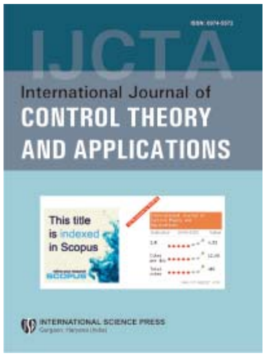

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Finite Element Analysis of An Edge Cracked Plate Subjected to Mixed Mode Loading (I/II) and Combined Tension-Bending – A Comparative Study

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Abstract: In most of the applications, the components are subjected to complex loading conditions during their service which are unpredictable. Hence the kind of load acting and its criticality is to be studied for the safe guarding of the machine components or structures. We therefore performed finite element analysis using ANSYS, to study the response of an edge cracked plate subjected to various kinds of loads. The investigation aimed to present the stress intensity factor, stress triaxiality and maximum shear tress of an edge cracked plate subjected to mixed mode loading (I/II) and combined tension – bending conditions using finite element analysis. The effect of various parameters like orientation of the crack, crack length to plate width ratio, and the nature of load are discussed. The study reveals that with increase in crack orientation, shear stress and bending stress becomes dominant due to mode mixity. In addition, increase in crack size resulted in increase of stress field intensity as well as stress triaxiality.

Keywords: *Bending, Finite element analysis, Mixed mode fracture, Shear.*

1. INTRODUCTION

Most of the mechanical systems in actual practice are subjected to complex and variable loads which may lead to the change in mechanical behavior of the material. This results in the formation of discontinuities and enhancing their growth leading to the deterioration of strength as well as life. Lot of research had been carried out by many researchers in the past to study the response of the cracked components. Dibakar Datta et. al [1] discussed the effect of mixed mode loading on crack propagation in graphene based nanodevices and found that the flaw size plays an important role in crack propagation. G.S.Palani et. al [2] proposed a generalized technique for the study of fracture analysis of cracked plates subjected to combined tension, shear and bending. T. Kevin O’brien et. at [3] modeled tapered laminate configurations using simple finite element beam subject to combined tension and bending and claimed that the results are in good agreement.

N. Ben Salem et. al [4] performed experiments to study the crack propagation in Hysol EA9321resin under mixed mode bending loading conditions. The results revealed that the sensitivity of the experimental data is not sufficient to identify the interface separation law more precisely. S.A.Fawaz et. al [5] performed experiments on thin specimens made of 2024-T3 aluminum to study the combined tension and bending. Orestes et. al[6] performed experiments on borosilicate glass rods subjected to tension and bending loads. They found that the Weibull exponent is same in both the cases and concluded that the failure in the materials tested is not satisfactorily described by Weibull statistics. Lanhui Guo et. al [7] performed experiments on composite joints subjected to various loads. They found that the rigid composite joint possesses good ductility and rotation capacity which meet the requirements for the formation of "catenary action". E.E.Gdoutos et.al [8] discussed various problems related to fracture mechanics and had given analytical solutions.

Literature reveals that effect of various kinds of loads and their criticality is not compared. Hence the present work has been carried out in this direction. This paper discusses the response of a cracked plate to combined tension-shear and combined tension-bending loads. In section-2, presents the development of finite element model – for varying geometry of the plates and the parameters such as loading conditions, a/w ratio, crack inclination angle. The methodology adopted in the finite element analysis of the plate is discussed in detail. Section-3 describes the mathematical relations for evaluation of stress triaxiality and maximum shear stress. Finally, the results obtained from the finite element analysis are presented in graphical form and are discussed in detail in section-4.

2. FINITE ELEMENT ANALYSIS

2.1. Geometry and Meshing

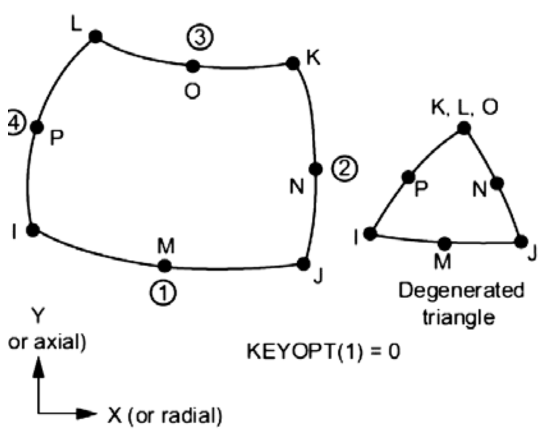


Figure 1: Shape of the element – Solid 183

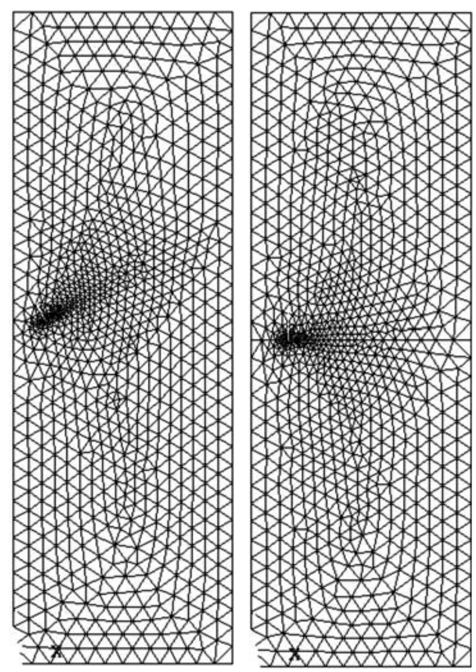


Figure 2: Meshed views of plates for β = 30°, 0°

A rectangular plate of width 65 mm and 180 mm long is considered to have an edge crack of length (*a*) related to the width of the plate (*w*) as a/w ratio. The a/w ratio is varied over a span of 0.5 with an interval of 0.1. The specimen is analyzed using commercial finite element package ANSYS 15.0. By performing the mesh convergence test, the element used in this analysis is finalized as solid 183, which is a higher order element with two degrees of freedom (u_x, u_y – translation in x and y) at each node. It exhibits quadratic (8 node) and triangular (6 node) behavior, represented in Fig 1. This element is well suited for the modeling of irregular meshes. The size of the element is increased from the crack tip to the free edges of the plate. Free meshing is performed on the cracked plate, with triangular shaped elements which generated 5194 nodes and 2154 elements. Meshed model of the cracked plate is presented in the Fig 2.

The material considered in the analysis is 2024-T3 Aluminum, which is used in aircraft applications. It exhibits isotropic behavior and the material properties are presented in Table-1.

Table 1
Properties of 2024-T3 Aluminum

S. No	*Property*	*Value*
1.	Young's Modulus (E)	71.2 Gpa
2.	Poisson's ratio (μ)	0.3
3.	Yield strength	344.5 Mpa

2.2. Loads Applied

Three kinds of loads are applied in the plate to study which is more predominant.

1. **Load of kind-I:** A uniformly distributed load equal to the pressure of 334.5 MPa equal to the yield strength is applied on the top surface of the plate. This creates an effect of *combined tension and shear*.
2. **Load of kind-II:** If the member is subjected to an application of uniformly varying load such that the magnitude is decreasing from left to right, it produces the effect of *combined tension and clockwise moment*.
3. **Load of kind-III:** If the member is subjected to an application of uniformly varying load such that the magnitude is decreasing from right to left, it produces the effect of *tension and counter clockwise moment*.

At the top surface of the plate, loads of various kinds are applied as mentioned in section-2.2, while the bottom surface is restricted in the *y*-direction and one node at $x = 0$ and $y = 0$, is restricted in x and y direction, for analyzing the cracked plate.

3. VALIDATION OF THE MODEL

3.1. Stress Intensity Factor

E.E. Gdoutos et. al.[8] developed the mathematical models for evaluating SIF in an edge cracked plate subjected to triangular loading.

