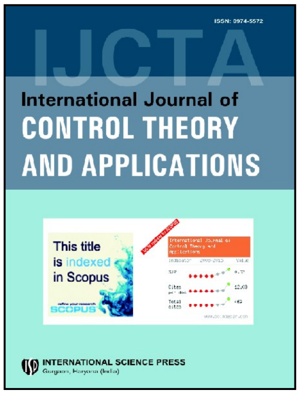

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Analysis of the Companding Transform for PAPR Reduction in OFDM system

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***Abstract*:** This paper addresses the problem of high peak to average power ratio (PAPR) in orthogonal frequency division multiplexing (OFDM) systems and investigates the performance of companding transform (CT) with single and double inflection point. The CT with single inflection point compresses the large and emphasizes the small OFDM samples compared to the inflection point. The CT with two inflection points enhances and reduces the samples of OFDM, which are lower than the smallest inflection point and higher than the largest inflection point, respectively. The choice of inflection point/s allows more flexibility to achieve a significant performance relating to PAPR and bit error rate (BER) of the OFDM system. Simulation result signifies that the degradation in both PAPR and BER by the CT method.

Keywords: Companding transform; OFDM; PAPR; SSPA

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is an eminent approach for transmitting high data rate information [1], [2]. The higher data rate is achieved by the combination of digital mapping/modulation technique such as BPSK, QPSK, QAM, etc., and multiplexing with a large number of subcarriers which is constructed by the inverse fast Fourier transform (IFFT). Due to high spectral efficiency, power efficiency, immunity to the frequency selective fading, and inexpensive implementation using FFT/IFFT, OFDM has been widely used in various rapidly growing applications such as digital audio/video broadcasting (DAB/DVB), worldwide interoperability for microwave access (WiMAX), long-term evolution (LTE) standards and also a promising technology for true 4G wireless systems.

However, the data rate of the OFDM system is proportional to the number of subcarriers in multiplexing, resulting in high peak to average power ratio (PAPR) when all the carriers are added in the coherent phase. This high PAPR causes the degradation in performance due to the nonlinear characteristics of the high power amplifier (HPA), which is used to amplify the signal before radiating into the channel. Alternatively, to transmit this high PAPR signal, the HPA requires a wide dynamic range which increases the complexity and the cost of the overall system. Thus, it is a vital role to reduce the high PAPR and has been investigated by many researchers [3]-[11].

The fundamental method for reducing high PAPR is clipping and filtering [6], which improves the performance when the order of the modulation is small. As the modulation order increases the clipping error becomes more significant and reduces the system performance. In [7]-[11], transform based methods have been extensively used for the PAPR reduction in OFDM systems. A series of companding transform (CT) approaches such as linear symmetrical, linear nonsymmetrical, nonlinear symmetrical, and nonlinear nonsymmetrical has been proposed in [9]. The resulting analysis in [9] implies that the linear symmetrical transform has a superior performance in terms of reducing PAPR and BER. The improvement in performance is obtained by introducing an inflection point, and hence, the small and large amplitude values are expanded and compressed, respectively. This inflection point allows more flexibility and freedom in designing a CT to meet the system requirements such as PAPR reduction, high power amplifier characteristics, and BER.

In [11], proposed a CT with two inflection points (CT-2) and the experiments demonstrate that the performance is significantly improved compared to the CT with a single inflection point (CT-1). The idea of CT-2 is, which enhances the OFDM samples that are lower than the smallest inflection point, reduces the samples that are higher than the largest inflection point, and does not manipulate the samples that occur between the two inflection points. The companding operation in [11] is independently applied to both the real and imaginary values of the OFDM modulated data. Thus, the method requires two separate index sets, for real and imaginary samples, to reconstruct the data at the receiver. This paper implements and investigates the performance of the CT-1 and CT-2, by applying the companding operation on the absolute value of the OFDM symbol, and hence, requires only one index set. Moreover, the implemented approach reduces the PAPR significantly.

