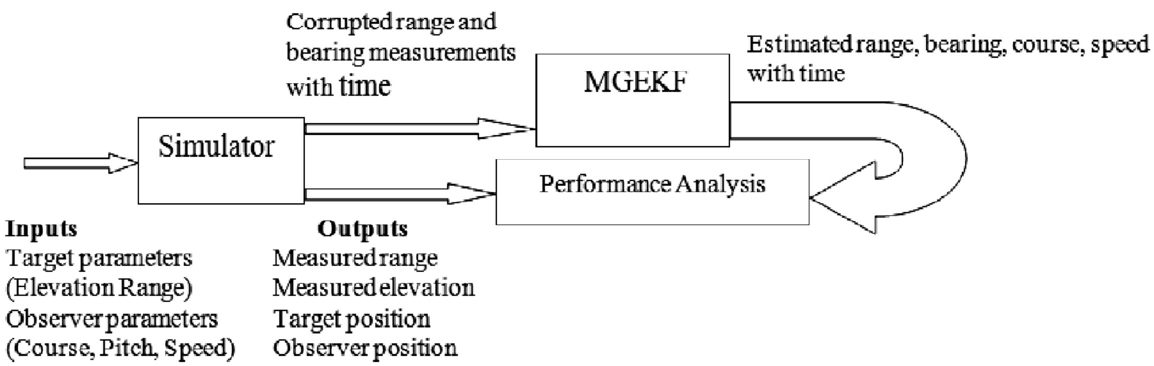

Figure 6: Block diagram of TMA in simulation mode

4. SIMULATION AND RESULTS

It is assumed that experiment is conducted at favorable environmental conditions and hence the angle measurements are available continuously. Simulation is realized on a personal computer using Matlab. The scenarios chosen for evaluation of algorithm are shown in Table.2. For example, scenario1 describes a target moving with bearing of 45^0 with course and speeds of 225^0 and 10m/s respectively. The elevation angle is 45^0. The bearing and elevation measurements are corrupted with 0.5^0(1ó) and 0.33^{-0} (1σ) respectively.

The ownship carries out S-maneuver for observability of the process [3,4], as shown in Figure 2.Ownship moves initially at 90^0, perpendicular to line of sight for a period of 120 seconds and then turns to 270^0 with a turning rate of 0.5deg/s towards target. Then it moves in straight line for a period of 240 seconds. Afterwards again it turns to 90^0 towards target for a period of 240 seconds and so on.

In simulation mode, estimated and actual values are available and hence the validity of the solution based on certain acceptance criterion is possible. The following acceptance criterion is chosen. The solution is converged when error in course estimate <= 3^0 and error in speed estimate <= 5m/s and range estimate<=8%.

The solution is converged when the course, speed and range are converged.

In UKF, for Scenario1, it is observed that the estimated course, speed and range of the target are converged at 152nd, 33thand 292th sample respectively. So, for scenario 1 in UKF the total solution is obtained at 292nd sample. Similarly for the other scenario the convergence time is shown in Table.3. For MGAEKF in scenario 1, range, course and speed are converged at 250th, 197th and 172nd sample. So, the total solution is converged at 250th sample. But it is observed for the 2nd scenario, in MGAEKF, the range and course are not converged and speed is converged at 16th sample. The errors in estimated range, speed and course for scenario 1 are presented in Figure7, Figure 8 and Figure 9 respectively for clarity of the concepts.

The angles-only process is evaluated against many scenarios and it is observed UKF is getting converged for every scenario and MGAEKF is diverging for many scenarios. So we conclude, UKF is better than MGAEKF.

Table 2
Input parameters chosen for the MGAEKF and UKF algorithms

Scenario	*Initial range(m)*	*Bearing(deg)*	*Elevation(deg)*	*Pitch(deg)*	*Course(deg)*	*Speed(m/s)*
1	3000	45	45	110	225	10

Table 3
Convergence time in samples for the chosen scenarios

Scenario	*Unscented Kalman Filter*				*Modified Gain Angles-only Extended Kalman Filter*			
	Range	*Course*	*Speed*	*Total solution*	*Range*	*Course*	*Speed*	*Total solution*
1	292	152	33	292	250	197	172	250
2	363	354	13	363	NC	NC	16	NC