The stress intensity factor K_T^I for a single edge cracked plate under uniform tension σ is given by the

$$K_T^I = \sigma_0\sqrt{\pi a}\left[1.12 - 0.23\left(\frac{a}{w}\right) + 10.55\left(\frac{a}{w}\right)^2 - 21.72\left(\frac{a}{w}\right)^3 + 30.94\left(\frac{a}{w}\right)^4\right], a/w < 0.6$$

The stress intensity factor K^B_I for a finite width strip with an edge crack uniform bending σ is given by the

$$K_I^B = \frac{6M_0}{b^2}\sqrt{\pi a}\left[1.12 - 1.40\left(\frac{a}{b}\right) + 7.33\left(\frac{a}{b}\right)^2 - 13.08\left(\frac{a}{b}\right)^3 + 14.0\left(\frac{a}{b}\right)^4\right]$$

The stress intensity factor K_I for the triangular load of fig (*a*) is obtained by adding the stress intensity factors for the uniform and bending loads. We have

$$K_I = K_I^T + K_I^B$$

In this case, we have yield strength of the material, $\sigma = 344.5$ Mpa

$$\sigma_0 \;=\; \frac{\sigma}{2} = 172.25 \text{ Mpa}$$

$$M_0 \;=\; \frac{1}{2}\frac{\sigma}{2}\frac{w}{2}\frac{2w}{3} = \frac{\sigma w^2}{12}$$

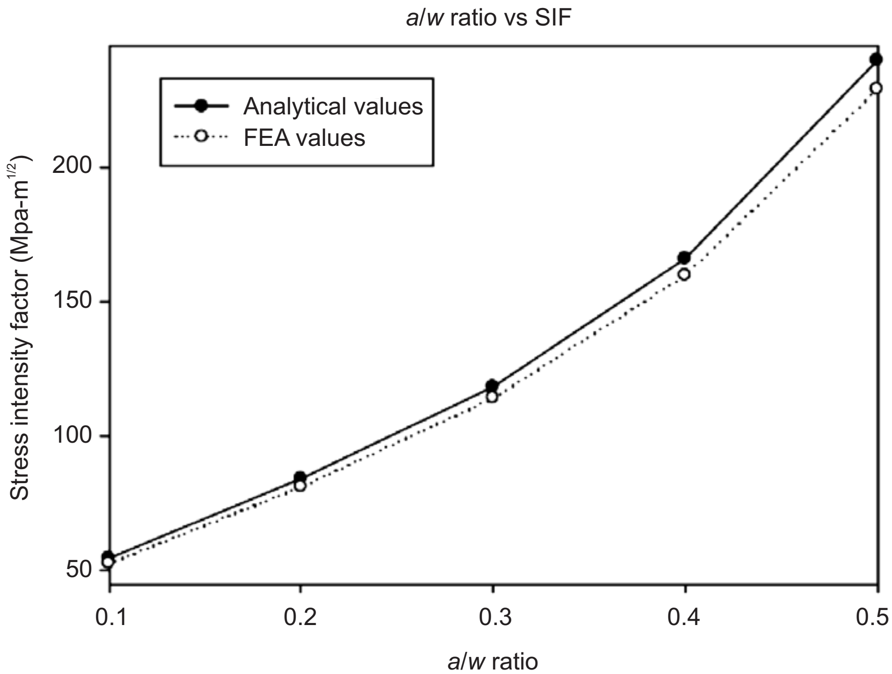


Figure 3: Comparison of Stress Intensity factor obtained Analytically and FEA

In view to assess the validity of the developed FE model for the present problem, the FE simulation results were compared with the analytical results according to the investigation of E.E. Gdoutos et. al.[8]. Fig. 3 presents the variation of SIF (K_I) values obtained from FE simulations to that of the analytical models for varying *a*/*w* ratio. Considerably a slight deviation of 3.4-4.4% was observed in the FEA results from the analytical values. Hence the developed model could be used for further analysis in the present investigation.

3.2. Stress Triaxiality Factor

It is known to have a great effect on plastic deformation and is defined as the ratio of average stress to hydrostatic stress to the equivalent stress. In this work, ductile material is considered and hence Von-Mises stress is calculated for equivalent stress.

$$M \;=\; \frac{\text{Average or hydrostatic stress}}{\text{von-mises stress}}$$

$$= \frac{\dfrac{\sigma_1+\sigma_2+\sigma_3}{3}}{\dfrac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2}}$$

Where σ_1 ,σ_2, σ_3 represents the principal stresses with decrease order in magnitude. The maximum shear stress induced can be calculated by knowing the magnitude of principal stresses.

4. RESULTS AND DISCUSSION

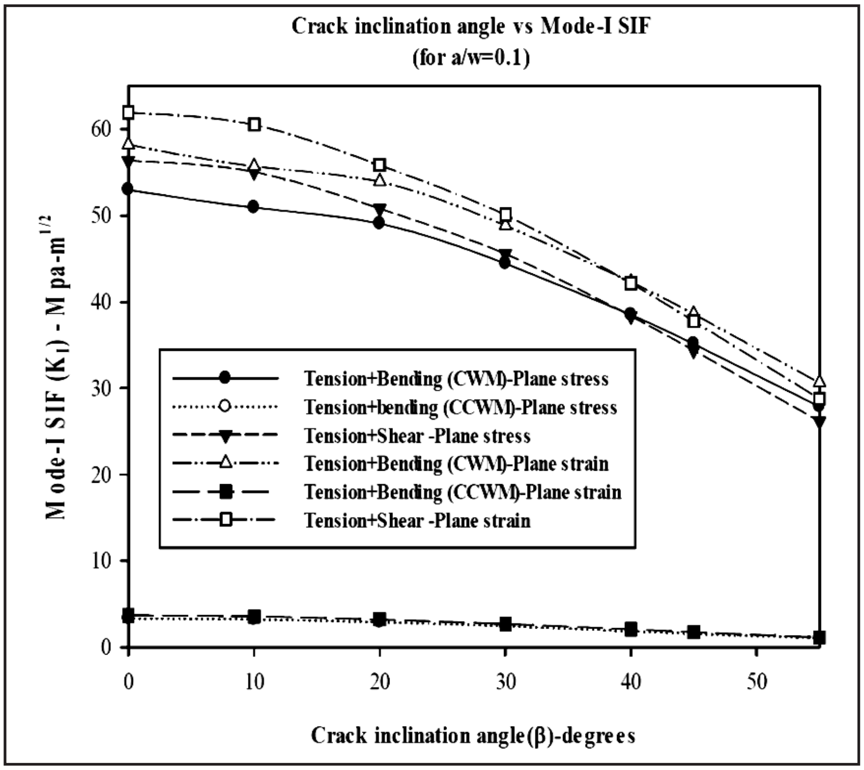
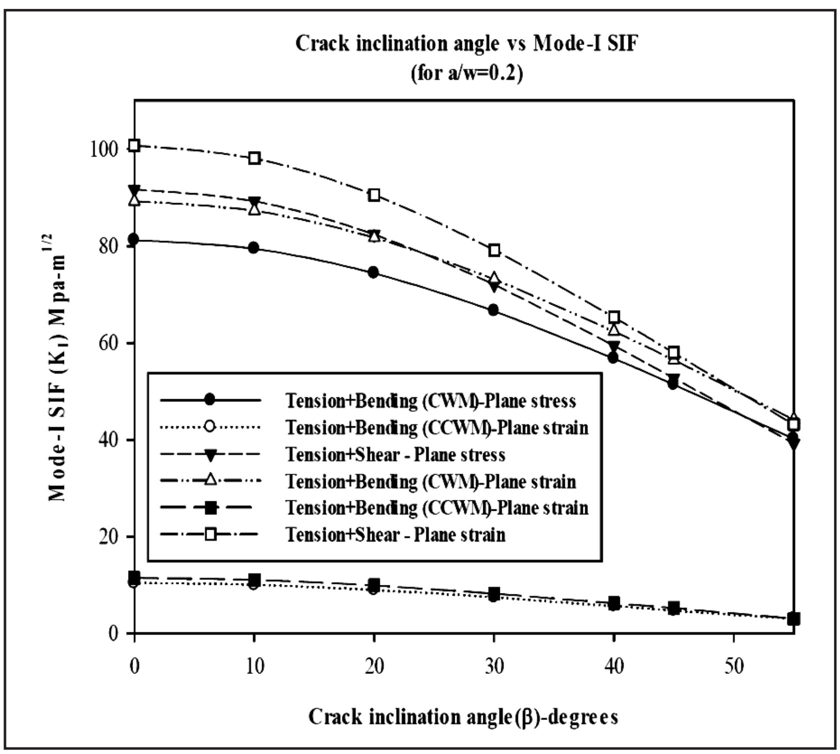