The paper is organized as follows: Section II presents the conventional OFDM system. Section III describes the mathematical model of the companding transform with single and double inflection point. Section IV evaluates the PAPR and BER performance using Monte-Carlo simulations. Finally, section V concludes the paper.

II. OFDM SYSTEM DESCRIPTION

In a typical OFDM system shown in Fig. 1, the high data (bit) rate input is modulated by the digital mapping/modulation technique such as BPSK, QPSK, QAM, etc., and converted into N parallel lower data rate symbols, $Y(k), k = 0, \ldots, N-1$. These symbols are then multiplexed using the IFFT operation to generate the OFDM symbol, which can be expressed as

$$y(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} Y(k)\, e^{j\frac{2\pi}{N}kn}, \; 0 \le n \le N-1. \tag{1}$$

After the IFFT block, a companding transform (CT) is deployed to reduce the high PAPR and described in the following section. After CT, a cyclic prefix (CP) is added at the beginning of each companded OFDM symbol, $z(n)$, to reduce the effect of channel delay spread, then the digital data $\tilde{z}(n)$ is converted into the analog signal, $\tilde{z}(t)$, and then amplified by the HPA before radiating the signal into the channel. The widely used memory less Rapp SSPA [3] for the OFDM system is considered in this paper. Thus, the output of the SSPA can be expressed as

$$s(t) = \frac{G_a \left|\tilde{z}(t)\right|}{\left[1 + \left(\frac{\left|\tilde{z}(t)\right|}{A_{sat}}\right)^{2p}\right]^{1/2p}}\, e^{j\angle\tilde{z}(t)} \tag{2}$$

where G_a is the gain of the amplifier, A_{sat} is the saturation level of the SSPA, and p is a positive number to control the nonlinear characteristics of the HPA. $|\tilde{z}(t)|$ and $\angle\tilde{z}(t)$ are the magnitude and phase of the input signal, respectively.

At the receiver, the received signal after the analog to digital converter (ADC) is represented by $r(n) = z(n) + w(n)$, where $w(n)$ is white Gaussian noise. The operation of the receiver is an undo of the transmitter.

The data rate of the OFDM system is proportional to the number of subcarriers in the multiplexing. However, as the number of subcarrier increases the peak amplitude of the multiplexed signal increases to a great extent at a particular instant and shown in Fig. 2. Fig. 2 shows the multiplexing of five BPSK modulated signals using the sinusoids with different frequencies, i.e., the imaginary part of the subcarriers in IFFT. The high peak amplitudes could result in high PAPR. The PAPR of the discrete signal $\tilde{z}(n)$, in fact at least four times over sampling of $\tilde{z}(n)$ is used to approximate to the continuous OFDM signal [12], can be expressed as

$$\text{PAPR} = \max_{0\le n\le N-1} \frac{|\tilde{z}(n)|^2}{\mathrm{E}\left[|\tilde{z}(n)|^2\right]} \tag{3}$$

where $\mathrm{E}[\cdot]$ is the expectation operator.

