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Active Control of Stall in Axial Flow Compressors

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Abstract: In this Paper, active stabilization of stall instabilities in axial flow compressors is pursued using a combination of bifurcation analysis and nonlinear control. A low order discretization of a PDE compressor modep is found to exhibit a stationary (pitchfork) bifurcation at the inception of stall. Using throttle opening as a control, analysis of the linearized System at stall shows that the critical mode (zero eigenvalue) is unaffected by linear feedback. Hence, nonlinear stabilization techniques are necessary. A nonlinear (quadratic) feedback control of the first mode amplitude is proposed based on the lower-order model and is found to eliminate or reduce the hysteresis for both the low order and high order discretizations. This improves the nonlinear stability of the compression system near the stall limit. Furthermore, the issue of designing non smooth feedback control laws is addressed. the merits of employing no smooth feedback are illustrated by bifurcation analysis of both the low order and high order and high order discretizations. A possible mechanism for the no smooth feedback is suggested.

Keywords: Hopf bifurcation, Galerkin's method, Moore Greitzer model, Bifurcation phenomena

1. INTRODUCTION

Recent Years have witnessed an increasing interest in axial flow compressor dy namics , both in terms of analysis of stall phenomena and their control. This interest is due to the increased performance that is potentially achievable in modern gas turbine jet engines by operation near the maximum pressure rise. The increased performance comes at the price of a significantly reduced stability margin, since the steady, spatially uniform gas flow loses stability when the system is operated near peak pressure-rise conditions. The resulting post-instability behavior leads to decreased operation performance of the compressor and to mechanical damage of the compression system.

In general there are two fundamentally different post-instability behaviours: surging flow and rotating stall compressor surge occurs when the plenum gas pressure exceeds the compressor pressure rise and so low frequency (in time) oscillations of the mean gas flow rate develop. Rotating stall is a local aerody namic phenomenon that occurs when the gas passing through the rotor disen gages from the blade surface, reducing the local gas flow rate. In this case, the bulk gas flow remains constant in time, but flow measurements taken along the circumferential coordinate (*0* of Fig. 6.1) of the compressor rotor will reveal spatial variations of the local gas flow. This means the local gas velocity takes the form of a traveling wave, rotating about the compressor annulus.

Greitzer developed a nondimensional fourth-order compression system model and introduced a nondimensional parameter, B, which he found to be a determinant of the nature of post-instability behavior. A global bifurcation of periodic solutions and other bifurcations were found for this mode, and were used to explain the observed dependence of the dynamical behavior on the B parameter. Moore and Geritzer introduced a refined model to describe stall phenomena in axial flow compression systems. This model accounts for nonaxisymmetric flow patterns, whereas the model of Greitzer had no means of explicitly describing spatial effects.

Our work begins with a modification to the 2-dimensional partial differential equation model of moore and Geritzer to include viscous dissipative forces is the unsteady performance of a compressor blade row. The resulting compression system model, while somewhat more complicated that the original Moore-Greitzer model, is still amenable to formal local stability and bifurcation analysis. Detailed studies about the transition from steady, spatially-uniform flow to nonuniform and time-dependent gas axial velocity profiles in this modified model are presented. It is found that the first stalled-flow solution is born through a subcritical bifurcation, meaning the bifurcating solution is born unstable. The practical importance of the subcritical stall bifurcation, however, is that when the uniform-flow operating point is subject to perturbations, the system will jump to large amplitude, fully developed stall call. Subcritical bifurcations also imply hysteresis, and so returning the throttle to its original position may not bring the system out of stall.

Several techniques have been proposed for active control of stall instabilities in axial flow compressors. From an analytical point of view, these methods employ linear control for avoiding or delaying the occurrence of stall. Of course the physical mechanisms of a proposed control implementation differ among the proposed active control schemes. The present work and that of however, begin with the recognition of the importance of local bifurcations as determinants of the nature of post-instability behavior of axial flow compression systems. In a nonlinear state feedback law was proposed for the Moore and Greitze model simplified to three ordinary differential equations with a low-order Galerkin discretization. The control philosophy of this work and that is similar to that of abed and fu. This entails determining feedback control laws which ensure the stationary bifurcation result is only stable bifurcated solutions. Thus, even though the nominal equilibrium is not stabilizable within the framework of linear theory, it may be possible to stabilize a neighborhood of the nominal solution for a range of parameter values including the stall value of the throuttle opening parameter, to finite amplitude perturbations. The control law is designed analytically based on the low-order discretization and is applied to both the low-order and high-order spectral discretizations of the full system Bifurcation analysis of a high-order discretization is used to assess the effectiveness of the controller.

In addition to the smooth feedback control design, it is found that some nonsmooth feedback controllers render surprisingly superior performance over smooth feedback designs. The merits of the nonsmooth feedback controllers are judged by difurcation studies though theoretically bifurcation analysis of nonsmooth systems is still a largely open area; the results revealed in this study appear to point out a new avenue in terms of control of bifurcating systems and critical system stabilization.

The chapter proceeds as follows. In Section 6.2, Modification of the moore Geritzer model is presented. In Section 6.3 analytical computations useful in the analysis of stationary bifurcations are reviewed. These results are applied to study the stability of a low order discretization of the axial flow compressing system in the vicinity of the stall point. The low order model is obtained by

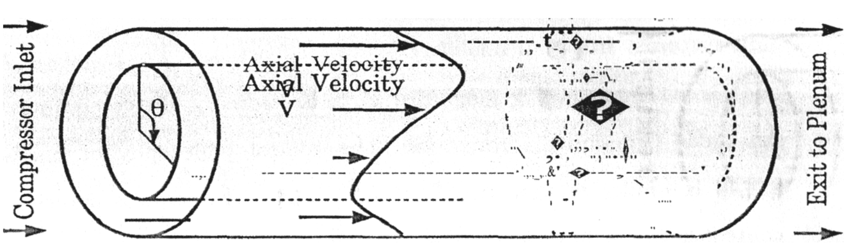


Figure 6.1: A schematic of the compressor geometry

applying Galerkin's method to the full PDE model. A Pitchfork-type stationary bifurcation is observed in the model at the stall point. In Section 6.4, a throttle opening control law is given. This nonlinear control law circumvents the uncontrollability of the zero eigenvalue of the linearized model at stall. A purely quadratic state feedback using measurement of the asymmetric flow disturbance amplitude is given, and found to result in local stabilization of the bifurcation leading to stall not only in the low order model, but also in high-order discretizations of the compression system model. In Section 6.5, the use of nonsmooth feedback is discussed.

