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DESIGN OF OPTIMAL ALLOCATION ALGORITHM FOR MATCHING MARKETS IN ADVERTISING NETWORKS

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Abstract: *Advertising plays a crucial role in 21st century's domestic and global markets. Advertising networks are playing an increasingly prominent role in the advertising market. The importance of finding the best publisher for an advertiser is obvious, and it can save a significant amount of money for advertising companies each year. Advertising networks differ in many cases such as allocation algorithms, pricing scheme, etc. The purpose of this paper is to determine the most efficient advertising slot for advertisers by considering factors such as economic factors, efficiency, previous advertisements experience, etc. Finally, we simulate a network from publisher's available advertisement slots and advertisements. The outcome will be the most optimal choice of publisher's slots for advertisements. After all, we have been through an optimization problem defined with two purposes: first, controlling risk of these choices and second, take account of penalties for choosing multiple slots of the same publisher for an advertisement.*

Keywords*: Advertising network, Allocation algorithm, Decision making, Operational research, Optimization Problem, a priori knowledge*

1. INTRODUCTION

Seizing the opportunity created by the radical developments in communication technologies that took place during the past two decades, firms increasingly use direct advertising to promote their products(Piwowar & Wei, 2006). For instance, internet commerce and services assisted by computer databases (e.g. telecom subscriptions) allow marketers to identify individual consumers and provide customized offers for them (Chen and Iyer 2002).

Advertising makes it possible for companies to be seen by a considerable number of people not only in domestic market, but also in global markets too(Alcalde, 1994). Nowadays, there are plenty of companies wanting their advertisements to be published. Also, we will have considerable number of publishers on the other side. The ones who make a relation between these two are advertising agencies(Postma, 1999).

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Traditionally, economic analysis treats the economic system as one of the givens(C&ela, Gaid, Li, & Niculescu, n.d.). The term "design" in the title is meant to stress that the structure of the economic system is to be regarded as an unknown(Rustem & Howe, 2002). An unknown in what problem? Typically, that of finding a system that would be, in a sense to be specified, superior to the existing one. The idea of searching for a better system is at least as ancient as Plato's Republic, but it is only recently that tools have become available for a systematic analytical approach to such search procedures (Leonid Hurwicz, 1973).

Nowadays, it is proven that the effect of advertising is much more than the quality of the product(Cook & Schulz, 2002). Factors such as designing a good trademark, choosing a great name, making a good slogan, and etc. are as important as publishing your advertisement on the best publishers' slots in the most efficient manner (e.g.: considering advertising's budget limit, assure the company that their advertisements will be seen by as many people as possible, etc.). It is the time that advertising agencies do their best to first create and design what was mentioned above and second, publish it on the most possible number of publishers' slots and in the most efficient way(Ferguson, 1989).

First series of actions named above are mostly depending on a combination of great taste (in choosing colors, design), psychological knowledge of the staff and head managers, a good knowledge of beliefs and culture of people in the design region, etc(Fremuth-Paeger & Jungnickel, 1999). Second, choosing the best publishers and slots, choosing the number of purchased slots in each publisher, taking advantage of previous advertisement experience, managing the amount of budget, level of expected risk, etc(Jackson, 2013). We are not going to suggest any algorithm for the first series of actions relating to other vast areas of science, art, and computer science.

In this paper, we are going to suggest an algorithm to take second series of actions by exact analysis of existing data in the agencies. The only task that the agency needs to do is to input those introduced data in the next sections. The result will be fascinating. A matrix showing the optimum choice of publishers and their corresponding slots while factors such as risk, dedicated budget, predicted cost, expecting profit, and other factors are considered exact and clear. In addition, this algorithm also suggests methods to estimate the amount of profit made as well as advertising costs based on effective factors.

