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A MATHEMATICAL MODEL FOR TWO POLITICAL PARTIES

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Abstract: *In this paper, a simple model for two political parties has been proposed. This model involves system of linear differential equations which is solved by using Laplace Transform. Whole population is assumed as constant and homogeneously mixed. Our model is capable of providing numeric results.*

Keywords: *Mathematical Modelling, Differential Equations, Laplace Transform.*

Mathematics Subject Classification: *00A71, 81T80, 93C15, 44A10.*

1. INTRODUCTION

Democracy is a system of government by the whole population or all the eligible members of a state, typically through elected representatives. A democracy is a type of political system, or a system of decision making within an institution, or organization, in which all members have the equal share to power. Systems of democracy stand in contrasts to other forms of government, including monarchy and oligarchy.

A political party is a group of people who come together to contest elections and hold power in the government. The party agrees on some proposed policies and programmes, with a view to promoting the collective the good or furthering their supporter's interests. Many political parties have an ideological core, but some do not. In democracies, political parties are elected by the electorate to run a government. Many countries have numerous powerful political parties, such as Germany and India and some nations have oneparty system, such as China and Cuba. The United States is in practice a two-party system, with many smaller parties participating. Its two most powerful parties are the Democratic Party and the Republic Party.

In some studies a non-linear mathematical model for the spread of two political parties has been proposed and analysed by using epidemiological approach [7]. In most of modelling studies statistical methodology is being used by considering that two political parties are competing for the voter's class [3].

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In this paper we will discuss the growth and decrease in development of two political parties using mathematical model. In this system we assume that the individuals have right of voting at the age of 18 and all eligible population using the right of voting. We also considered two political parties B and C only, it means an individual must have vote a party B or Party C.

2. MATHEMATICAL MODEL

Let N be total population considered in the system. And it is divided into three classes: $U(t)$ = Class containing the population below 18 years old, $B(t)$ = Class containing population which votes to party B, $C(t)$ = Class containing the population which votes to party C. The entire above are functions of time t, from above information we write $N(t) = U(t) + B(t) + C(t)$.

As we know that most worldwide constitutions gives the right of voting to the individuals when they become 18 years old, hence we assume that individuals enters in the class $U(t)$ at the rate μN. Due to inactiveness and death the individuals leaves the class $U(t)$, $B(t)$, $C(t)$ at the rate μU, μB, μC respectively. Let us assume that the individuals enter in the class B at the rate λ_{1v} and in the class C at the rate λ_{2v}. The individuals in class B leaves the class and enter in to the class C at the rate $\beta 1$ and the individuals in class C leaves the class and enters into the class B at the rate of β_2. The ?ow diagram of the model is as shown in the fig. 1.

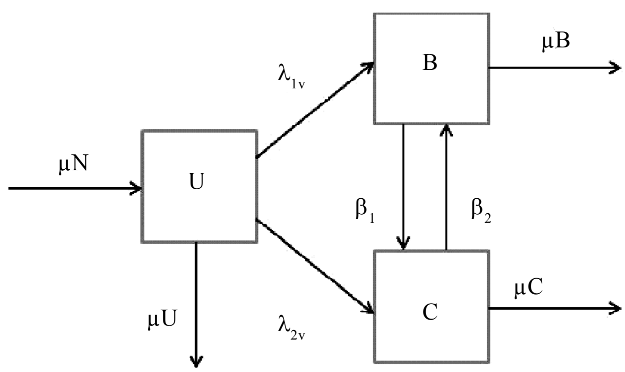

Figure 1

The model then consists of three ordinary differential equations of the form

$$\frac{dU}{dt} = \mu N - \lambda_{1v} U(t) - \lambda_{2v} U(t) - \mu U, \tag{2.1}$$

$$\frac{dB}{dt} = \lambda_{1v} U(t) - \beta_1 B(t) + \beta_2 C(t) - \mu B, \tag{2.2}$$

$$\frac{dC}{dt} = \lambda_{2v}U(t) - \beta_2 C(t) + \beta_1 B(t) - \mu C. \tag{2.3}$$