an, bn, An mode amplitude coefficients

Nomenclature

B	Plenum/compressor volume
c	wave speed
f	axisymmetric compressor characteristic
fo	Shut- off head
F	throttle characteristic
H	Pressure rise scaling factor
k	controller gain
le	overall compressor length
m	exit duct length factor
n	mode number
u	control input
v	axial velocity perturbation
vo	perturbation at T| = 0
V	mean axial velocity
Vioc	total local axial velocity
w	mean velocity scaling factor
a	internal compressor lag
Y	throuttle opening
Yo	nominal throttle opening
Δp	plenum-atmosphere pressure rise

n	axial coordinate
θ	circumferential coordinate
λ*n*	n th eigenvalue
μ	viscosity
T	time

2. MODIFICATION OF THE MOORE-GREITZER MODEL

The model is based on Moore and Greitzer's model, but a term which accounts for momentum transfer in the compressor section by viscous transport is also included. A local momentum b alance describing the two-dimensional flow in the compressor and its associated ducting gives the partial differential equation:

$$\Delta_p = f\left(V + vo\right) - lc\frac{dV}{dr} - m\frac{\partial}{\partial r}\int_{-\infty}^{o} vdn - \frac{1}{2a}\left[2\frac{\partial V_0}{\partial r} + \frac{\partial V_0}{\partial \theta} - \mu\frac{\partial^2 V_0}{\partial \theta^2}\right] \quad (6.1)$$

Note the our notation differs considerably from the original notation of moore and Greitzer: V denotes the annulus-averaged (mean) gas axial velocity; v_0 is the axial velocity perturbation evaluated at $x = 0$ (the inlet face of the compressor); Δp is the plenum-to- atmosphere pressure rise; x, θ are the axial and angular coordinates, respectively; and μ is the gas viscosity.

The compressor pressure rise *F(Vioc)* is particular to each compressor and is obtained from experiments in the stable operating range and estimated in the non-uniform-flow range. Following Moore and Greitzer (72) we use a cubic equation in axial velocity

$$f(Vioc) = f_0 + H[1 + \tfrac{3}{2}(\tfrac{Vioc}{W} - 1) - \tfrac{1}{2}(\tfrac{Vioc}{W} - 1)^3 1] \quad (6.2)$$

Where *Voic* =*V*+*vo* (the total local axial flow) and the characteristic parameters used throughout this work are given in Table 6.1, with le fixed at a representative value [72, 73]. If there are no spatial variations of gas density and pressure is the plenum, an overall material balance on the gas over the plenum gives:

$$lc\,\frac{d\Delta_p}{dr} = \frac{1}{2B^2}\left[V(r) = F^{-1}\left(\Delta_p\right)\right] \quad (6.3)$$

Where the throttle characteristic is given by the orifice equation

$$f^{-1}\left(\Delta_p\right) = r\sqrt{\Delta_{p}.} \quad (6.4)$$

The parameter / is proportional to the throttle opening.

Parameter	Value	Description
$\propto$	1/3.5	Internal Compressor lag.
l_c	8.0	Overall compress or length
m	1.74	Exit duct length Feature
H	0.18	Compressor characteristic height Featur
w	0.25	Compressor characteristic Width factur.
f_0	0.3	Shut-off Head
μ	0.01	Fluid Viscosity

Table 6.1: Values of Compressor Parameters

3. STABILITY ANALYSIS

In the section we first recall some bifurcation-theoretic result on stability of one-parameter families of nonlinear systems. They then will be applied to study the dynamic behavior of axial flow compression systems.

3.1 Bifurcation Formulae

Consider a one-parameter family of nonlinear autonomous systems

$$\dot{x} = f\mu\{x\}, \tag{6.5}$$

with $J\mu(xe, \mu)$= *0*, where $x\ E\ JR/$ pis a real-valued parameter, $J\mu$ is sufficiently smooth in x and μ, and Xe, μ is the nominal equilibrium point of the system as a function of the parameter μ Suppose the following Hypothesis holds:

(S) The Jacobian matrix of system (6.5) at the equilibrium Xe, μ possesses a simple eigenvalue $\lambda(\mu)$ with λ(0) = 0, , λ (0)=/= 0, with the remaining eigen-values, X2(0), ..., $\lambda n(0)$ lie in the open left-half complex plane for μ within a neighborhood of μc = 0.

Theorem 2.1 asserts that hypothesis (s) leads to a stationary bifurcation from Xe, o at μ = 0 for Eq (6.5). That is, new equilibrium points bifurcate from Xe,o at μ = 0. Recall that near the point $(Xe, o, 0)$ of the $(n$ +1)-dimensional (n, μ)-space, there exists a parameter e and a locally unique curve of critical points $(x(e), \mu(t))$, distinct from Xe, μ and passing throuth (Xe,o 0), such that for all sufficiently small [e], *X(t)* is an equilibrium point of (6.5) when μ=μ(t).

The parameter t may be chosen so that x(e), μ (t) are smooth. The series expansion of x(t), $\mu(\in)$ can be written as

$$\mu(\in) = \mu_1 \in +\mu_2 \in^2 \ldots \tag{6.6}$$

$$x(\in) = x_{e,}o + x_1 \in +x_2 \in^2 + \cdots \tag{6.7}$$

If $\mu_1 = 0$, the system undergoes a transcritical bifurcation from Xe, l, at μ = 0. That is, there is a second equilibrium point besides Xe, μ for both positive and negative values of *ll* with |lll small. If $\mu_1 = 0$ and μ_2 = 0, the system undergoes a pitchfork bifurcation for |lll sufficiently small. That is, there are two new equilibrium points for either positive or negative values of *ll* with |μ| small. The new equilibrium points have an eigenvalue /3 which vanishes at *ll*=0. The series expansion of /3 in E is given by

$$\beta(\epsilon) = \beta_1\epsilon + \beta_2\epsilon^2 + \cdots \tag{6.8}$$

With

$$\beta_1 = -\,\mu_1\,\lambda'(0), \tag{6.9}$$

and, in case /3$_1$ = 0, {3$_2$ is given by

$$\beta_2 = -\,2\mu_2\,\lambda'(0), \tag{6.10}$$

Thus, the system exhibits an exchange of stabilities at the bifurcation point $x_e, o(at\,\mu = 0)$.