1.1. Model Explanation

We start demonstrating our model by presenting some simple definitions for ~~a~~ better interpretation. The word "Publisher" means the one that publishes something, especially, a person or corporation whose business is publishing (Merriam-Webster, 1828). $\mathcal{P} = \{P_1, ..., P_m\}$ refers to the publishers. Number of slots is indicated by $\mathcal{S}$ and for each publisher the number of publisher's slots is shown by $\mathcal{S} = \{S_1, ..., S_k\}$. Finally, the model relating publishers and publishers' slot looks like this: we will have a specific number of "Publishers" each with a specific number of "Slots".

Table 1
Model Relating Publishers And Publisher's Slots

	Slot 1	Slot 2	Slot 3	...	Slot k^{th}
Publisher 1					
⋮					
Publisher m^{th}					

The word "Advertising" means the nonpersonal communication of information usually paid for and usually persuasive in nature about products, services, or ideas by identified sponsors through the various media (Bovee, 1992, p. 7). Next is "Advertisement" which means something (such as a short film or a written notice) that is shown or presented to the public to help sell a product or to make an announcement (Merriam-Webster, 1828).

Advertisement history is the previous experience of advertising product α in a specific number of publishers by purchasing a specific number of slots based on budget, risk factor, etc. Advertisements' histories are indicated by $\mathcal{A} = \{A_1, ..., A^n\}$ with each A^i showing the advertisement's history. For instance, an agency's published advertisements A^α for a specific product are as below:

Table 2
Example of Advertisement History

	Slot 1	Slot 2	Slot 3	...	Slot k^{th}
Publisher 1	0/1	0/1	0/1	...	0/1
Publisher 2	0/1	0/1	0/1	...	0/1
⋮	⋮	⋮	⋮	...	⋮
Publisher m^{th}	0/1	0/1	0/1	...	0/1

Furthermore, as you see in Table 2, for each $A^\alpha (P_j, S_i)$ we expect a corresponding value of 1 or 0, which means whether that slot of the related publisher is purchased for advertising or not. What we have explained here so far is enough to understand future parts. Next section will demonstrate the mathematical operations we will do on each cell; also, more parameters will be introduced.

1.1.1. Mathematical Model

We start modeling the market transactions of advertising and available advertisement's slots, in which: $\mathcal{A} = \{A_1, ..., A^n\}$ and $\mathcal{P} = \{P_1, ..., P_m\}$ and $\mathcal{S} = \{S_1, ..., S_k\}$, to assure that the reader has a good understanding of the model. We will add a short example after introducing each parameters or functions.

First, we need to define a function to relate each slot to its corresponding publisher. The function drawn in Table 1 relates slots to their corresponding publishers.

$$\mathcal{J}: \mathcal{S} \to \mathcal{P}$$

As an example, an advertising agency has revealed one of its previous advertisement's data sheets. We are going to see the number of publishers in different mass media parts and visualize what we meant for publishers' slots. Therefore, we will have a better interpretation about the functions and other functions $\mathcal{J}$ and parameters in the future.