Figure 3



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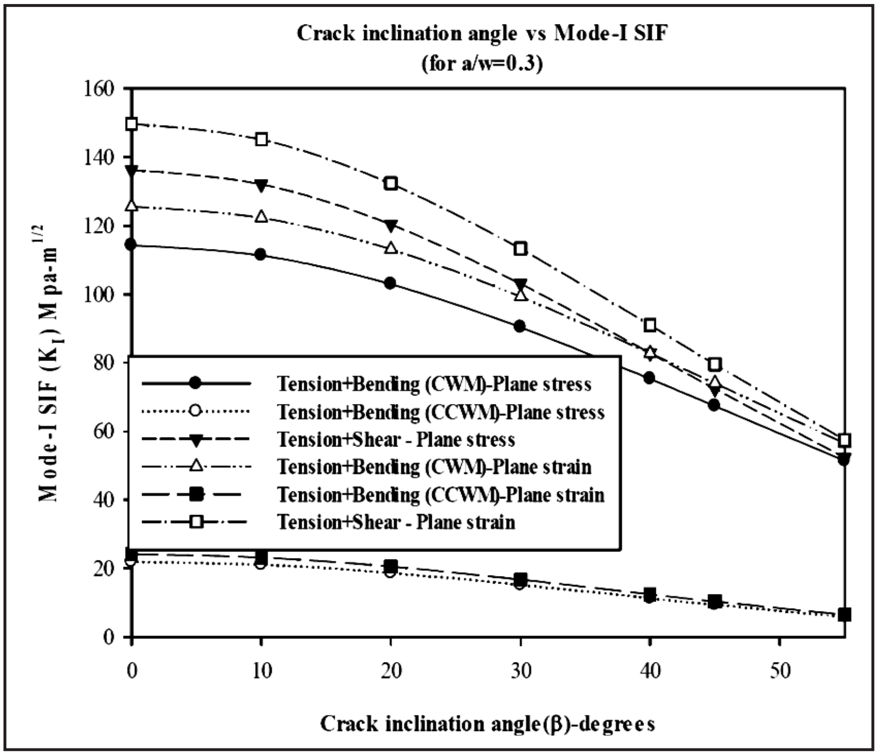

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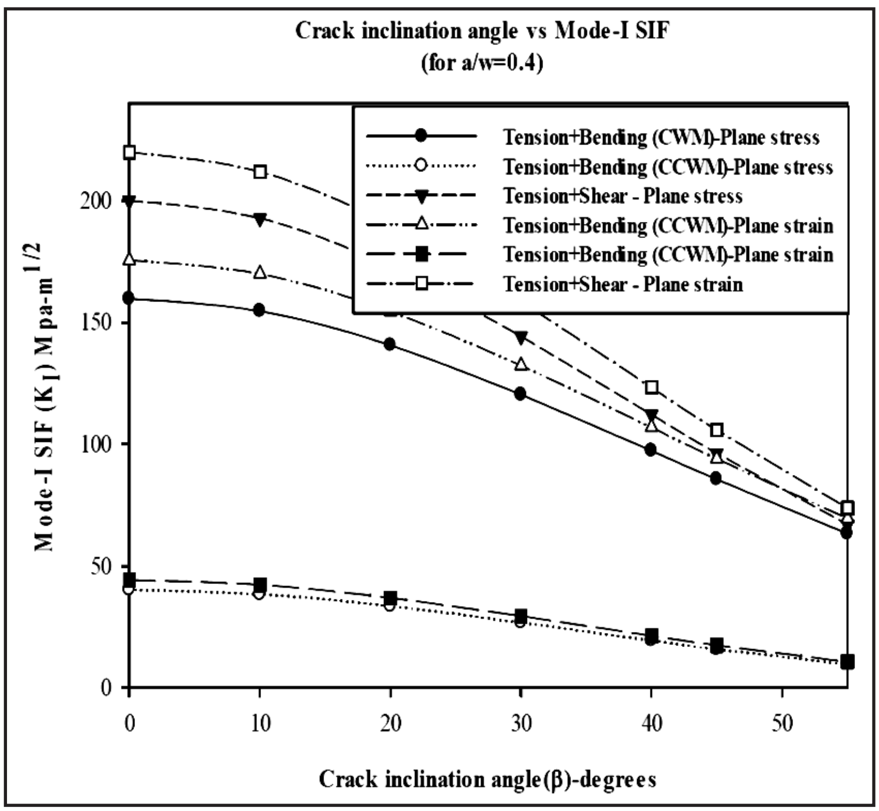

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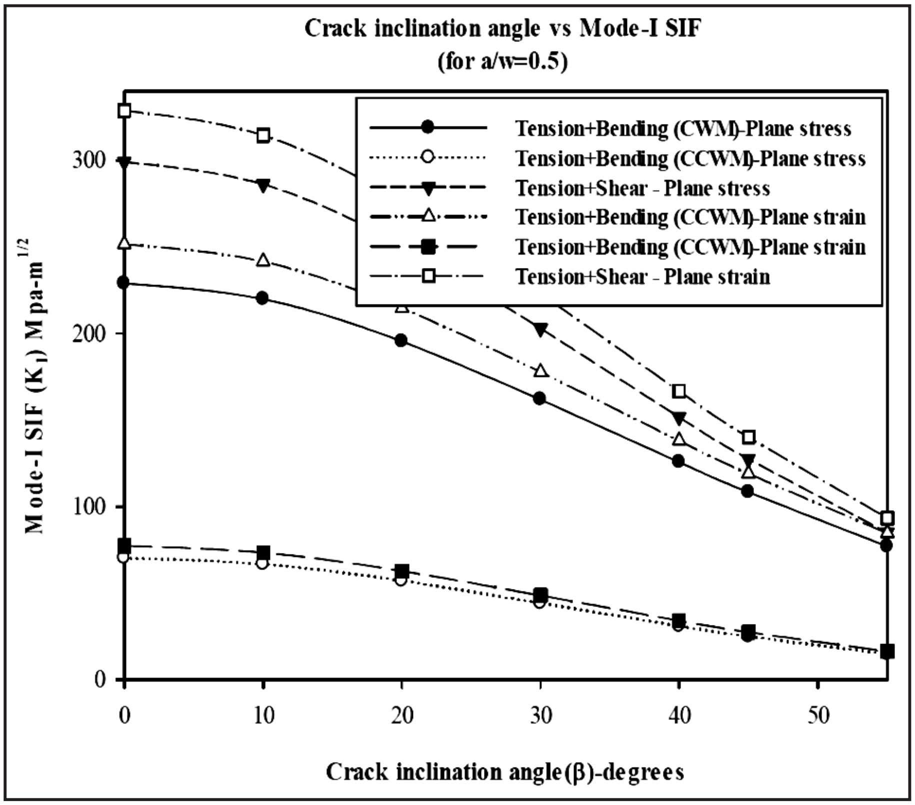

