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A Novel Asymmetric Hyperchaotic System and its Circuit Validation

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Abstract: The Objective of the paper is to develop a new asymmetric hyperchaotic system with more complexity. The system has two dissimilar equilibrium points. Non-uniform contraction and expansion of volume in phase space is also an important property of the proposed system. A hyperchaotic system with above characteristics is first time reported here. Details theoretical and numerical analyses of the proposed hyperchaotic system are presented. The hyperchaotic system is analyzed using phase portrait, Poincare Map, waveform, frequency spectrum. Lyapunov spectrum analyses depict that system has large range of parameter for chaos and hyperchaos. The system has comparatively more frequency spectrum and is applicable for secure communication. Circuit design of the system is also presented.

***Keywords**:* Hyperchaotic system; Lyapunov spectrum; circuit design; asymmetric.

1. INTRODUCTION

Now a days requirement of more complex behaviour in the field of chaos leads to the development of new hyperchaotic systems. Because of more random and unpredictable behaviour, application of hyperchaotic systems are explored in many fields like electronics, Information theory, computer science, security (Banerjee et al. 2011). So, constructing a new hyperchaotic system is the motivation of this paper.

In the last two decades, many hyperchaotic systems have been reported. But construction of more complex and unpredictable hyperchaotic system is still a challenging problem [3]. Chaotic systems with asymmetric nature and non-similar equilibrium points are more complex than symmetrical and similar equilibrium points chaotic systems. Most of the reported hyperchaotic systems have symmetric have symmetric nature like hyperchaotic Liu [4], hyperchaotic Chen [5], hyperchaotic Bao [6], hyperchaotic Lorenz [7], hyperchaotic Lorenz [8, 9], hyperchaotic Lu [10], hyperchaotic Qi [11], Hyperchaotic Xu [12], Modified Lu [13], etc. Very few hyperchaotic systems have asymmetric behaviour like Rossler system [1] etc. Thus, forming a asymmetry hyperchaotic system is a new problem.

In this paper a new 4-D, twelve terms hyperchaotic system with asymmetric property, non-similar equilibrium point is reported. Detailed theoretical and numerical analyses of the system are presented. Attractor, Poincare map, frequency spectrum, waveform, Lyapunov spectrum, are the tools used to analyze the system. The novelty of the proposed hyperchaotic system is validated using the following points:

1. The proposed hyperchaotic system has non-uniform contraction and expansion of volume in phase space, which is not in the case in earlier reported systems like in [5-16].
2. The proposed system is not symmetric about its axis, plane, and space. But the reported systems like in [5-14, 16] have symmetrical nature of attractor.

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3. The novel system has dissimilar and asymmetric equilibria whereas many earlier reported hyperchaotic systems have similar and symmetrical equilibria like [3, 5, 6, 9, 10].
4. The proposed system is comparatively more complex with above three properties compared to the systems in [5, 6, 9, 10, 16].
5. The proposed system has relatively larger bandwidth compared to the systems like in [1, 5, 8, 9].

Section 2 describes the dynamics of the proposed hyperchaotic system. Theoretical analyses and description of the proposed hyperchaotic system are presented in Section 3. Section 4 describes the numerical analyses and discussions of the proposed hyperchaotic system; finally conclusions are given in Section 5.

2. DYNAMICS OF THE PROPOSED HYPERCHAOTIC SYSTEM

Considering the conditions for hyperchaotic systems (dynamical system must be of 4-D, it must have two positive Lyapunov exponents), a new system with hyperchaotic behaviour is described as:

$$\begin{gathered} \dot{x}_1 = (d-1)x_2 - dx_1 - x_3 \\ \dot{x}_2 = rx_1 - (1-a)x_2 - 10x_1x_3 + x_4 \\ \dot{x}_3 = 10x_1x_2 - bx_3 + x_1(x_3 - c) \\ \dot{x}_4 = px_2 \end{gathered} \tag{1}$$

where x_1, x_2, x_3, x_4, are the state variables and *a, b, d, r* are the positive parameters of the system (1) and $c \in R$, $p = -4$. Range and value of the parameters are founded using Lyapunov spectrum which is discussed in Section 4.4. When $d = 33$, $r = 46.6$, $a = 4$, b = 10, $c = 11$, $p = -4$ Lyapunov exponents of the system are L_i = 1.8599, 0.13314, –0.00088, –42.067 and Lyapunov dimension is L_D = 3.046. Detailed theoretical description of the system are given in the upcoming section.