The stability coefficients β_1 and β_2 can be determined solely by eigenvector computations and the coefficients of the series expansion of the vector field. System (6.5) can be rewritten in the series form

$$\begin{aligned}\dot{\hat{x}} &= L_\mu\hat{x} + Q\mu\,(\hat{x},\hat{x}) + C\mu(\hat{x},\hat{x}) + \cdots \\ &= L_0\hat{x} + \mu L_1\,\hat{x} + x^2 L_2\hat{x} + \cdots \\ &\quad + Q_0(\hat{x},\hat{x}) + \mu Q_1(\hat{x},\hat{x}) + \cdots \\ &\quad + C_0(\hat{x},\hat{x},\hat{x}) + \cdots \end{aligned} \tag{6.11}$$

Here, $\hat{x} := x - x_{e,0,}\, L_{\mu,}L_0, L_1, L_2$ are $n \times n$ matrices, $Q_\mu(x,x), Q_0(x,x)$, $Q_1(x,x)$ are vector-valued quadratic forms generated by symmetric bilinear forms, and $C_\mu(x,x,x), C_0(x,x,x)$ are vector-valued cubic forms generated by symmetric trilinear forms.

By assumption, the Jacobian matrix L_o *had only* one simple zero eigenvalue with the remaining eigenvalues stable. Denote by l and r and left (row) and right (column) eigenvectors of the matrix L_0 corresponding to the simple zero eigenvalue, respectively, where the first component of r is set to l and the left eigenvectr l is chosen such that $lr = 1$. It is easy to check [51, 65] that

$$\lambda'(0) = lL_1 r \tag{6.12}$$

Stability criteria for system (6.11) can be summarized in the following two lemmas. For details, see [3].

Lemma 3.1 The bifurcated solutions of (6.11) for μ near 0, which appear only for $\mu > 0 (resp. \mu < 0)$ when $lL_1 r > 0$ $(resp - lL_1 r < 0)$, are asymptotically

Stable if $\beta_1 = 0$ *and* $\beta_2 < 0$, and are unstable if $\beta_1 = 0$ *and* $\beta_2 > 0$. *Here,*

$$\beta_1 = 1Q_0(r,r) \tag{6.13}$$

and
$$\beta_2 = 2l\,\{2Q_0(r,x_2) + C_0(r,r,r)\} \tag{6.14}$$

With x_2 satisfying the following equation:

$$L_0 X_2 = -Q_0(r,r). \tag{6.15}$$

Lemma 3.2 Suppose the value of β_1 given in (6.13) above is negative.Then the bifurcated solution occurring for $\mu > 0$ (resp. $\mu < 0$) is asymptotically stable when $lL_1 r > 0 (resp. lL_1 r < 0)$.

The criterion given in Lemma 3.1 corresponds to the pitchfork (stationary) bifurcation, while the one in Lemma 3.2 is for the transcritical (stationary) bifurcation. Examples of these bifurcation diagrams can be found in many books on bifurcation theory.

3.2 Stability Analysis of Rotating Stall

The rotating stall equilibria born at the stall birurcation point are spatial waves of local axial velocity, rotating at a constant speed around the annulus. Rather than computing these traveling wave solution as limit cycles in the Fourier coefficient space, a more efficient method is to introduce a fotating coordinate frame $\theta \rightarrow \theta +$

CT so that amplitude coefficients of the traveling waves can be bound as fixed points. Making this coordinate change affects the PDE (6.1) only:

$$\Delta_p = (V + u_0) - l_c \frac{dV}{dr} - m \int_{-\infty}^{0} \left[c\frac{\partial v}{\partial \theta} + \frac{\partial v}{\partial t} \right] dn$$

$$-\frac{1}{2a}[2c\frac{\partial u_0}{\partial \theta} + 2\frac{\partial u_0}{\partial r} + \frac{\partial u_0}{\partial \theta} - \mu \frac{\partial^2 u_0}{\partial \theta^2}] \quad (6.16)$$

Near the bifurcation point , the amplitude of the bifurcating mode will dominate the shape of the stall cell. Thus, in the neighborhood of the bifurcation point we can approximate u by the eigenfunction associated with the critical eigenvalue

λ_n

$$u = \exp(nn)\,[a_n \cos(n\theta) + b_n sin(n\theta)] \quad (6.17)$$

(and so $v_0 = a_n \cos(n\theta) + a_n \sin(n\theta)$), substitute (6.17) into (6.16) to form the residual, use Galerkin's method to determine the amplitude coefficients, and constrain the Fourier coefficients by the relationship

$$A_n^2 = a_n^2 + b_n^2, \quad (6.18)$$

we obtain the greatly simplified, third-roder set of ODEs (c.f.eqns of Moore and Greitzer (1986)

$$\frac{ma+n}{na}\ \frac{dA_n}{dr1} = \left[\frac{3HV}{2\omega^3}(2\omega - V) - \frac{\mu n^2}{2a}\right] A_n - \frac{3H}{8\omega^3}A_n^3, \quad (6.19)$$

$$l_c\frac{dV}{dr} = -\Delta_p + fo + \frac{HV^2}{2\omega^3}(3\omega - V) + \frac{3H}{4\omega^3}(\omega - V)A_n^2, \quad (6.20)$$

$$l_c\frac{d\Delta_p}{dr} = \frac{1}{4B^2}[V - \curlyvee\sqrt{\Delta_p} \quad (6.21)$$

Our control design will be carried out based on the stability analysis of system (6.19)-(6.12) for the case n = 1 (the first harmonic of the flow disturbance). The controller, however, will be applied to a high-order discretization in the ensuing numerical analysis. To solve for an equilibrium point of (6.19)-(6.21), it is easy to see that $A_1 = 0\ is$ always a solution of $\frac{A_1}{dr}$=0 for the right hand side of Eq. (6.19). However, $A_1 = 0$ may not be the necessary condition for the existence of equilibrium point for (6.19)-(6.21). Denote $\hat{x} = \left(0, \hat{V}, \widehat{\Delta_p}\right)^T$ as an equilibrium point for (6.19)-(6-21) at $\curlyvee = \curlyvee^0, \hat{V}$ and $\widehat{\Delta_p}$ should then satisfy the relationships $\hat{V} = \curlyvee$

$\sqrt{\widehat{\Delta}_p}\, and\, \widehat{\Delta_p} = f(\widehat{V})$. Under the assumption $A_1 = 0$, $\hat{x}$ denotes a uniform flow equiblbrium point for the axial flow compression model (6.19)-(6.21), which depends on the throttle control parameter $\curlyvee^0$. In the following, we consider the stability conditions for the uniform flow equilibrium $\hat{x}$ and treat $\curlyvee$ as a bifurcation parameter to seek possible bifurcating stalled-flow solutions emanating from $\hat{x}$ such that $A_1 \neq 0$.