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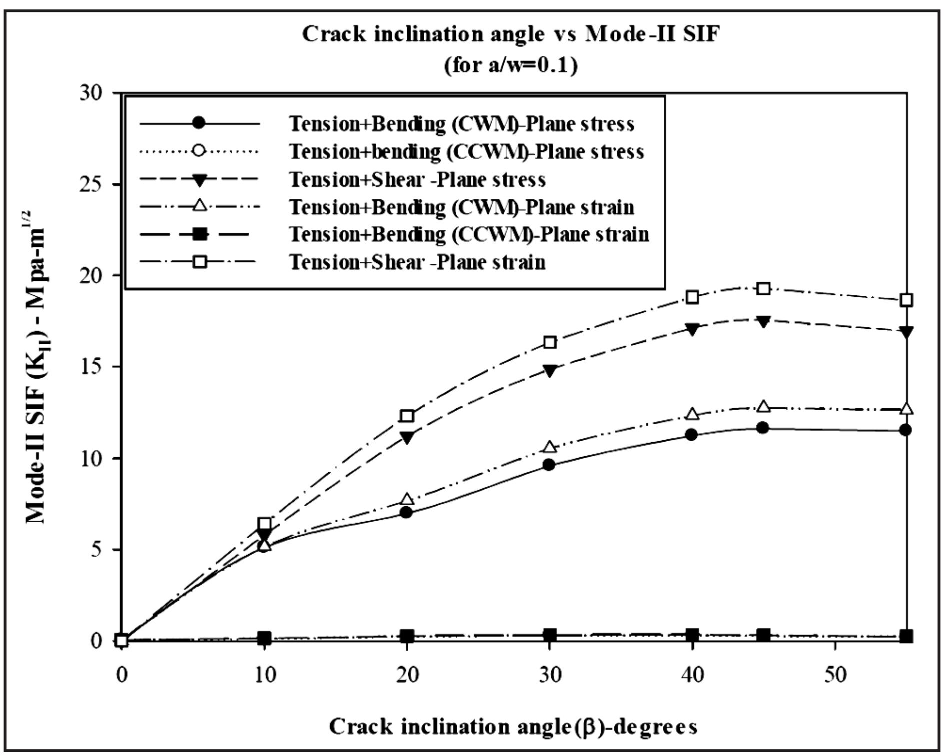

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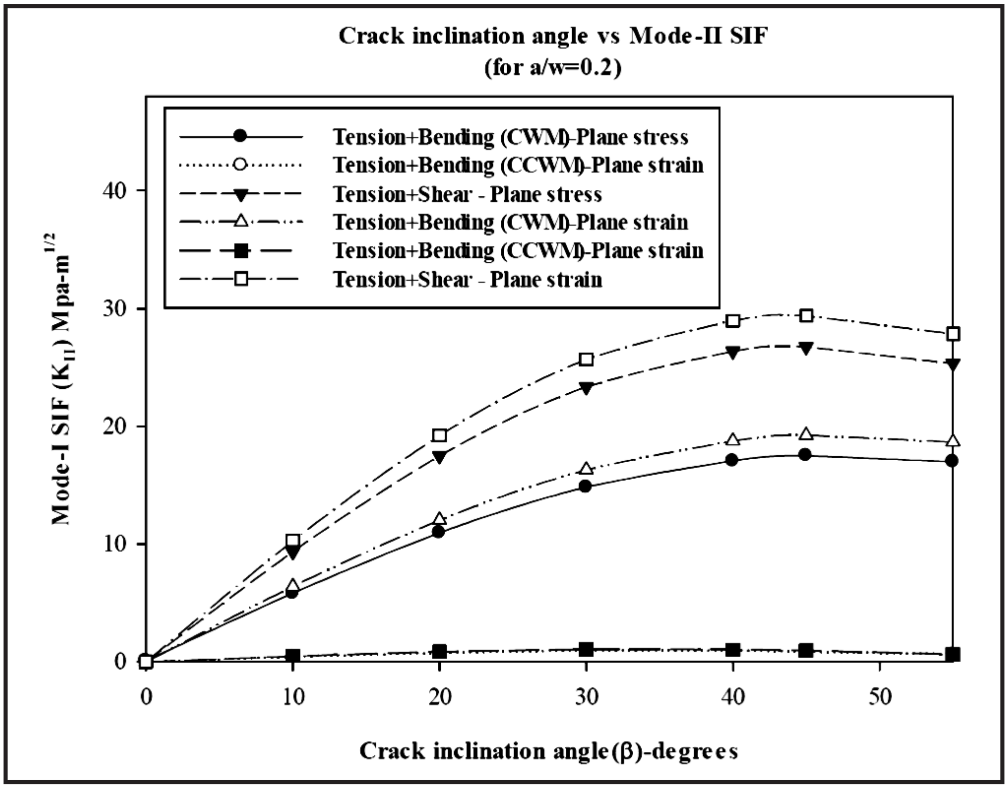

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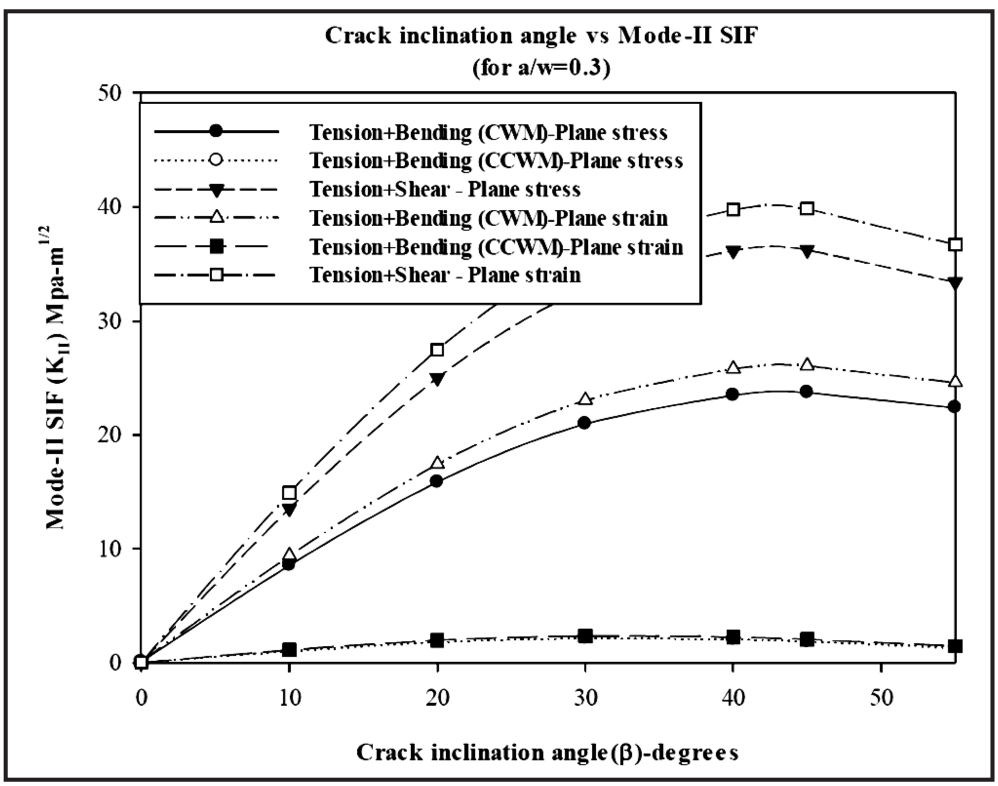