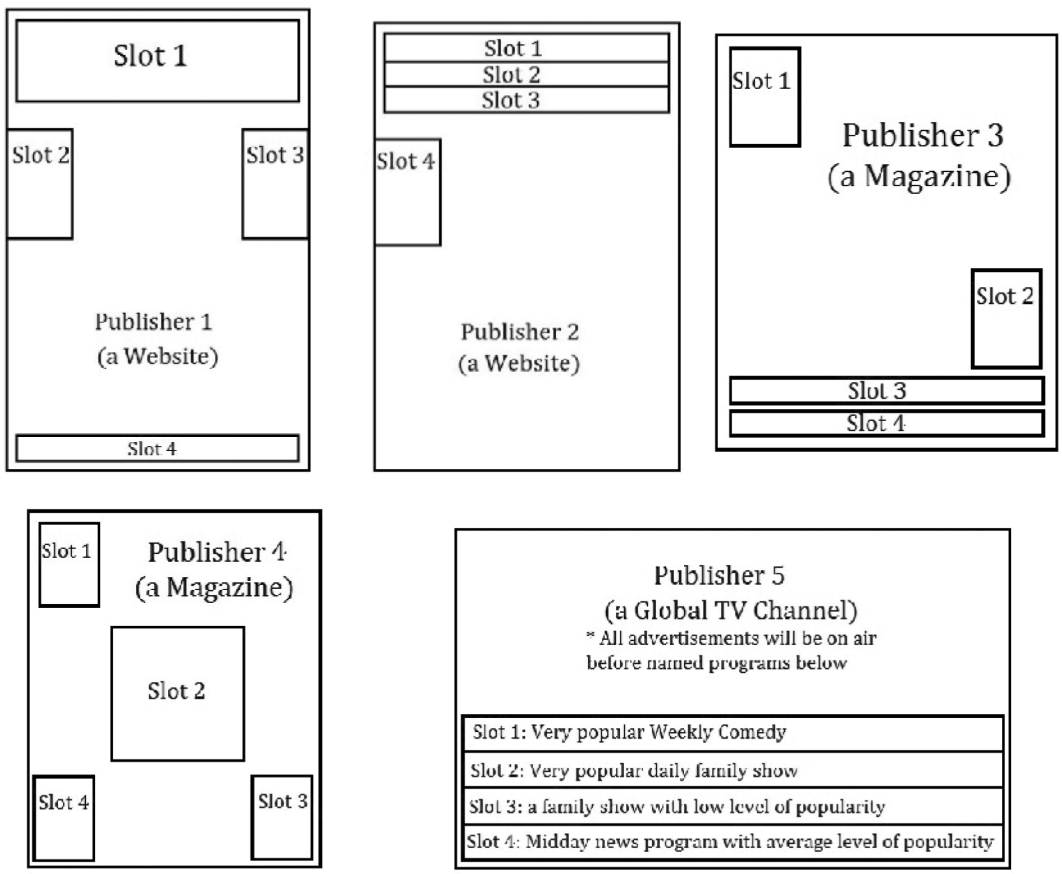

Figure 1: Instance showing four publishers and their slots

Each member of $\mathcal{A}$ like $\mathcal{A}^i$ shows one of the previous advertisement experience in Table 2 and we should note that

$$\mathcal{A}^i \subseteq \mathcal{P} \times \mathcal{S}$$

Where each advertisement A^i should satisfy the condition below:

$$\textit{if } (P_j, S_t) \in A^i \textit{ then } \mathcal{J}(S_t) = P_j$$

To specify purchase fee for each advertisement's slot in each publisher we define functions as below:

$$p^i : A^i \rightarrow \mathcal{N}$$

Such that $\mathcal{N} = \{0, 1, 2, ...\}$ is the set of natural numbers indicating purchasing items' cost for each advertisements' slot. We associate $p^i(P_j, S_t) = p^i_{jt}$. Now, we will see an instance of what this statement is going to say. In addition, we know that advertisement-purchasing fee for websites differs by "Pay per Click" or other estimating methods, or fee schedule will be much different for TV programs and so on (Kahng & Goto, 2004). We assumed schedule based purchasing price method for all slot. Prices for newspapers and magazines are for 6 months advertising and prices in TV programs are for a 5 minutes advertisement shown five times (Note that none of them are based on reality). By looking at figure 2, on the next page we will see the pricing for publishers and slots below.

Therefor, $p^i(P_j, S_t) = p^i_{jt}$ for the Figure 2, will be like (Assume it is the 10th experience of agency):