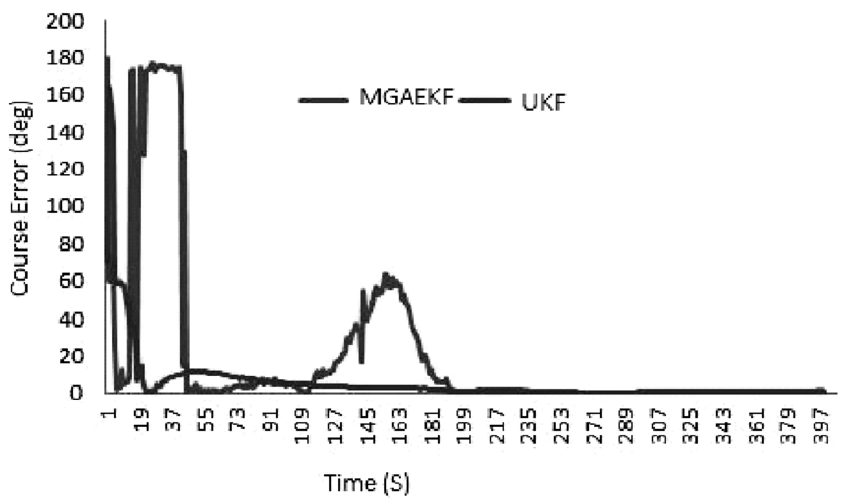

Figure 7: Error in estimated course

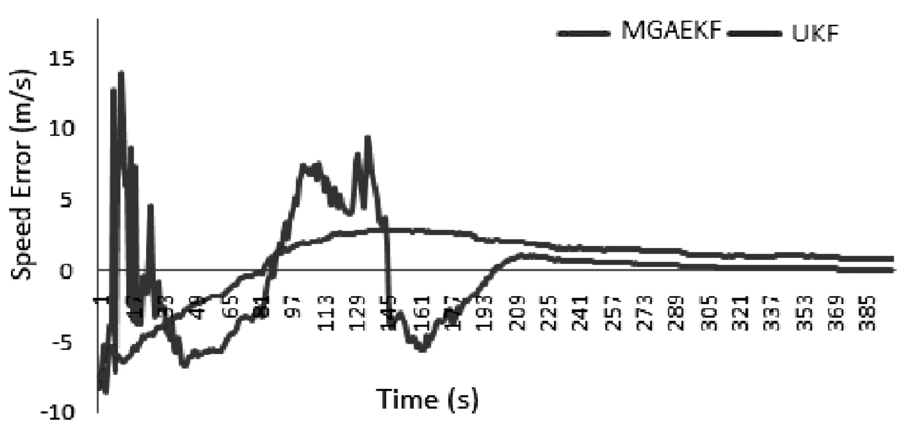

Figure 8: Error in estimated speed

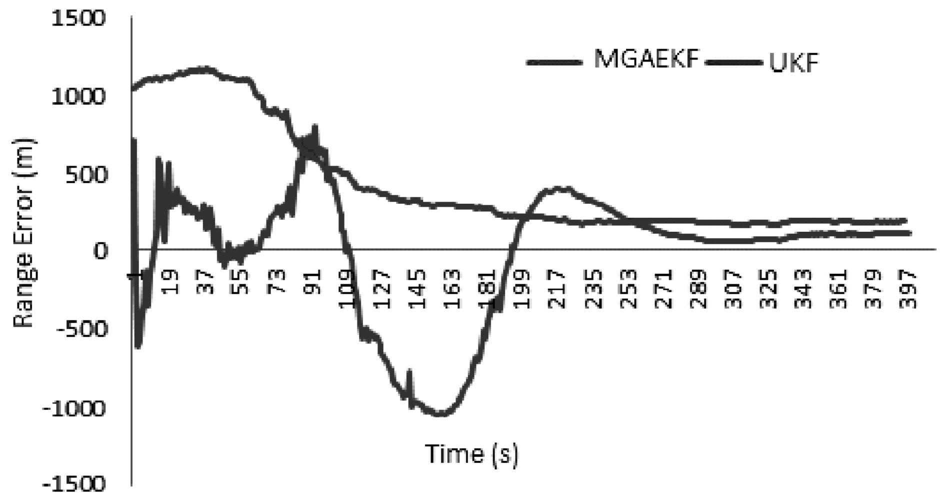

Figure 9: Error in estimated range

5. CONCLUSION

Based on these results, UKF is recommended for passive target tracking and in particular, submarine to submarine scenario, when elevation measurements are also available along with bearing measurements.

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