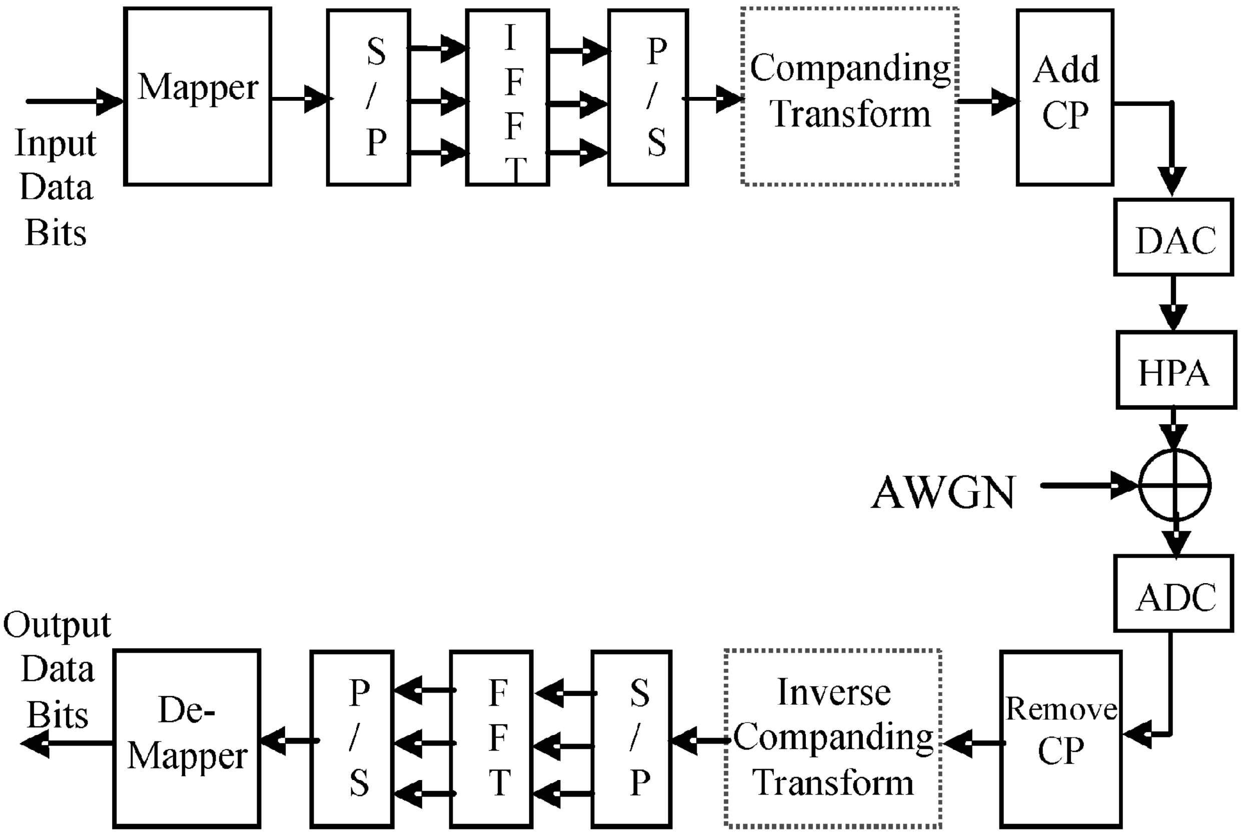

Figure 1: Conventional FFT based OFDM system with companding transform

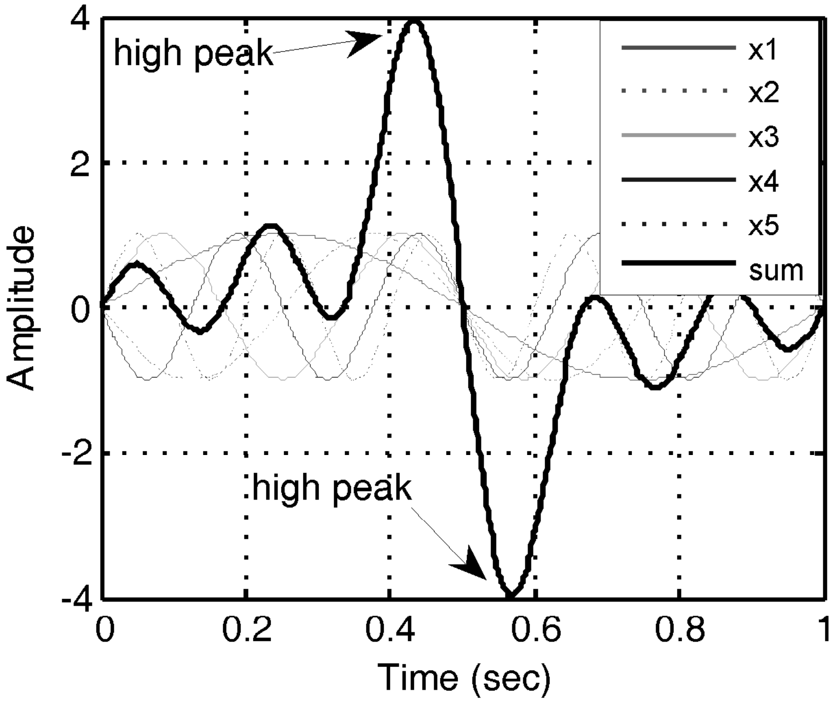

Figure 2: High peaks in the OFDM signal generated by summing five BPSK modulated sinusoids