Taking Laplace Transform of equation (2.1) we get,

$$sL[U(t)] - U(0) = \frac{\mu N}{s} - \lambda_{1v}L[U(t)] - \lambda_{2v}L[U(t)] - \frac{\mu U}{s},$$

here we assume, $U(0) = 0$

therefore,

$$(s + \lambda_{1v} + \lambda_{2v})L[U(t)] = \frac{\mu N - \mu U}{s},$$

$$L[U(t) = \frac{\mu N - \mu U}{s(s + \lambda_{1v} + \lambda_{2v})}. \tag{2.4}$$

Taking Inverse Laplace Transform of equation (2.4) we get,

$$U(t) = \frac{1}{a}(1 - e^{-at}), \tag{2.5}$$

where, $a = \lambda_{1v} + \lambda 2v.$

Now taking Laplace Transform of equation (2.2) we get,

$$s\,L[B(t)] - B(0) = \lambda_{1v}L[U(t)] - \beta_1 L[B(t)] + \beta_2 L[C(t)] - \frac{\mu B}{s},$$

here we assume, $B\,(0) = 0$

$$s\,L[B(t)] = \lambda_{1v}L[U(t)] - \beta_1 L[B(t)] + \beta_2 L[C(t)] - \frac{\mu B}{s},$$

$$(s + \beta_1)L[B(t)] = \lambda_{1v}\frac{\mu N - \mu U}{s(s + \lambda_{1v} + \lambda_{2v})} + \beta_2 L[C(t)] - \frac{\mu B}{s},$$

$$L[B(t)] = \lambda_{1v}\frac{\mu N - \mu U}{s(s + \lambda_{1v} + \lambda_{2v})(s + \beta_1)} + \frac{\beta_2}{(s + \beta_1)}L[C(t)] - \frac{\mu B}{s(s + \beta_1)}. \tag{2.6}$$

Now taking Laplace Transform of equation (2.3) we get,

$$s\,L[C(t)] - C(0) = \lambda_{2v}L[U(t)] - \beta_2 L[C(t)] + \beta_1 L[B(t)] - \frac{\mu C}{s},$$

here we assume, $C(0) = 0$

$$sL[C(t)] = \lambda_{2v} L[U(t)] - \beta_2 L[C(t)] + \beta_1 L[B(t)] - \frac{\mu C}{s},$$

We get,

$$L[C(t)] = \frac{\lambda_{2v}}{(s+\beta_2)} L[U(t)] + \frac{\beta_1}{(s+\beta_2)} L[B(t)] - \frac{\mu C}{s(s+\beta_2)}. \tag{2.7}$$

Using equations (2.4) and (2.6) in equation (2.7) we get,

$$\begin{aligned} L[C(t)] = &\frac{\lambda_{2v}}{(s+\beta_2)} \frac{\mu N - \mu U}{s(s+\lambda_{1v}+\lambda_{2v})} \\ &+ \frac{\beta_1}{(s+\beta_2)} \left(\lambda_{1v} \frac{\mu N - \mu U}{s(s+\lambda_{1v}+\lambda_{2v})(s+\beta_1)} + \frac{\beta_2}{(s+\beta_1)} L[C(t)] - \frac{\mu B}{s(s+\beta_1)} \right) \\ &- \frac{\mu C}{s(s+\beta_2)}, \end{aligned}$$

$$\begin{aligned} = &\frac{\lambda_{2v}\,(\mu N - \mu U)}{s(s+\lambda_{1v}+\lambda_{2v})(s+\beta_2)} + \frac{\beta_1 \lambda_{1v}\,(\mu N - \mu U)}{s(s+\lambda_{1v}+\lambda_{2v})(s+\beta_1)(s+\beta_2)} \\ &+ \frac{\beta_1 \beta_2}{(s+\beta_1)(s+\beta_2)} L[C(t)] - \frac{\beta_1 \mu B}{s(s+\beta_1)(s+\beta_2)} - \frac{\mu C}{s(s+\beta_2)}, \end{aligned}$$

(2.8)