Let X = $(x_1, x_2, x_3)^T$ denote the state variation of the third order model above near the uniform flow equilibrium point $\hat{x}$, where $x_{1,}A_1, x_2 = V - \widehat{V} and\, x_3 = \Delta_p - \widehat{\Delta}_{p.}$ The linearization of (6.19)-(6.21) at $\hat{x}$ for $\curlyvee$=$\curlyvee^0$ is

$$\frac{dX}{dt} = L_0 X, \tag{6.22}$$

Where

$$L_0 = \begin{pmatrix} \left[\frac{3H\widehat{V}}{2\omega^3}\left(2\omega - \widehat{V}\right) - \frac{\mu}{2a}\right]\frac{a}{ma+1} & 0 & 0 \\ 0 & \frac{3H\widehat{V}}{2\omega^3}\left(2\omega - \widehat{V}\right)\frac{1}{1_c} & -\frac{1}{1_c} \\ 0 & \frac{1}{4B^2 l_c} & -\frac{\curlyvee^0}{8B^2\sqrt{\widehat{\Delta}_p l_c}} \end{pmatrix} \tag{6.23}$$

Form (6.23), the linearization of (6.19)-6.21) has one zero eigenvalue when$\frac{3H\widehat{V}}{2\omega^3}\left(2\omega - \widehat{V}\right) = \frac{\mu}{2a}$. This implies a stationary bifurcation may occur from the equilibrium point $\hat{x}$ for some value of $\curlyvee o$. The bifurcation calculations of the preceding subsection will now be applied to derive the conditions for existence and stability of such a bifurcation.

Let $\hat{x}$ be the equilibrium point at which $\frac{3H\widehat{V}}{2\omega^3}\left(2\omega - \widehat{V}\right) = \frac{\mu}{2a} for\, \gamma = \gamma^0$. Taking the Taylor series expansion of (6.19)-(6.21) at the point $(\hat{x}, \gamma^0)$, we have

$$\frac{dX}{dt} = L_0 X + Q_0(X, X) + C_0(X, X, X) + (\gamma - \gamma^0) L_1 X + \cdots \tag{6.24}$$

where $L_0\, is$ as in (6.23) and

$$Q_0(X, X) = \begin{pmatrix} \frac{a}{ma+1}\frac{3H}{\omega^3}\left(\omega - \widehat{V}\right)x_1 x_2 \\ \frac{3H}{4\omega^3}\left(\omega - \widehat{V}\right)(x_1^2 + 2x_2^2)\frac{1}{l_c} \\ \frac{\gamma^0}{32B^2 l_c \widehat{\Delta_p}\sqrt{\widehat{\Delta}_p}} x_3^2 \end{pmatrix} \tag{6.25}$$

$$C_0(X,X,X) = \begin{pmatrix} -\frac{a}{ma+1}\frac{3H}{8\omega^3}(x_1^3 + 4x_1x_2^2) \\ -\frac{3H}{\omega^3}\left(\frac{1}{4}x_1^2x^2 + \frac{1}{6}x_2^3\right)\frac{1}{l_c} \\ -\frac{\gamma^0}{64B^2 l_c \widehat{\Delta_p^2}\sqrt{\hat{\Delta}_p}}x_3^3 \end{pmatrix}, \tag{6.26}$$

$$L_1 = \begin{pmatrix} \frac{a}{ma+1}\frac{3H}{\omega^3}(\omega - \hat{V})\frac{\partial\hat{V}}{\partial\gamma} & 0 & 0 \\ 0 & \frac{1}{l_c}\frac{3H}{\omega^3}(\omega - \hat{V})\frac{\partial\hat{V}}{\partial\gamma} & 0 \\ 0 & 0 & -\frac{1}{8B^2\sqrt{\hat{\Delta}_p l_c}}\left[1 - \frac{\gamma^0}{2\hat{\Delta}_p}\frac{\partial\widehat{\Delta_p}}{\partial\gamma}\right] \end{pmatrix} \tag{6.27}$$

Choose $l = (1,0,0) and\ r = l^T\ as$ The left and right eigenvectors corresponding to the zero eigenvalue of L_0. The dynamical behavior of (6.19)-6.21_ with respect to variation of γ near the unstalled point $\hat{x}$ is obtained as folows. The transversality condition $lL_1r \neq 0$ is obtained as

$$lL_1r = \frac{a}{ma+1}\frac{3H}{\omega^3}(\omega - \hat{V})\frac{\partial\hat{V}}{\partial\gamma} \neq 0, \tag{6.28}$$

and the bifurcation stability coefficients are calcuolated, using Eqs. (6.13) and (6.14), as

$$\beta_1 = 1Q_0(r,r) = 0 \tag{6.29}$$

$$\beta_2 = 2l\{2Q_0(r,x_2) + C_0(r,r,r)\}$$

$$= \frac{a}{ma+1}\frac{3H}{4\omega^3}\left(8(\omega - \hat{V})\varsigma - 1\right) \tag{6.30}$$

Where

$$\varsigma = \frac{-\frac{3H}{4\omega^2}+\frac{3H\hat{V}}{4\omega^3}}{\frac{\vartheta}{2a}-\frac{2\sqrt{\hat{\Delta}_p}}{\gamma^0}} \tag{6.31}$$

$$= \frac{3H(\hat{V}-w)\gamma^0}{6H\hat{V}(2\omega-\hat{V})\gamma^0-8\omega^3\sqrt{\hat{\Delta}_p}} \tag{6.32}$$

Note that at $\gamma = \gamma^0$

$$= \frac{3H\hat{V}}{2\omega^3} - 2\omega - \hat{V}) = \frac{\mu}{2a} \tag{6.33}$$

The next result follows readily from Lemma 6.1 and the discussions above.

Theorem 6.3 (Stability) Suppose the quantily lL_1r given in (6.28) is nonzero as is the stability coeffcient β_2 given in (6.30). Then system (6.19)-(6.21) exhibits a pitchfork-type stationary bifuracation withrespect to the small variation of γ at the point $(\hat{x}, \gamma^0)$, where Eq.(6.33) holds. Moreover, if $\beta_2 < 0$ (resp. $\beta_2 > 0$) the local bifurcated solutions near $\hat{x}$ will be asymptotically stable (resp. unstable).

The uniform-flow equilibrium point becomes unstable after the parameter γ crosses the critical value γ^0, the stall bifurcation point. Moreover, according to Theorem 6.3 the local bifurcated solutions, near the stall point, may not be stable. If such a condition occurs, the compression system will exhibit a jump from the stable nominal equilibrium when the parameter γ crosses the critical value γ^0. Also the subcritical nature of the stall bifurcation along with secondary limit-point bifurcations leads to operating conditions featuring multistability: conditions where the locally stable uniform flow solution coexists with a locally stable fully developed stall cell. This results in a hysteresis loop of the stable equilibria with respect to the parameter γ near the stall point. Practically this means that the system will jump from the uniform-flow operationg point to a fully developed stall cell under perturbations in the range of multistability.