3. THEORETICAL DESCRIPTION OF THE PROPOSED SYSTEM

This section describes some common basic properties of the system (1) using theoretical analyses.

3.1 Equilibrium Point and their Behaviour

Equilibrium points of the system are given as: E_0 = (0,0,0,0),

$E_1 = \left(\frac{-(c-bd)}{d}, 0, \frac{(c-bd)}{d}, \frac{((c-bd)(r-10c+10bd))}{d}\right)$ Eigenvalues corresponding to each equilibrium point is given in Table 1.

Table 1

Eigen value of the system (1) with

Sl. no	*E0 = (0, 0, 0, 0)*	*E_1 = (9.66, 0 – 319, –31287.133*
1.	$\lambda_1 = -57.7492$	$\lambda_1 = -321.00$
2.	$\lambda_2 = 27.5870$	$\lambda_2 = 259.238$
3.	$\lambda_3 = 0.0808$	$\lambda_3 = -4.5725$
4.	λ_4 = –9. –185	$\lambda_4 = -0.0029$

It is clear from Table 1 that all the equilibrium points are unstable and dissimilar and hence increases unpredictability of the system. Both equilibria of the system have saddle node behaviour.

3.2 Asymmetry Property

The proposed system (1) is asymmetric about its principle axis and plane. Asymmetric nature of the system can be easily verified by changing the sign of each state variable. Asymmetry increases random and complex property of a system.

3.3 Dissipativity

The proposed system (1) is a dissipative hyperchaotic system. Dissipative nature of the system is evaluated by calculating divergence of the vector field $V(x)$ on R^4 as:

$$\nabla V = \frac{\partial \dot{x}_1}{\partial x_1} + \frac{\partial \dot{x}_2}{\partial x_2} + \frac{\partial \dot{x}_3}{\partial x_3} + + \frac{\partial \dot{x}_4}{\partial x_4} = -(1-a) - b - d + x_1, \quad V(t) = V(0)e^{-(1-a+b+d)t + x_1(\mathrm{t})} \quad (2)$$

For, $x_1(t) = 0$, $\Delta V < 0$, since $d = 33$, $a = 4$, $b = 10$. So, volume in phase space contract exponentially and system exhibits dissipative hyperchaotic system. For $x_1(t) = 0$, and the system has non-uniform shrinking and expansion of volume in phase space. This property also increases the complexity of the system.

4. NUMERICAL ANALYSES OF THE PROPOSED SYSTEM

This section deals with the numerical analyses and discussions of the system (1).

4.1 Hyperchaotic Attractor

Fig. 1 (a-d) shows the hyperchaotic attractors of the system (1). The following observations can be made from Fig.1:

System has attractors similar to Lorenz type hyperchaotic system double scrolls, butterfly shape. But the attractors of the proposed system are different because of asymmetric nature. Attractors of the system are more complex and dense. Complexity of the system increases because of non-uniform contraction and expansion of volume.