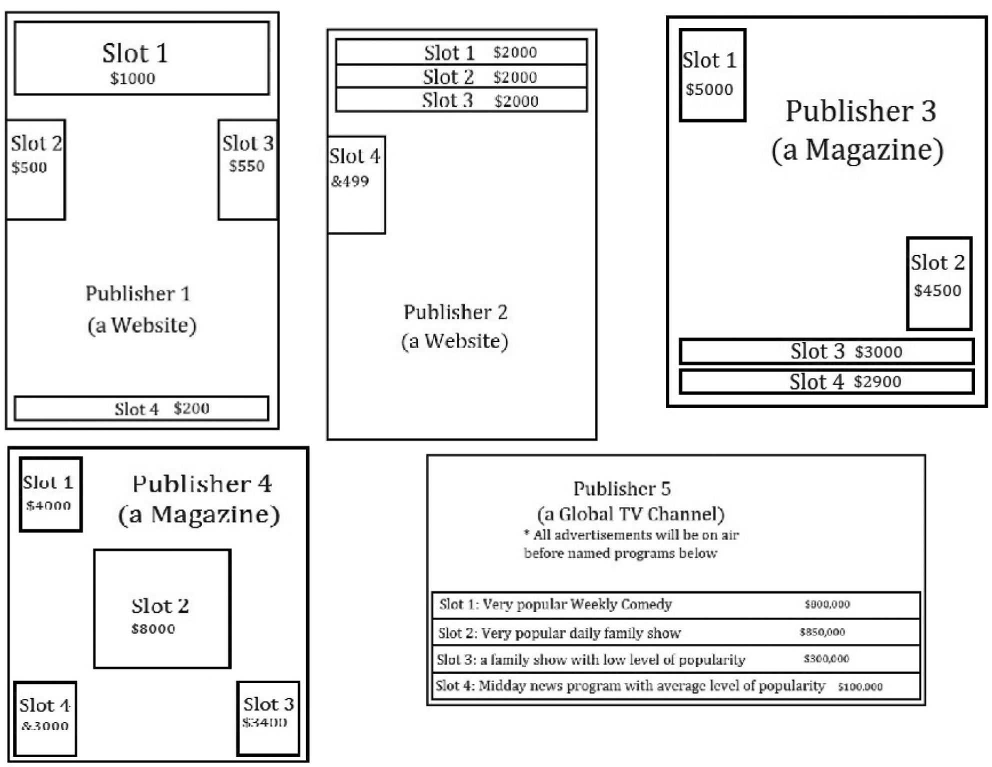

Figure 2: Instance showing four publishers and their slots

$$\begin{cases} P_{11}^{10}=1000, P_{12}^{10}=500, P_{13}^{10}=550, P_{14}^{10}=200 \\ P_{21}^{10}=2000, P_{22}^{10}=2000, P_{23}^{10}=2000, P_{24}^{10}=499 \\ P_{31}^{10}=5000, P_{32}^{10}=4500, P_{33}^{10}=3000, P_{34}^{10}=2900 \\ P_{41}^{10}=4000, P_{42}^{10}=8000, P_{43}^{10}=3400, P_{44}^{10}=3000 \\ P_{51}^{10}=800,000, P_{52}^{10}=850,000, P_{53}^{10}=300,000, P_{54}^{10}=100,000 \end{cases}$$

Again, we insist that none of the figures and data in this section are real and are assumed only for a better understanding of the reader.

To indicate if the couple (P_j, S_t) is used for advertisement A^i we define functions

$$\delta^i : \mathcal{P} \times \mathcal{S} \to \{0, 1\}$$

Where

$$\delta_{jt}^i = \delta^i (P_j, S_t) = 1 \text{ if and only if } \mathcal{J}(S_t) = P_j \text{ and } (P_j, S_t) \in A^i$$

We introduce a matrix relating (P_j, S_t) to a value of zero or one on the next page. It means whether we have used that slot of the corresponding publisher in our 10th advertisement experience or not.

$$\delta^{10} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Now, to estimate the total cost of advertisement A^i we define C^i as:

$$C^i = \sum_j \sum_t p_{jt}^i \delta_{jt}^i$$

This parameter for our special case discussed as an example would be:

$$C^{10} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} * \begin{bmatrix} 1000 & 500 & 550 & 200 \\ 2000 & 2000 & 2000 & 499 \\ 5000 & 4500 & 3000 & 2900 \\ 4000 & 8000 & 3400 & 3000 \\ 800000 & 850000 & 300000 & 100000 \end{bmatrix}$$

$$C^{10} = 1000 + 200 + 2000 + 5000 + 8000 + 300000 = 316200$$

Therefore, this advertisement costs the agency \$316200.