Solutions outside the range of validity of the local analysis. The results are shown in Fig. 6.2. In Fig. 6.2, a solid curve represents a locus of locally asymptotically stable equilibrium points, while dashed curves correspond to unstable branches. There are two pitchfork stationary bifurcations, one stable and the other unstable. The numerical analysis vividly demonstrates the large magnitude of the jump resulting from the hysteresis loop associated with the unstbale bifurcation point. It is not possible to observe the pitchfork bifurcation phenomena in Figs. 6.2 (b), 6.2 (c), and 6.2 (d) since the stalled -flow solutions $(A_1, \hat{V}, \hat{\Delta}_p)$ Possess the symmetry $\hat{V}(A_1) = \hat{V}(-A_1)$ and $\hat{\Delta}\,p(A_1) = \hat{\Delta}\,p(-A_1)$.

Numerical bifurcation analysis was carried out to verify the predicted compression system behavior and to study the bifurcation behavior of the stalled-flow

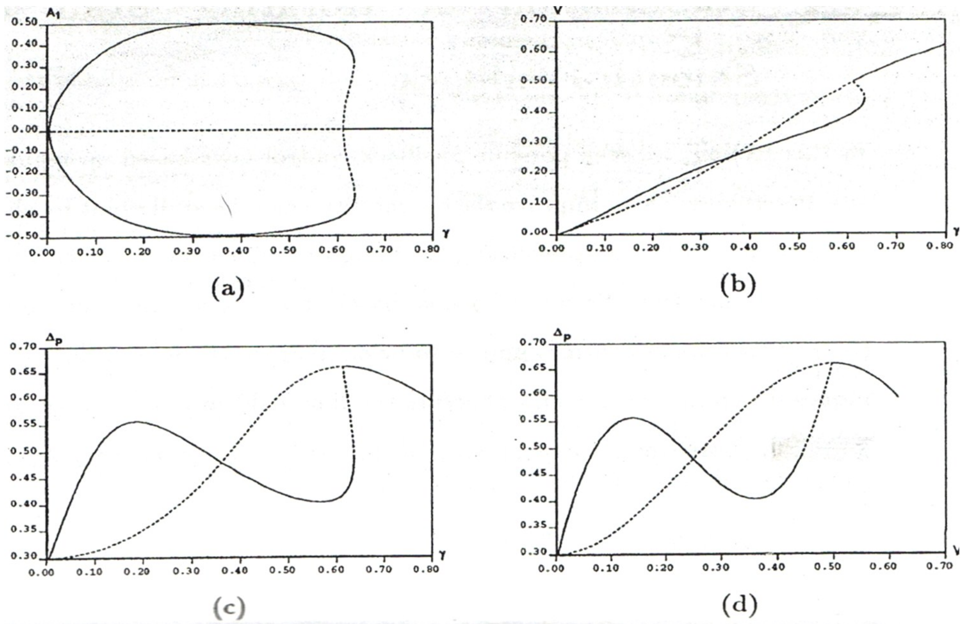


Figure 6.2: Subcritical pitchfork stationary bifurcations in the open loop axial flow compression system

4. STABILIZATION OF ROTATIONG STALL USING SMOOTH FEEDBACK

In this section, we seek possible feedback control laws based on regulating the throttle setting which improve the operability near the stall point by eliminating the hysteresis loop and preventing the jump behavior associated with the unstable stall bifurcation. So $\gamma = \gamma^0 + \mu$ is substituted into the plenum mass balance (6.3). This type of control appears to be simpler to implement than those techniques depending on directly affecting the flow field in the compress inlet duct. Moreover, successful experimental results have been reported.

4.1 Low-Order Model

Denote $(\hat{x}, \gamma^0)$ as the stall point and let $\gamma{:=}\gamma^0 + u$, where u is the control imput. We then can rewrite systerm (6.19)-(6.21) as a throttle control system given by

$$\frac{ma+n}{na}\frac{dA_n}{dr} = \left[\frac{3HV}{2\omega^2}(2\omega - V) - \frac{\mu n^2}{2a}\right] A_n - \frac{3H}{8\omega^3}A_n^3, \tag{6.34}$$

$$l_c\frac{dV}{dr} = -\Delta_p + f_0 + \frac{HV^2}{2\omega^2}(3\omega - V) + \frac{3H}{4\omega^3}(\omega - V)A_n^2, \tag{6.35}$$

$$l_c\frac{d\Delta_p}{dr} = \frac{1}{4B^2}\left[V - (\gamma^0 + u)\sqrt{\Delta_p}\right]. \tag{6.36}$$

We observed from (6.34) that $A_1 = 0$ is an invariant submanifold of (6.34)-(6.36) is uncontrollable. Moreover, it is not difficult to check that system (6.34)-(6.36) unaffected by the value of control input u, which implies that (6.34)-(6.36) possesses an uncontrollable zero eigenvalue at the stall point. This means that we can not extend the range γ where the uniform-Flow solution is locally asymptotically stable with any type of state feedback in (6.34)-(6.36).

Next, we consider the design of a control law which guarantees stability of the local bifurcated solutions near the stall point. From the property of the exchange of stability, the system will not jump from the stable uniform-flow equilibrium near the stall point, if the local bifurcated solutions are stable. In the following, for simplicity, we consider the control input u to be a purely nonlinear feedback of the state variations as given by

$$u = k_1x_1^2 + k_3x_1x_2 + k_3x_1x_3 + k_4x_2^2 + k_5x_2x_3 + k_6x_3^2 + U(x_1x_2, x_3) \tag{6.37}$$

Where U is a high order function. Using formulae (6.13) and (6.14) to calculate the stability coefficients β_1^* and β_2^* for the controlled model (6.34)-(6.36) at $\hat{x}$, we obtain

$$\beta_1^* = \beta_1 = 0 \tag{6.38}$$

$$\beta_2^* = \beta_2 + 4\frac{a}{ma+1}\frac{3H}{\omega^3}\left(\omega - \hat{V}\right)\frac{K_1}{2}\frac{2\hat{\Delta}_p}{\frac{\mu\gamma^0}{2a}} - 2\sqrt{\hat{\Delta}_p}$$

$$= \beta_2 + Ck_1 \tag{6.39}$$

Where β_1 and β_2 are the stability coefficients of the uncontrolled version of (6.34)-(6.36). It is observed from the expression of β_2^* that only the quadratic feedback $k_1A_1^2$ contributes to the determination of system stability. we have the following result

Theorem 6.4 Stabilization) The stationary bifurcation of (6.34)-(6.36) at the point $(\hat{x}, \gamma^0)$ can be guaranteed to be a supercritical pitchfork bifurcation by a purely quadratic feedback control if C≠0. The feedback control is of the form $u = k_1 A_1^2$.