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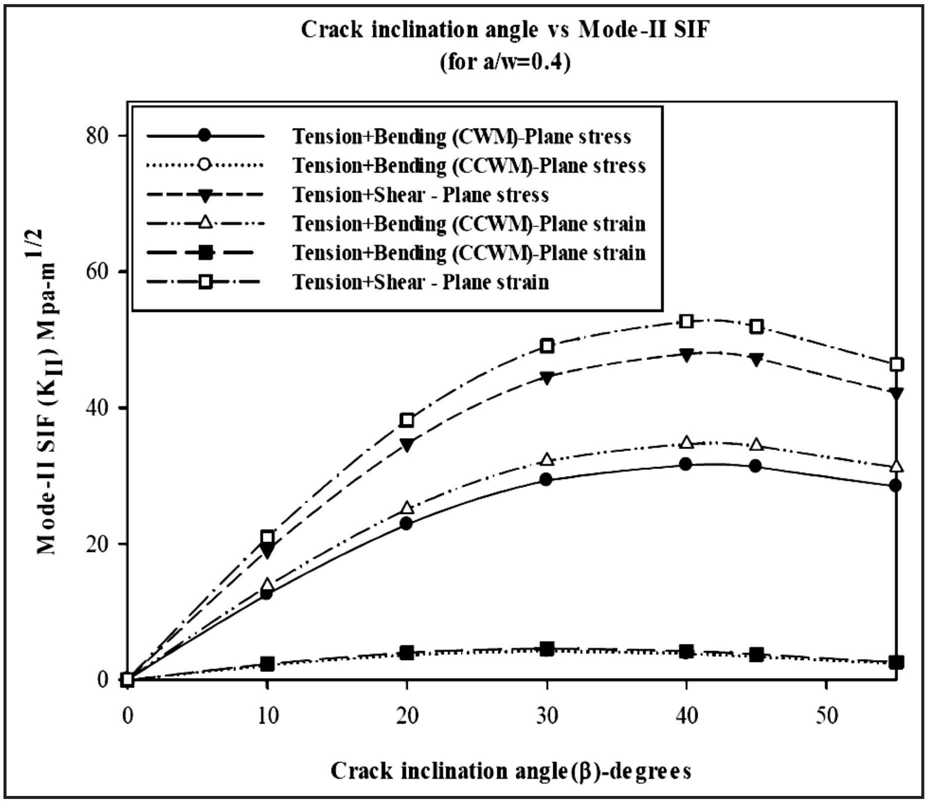

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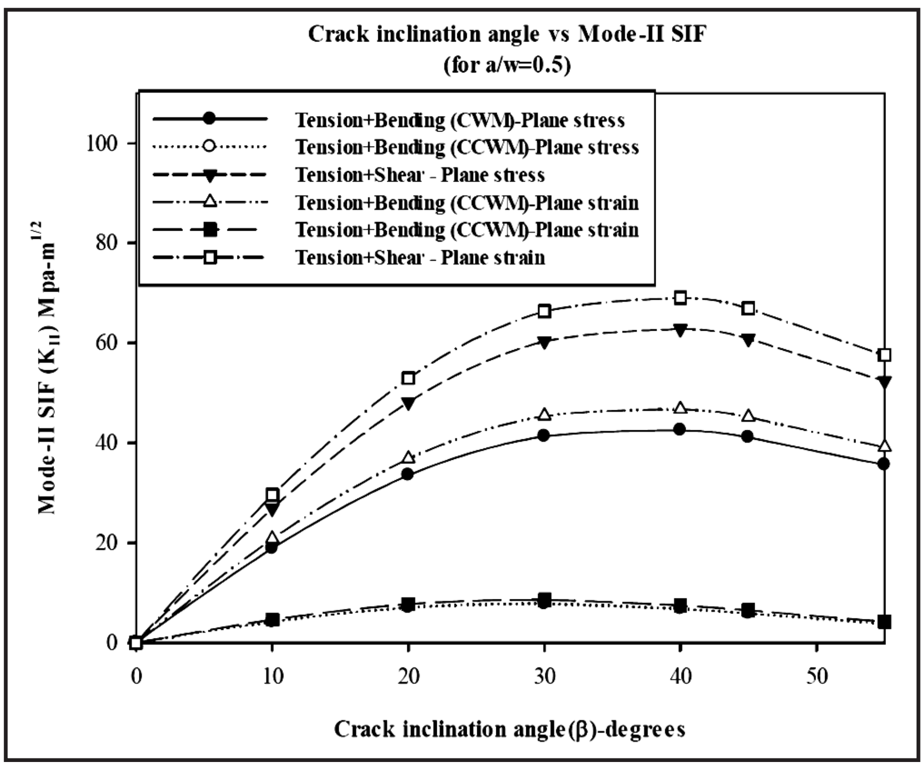

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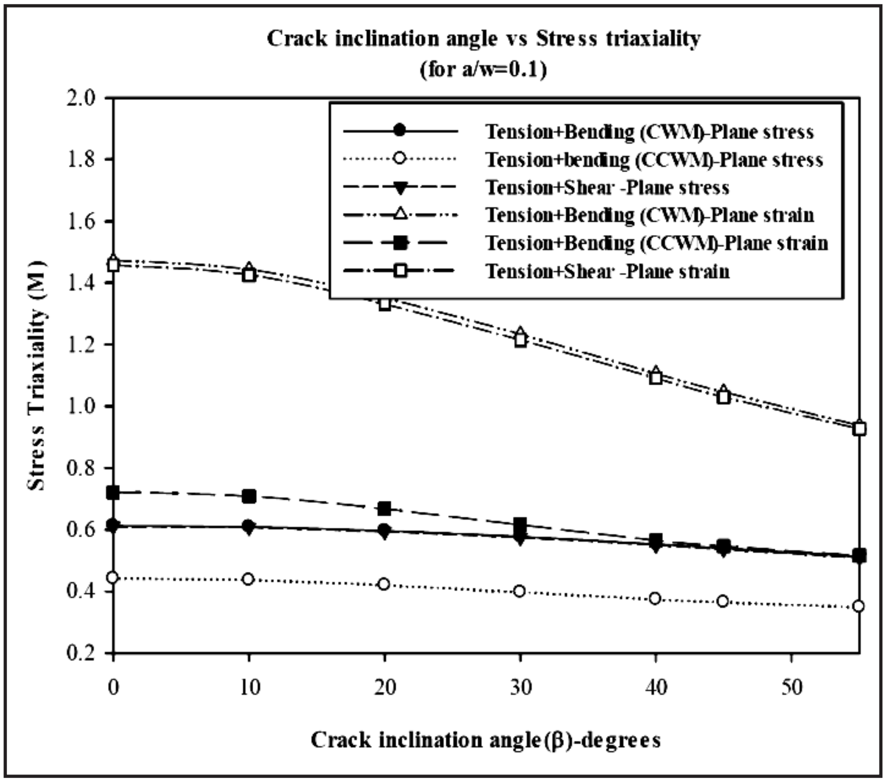