It is time to find the income from advertisement A^i with a new function R by:

$$R : \mathcal{A} \to \mathcal{N}$$
$$R^i : = R(A^i)$$

In addition, the profit from advertisement A^i is equal to

$$B^i = R^i - C^i$$

Another vector we assign to calculate the profit margin of each advertisement history is W^i which calculated as below

$$W^i = \frac{B^i}{C^i}$$

Since profit margin of for each slot is not known, we instead use profit margin for every advertisement history q^i_{jt} with $q^i_{jt} = W^i \delta^i_{jt}$

To indicate how many times each slot is used for advertising, we assign

$$U_{jt} = \sum_i \delta^i_{jt}$$

For average of profit margin for each slot we define

$$M_{jt} = \left(\sum_i q^i_{jt} \delta^i_{jt} \right) / U_{jt}$$

For risk analysis of each slot, we need to calculate the variance of profit margins as below:

$$V_{jt} = \left(\sum_i \left(q^i_{jt} - M_{jt} \right)^2 \delta^i_{jt} \right) / U_{jt}$$

Finally, we reached the end of section one. We did not extended our example for all the parameters since they are just simple mathematical operations and easily understandable. A complete example is done in section three.

1.2. Optimization Problem

First step is to create all possible functions for our special cases; next, we need a function

$$f : \mathcal{N} \to \mathcal{N}$$

Which associating penalty for using multiple slots of a publisher with the following properties:

1. $f(0) = f(1) = 0$

This statement suggests that, there is no punishment only if one or no slot is purchased in a publisher.

2. $f(t + 1) \geq f(t)$ *for* $t \in \mathcal{N}$

The more slots purchased in a publisher the more punishment provided, exerting negative effects of using more than one slots of a publisher in an advertisement. With parameters $C, D \geq 0$, we can define following optimization problem:

$$Max\left(\sum_{j=1}^{n}\sum_{t=1}^{k}P_{jt}M_{jt}\delta_{jt} - C\sum_{j=1}^{n}\sum_{t=1}^{k}P_{jt}\delta_{jt}\sqrt{V_{jt} - D\sum_{j=1}^{n}f\left(\varepsilon_{j}\right)}\right)$$

such that:

$$\delta_{jt} \in \{0, 1\}, 1 \leq j \leq n, 1 \leq t \leq k$$

$$\sum_{j=1}^{n}\sum_{t=1}^{k}p_{jt}\delta_{jt} \leq T$$

The inequality puts a restriction on available matrices so the cost does not exceed the allocated budget to advertisement.

$$\varepsilon_{j} = \sum_{t=1}^{k}\delta_{jt}, 1 \leq j \leq n$$

Where $T > 0$ is the budget and P_{jt} is the price of purchasing slot S_t in publisher P_j. Our optimization problem intends to estimate the most efficient matrix from available matrices by taking factors such as risk and punishment for choosing more than one slot from a publisher. The result is a fascinating matrix showing the best possible choices while the purchase fee does not exceed the predicted budget.

1.3. Simulation and Numerical Example

It is time to prove what we claimed from the beginning up to this part. We have coded a program to solve all above equations, and all we need is to insert data. First, we assumed five number of publishers so$\mathcal{P} = \{P_1, P_2, P_3, P_4, P_5\}$. We assume each publisher has four publisher's slots so $\mathcal{S} = \{S_1, S_2, S_3, S_4\}$. Presuming that the agency has done fifteen similar tasks for a similar product before, then fifteen advertisement's history would be available or $\mathcal{A} = \{A^1, A^2, A^3, ..., A^{15}\}$. Now, you can see each advertisement's history matrix in table 3:

One introduced parameter is the purchasing fee of these experiences $p^i(P_j, S_t) = p_{jt}^{i}$. We assume constant-fee schedule for simplification in our model to avoid complexity and confusion of main body because of too many data. We presumed each slot has a fixed fee schedule while all the advertisements published. We repeat one