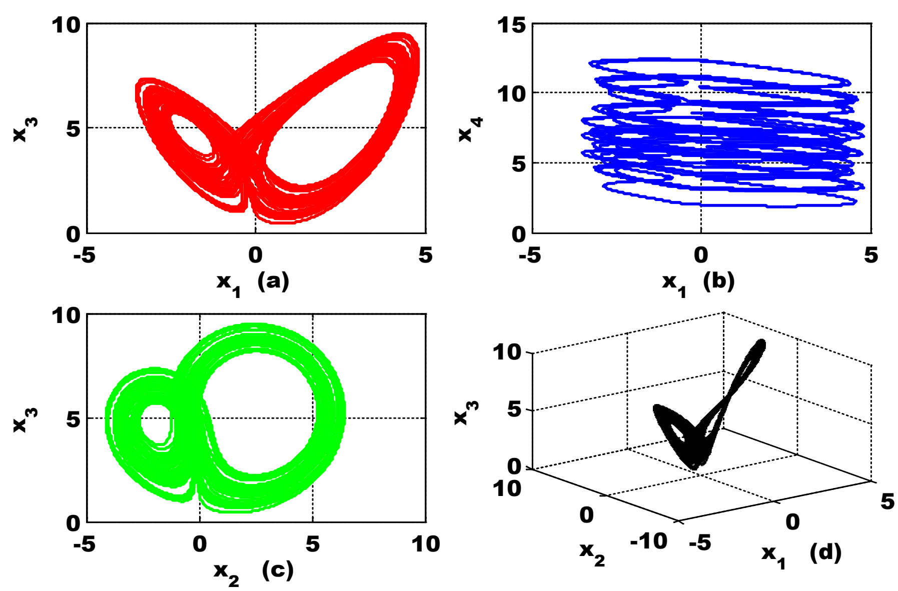

Figure 1: Orbits of the system (1) with $d = 33$, $r = 46.6$, $a = 4, b = 10, c = 11, p = -4$ (a) on $x_1 - x_3$ plane, (b) on $x_1 - x_4$ plane, (c) on $x_2 - x_3$ plane, (d) in $x_1 - x_2 - x_3$ space.

4.2 Poincare Maps and Waveform

The dynamics of the system (1) are also analysed with Poincare map. Fig. 2 (a, b) shows the Poincare map of the system (1) in different planes. It is observed from the figure that several sheet of attractors are separated asymmetrically and folded. Waveform plot is given in Fig. 2 (c). Results show that the system has complex and dense structure.

4.3 Frequency Spectrum

Normalized frequency spectrum of $x_2(t)$ signal of the system (1) is shown in Fig. 3. One can see from Fig. 3 that system has large bandwidth. Therefore, the signal of the system (4) may be better for secure communication. The system has [0-13] Hz bandwidth with 0.1 normalized spectral value as cut-off.

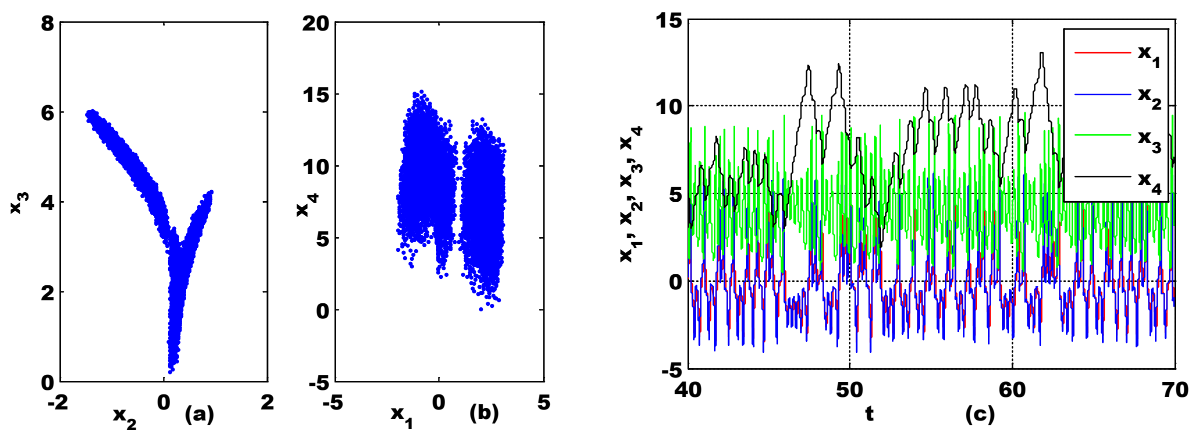

Figure 2: (a) Poincare map in $x_2 - x_3$ plane for $x_1 = 0$ (b) Poincare map in $x_1 - x_4$ plane for $x_2 = 0$ and (c) waveform of system (1), with $d = 33,\ r = 46.6,\ a = 4, b = 10, c = 11, p = -4$.