From above equation (2.8),

$$\begin{aligned} L[C(t)] - \frac{\beta_1 \beta_2}{(s+\beta_1)(s+\beta_2)} L[C(t)] \\ = &\frac{\lambda_{2v}\,(\mu N - \mu U)}{s(s+\lambda_{1v}+\lambda_{2v})(s+\beta_2)} + \frac{\beta_1 \lambda_{1v}\,(\mu N - \mu U)}{s(s+\lambda_{1v}+\lambda_{2v})(s+\beta_1)(s+\beta_2)} \\ &- \frac{\beta_1 \mu B}{s(s+\beta_1)(s+\beta_2)} - \frac{\mu C}{s(s+\beta_2)}, \end{aligned}$$

$$\begin{aligned} \left(1 - \frac{\beta_1 \beta_2}{(s+\beta_1)(s+\beta_2)}\right) L[C(t)] \\ = &\frac{\lambda_{2v}\,(\mu N - \mu U)}{s(s+\lambda_{1v}+\lambda_{2v})(s+\beta_2)} + \frac{\beta_1 \lambda_{1v}\,(\mu N - \mu U)}{s(s+\lambda_{1v}+\lambda_{2v})(s+\beta_1)(s+\beta_2)} \\ &- \frac{\beta_1 \mu B}{s(s+\beta_1)(s+\beta_2)} - \frac{\mu C}{s(s+\beta_2)}, \end{aligned}$$

$$\left(\frac{(s+\beta_1)(s+\beta_2)-\beta_1\beta_2}{(s+\beta_1)(s+\beta_2)}\right)L[C(t)] = \frac{\lambda_{2v}\,(\mu N-\mu U)}{s(s+\lambda_{1v}+\lambda_{2v})(s+\beta_2)} + \frac{\beta_1\lambda_{1v}\,(\mu N-\mu U)}{s(s+\lambda_{1v}+\lambda_{2v})(s+\beta_1)(s+\beta_2)} - \frac{\beta_1\mu B}{s(s+\beta_1)(s+\beta_2)} - \frac{\mu C}{s(s+\beta_2)},$$

This gives,

$$L[C(t)] = \frac{(s+\beta_1)(s+\beta_2)}{(s+\beta_1)(s+\beta_2)-\beta_1\beta_2}\frac{\lambda_{2v}(\mu N-\mu U)}{s(s+\lambda_{1v}+\lambda_{2v})(s+\beta_2)} + \frac{(s+\beta_1)(s+\beta_2)}{(s+\beta_1)(s+\beta_2)-\beta_1\beta_2}\frac{\beta_1\lambda_{1v}(\mu N-\mu U)}{s(s+\lambda_{1v}+\lambda_{2v})(s+\beta_1)(s+\beta_2)} - \frac{(s+\beta_1)(s+\beta_2)}{(s+\beta_1)(s+\beta_2)-\beta_1\beta_2}\frac{\beta_1\mu B}{s(s+\beta_1)(s+\beta_2)} - \frac{(s+\beta_1)(s+\beta_2)}{(s+\beta_1)(s+\beta_2)-\beta_1\beta_2}\frac{\mu C}{s(s+\beta_2)},$$

$$= \frac{\lambda_{2v}(\mu N-\mu U)(s+\beta_1)}{s(s+\lambda_{1v}+\lambda_{2v})[(s+\beta_1)(s+\beta_2)-\beta_1\beta_2]} + \frac{\beta_1\lambda_{1v}(\mu N-\mu U)}{s(s+\lambda_{1v}+\lambda_{2v})[(s+\beta_1)(s+\beta_2)-\beta_1\beta_2]} - \frac{\beta_1\mu B}{s[(s+\beta_1)(s+\beta_2)-\beta_1\beta_2]} - \frac{\mu C(s+\beta_1)}{s[(s+\beta_1)(s+\beta_2)-\beta_1\beta_2]},$$

$$= \frac{\lambda_{2v}(\mu N-\mu U)(s+\beta_1)}{s(s+\lambda_{1v}+\lambda_{2v})[s^2+(\beta_1+\beta_2)s]} + \frac{\beta_1\lambda_{1v}(\mu N-\mu U)}{s(s+\lambda_{1v}+\lambda_{2v})[s^2+(\beta_1+\beta_2)s]} - \frac{\beta_1\mu B}{s[s^2+(\beta_1+\beta_2)s]} - \frac{\mu C(s+\beta_1)}{s[s^2+(\beta_1+\beta_2)s]},$$

$$= \frac{\lambda_{2v}(\mu N-\mu U)(s+\beta_1)+\beta_1\lambda_{1v}(\mu N-\mu U)}{s(s+\lambda_{1v}+\lambda_{2v})[s^2+(\beta_1+\beta_2)s]} - \frac{\beta_1\mu B+\mu C(s+\beta_1)}{s[s^2+(\beta_1+\beta_2)s]}. \tag{2.9}$$