Table 3
Advertisements done for a similar product up to this time

A^1				A^3				A^3			
1	1	1	1	0	1	0	0	0	0	0	0
1	0	0	1	1	1	1	0	1	0	1	1
0	1	0	0	0	0	0	0	1	0	1	0
1	0	0	0	1	1	0	1	1	0	1	0
0	1	0	0	1	0	1	1	0	1	0	1
A^4				A^5				A^6			
1	0	0	1	1	1	0	1	1	1	0	1
0	0	0	0	0	1	0	0	1	1	0	0
1	1	0	0	1	0	1	1	0	1	0	1
0	1	0	0	1	0	0	0	1	0	0	1
0	1	0	1	1	1	1	0	0	0	1	0
A^7				A^8				A^9			
0	0	0	1	1	1	0	0	1	1	0	1
1	0	1	1	0	1	1	1	0	1	0	1
1	1	1	0	0	0	1	0	1	1	1	0
0	0	1	0	0	1	1	0	0	0	1	0
0	0	1	1	1	0	1	1	0	0	1	0
A^{10}				A^{11}				A^{12}			
1	1	1	0	0	1	1	0	1	1	0	1
0	0	1	0	0	1	0	1	1	1	0	0
0	0	0	1	1	1	1	1	0	0	0	1
1	0	0	1	1	0	1	0	1	1	0	1
1	0	0	0	1	1	0	1	1	1	0	0
A^{13}				A^{14}				A^{15}			
1	1	1	0	0	0	1	0	1	1	0	1
1	0	0	0	1	0	1	1	0	1	0	1
0	0	0	1	1	0	0	1	0	0	0	1
0	0	1	1	0	1	0	0	0	1	1	0
1	1	0	0	1	1	1	1	1	0	0	1

more time that the prices are for the period that advertisements were published. Table 4 shows function $p^i(P_j, S_t)$ which in our special case $p^1 = p^2 = ... = p^{15}$.

Table 4
Purchasing Fee of Each Slot

	S1	S2	S3	S4
P1	$960	$947	$721	$898
P2	$1278	$244	$1005	$1040
P3	$414	$1059	$745	$809
P4	$384	$1699	$1704	$1503
P5	$640	$838	$913	$656

To follow our mathematical modeling in section one, next we should estimate cost functions for all fifteen pervious advertisement history.

Table 5
Cost function estimated for each Advertisement Experience

C^1	C^2	C^3	C^4	C^5	C^6	C^7	C^8
$8125	$9269	$8064	$6524	$7792	$8995	$9712	$10553

C^9	C^{10}	C^{11}	C^{12}	C^{13}	C^{14}	C^{15}
$8924	$6969	$10201	$10200	$9400	$10013	$9597

"Revenues in a previous Advertisement experience" are the money that the company paid after contracting with the advertisement agency.

Table 6
Revenues in an Advertisement History

R^1	R^2	R^3	R^4	R^5	R^6	R^7	R^8
$16753	$17376	$15080	$9663	$13347	$14983	$19457	$1661

R^9	R^{10}	R^{11}	R^{12}	R^{13}	R^{14}	R^{15}
$11340	$18964	$10649	$19211	$16723	$11550	$9687

More analysis could be done to better understand the data. The following relation calculates profit made by each advertisement:

$$B^i = R^i - C^i$$

Table 7
Profit Made by Each Advertisement Experience

B^1	B^2	B^3	B^4	B^5	B^6	B^7	B^8
$8628	$8107	$7016	$3139	$5555	$5988	$9745	$6058

B^9	B^{10}	B^{11}	B^{12}	B^{13}	B^{14}	B^{15}
$2416	$11995	$448	$9011	$7323	$1537	$90

Margin of profit, W^i for each advertisement's slot is calculated to give an accurate analysis of costs, revenues, and profits respect to monitor deviations of average profit and to give a insight to the experiences and their corresponding slots that made us the most profit. Therefore:

$$W^i = \frac{B^i}{C^i}$$

Table 8
Profit Margin of Each Advertisement's experience

W^1	W^2	W^3	W^4	W^5	W^6	W^7	W^8
1.061908	0.874636	0.87004	0.481147	0.712911	0.665703	1.003399	0.574055

W^9	W^{10}	W^{11}	W^{12}	W^{13}	W^{14}	W^{15}
0.270731	1.721194	0.0439173	0.883431	0.779043	0.153501	0.0093779

Sticking to the mathematical model, it is time to calculate the profit margin of each slot with the function $q^i_{jt} = W^i \delta^i_{jt}$ so the outcome is clearly a simple multiplication. One other parameter helping us in this analysis is $U_{jt} = \sum_i \delta^i_{jt}$. U_{jt} is a function that finds the total times used for advertisement by all slots. Table 9 shows the output of this function:

Table 9
Indicting How Many Times a Slot Used for an Advertisement

	S1	S2	S3	S4
P1	10	11	5	8
P2	8	8	6	8
P3	7	6	6	8
P4	8	6	7	5
P5	9	8	7	8

After estimating average margin of profit for each slot, to we establish a standard reference, M_{jt}, to compare measure each slot's deviation from the reference.

$$M_{jt} = \left(\sum_i q^i_{jt} \delta^i_{jt} \right) / U_{jt}$$

The result will be like:

Table 10
Average of Profit Margin

	S1	**S2**	**S3**	**S4**
P1	**0.715949917430987**	**0.690627798591524**	**0.751912363288932**	**0.636075731220822**
P2	**0.786457332475148**	**0.504345210061412**	**0.866137083832914**	**0.498365782678626**
P3	**0.505091868692645**	**0.587800522001600**	**0.579175144453386**	**0.621134659166784**
P4	**0.854217449752765**	**0.496024490440302**	**0.507222953271007**	**0.984801367146946**
P5	**0.639118306568169**	**0.623237028312839**	**0.607847631722985**	**0.501258796745689**

Using the values of profit margin and average profit margin form Tables 8 and 10, the variance of profit margin is calculated in Table 11 by the function $V_{jt} = \left(\sum_i \left(q^i_{jt} - M_{jt} \right)^2 \delta^i_{jt} \right)$.

Table 11
Variance of Profit Margin for Each Slot

	S1	S2	S3	S4
P1	0.1937497915527	0.213829191372205	0.379138729777449	0.116806856869247
P2	0.07029656073210	0.108347393825089	0.223868853017008	0.166611204590528
P3	0.1166402204479	0.135214162648195	0.110686254915672	0.280047733999319
P4	0.1885494587129	0.108981391673582	0.139229765419794	0.141769312196549
P5	0.2557738002420	0.116226859422733	0.0804632375187873	0.138162639685280

Up to here, we analyzed previous data and estimated what we needed as well as storing them. It is time to start the optimization problem. There are really critical decisions that agencies need to make on dedicating budget for advertisement and estimate degree of risk and the amount of penalty due to choosing two slots from one publisher. (We assumed it is a waste of money and resources to purchase two slots of one publisher in one advertisement experience).

Note that negative effect of risk, $C\%$, is the probability of being a looser and the rest $(100 - C)\%$ is the probability of being a winner. Therefore, it is time to insert the amount of budget predicted for advertisement which is \$7,000 in our special case. We also assume negative effect for risk factor is $C = 15\%$ $(100 - C)\% = (100 - 15)\% = 85\%$ and $D = \$100$ as the punishment fee.