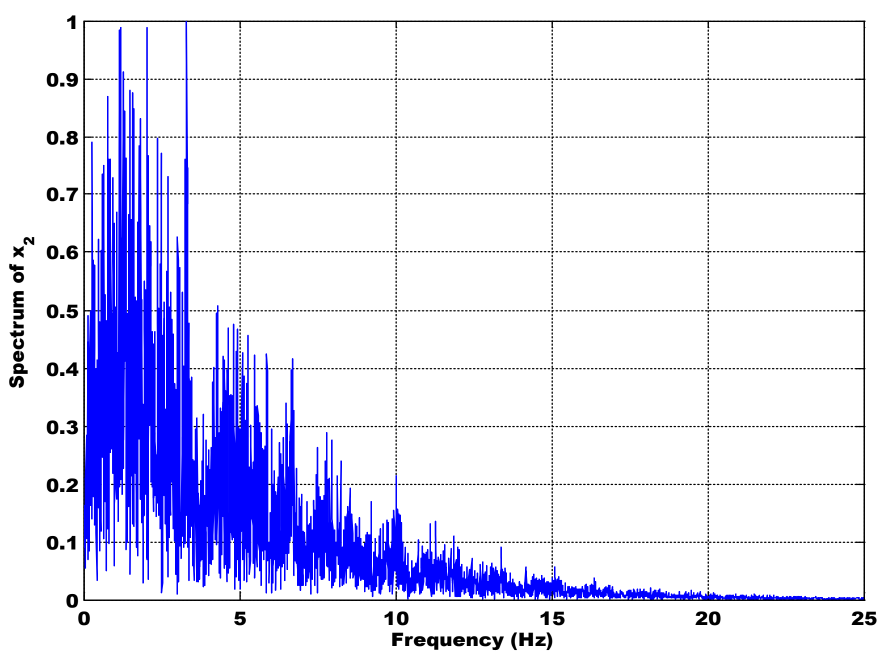

Figure: 3 Normalized frequency spectra of the system (1) for $d = 33,\ r = 46.6,\ a = 4, b = 10, c = 11, p = -4$.

4.4 Lyapunov Spectrum Analysis

Lyapunov spectrum of the system (1) is obtained by fixing five parameters and varying one parameter.

4.4.1 Fix Parameters r = 45, a= 4, b = 10, c = 11, p – 4 and Vary Parameter 4

Fig. 4 (a, b) shows the Lyapunov spectrum (LS) of the system (1) with variation of parameter *d*. It is observed from Fig. 4 that the system produces hyperchaotic, periodic, and chaotic behaviour. Table 2 (a) shows the summary of behaviour of the system (1) for diferent values of *d*.

Table 2 (a)

Behaviour of the proposed system (1) with variation of parameter

Range of parameter	*Behaviour*	*Range of parameter*	*Behaviour*
$0 < d < 15$	Periodic	$47.7 \le d < 49.1$	Periodic
$15 \le d \le 45.5$	Hyperchaotic	$49.2 \le d \le 55$	Chaotic but some periodic window
$45.6 < d \le 47.6$	Chaotic	$55.1 \le \mathrm{d} \le 60$	Periodic

4.4.2 Fix Parameters d = 33, a= 4, b = 10, c = 11, p = – 4 and Vary Parameter r

Fig. 5 (a, b) shows the LS of the system (1) for variation of parameter. Fig. 5 depicts that the system produces hyperchaotic orbits, chaotic orbits and periodic orbits. Table 2 (b) summarizes of dynamical behaviour of the proposed system (1) with variation of parameter.

Table 2 (b)

Different dynamics of the system (1) with variation of parameter

Range of parameter	*Behaviour*	*Range of parameter*	*Behaviour*
$20 < r \le 31.6$	Chaotic	$77.2 \le r \le 77.6$	Chaotic
$31.7 < r < 34.8$	Chaotic; but some periodic window	$77.7 \le r \le 79.6$	Periodic
$34.9 \le r \le 77.2$	Hyperchaotic	$80 \le r \le 90$	Different behaviour

4.4.3 Fix Parameters r = 45.6, d = 33, b = 10, c = 11, p – 4 and Vary Parameter a

Fig. 6 (a, b) shows the LS of the dynamic (1) with variation of parameter. It is observed from Fig. 6 that the system produces different behaviours. Table 2 (c) shows the summary of dynamical behaviour of the proposed system (1).