Let

$$\lambda_{2v}(\mu N-\mu U)=a,\quad \beta_1\lambda_{1v}(\mu N-\mu U)=b,\quad \lambda_{1v}+\lambda_{2v}=c,\ \ \beta_1+\beta_2=d,\quad \beta_1\mu B=e,\quad \mu C=f$$

$$L[C(t)] = \frac{a(s+\beta_1)+b}{s(s+c)[s^2+ds]} - \frac{e+f(s+\beta_1)}{s[s^2+ds]}. \tag{2.10}$$

Taking Inverse Laplace Transform of equation (2.10) we get,

$$C(t) = \frac{(2e - 2df + 2f\beta_1)\sinh\left(\frac{dt}{2}\right)}{d^2} e^{\left(\frac{-dt}{2}\right)} + \frac{(-ad + a\beta_1 + b)}{d^2(c-d)} e^{-dt} + \frac{(ac - a\beta_1 - b)}{c^2(c-d)} e^{-ct} + \frac{-cd(ce + cf\beta_1 - a\beta_1 - b)t + acd - ac\beta_1 - bc - ad\beta_1 - bd}{c^2 d^2}, \tag{2.11}$$

where,

$$a = \lambda_{2v}(\mu N - \mu U), \quad b = \beta_1\lambda_{1v}(\mu N - \mu U), \quad c = \lambda_{1v} + \lambda_{2v}, \quad d = \beta_1 + \beta_2, \quad e = \beta_1\mu B, \quad f = \mu C.$$

Using equation (2.7) in equation (2.6) we have,

$$L[B(t)] = \lambda_{1v}\frac{\mu N - \mu U}{s(s + \lambda_{1v} + \lambda_{2v})(s + \beta_1)} + \frac{\beta_2}{(s+\beta_1)}\left(\frac{\lambda_{2v}}{(s+\beta_2)} L[U(t)] + \frac{\beta_1}{(s+\beta_2)} L[B(t)] - \frac{\mu C}{s(s+\beta_2)}\right) - \frac{\mu B}{s(s+\beta_1)}. \tag{2.12}$$

Using equation (2.4) in equation (2.12) we get,

$$L[B(t)] = \lambda_{1v}\frac{\mu N - \mu U}{s(s + \lambda_{1v} + \lambda_{2v})(s + \beta_1)} + \frac{\beta_2}{(s+\beta_1)}\left(\frac{\lambda_{2v}}{(s+\beta_2)}\frac{\mu N - \mu U}{s(s + \lambda_{1v} + \lambda_{2v})} + \frac{\beta_1}{(s+\beta_2)} L[B(t)] - \frac{\mu C}{s(s+\beta_2)}\right) - \frac{\mu B}{s(s+\beta_1)},$$

$$= \frac{\lambda_{1v}(\mu N - \mu U)}{s(s + \lambda_{1v} + \lambda_{2v})(s + \beta_1)} + \frac{\beta_2\lambda_{2v}(\mu N - \mu U)}{s(s+\beta_1)(s+\beta_2)(s + \lambda_{1v} + \lambda_{2v})} + \frac{\beta_1\beta_2}{(s+\beta_1)(s+\beta_2)} L[B(t)] - \frac{\beta_2\mu C}{s(s+\beta_1)(s+\beta_2)} - \frac{\mu B}{s(s+\beta_1)}. \tag{2.13}$$

This implies,

$$\left(1 - \frac{\beta_1\beta_2}{(s+\beta_1)(s+\beta_2)}\right) L[B(t)] = \frac{\lambda_{1v}(\mu N - \mu U)}{s(s + \lambda_{1v} + \lambda_{2v})(s + \beta_1)} + \frac{\beta_2\lambda_{2v}(\mu N - \mu U)}{s(s+\beta_1)(s+\beta_2)(s + \lambda_{1v} + \lambda_{2v})} - \frac{\beta_2\mu C}{s(s+\beta_1)(s+\beta_2)} - \frac{\mu B}{s(s+\beta_1)}. \tag{2.14}$$