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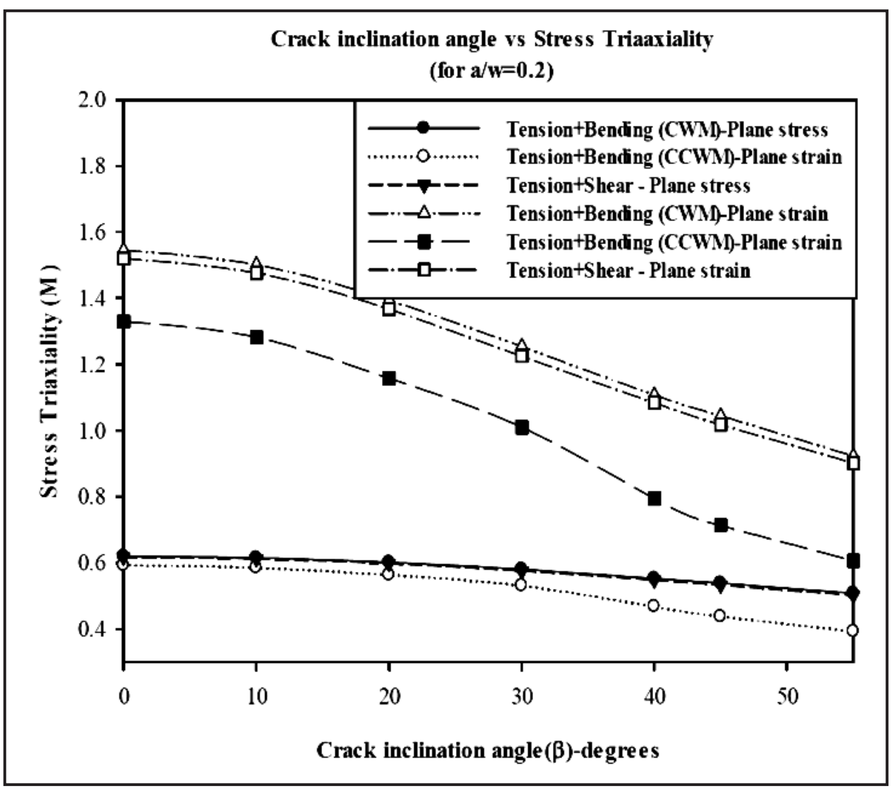

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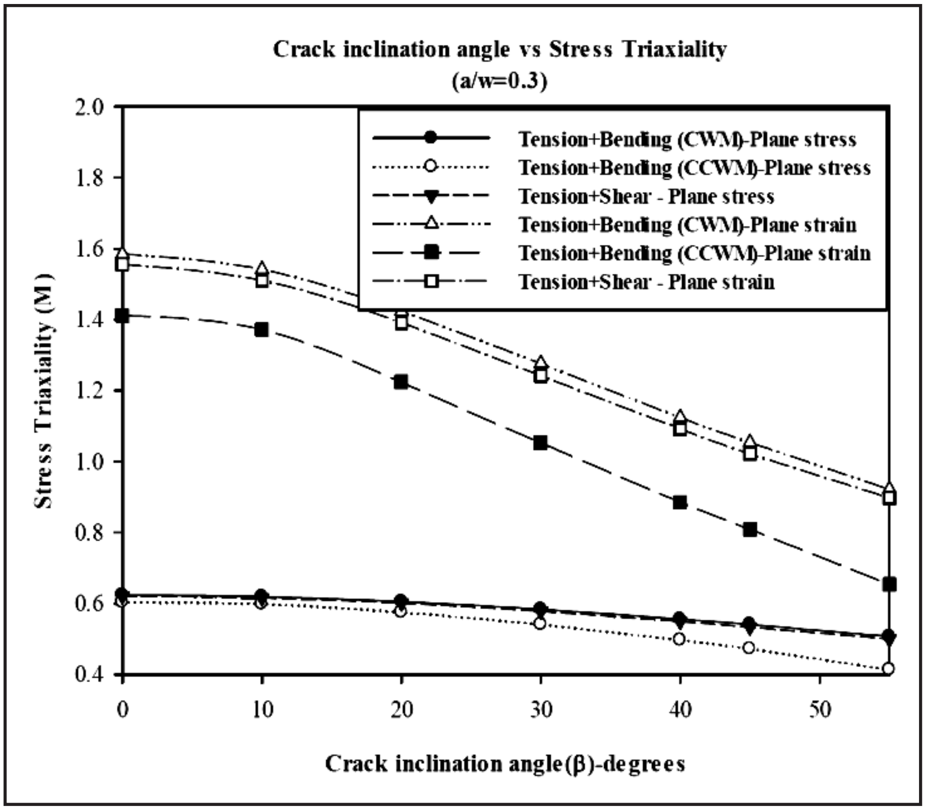

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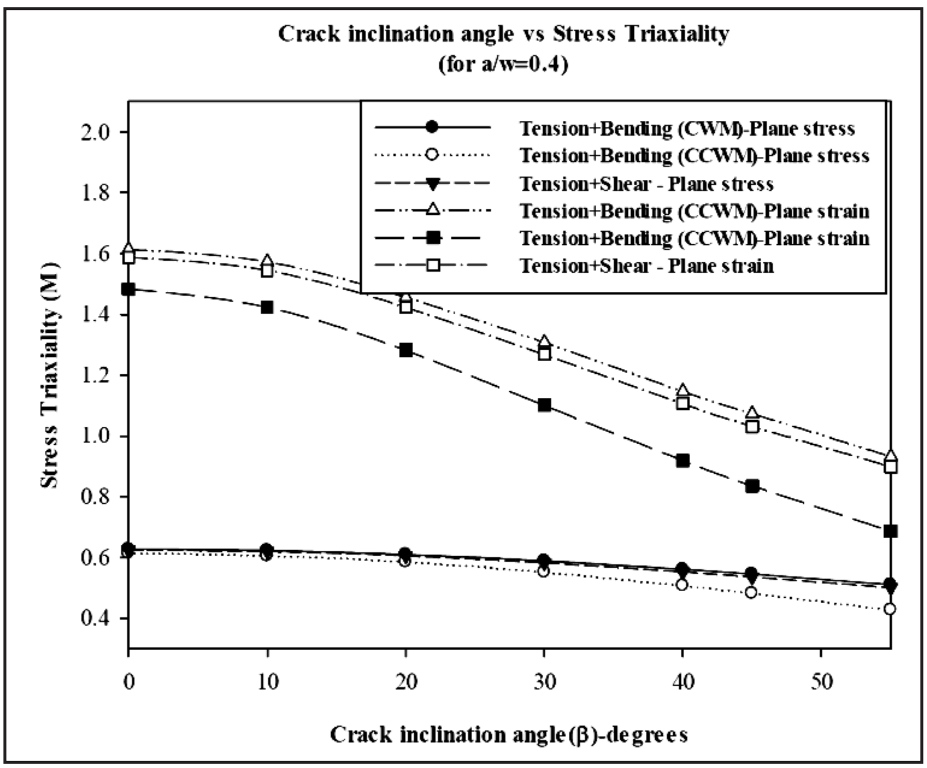

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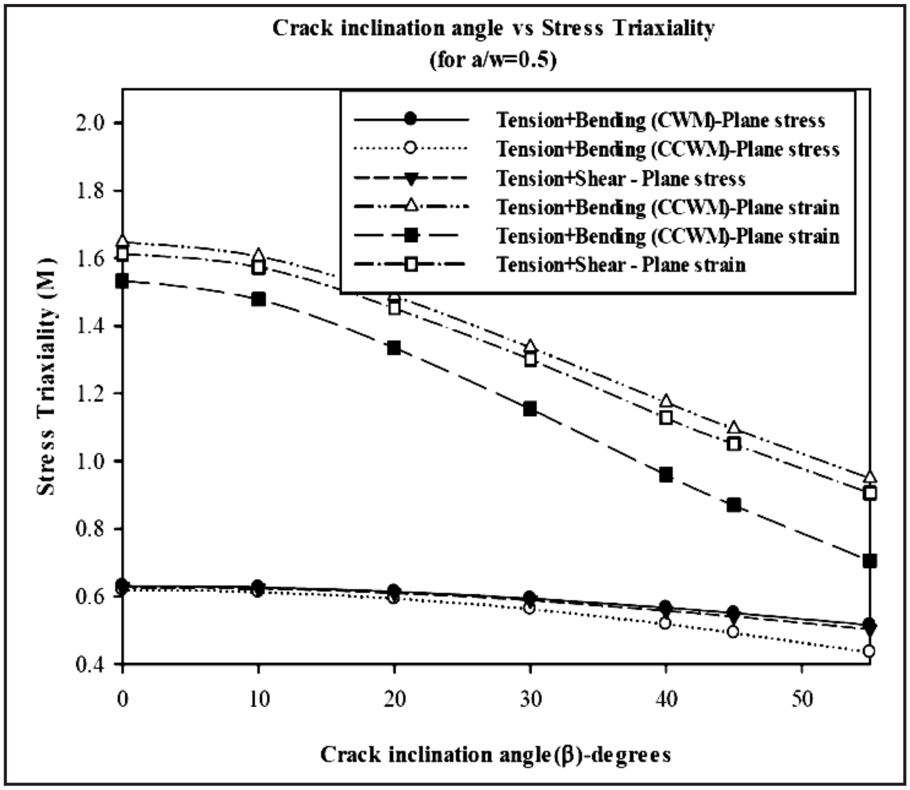