The numerical results for the controlled case are given in Fig. 6.3. By theorem 6.4, we find that purely quadratic feedback control laws will stabilize bifurcation-

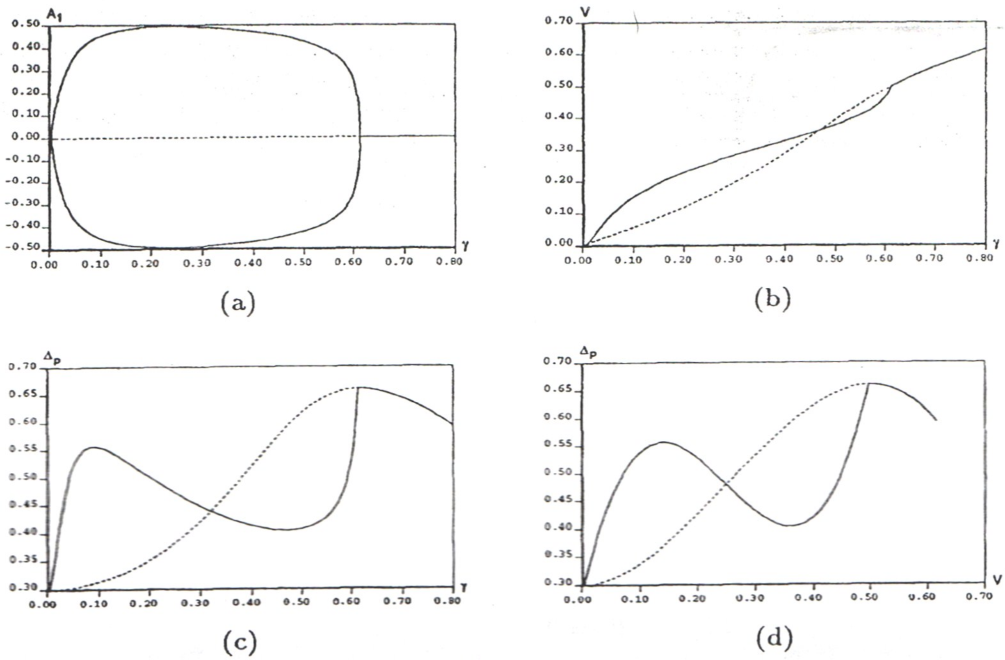


Figure 6.3: Supercritical pitchfork bifurcations in the closed-loop axial flow compression system

cated equilibrium solutions. The control input is given by

$$u = k_1 A_1^2 \tag{6.40}$$

with k_1=0.5. It is not difficult to see that the hysteresis loops of the stable system equilibria shown in Fig. 6.2 no longer exist in Fig. 6.3.

4.2 High-Order Models

We have designed the control law based on a highly truncated discretization of the flow field perturbation. To test the controller in a more realistic manner, the gas axial velocity perturbation u is approximated by

$$u = \sum_{n=1}^{N} \exp(n\eta)\, [a_n \cos(n^{\theta}) + b_n\, sing(\, n^{\theta})] \tag{6.41}$$

and, as before, the axial velocity profile at the inlet guide vanes is denoted $u_0(\, at\, \eta = 0)$. Substituting the Fourier expansion into the local momentum balance PDE and using Galerkin's method to determine the amplitude coefficients, we obtain

$$\left\{\frac{m}{n} + \frac{1}{a}\right\} \dot{a}_n = \frac{1}{\pi} \int_0^{2\pi} f \cos(n\theta)\, d\theta - \frac{\mu n^2}{2a} a_n - \frac{n}{2a}\, b_n \tag{6.42}$$

$$\left\{\frac{m}{n} + \frac{1}{a}\right\} \dot{b}_n = \frac{1}{\pi} \int_0^{2\pi} f\, sin(n\theta)\, d\theta - \frac{\mu n^2}{2a} b_n + \frac{n}{2a}\, a_n \tag{6.43}$$

Along with the ODEs

$$l_c \frac{dV}{dr} = -\Delta_p + \frac{1}{2\pi} \int_0^{2\pi} f\, d\theta \tag{6.44}$$

$$l_c \frac{dV}{dr} = \frac{1}{4B^2} \left[V(r) - F^{-1}(\Delta_\mathrm{p})\right] \tag{6.45}$$

The only nonlinearities in the ODEs above are the throttle and compressor performance characteristics. One of the advantages of using the cubic compressor characteristic is that the integrals can be evaluated explicitly. These results are discussed in detail by Adomaitis and are given in Appendix 6.A. Thus, we obtain a large set of ordinary differential equations in time describing the dynamics of the Fourier mode amplitude coefficients.

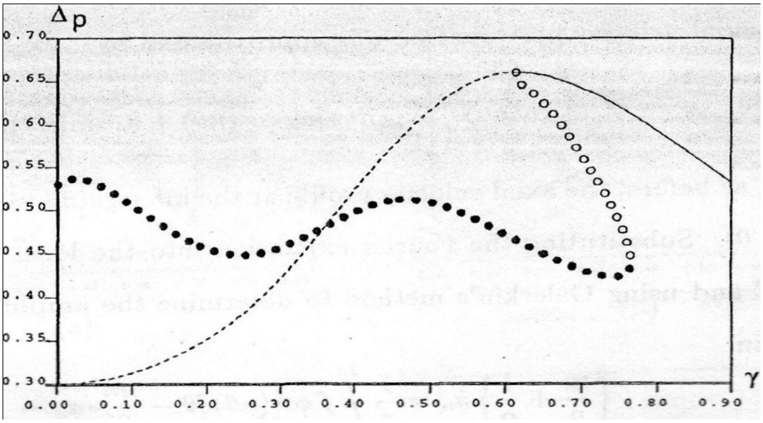


Figure 6.4: Bifurcation diagram of open loop high-order system

Denoting γ^0 as the nominal stall point of the throttle opening parameter and letting $\gamma := \gamma^0 + u$, where u is the control input, we consider feedback of only the amplitude of the first Fourier mode of the flow disturbance in the controller:

$$u = k(a_1^2 + b_1^2). \tag{6.46}$$

Using the same numerical bifurcation analysis techniques discussed in the previous section and extensive simulation studies, we find controller (6.46) is effective in eliminating the hysteresis loop in the vicinity of stall point for high-order discretizations (large N in (6.41)). Thus the occurrences of jump behavior of the stable system equilibria are prevented.