Therefore,

$$L[B(t)] = \frac{\lambda_{1v}(\mu N - \mu U)(s + \beta_2)}{s(s + \lambda_{1v} + \lambda_{2v})[s^2 + s(\beta_1 + \beta_2)]} + \frac{\beta_2 \lambda_{2v}(\mu N - \mu U)}{s(s + \lambda_{1v} + \lambda_{2v})[s^2 + s(\beta_1 + \beta_2)]} - \frac{\beta_2 \mu C}{s[s^2 + s(\beta_1 + \beta_2)]} - \frac{\mu B(s + \beta_2)}{s[s^2 + s(\beta_1 + \beta_2)]}. \tag{2.15}$$

Let λ_{1v} $(\mu N – \mu U) = a$, β_2 λ_{2v} $(\mu N – \mu U) = b$, $\beta_1 + \beta_2 = c$, $\lambda_{1v} + \lambda_{2v} = d$, β_2 $\mu C = e$, $\mu B = f$.

$$L[B(t)] = \frac{a(s + \beta_2)}{s(s + d)[s^2 + sc]} + \frac{b}{s(s + d)[s^2 + sc]} - \frac{e}{s[s^2 + sc]} - \frac{f(s + \beta_2)}{s[s^2 + sc]},$$

$$= \frac{a(s + \beta_2) + b - e(s + d) - f(s + \beta_2)(s + d)}{s(s + d)[s^2 + sc]}. \tag{2.16}$$

Taking inverse Laplace Transform of equation (2.16) we get,

$$B(t) = \frac{(de - a\beta_2 - b + df\beta_2 - cdf - ce - cf\beta_2 + c^2 f + ac)}{c^2(c - d)} e^{-ct} + \frac{(a\beta_2 + b - ad)}{d^2(c - d)} e^{-dt} + \frac{(a\beta_2 + b - de - f\beta_2 d)t}{cd} + \frac{acd - ac\beta_2 - bc - d^2 cf + ed^2 - ad\beta_2 - bd + f\beta_2 d^2}{c^2 d^2}, \tag{2.17}$$

where,

$$a = \lambda_{1v}(\mu N - \mu U), \quad b = \beta_2 \lambda_{2v}(\mu N - \mu U), \quad c = \beta_1 + \beta_2,$$
$$d = \lambda_{1v} + \lambda_{2v}, \quad e = \beta_2 \mu C, \quad f = \mu B.$$

The equations (2.5), (2.11), (2.17) are the required solution of the system of linear differential equations (2.1), (2.2), (2.3).

3. NUMERICAL EXPERIMENTS

In this section, numerical results are presented. Our aim is to show which factor is more important as compared to other by considering some parameters. We consider three different examples.

3.1. Example 1

Let us assume that, $\lambda_{1v} = 0.59$, $\lambda_{2v} = 0.35$, $\beta_1 = 0.142$, $\beta_2 = 0.1$, $\mu N = 0.20$, $\mu U = \mu B = \mu C = 0.008$