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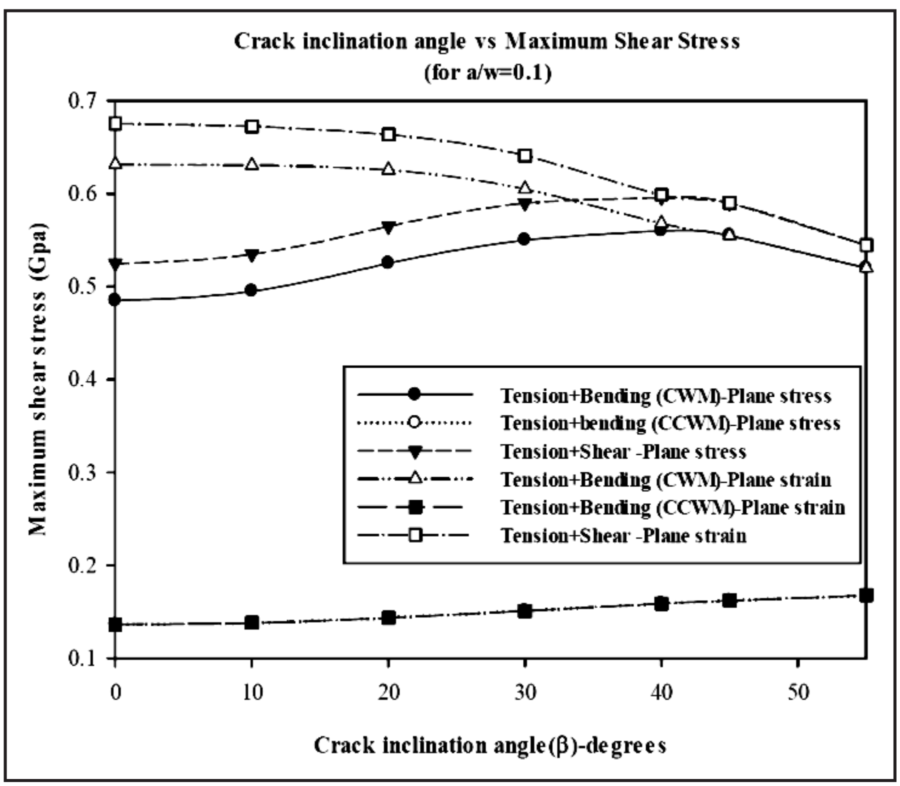

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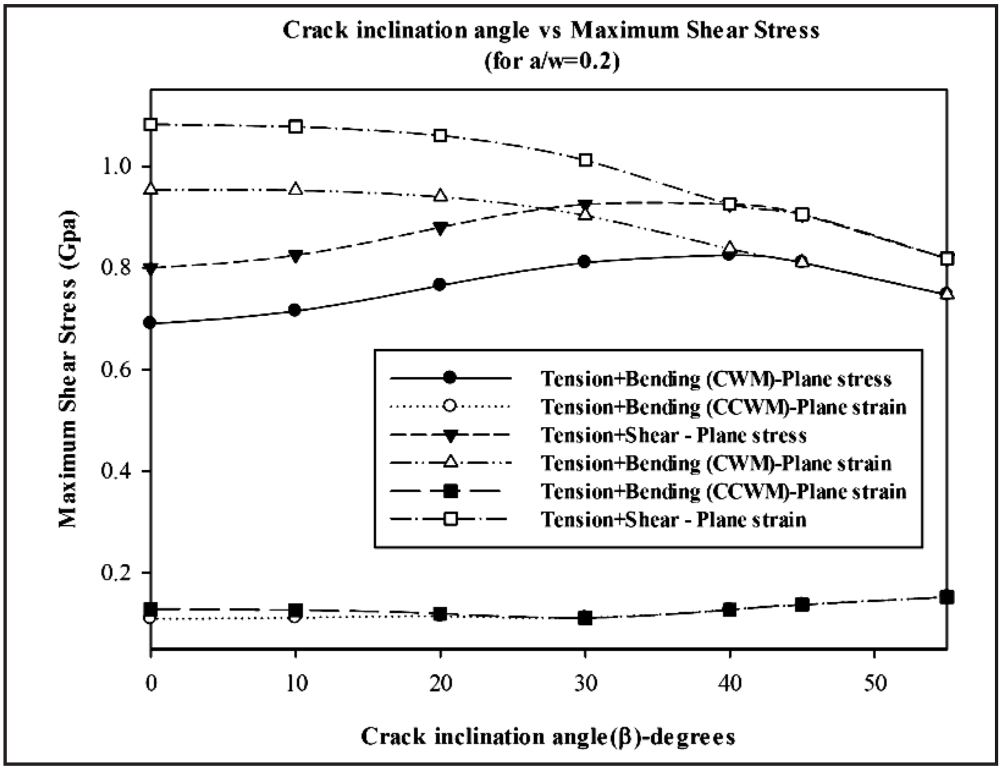

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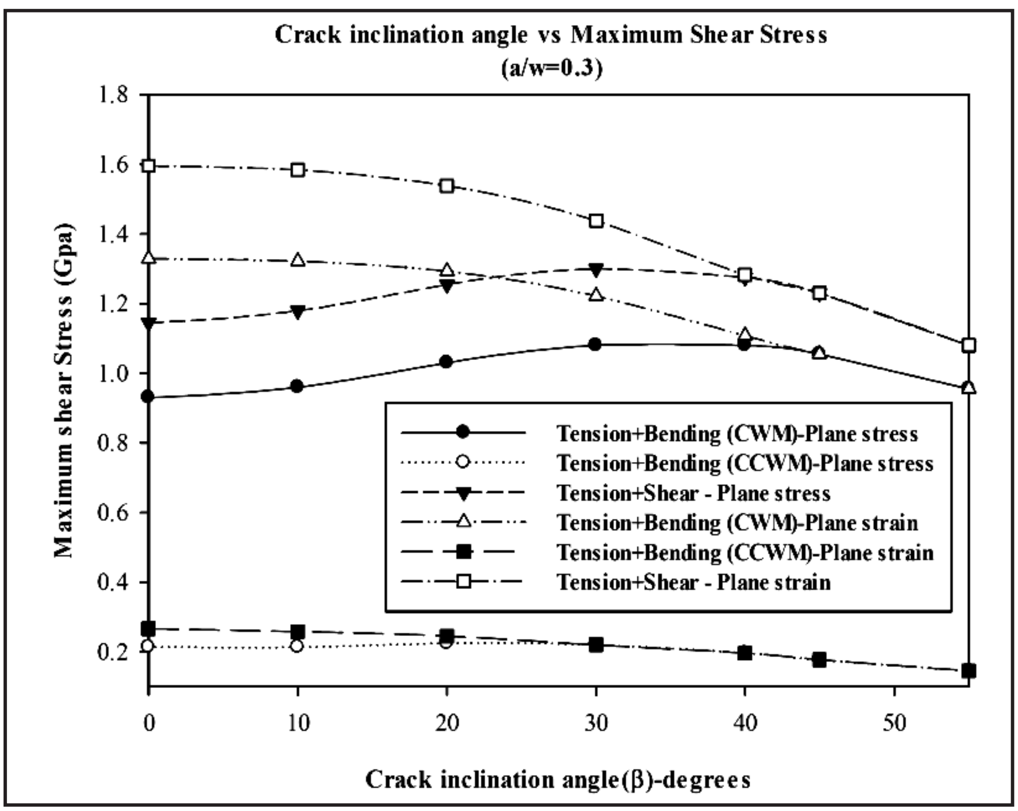