For the result reported in this work, we take N=2. See [14] for representative results on N=6. Figure 6.4 is a bifurcation diagram of the open-loop system. Note that the stall bifurcation is a subcritical Hopf bifurcation. The bifurcated solution becomes stable through a cyclic fold bifurcation, resulting a hysteresis loop near the stall point. Since the Hopf bifurcation is linearly uncontrollable, results, from section 3.3.2, especially Theorem 3.2 show that only quadratic terms in the feedback control can influence the value of stability coefficient β_2. Thus control function (6.46), is a logical choice.

To see the effect of the nonlinear controller (6.46), bifurcation analysis of the high-order discretization under control is carried out. In Fig.6.5 a bifurcation

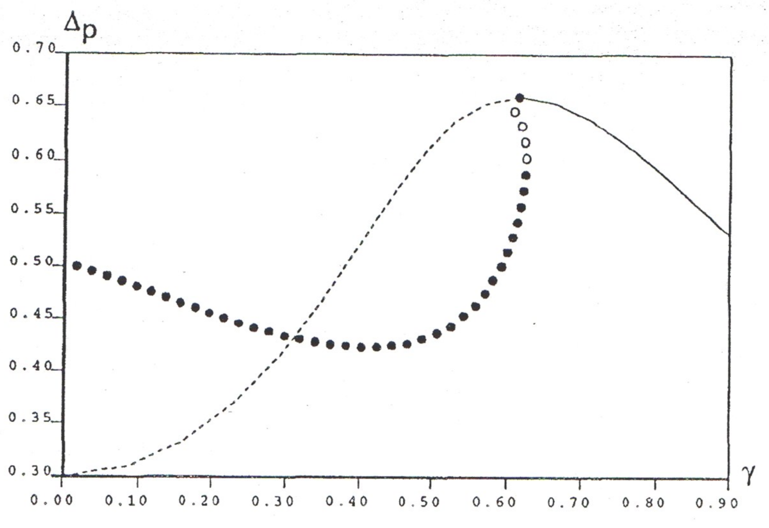


Figure 6.5: Bifurcation diagram of closed-loop high-order system (k=2.5)

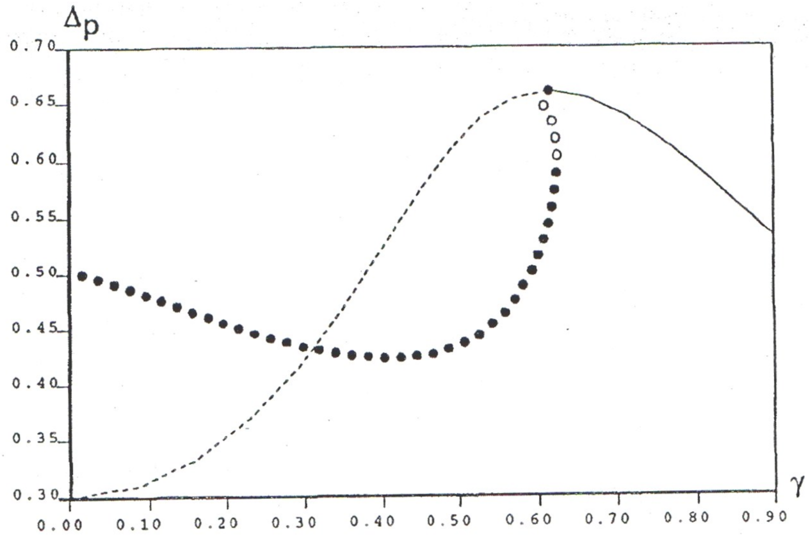


Figure 6.6: Bifurcation diagram of closed-loop high-order system (k=5.0)

Diagram of the closed-loop system with k=2.5 is shown. Locally the stall bifurcation is now a supercritical Hopf bifurcation. However, the bifurcating stalled flow solution becomes unstable giving rise to a hysteresis loop. Even with increased controller gain (fig. 6.6 k=5), the hysteresis loop still persists (not discernible from Fig. 6.6, but can be seen if the region near the stall point is magnified) but with a reduced range in parameter space. While hysteresis is not completely eliminated in this case, the stability of the system to finite sized perturbation in the axial velocity profile is still improved and the range of the hysteresis is also reduced under this control.

5. STABILIZATION OF ROTATING STALL USING NON-SMOOTH FEEDBACK

As shown in the last section, the hysteresis can not be completely eliminated by the controller (6.46) with even a rather large gain. So far in this dissertation, we have only considered smooth feedback, Our theory results in a quadratic controller. The topological characteristic of the controller of the form $u = kx^2$ is (See Fig. 6.7)

- Symmetric in x
- Monotone in $| x |$.

Based on this observation, we conjecture that any controller having these two properties at least locally around x=0 would locally stabilize the stall bifurcation.

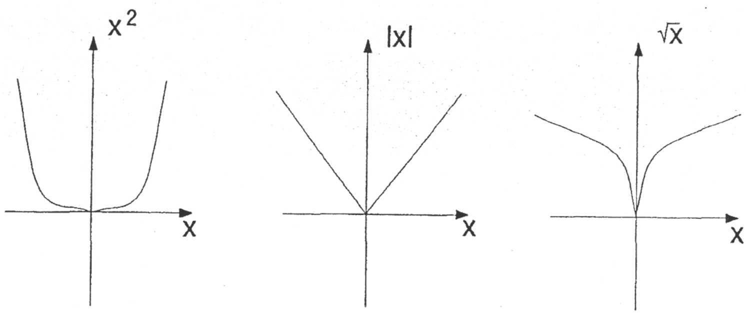


Figure 6.7: Qualitative characteristic of control functions

5.1 Low-Order Model

The conjecture above is supported by the bifurcation analysis of low-order system under the following nonsmooth feedback control:

$$u = k|A_1| \tag{6.47}$$

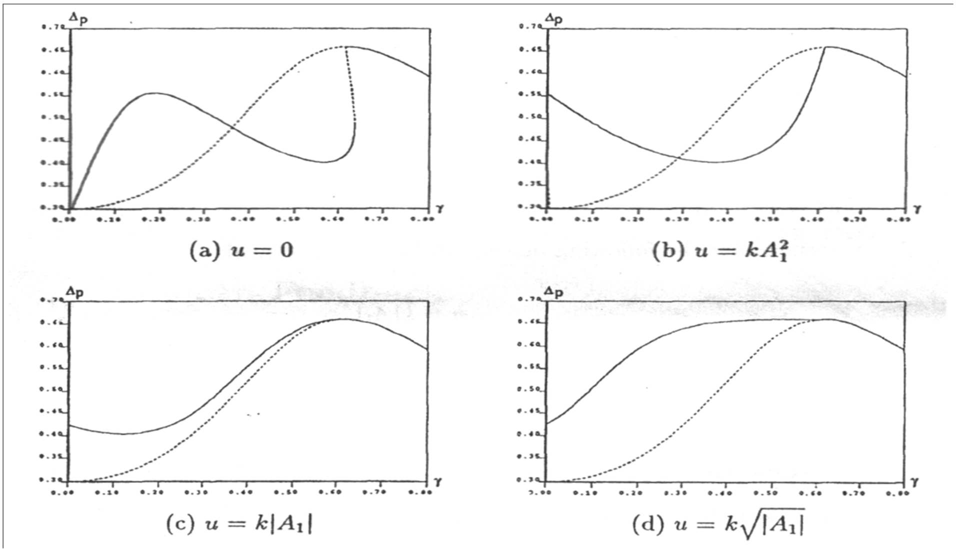
and

$$u = k\sqrt{|A_1|} \tag{6.48}$$

Figure 6.8 illustrates the effect of such nonsmooth controllers. It can be clearly seen that the use of such nonsmooth controllers has significant advantages. With less control energy, the nonsmooth controllers effectively eliminate the hysteresis loop. More significantly, the pressure rise can be kept at a high level beyond the stall point.

We remark that the two controllers above are just a subset of a family of controllers that will show such advantages over the smooth feedback design. For example any controller of the form belongs to such a family. Next we consider the use of nonsmooth feedback in the high-order discretization models.

$$u = k|A_1|^{\beta_n/\beta_d} \; \beta_n \leq \beta_d \tag{6.49}$$



(a) $u = 0$ (b) $u = kA_1^2$ (c) $u = k|A_1|$ (d) $u = k\sqrt{|A_1|}$

Figure 6.8: Bifurcation diagrams in low-order model:
smooth vs. nonsmooth feedback (k=1.0)

5.2 High-Order Models

Two representative controllers are applied to the high-order model:

$$u = k\sqrt{a_1^2 + b_1^2} \tag{6.50}$$

and

$$u = k(a_1^2 + b_1^2)^{\frac{1}{4}} \tag{6.51}$$

Figure 6.9 the illustrates the effect of the two controllers above vs. the open-loop case and that of smooth feedback. The nonsmooth controllers can completely eliminate the hysteresis loop with less control energy than smooth feedback. Also the pressure rise is kept at a high level beyond the stall point.

This brings up the natural question, presently under investigation, of the bifurcation mechanisms associated with nons-mooth systems. General theory no bifurcation of non-smooth systems is in general an open area. Some case studies [76] exist in the literature. In terms of controllers (6.48) and (6.51) etc., The common feature is that the derivatives of the control function at the origin are infinity (see Fig. 6.7 and 6.10). This indicates "infinite local stabilizing power." In this regard, there is a strong resemblance to the terminal attractor theory. In the case of controllers (6.47) and (6.50), though the local stabilizing power is not infinite, it is still larger than those of controllers (6.40) and (6.46). The use of non-smooth feedback in bifurcation control theory or even the general control theory is a worthy new avenue.

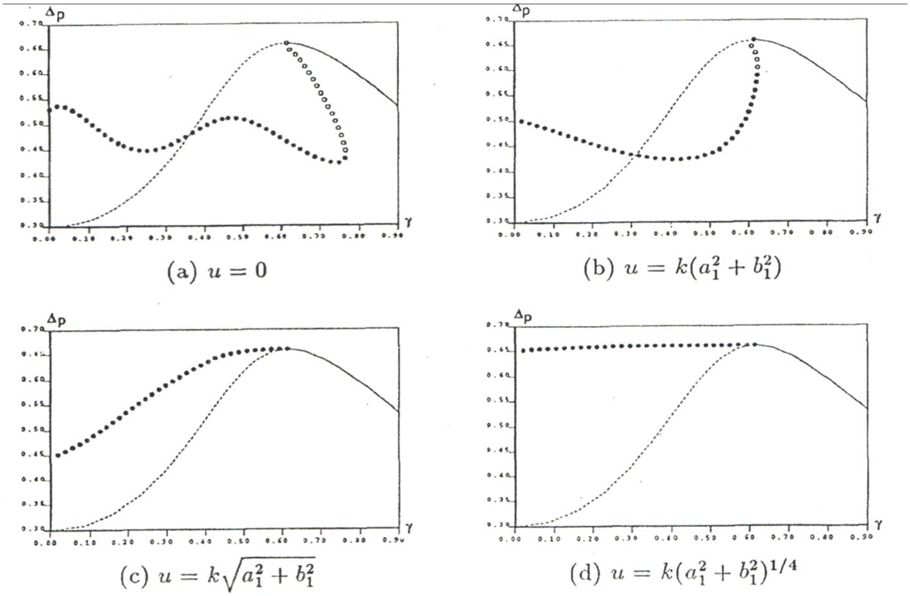


Figure 6.9: Bifurcation diagrams in high-order model (N=2): smooth vs. nonsmooth feedback (k=2.5)

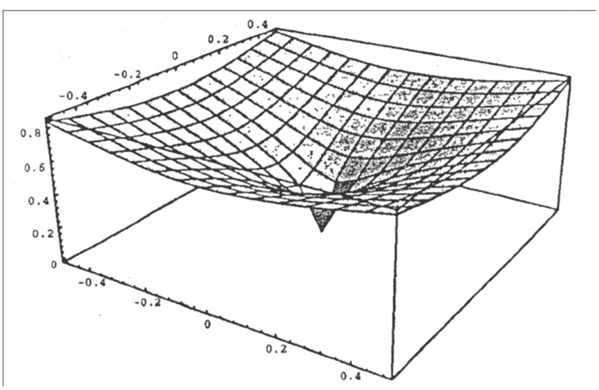


Figure 6.10: Plot of f(x,y)=$(x^2 + y^2)^{\frac{1}{4}}$

Controllers such as (6.46), (6.47), (6.50) have been successful implemented in experiments.

6. CONCLUDING REMARKS

Investigated the design of stabilizing feedback control laws for rotating stall in axial flow gas compressors. The proposed approach begins with recognizing the importance of local bifurcations in determining the nature of post-instability behavior of axial flow compression systems. The controllers are (analytically) designed based on a highly-truncated discretization of a compressor model, and then are applied to borty the low-order model and high-order discretizations. Although the results are obtained using a particular dynamical model, the bifurcation based control approach appears to be a viable technique for control of these systems.

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