III. COMPANDING TRANSFORM

This section describes the companding transform with single and double inflection point based on the absolute value of the OFDM signal.

(A) CT with Single Inflection Point

CT with single inflection point (CT-1) can be expressed as

$$z(n) = \begin{cases} \dfrac{y(n)}{\delta} & \text{if } |y(n)| \le \alpha \\ \delta y(n) & \text{if } |y(n)| > \alpha \end{cases} \tag{4}$$

where $0 < \alpha < \max\{|y(n)|\}$ is the location of the inflection point and $0 \le \delta \le 1$ is the companding factor.

At the receiver, the data can be recovered by the inverse CT-1 and can be expressed as

$$\hat{y}(n) = \begin{cases} \delta r(n); & n \in \phi_1 = \{n \ \forall |y(n)| \le \alpha\} \\ \dfrac{r(n)}{\delta}; & n \in \phi_2 = \{n \ \forall |y(n)| > \alpha\} \end{cases} \tag{5}$$

where ϕ_1 and ϕ_2 are the index sets of the OFDM samples and assumed that the receiver has the knowledge of ϕ_1 and ϕ_2. The method proposed in [11] requires two index sets for real values and two index sets for the imaginary samples, but the method described in this paper requires only two index sets.

(B) CT with Double Inflection Point

CT with two inflection points (CT-2) can be expressed as

$$z(n) = \begin{cases} \dfrac{y(n)}{\delta} & \text{if} \quad |y(n)| \le \beta_1 \\ y(n) & \text{if} \quad \beta_1 < |y(n)| \le \beta_2 \\ \delta y(n) & \text{if} \quad |y(n)| > \beta_2 \end{cases} \tag{6}$$

Where $0 \le \beta_1, \beta_2 \le 1$, and are the inflection points of the CT-2. The inverse CT-2 can be expressed as

$$\hat{y}(n) = \begin{cases} \delta r(n); & n \in \psi_1 = \{n \ \forall |y(n)| \le \beta_1\} \\ r(n); & n \in \psi_2 = \{n \ \forall \beta_1 < |y(n)| \le \beta_2\} \\ \dfrac{r(n)}{\delta}; & n \in \psi_3 = \{n \ \forall |y(n)| > \beta_2\} \end{cases} \tag{7}$$

where ψ_1, ψ_2, and ψ_3 are the index sets of OFDM samples and assumed that the receiver has the knowledge of these three index sets.

In CT-1 the small amplitude values are emphasized and large values are compressed as compared with the inflection point, whereas in CT-2, the OFDM sample values less than the lower inflection point are enlarged and the samples are greater than the upper inflection point are compressed. The selection of CT parameters, i.e., (δ, α) in CT-1 and $(\delta, \beta_1, \beta_2)$ in CT-2, consists of more flexibility and freedom to design the OFDM system in order to meet PAPR reduction, HPA characteristics and BER, and hence, leads to a significant performance.

IV. PERFORMANCE EVALUATION

This section presents the Monte-Carlo simulations to investigate the PAPR and bit error rate (BER) performance of the CT-1 and CT-2. In CT-1, we investigated two approaches, one is the CT-1 independently applied to the real and imaginary samples of OFDM (CT-1I) and the other is the CT-1 applied to the absolute values of the OFDM samples (CT-1A). The performance of these approaches is then compared with the conventional OFDM system. Throughout the simulation, we considered 16-QAM for mapping the input bit stream, $N = 1024$ subcarriers in the FFT/IFFT, $N/4$ for CP length, and $p = 1.2$ of the SSPA. All simulations are averaged over 10^3 OFDM symbols.