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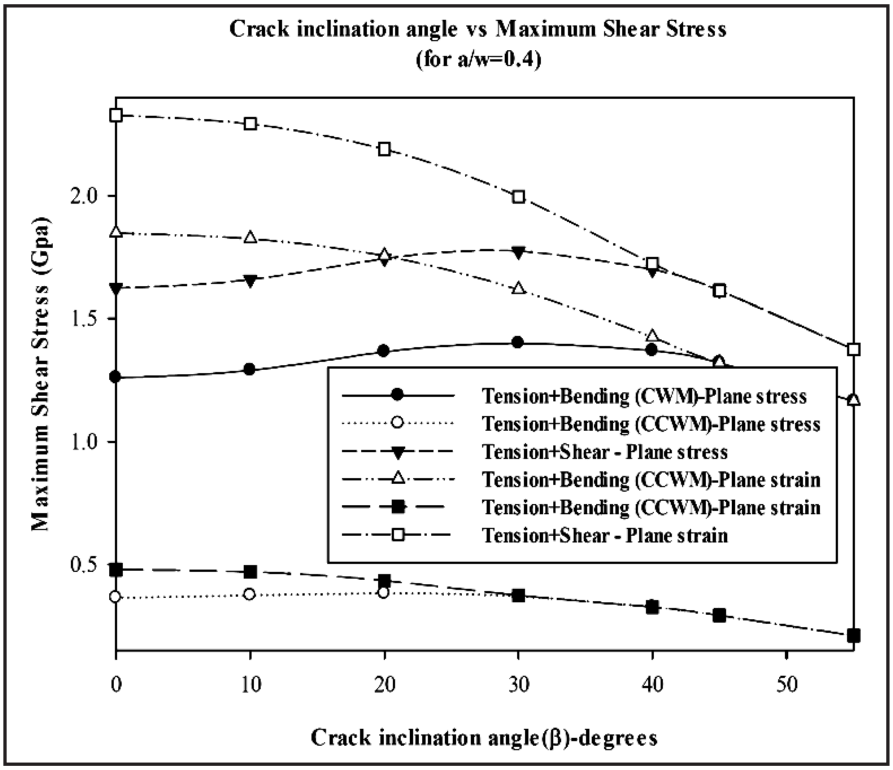

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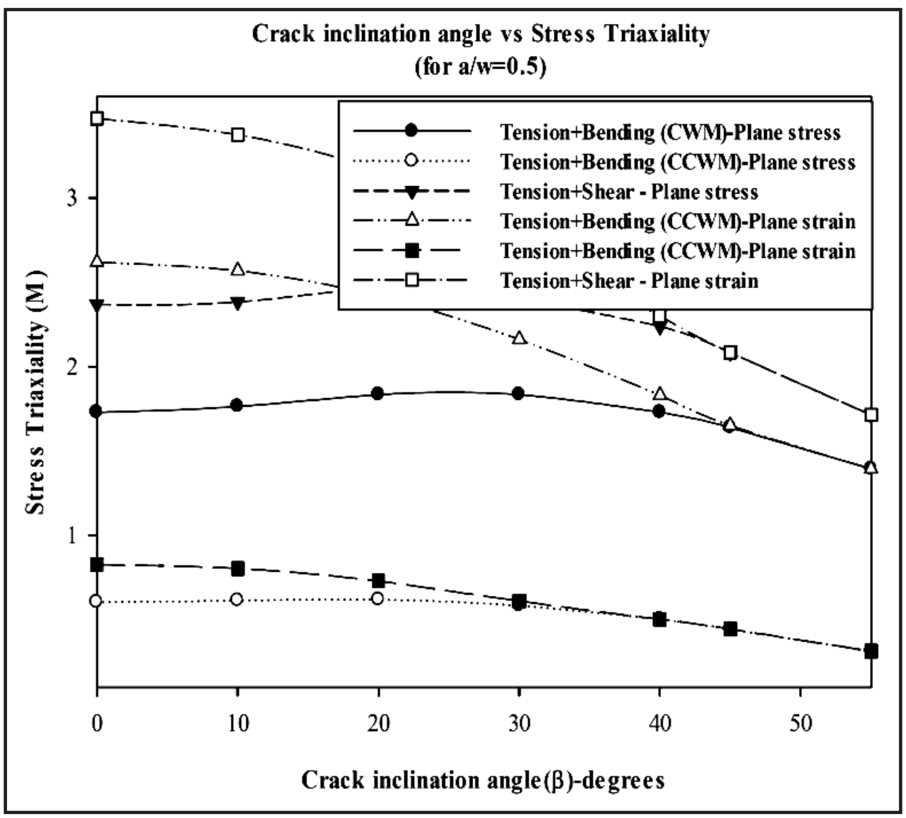

Figure 22

From the finite element analysis, stress intensity factors, components of normal stress, shear stress and principal stresses are computed numerically at different crack inclination angle (β) varied from 0° to 55°, subject to various kinds of loads. Fig. 3 to fig. 7 demonstrates the variation of mode-I SIF and Fig. 8 to fig. 12 demonstrates the comparison of the mode-II SIF, calculated at various crack inclination angles for various a/w ratios. It has been observed that increase in crack inclination angle (β) leads to decrease of mode-I stress field intensity and increase in mode-II stress field intensity. This occurred because of mode mixity as moving from mode I towards mode II, the shear stress component or bending stress dominate the normal stress components and thus, high load would be needed in order to propagate the crack. In support of this statement, fig. 13 to fig. 17 depicted that increase in β leads to decrease in stress triaxiality thus reduces the chance to propagate and thus plate containing inclined crack can sustain higher load without failure as compared to pure mode I condition.

Of all kinds of loading conditions, combined tension-shear mode and combined tension-clockwise moment tends to open the crack with larger plastic deformations whereas the combined tension-counter clockwise moment tends to close the crack with developing a new crack in the opposite direction. This needs higher loads and hence the SIFs obtained for the load of kind-III are very less. This mode of loading is not all desirable which leads to the formation of new cracks.

In addition, it has been observed that the SIFs reached higher values at higher a/w ratio, indicating that the plate cannot withstand higher loads due to lesser resistive area and hence these design conditions are to be omitted. When such service conditions are observed, suitable measures are to be taken for the closure of the crack, so that the propagation as well as huge damage may be avoided.

Fig, 18 to fig. 22 depicts the variation of maximum shear stress calculated based on the principal stresses. It has been observed that the maximum shear stress is increasing in plane stress with decrease in plane strain condition. This is due to the fact that the thickness is more in plane strain condition. Also the effect of kind-I and kind-II loads is same *i.e*. opening of the crack. But due to lesser shear stress developed in kind-II loading, higher loads are needed to make the crack critical. Even the stress triaxiality values reflect the same.

5. CONCLUSION

The focus of this article is to investigate the effect of a/w ratio, loading conditions on the reponse of the cracked plate.

1. Increase in crack inclination angle leads to decrease in stress triaxiality thus reduces the possibility of crack propagation and thus plate containing inclined crack can sustain higher load than pure mode I case.
2. Higher loads are loaded needed to make the crack critical for the load of kind-III (combined tension and counter clockwise moment), compared to other two kinds of loads, which is quite dangerous.
3. If conditions leading to higher a/w ratios are observed during the service, then suitable measures are to be suggested to avoid the sudden failure.

6. ACKNOWLEDGMENT

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