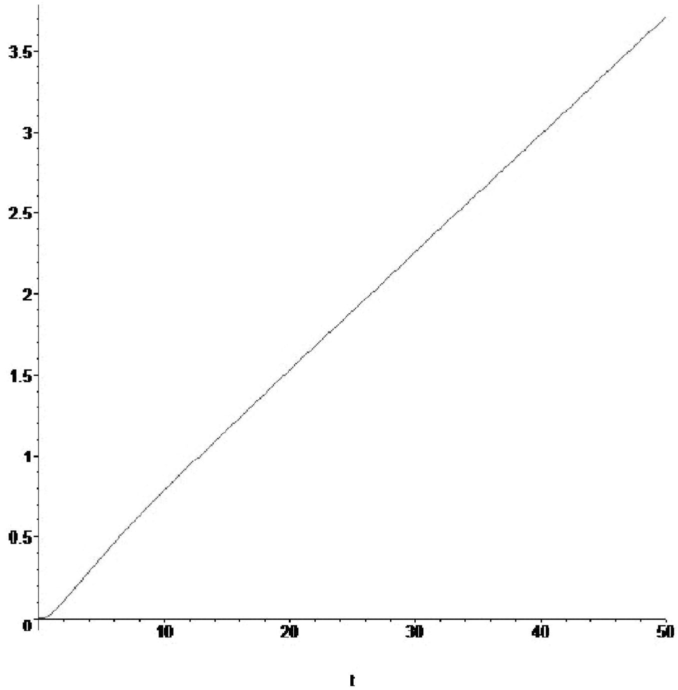

Figure 2: Graph for political party B

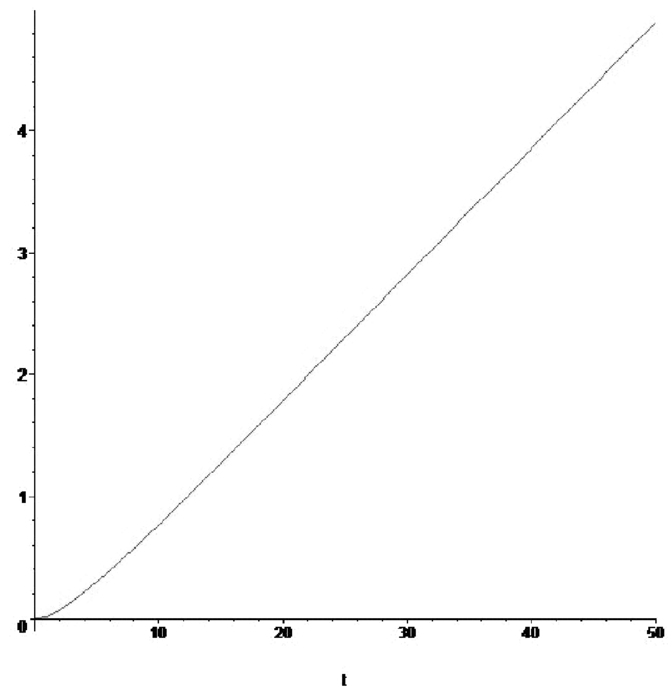

Figure 3: Graph for political party C

3.2. Example 2

Let us assume that, $\lambda_{1v} = 0.58$, $\lambda_{2v} = 0.42$, $\beta_1 = 0.1$, $\beta_2 = 0.1$, $\mu N = 0.30$, $\mu U = \mu B = \mu C = 0.008$

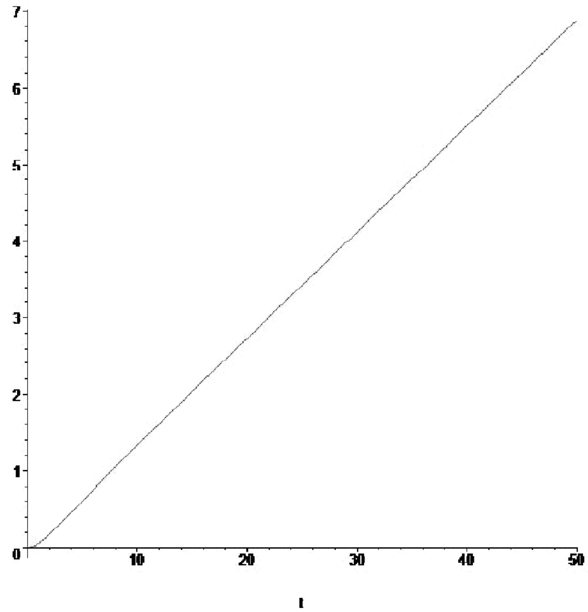

Figure 4: Graph for political party B

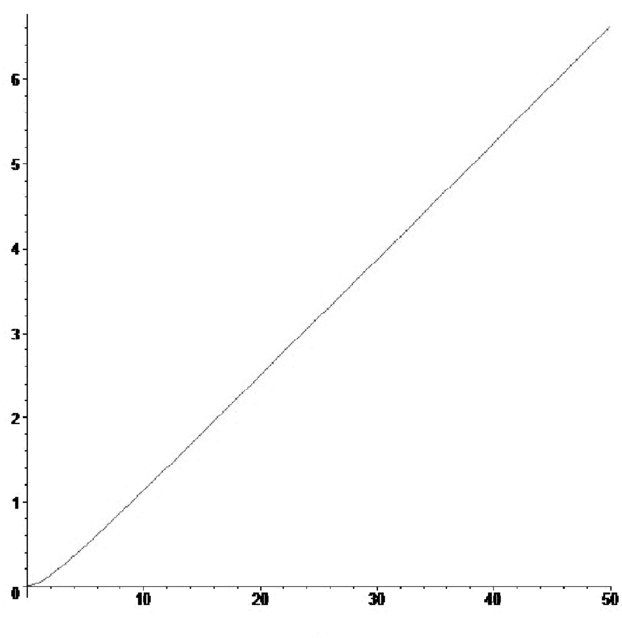

Figure 5: Graph for political party C

3.3. Example 3

Let us assume that, $\lambda_{1v} = 0.5$, $\lambda_{2v} = 0.5$, $\beta_1 = 0.2$, $\beta_2 = 0.1$, $\mu N = 0.30$, $\mu U = \mu B = \mu C = 0.008$