Table 2 (c)

Dynamical behaviour of the proposed system (1) with variation of parameter

Range of parameter	*Behaviour*	*Range of parameter*	*Behaviour*
$0 < a < 0.2$	Chaotic	$6 < a < 15$	Chaos with some periodic window
$0.23 < a < 0.8$	Periodic	$15 < a < 25$	Periodic
$0.9 < a < 6$	Hyperchaotic		

4.4.4 Fix Parameters r = 45.6, d= 33, a = 4, c = 11, p = – 4 and Vary Parameter b

Fig. 7 (a, b) shows the LS of the proposed system (1) with variation of parameter. It is observed from Fig. 7 that the system produces periodic orbits, and chaotic orbits, and hyperchaotic behaviour. Table 2 (d) shows the summary of performance of the system (1) with varying parameter.

Table 2 (d)

Dynamical performance of the system (1) with variation of parameter

Range of parameter	*Behaviour*	*Range of parameter*	*Behaviour*
$0 < b < 0.2$	Stable	$5.05 < b < 5.6$	Periodic
$0.2 < b < 1.6$	Periodic	$5.6 < b < 13.5$	Hyperchaotic
$1.6 \leq b < 5.05$	Hyperchaotic	$13.5 < b < 20$	Periodic

4.4.5 Fix Parameters r = 45.6, d = 33, a = 4, b = 10, p = – 4 and Vary Parameter c

Fig. 8 (a, b) shows the LS of the system (1) with variation of parameter c. Table 2(e) shows the summary of behaviour of the proposed system (1) with variation of parameter c.

Table 2 (e)

Dynamical performance of the system (1) with variation of parameter

Range of parameter	*Behavior*	*Range of parameter*	*Behaviour*
$-20 < c < -8$	Periodic	$-6 \leq c \leq 45.2$	Hyperchaotic
$-8 \leq c \leq -7.7$	Chaotic	$45.3 \leq c < 54$	Chaotic with some periodic window
$-7.8 \leq \leq -7$	Periodic	$54 \leq c < 60$	Periodic

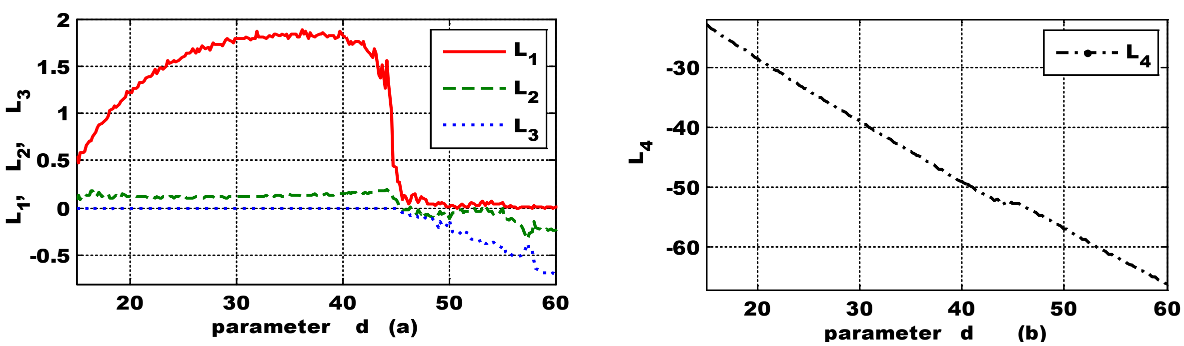

Figure 4: Lyapunov spectrum of the system (1) with $a = 4, r = 46.6, b = 10, c = 11, p = -4$ and $d \in [15, 60]$; (a) Lyapunov exponents L_1, L_2, L_3 and (b) Lyapunov exponent L_4.