Now, we define function f as following:

$$f(0) = 0$$
$$f(1) = 0$$
$$f(2) = 1$$
$$f(3) = 3$$
$$f(4) = 6$$

$$Max\left(\sum_{j=1}^{n}\sum_{t=1}^{k}P_{jt}M_{jt}\delta_{jt} - -.15\sum_{j=1}^{n}\sum_{t=1}^{k}P_{jt}\delta_{jt}\sqrt{V_{jt}} - 100\sum_{j=1}^{n}f\left(\varepsilon_j\right)\right)$$

such that:

$$\delta_{jt} \in \{0, 1\}, 1 \le j \le 5, 1 \le t \le 4$$

$$\sum_{j=1}^{n}\sum_{t=1}^{k}p_{jt}\delta_{jt} \le 7000$$

$$\varepsilon_j = \sum_{t=1}^{k}\delta_{jt}, 1 \le j \le 5$$

Finally, after inserting all values into the algorithm, the output in Table 12 shows the optimized number of slots that should be bought from each publisher.

Table 12
Optimized choices of publishers and slots

S1	S2	S3	S4	
P1	1	1	0	0
P2	1	0	1	0
P3	0	0	0	0
P4	1	0	0	1
P5	0	0	1	0

In addition, maximum estimated profit made by advertisement is:

$4880.10

Furthermore, estimation of revenues, budget, etc. claims that the net cost of purchasing the slots will be $6990, which is provable even by handy calculations. Note that it takes 290 seconds for anIntel® Core™ i5 CPU to do the calculations from the beginning up to here, for this short example. Number of calculations also is computable as an exponential function.

2. CONCLUSION

In 21st century, the importance of advertisement is not concealed from any one. Most of the academic research concentrate on artistic, graphical, or other aspects of this business, and there is not too much attention on these sorts of optimization problems. Our pioneering research opens a gateway to the new aspects of this business. What we did was analyzing the previous information and data, next using the results of processed data to predict the best choice of slots in each publisher in a way the cost would not exceed the suggested budget, exerting negative effects of risk, and choosing multiple slots in a publisher. Therefore, numerous challenges and problems are raised, for instance, effect of psychological, graphical, and other factors' effects on profit and cost, finding an optimized algorithm to reduce the number of calculations, remarking active slots of each publisher, evaluating exerting factors such as time needed to make the profit, and so on.

References

Chen, Y., Iyer, G. (2002). Consumer addressability and customized pricing. Marketing Science, 21(2), 197–208. Research note.

L. Hurwicz , The Design of Mechanisms for Resource Allocation, The American Economic Review, Vol. 63, No. 2, Papers and Proceedings of the Eighty-fifth Annual Meeting of the American Economic Association (May, 1973), pp. 1-30

Alcalde, J. (1994). Exchange-proofness or divorce-proofness? Stability in one-sided matching markets. Economic Design, 1(1), 275-287. http://dx.doi.org/10.1007/bf02716626

C&ela, A., Gaid, M., Li, X., & Niculescu, S. Optimal design of distributed control and embedded systems.

Cook, W. & Schulz, A. (2002). Integer programming and combinatorial optimization. Berlin: Springer.

Ferguson, D. (1989). The application of microeconomics to the design of resource allocation and control algorithms. New York, N.Y.: Dept. of Computer Science, Columbia University.

Fremuth-Paeger, C. & Jungnickel, D. (1999). Balanced network flows. I. A unifying framework for design and analysis of matching algorithms. Networks, 33(1), 1-28. http://dx.doi.org/10.1002/(sici)1097-0037(199901)33:1<1::aid-net1>3.0.co;2-m

Jackson, M. (2013). Economic Engineering and the Design of Matching Markets: The Contributions of Alvin E. Roth. Scand. J. Of Economics, 115(3), 619-639. http://dx.doi.org/10.1111/sjoe.12026

Kahng, H. & Goto, S. (2004). Information networking. Berlin: Springer.

Piwowar, M. & Wei, L. (2006). The Sensitivity of Effective Spread Estimates to Trade–Quote Matching Algorithms. Electronic Markets, 16(2), 112-129. http://dx.doi.org/10.1080/10196780600643803

Postma, P. (1999). The new marketing era. New York: McGraw-Hill.

Rustem, B. & Howe, M. (2002). Algorithms for worst-case design and applications to risk management. Princeton, N.J.: Princeton University Press.