Fig. 3 and Fig.4 respectively illustrates the PAPR and BER performance the CT-1 (CT-1I proposed in [11] and CT-1A) at different values of the inflection point versus the companding factor δ. The minimum PAPR achieved by the CT-1I is 5.627 dB at $\delta = 0.6$ with $\alpha = \sqrt{1.5P_a}$ and the CT-1A is 4.603 dB at $\delta = 0.625$ with $\alpha = \sqrt{1.5P_a}$, where P_a is the average power of the OFDM signal. The PAPR is reduced by a factor of 1.024 dB using the CT-1A. However, Fig. 4 shows that the BER performance is significant in the CT-1I. Signal to noise ratio per bit (Eb/N0) is 4 dB and IBO =2dB were considered for the evaluation of Fig. 4 and Fig. 6.

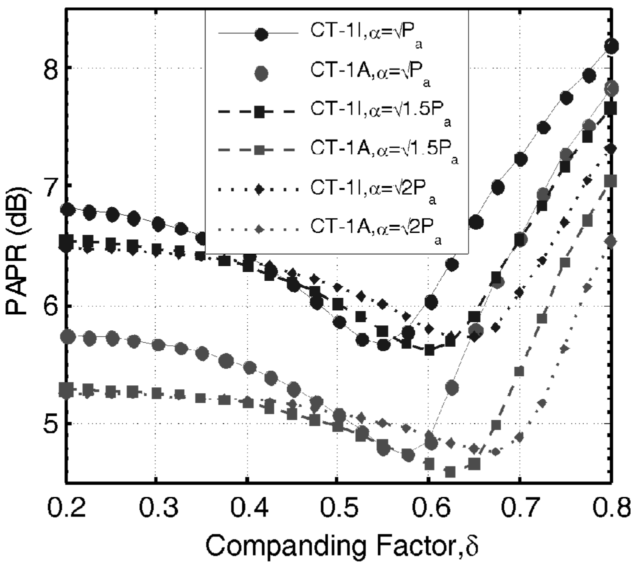

Figure 3: PAPR evaluation of CT-1I and CT-1A at different values of inflection point versus the companding factor

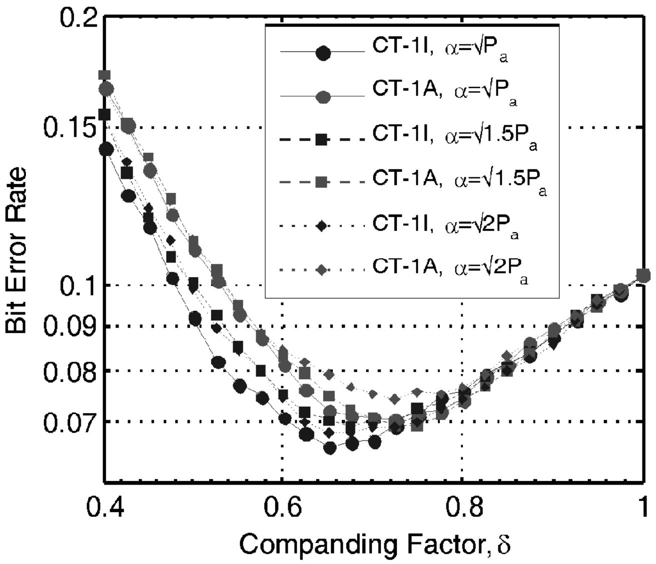

Figure 4: BER performance of CT-1I and CT-1A at different values inflection point versus the companding factor