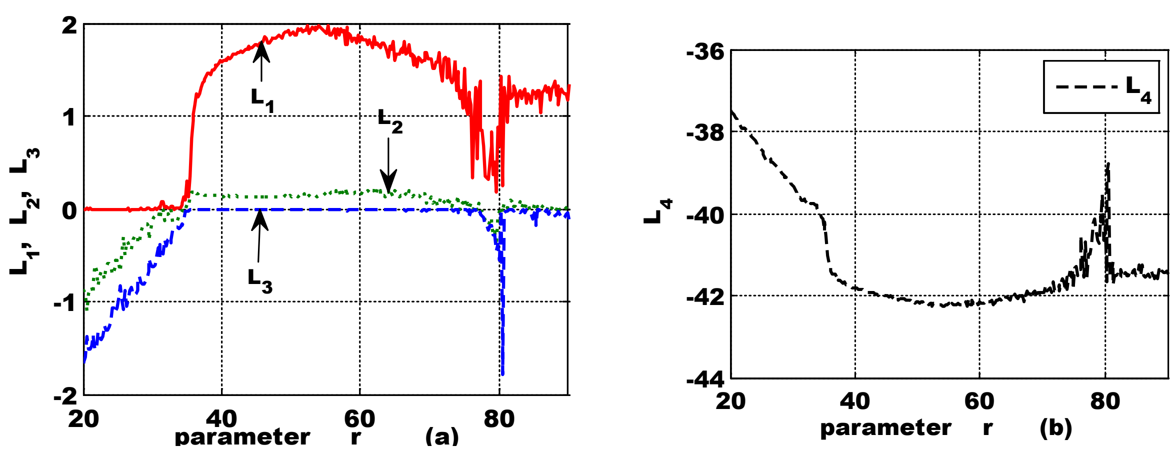

Figure 5: Lyapunov spectrum of the system (1) with $a = 4, d = 33, b = 10, c = 11, p = -4$ and $r \in [20, 90]$; (a) Lyapunov exponents L_1, L_2, L_3 and (b) Lyapunov exponent L_4.

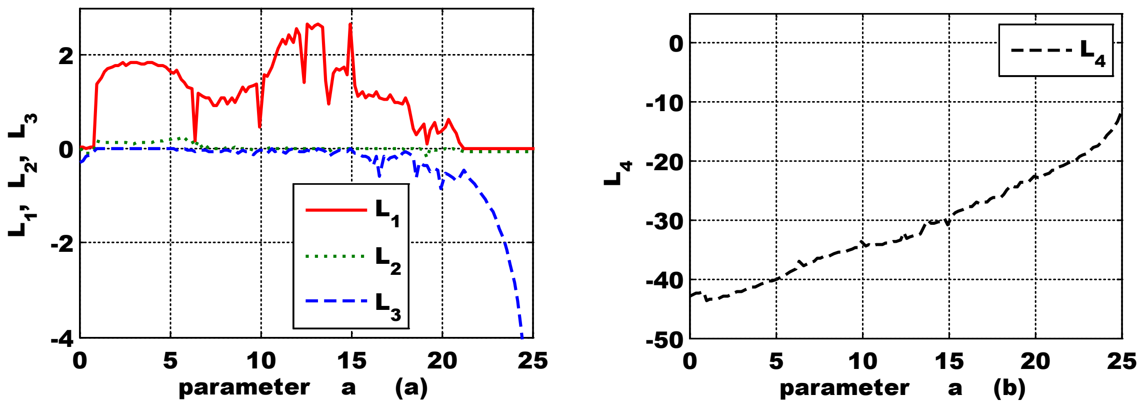

Figure 6: Lyapunov spectrum of the system (1) with $d = 33, r = 46.6, b = 10, c = 11$ and $a \in [0, 25]$; (a) Lyapunov exponents L_1, L_2, L_3 and (b) Lyapunov exponent L_4.