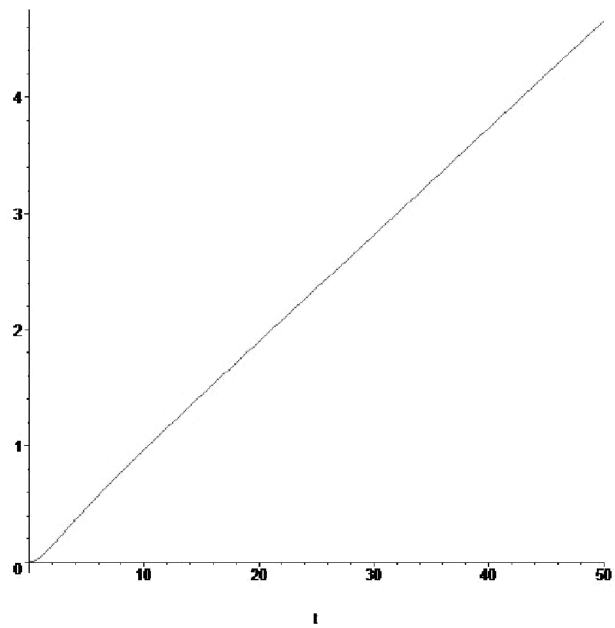

Figure 6: Graph for political party B

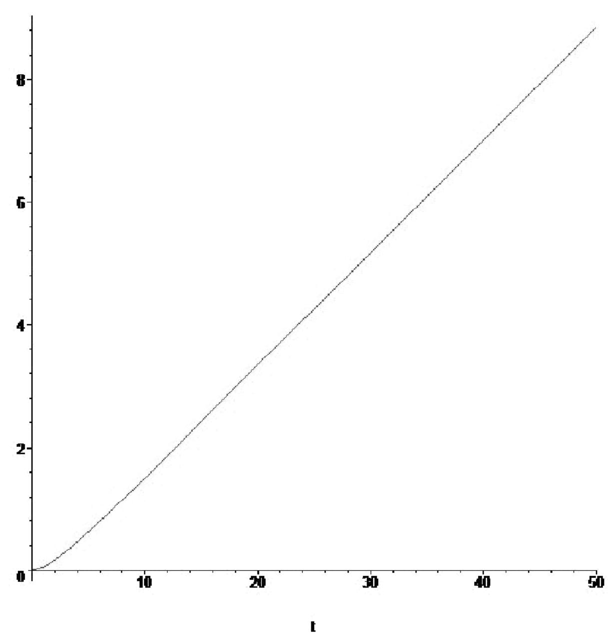

Figure 7: Graph for political party C

4. CONCLUSION

In this we have studied a mathematical model for two political parties, with numerical examples. It shows that the main affecting factor in growth of political party are (i) New voters (ii) Voters which changes party from one to another. Other factors (like death and inactiveness) are negligible.

In example 1 graph of party C increases faster than that of graph of party B. In this example λ_{1v} is greater than λ_{2v} by 0.24 but the factor β_1 is greaterthan β_2 by 0.042. In example 2 graph of party B increases faster than that of graph of party C. In this example λ_{1v} is greater than λ_{2v} by 0.16 but the factor $\beta_1 = \beta_2$. And In example 3 graph of party C increases faster than that of graph of party *B*. In this example $\lambda_{1v} = \lambda_{2v}$ and β_1 is greater than β_2 by 0.1.

The factors β_1, β_2 are more effective than the factors $\lambda_{1v,}$ λ_{2v} i.e. the rate at which the voters changes the party from one to another is more effective than the rate at which the new voters enters in political party. In order to increase no. of voters in political party the factors β_1, and β_2 must be considered.

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