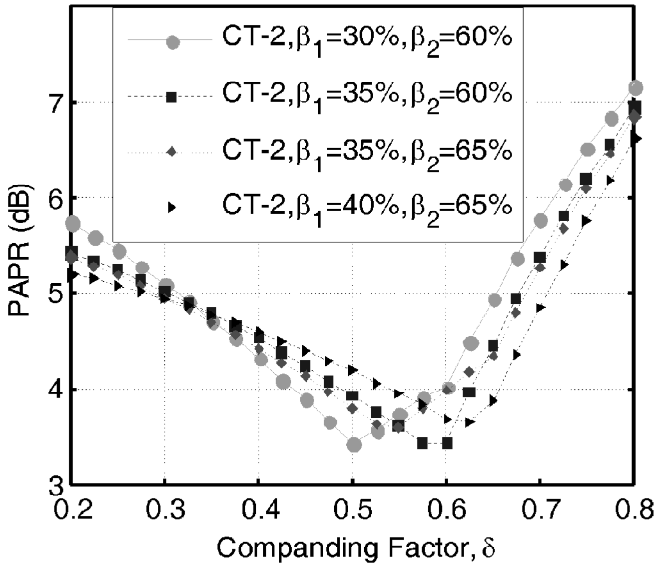

Figure 5: PAPR evaluation of CT-2 at different values of the two inflection points versus the companding factor

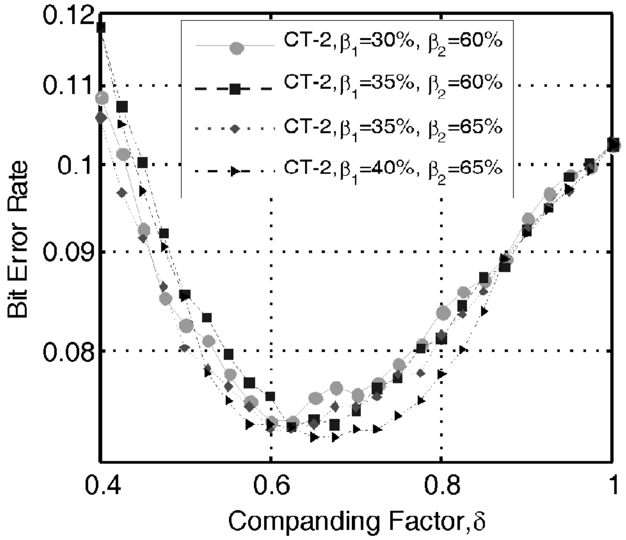

Figure 6: BER performance of CT-2 at different values of the two inflection points versus the companding factor

Fig. 5 shows the PAPR performance of CT-2 at different values (in percentage) of the two inflection points, (β_1, β_2), versus the companding factor. From Fig. 5, the optimum PAPR is 3.435 dB at $\delta = 0.575$ and $(\beta_1, \beta_2) =$ (0.35, 0.6). The PAPR of CT-2 is reduced by 2.192 dB and 1.168 dB compared with the CT-1I and CT-1A, respectively. Fig. 6 depicts the BER performance of CT-2 versus δ at different values of (β_1, β_2). From Fig. 6, the minimum BER of CT-2 is 0.07202 at $\delta = 0.65$ with $(\beta_1, \beta_2) = (0.4, 0.65)$, which is evaluated at Eb/N0 of 4 dB and IBO of 2 dB.

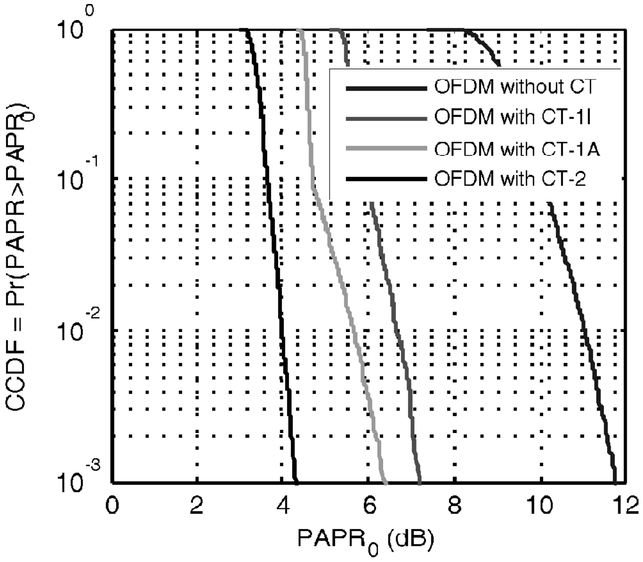

Figure 7: CCDF performance comparison of CT-1I, CT-1A, and CT-2 with the typical OFDM system