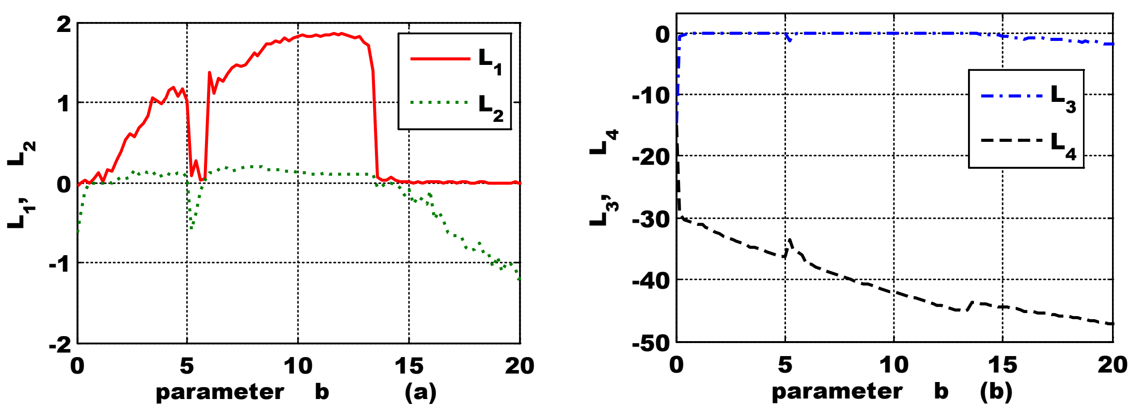

Figure 7: Lyapunov spectrum of the system (1) with $d = 33, r = 46.6, c = 11, a = 4, p = -4$ and $b \in [0, 20]$; (a) Lyapunov exponents L_1, L_2, L_3 and (b) Lyapunov exponent L_4.

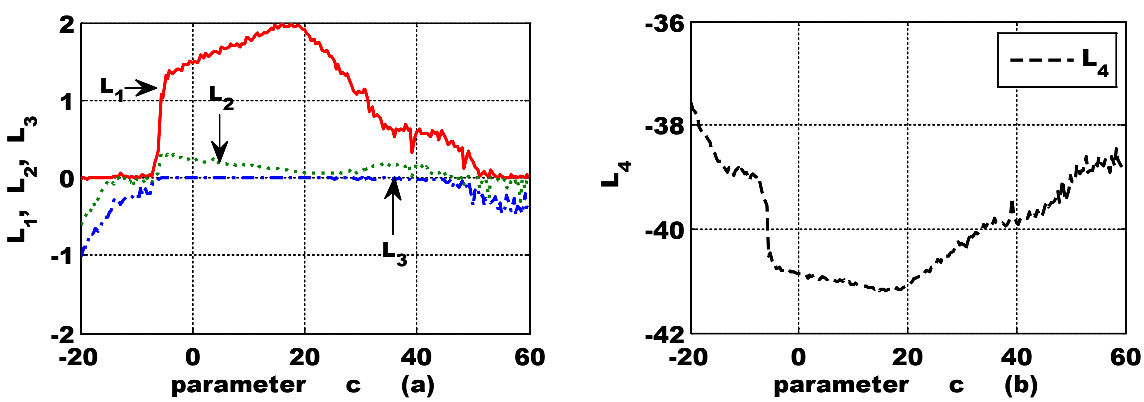

Figure 8: Lyapunov spectrum of the system (1) with $d = 33, r = 46.6, b = 10, a = 4, p = -4$ in region of $c \in [-20, 60]$; (a) Lyapunov exponents L_1, L_2, L_3 and (b) Lyapunov exponent L_4.

5. CIRCUIT DESIGN OF THE PROPOSED SYSTEM

Circuit design for implementation of the system (1) is shown in Fig. 9 (a). Three multipliers, many resistors, capacitors, and seven amplifiers are used to design the circuit. Resistors, capacitors values are shown in circuit itself. Attractors are shown in Fig. 9 (b-c) which matches with the MATALB simulation results.

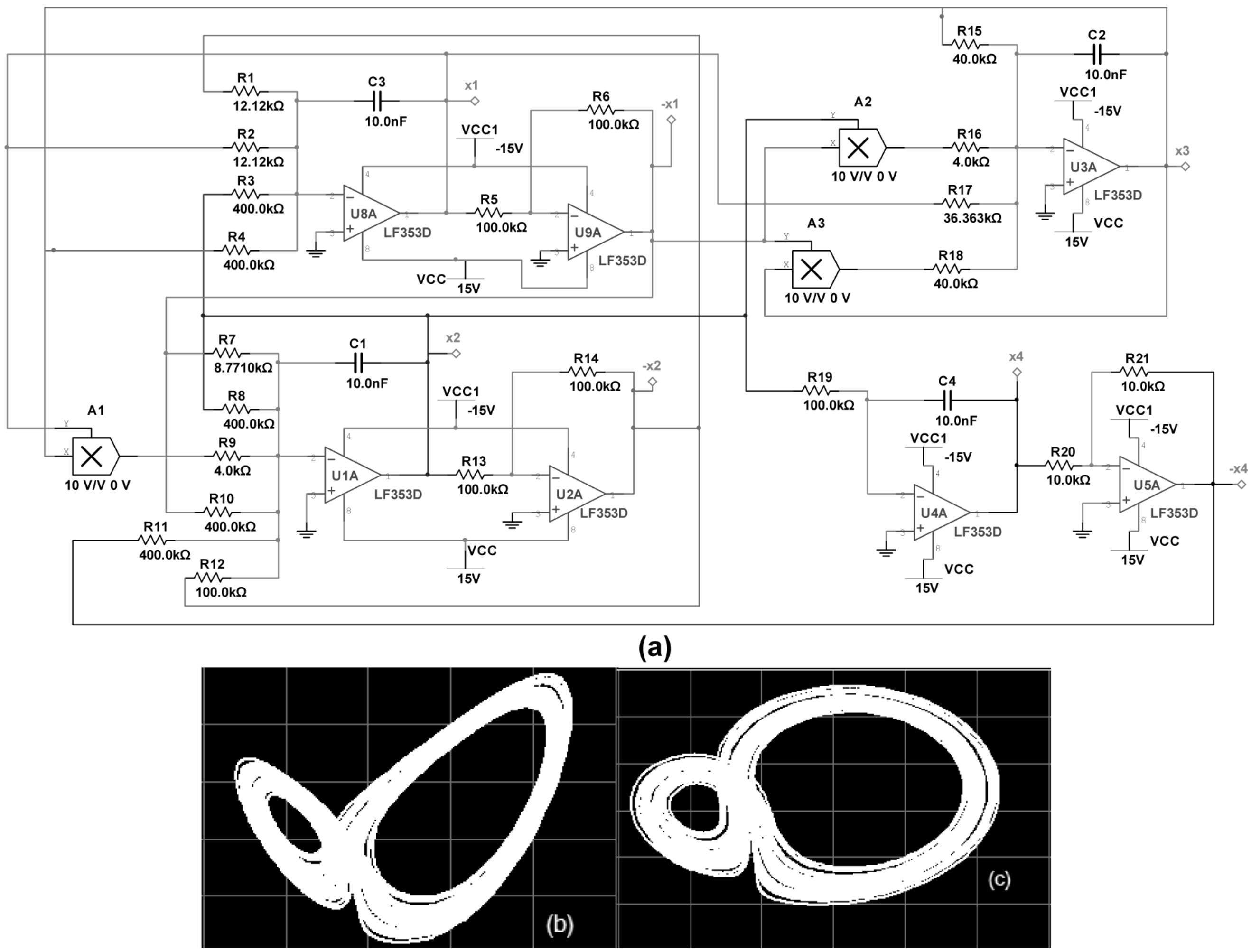

Figure 9: (a) Circuit design; attractor of the system (1) (b) on x_1–x_3 plane, (c) on x_2–x_3 plane.

6. CONCLUSIONS

A new hyperchaotic system is proposed in this paper. Asymmetry and non-similar equilibrium points are the important properties of the proposed system. Non-uniform contraction and expansion of volume in phase space is also another important property of the system. The complex nature of the systems is highlighted using attractor and time series plot. Poincare map validates the asymmetrical hyperchaotic nature of the systems. Bandwidth of the system is shown using frequency spectrum. Lyapunov spectrum is used to highlight the parameter range for different behaviours (hyperchaotic, chaotic, periodic, stable nature). The proposed system may be used for secure communication. Circuit simulation results of the system validating the numerical simulation results are also presented.

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