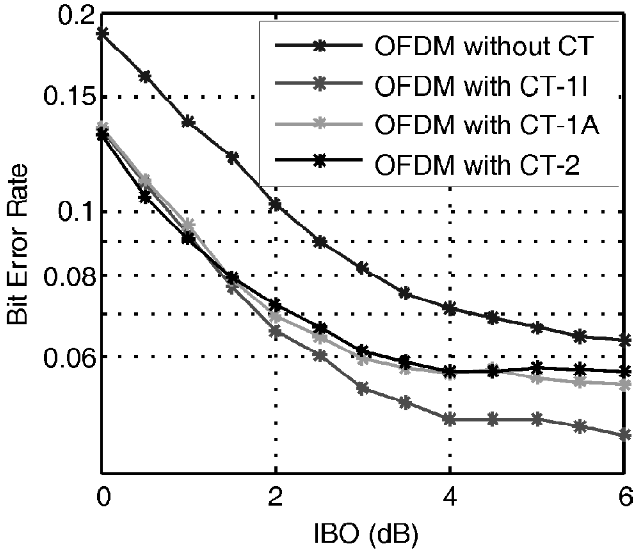

Figure 8: BER performance evaluation of OFDM system with CT-1I, CT-1A, CT-2, and no CT versus the IBO of the HPA

Fig. 7 shows the CCDFs of the PAPR using the optimum parameters of the CT with respect to the PAPR. At a particular CCDF of 10^{-2}, the PAPR of OFDM system without CT is 11.04 dB, OFDM with CT-1I is 6.674 dB, using CT-1A is 5.667 dB, and with CT-2 is 3.998 dB, and hence, the PAPR is reduced by the CT-1I, CT-1A, CT-2 is 4.366 dB, 5.373 dB, and 7.042 dB, respectively. Fig. 8 and Fig. 9 shows the BER performance as a function of, respectively, the IBO value of the HPA and signal to noise ratio per bit, Eb/N0. SNR of 4 dB and the parameters of the CT with respect to an optimum BER were considered in the evaluation of Fig. 8, and it shows the BER is reduced with the increase in IBO of HPA. In Fig. 9, the parameters of CT were optimum with respect to BER and the performance is significantly improved using the CT.

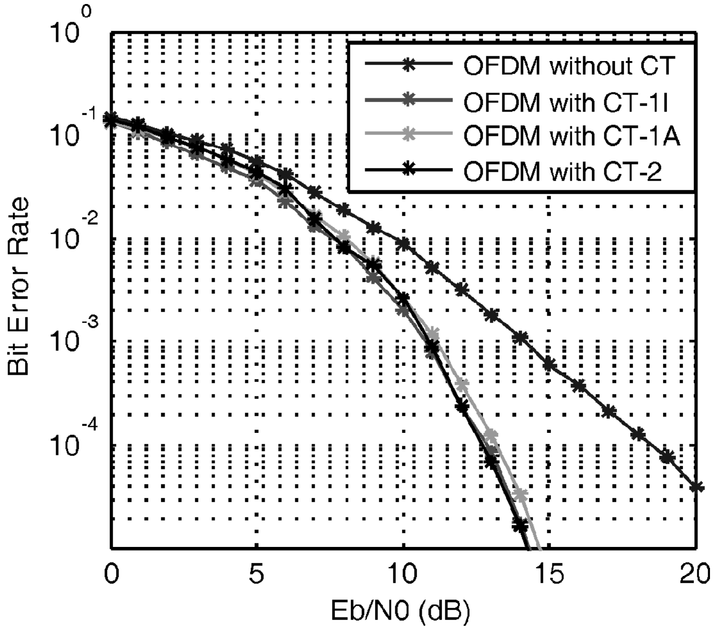

Figure 9: BER performance comparison of OFDM system with CT-1I, CT-1A, CT-2, and no CT versus the SNR

V. CONCLUSION

This paper implements the modified CT with single and double inflection point and presents the vital investigations on the selection of the transform parameters. CT on the absolute value of the OFDM symbol reduces the PAPR significantly relative to that applied independently on the real and imaginary values. Simulation results indicate that the CT with two inflection points reduces the PAPR and BER significantly compare with single inflection point and these two approaches work well at lower IBO of the HPA, and hence, reduces the complexity and cost of the HPA.

REFERENCES

[1] Yiyan Wu and W. Y. Zou, “Orthogonal frequency division multiplexing: a multi-carrier modulation scheme,” in *IEEE Transactions on Consumer Electronics*, vol. 41, no. 3, pp. 392-399, Aug 1995.

[2] M. Rohling, T. May, K. Bruninghaus and R. Grunheid, “Broad-band OFDM radio transmission for multimedia applications,” in *Proceedings of the IEEE*, vol. 87, no. 10, pp. 1778-1789, Oct 1999.

[3] Y. Rahmatallah and S. Mohan, “Peak-to-average power ratio reduction in OFDM systems: a survey and taxonomy,” *IEEE Commun. Surveys Tuts.,* vol.15, no.4, pp. 1567-1592, Fourth quarter 2013.

[4] T. Jiang and Y. Wu, “An Overview: Peak-to-Average Power Ratio Reduction Techniques for OFDM Signals,” in *IEEE Transactions on Broadcasting*, vol. 54, no. 2, pp. 257-268, June 2008.

[5] R. V. Nee and A. D. Wild, “Reducing the peak-to-average power ratio of OFDM,” in *Proc. IEEE Vehicular Technology Conference (VTC)*, vol. 3, New York, NY, USA, 1998, pp. 2072–2076.

[6] S. C. Thompson, J. G. Proakis, and J. R. Zeidler, “The effectiveness of signal clipping for PAPR and total degradation reduction in OFDM systems,” in *Proc. of the IEEE Global Telecommun. Conf.*, St. Louis, Dec. 2005, vol. 5, pp. 2807–2811.

[7] X. B.Wang, T. T. Tjhung, and C. S. Ng, “Reduction of peak-to-average power ratio of OFDM system using a companding technique,” *IEEE Trans. Broadcast.*, vol. 45, no. 3, pp. 303–307, Sep. 1999.

[8] S. A. Aburakhia, E. F. Badran, and D. A. E. Mohamed, “A comparison between signal clipping and μ-law companding schemes for the reduction of peak-to-average power ratio of OFDM signals,” in *Proc. 1st Int. Conf. on Digital Commun. & Computer Applications DCCA2007*, Jordan, Mar. 2007, pp. 423–431.

[9] X. Huang, J. Lu, J. Zheng, K. B. Letaief, and J. Gu, “Companding transform for reduction in peak-to average power ratio of OFDM signals,” *IEEE Trans. Wireless Commun.*, vol. 3, no. 6, pp. 2030–2039, Nov. 2004.

[10] Y. Rahmatallah, N. Bouaynaya, and S. Mohan, “Bit-error-rate performance of companding transforms for OFDM,” *IEEE Trans. Veh. Technol.,* vol.62, no.8, pp. 4116-4120, Oct. 2013.

[11] S. Aburakhia *et al.,* “Linear companding transform for the reduction of peak-to-average power ratio of OFDM signals,” IEEE Trans. Broadcast., vol. 55, no. 1, pp. 155–160, March 2009.

[12] C. Tellambura, “Computation of the continuous-time PAR of an OFDM with BPSK subcarriers,” *IEEE Commun. Lett.*, vol. 5, no. 5, pp. 185–187, May 2001.