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REFLECTION AND TRANSMISSION OF *P*-WAVES AT AN INTERFACE OF TWO MICRO-ISOTROPIC, MICRO-ELASTIC SOLID HALF-SPACES

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Abstract: In this paper, we investigated the reflection and transmission of *P*-waves at an interface of two micro-isotropic, micro-elastic solid half-spaces. We obtained the ratios of amplitudes of reflected *P*-wave, refracted *P*-wave, reflected *SV*-wave, and refracted *SV*-wave to incident *P*-wave. These amplitude ratios are shown graphically against the various incident angles. Further, comparative amplitude ratios of micro-isotropic, micro-elastic and classical cases are shown graphically.

Keywords: Reflection and Transmission of *P*-wave, Micro-isotropic, Micro-elastic solid half-spaces, Amplitude ratios.

1. INTRODUCTION

Eringen and Suhubi [3, 4] developed a theory of micromorphic materials and it is the generalization of the classical theory of elasticity. Koh [7] simplifies this theory and named it as micro-isotropic, micro-elastic materials. It consists of three macro-displacement components, three micro-rotation components and six components of micro-deformation and are all independent. Further, the number of elastic constants in it is ten. In the classical theory of elasticity, we assume the matter is continuous and the density is constant. But the experimental results given in the book by Eringen [2] reveal that the density may vary in a volume that is less than a critical volume. Thus, the classical theory of elasticity is inadequate to describe the behavior of such materials. This lead to develop the theory of micromorphic materials, which includes the micro-structure of the materials. The reflection and refraction of plane waves studied by Knott [8], Jeffreys [5, 6]. Tomar and Garg [12] and Singh and Kumar [10] discussed some problems on reflection and transmission of waves from a plane interface between two micropolar solid half-spaces.

In the book by Achenbach [1], the problem of reflection and refraction of *P*-waves was discussed in classical theory of elasticity. In the present paper, an attempt is made to study the same in micro-isotropic, micro-elastic medium. We obtained the amplitude ratios and they are computed for various angles of incidence by assuming certain values to non-dimensional quantities and they are shown graphically. Allowing the elastic constants expect λ and μ tend to zero the classical result is obtained [1].

2. BASIC EQUATIONS

The equations of motion and the constitute equations of micro-isotropic, micro-elastic solid under the absence of body forces and body couples are given by Parameshwaran and Koh [9] The stress, couple-stress and stress moment are as follows.

$$t_{(km)} = A_1 e_{pp} \delta_{km} + 2A_2 e_{km} \tag{1}$$

$$t_{[km]} = \sigma_{[km]} = 2A_3 \varepsilon_{pkm} (r_p + \phi_p) \tag{2}$$

$$\sigma_{(km)} = -A_4 \phi_{pp} \delta_{km} - 2A_5 \phi_{(km)} \tag{3}$$

$$t_{k(mn)} = B_1 \phi_{pp,k} \delta_{mn} + 2B_2 \phi_{(m,n),k} \tag{4}$$

$$m_{kl} = -2(B_3 \phi_{l,k} + B_4 \phi_{k,l} + B_5 \phi_{p,p} \delta_{kl}) \tag{5}$$

where

$$\begin{aligned} &A_1 = \lambda + \sigma_1, && B_1 = \tau_3, \\ &A_2 = \mu + \sigma_2, && 2B_2 = \tau_7 + \tau_{10}, \\ &A_3 = \sigma_5, && B_3 = 2\tau_4 + 2\tau_9 + \tau_7 - \tau_{10}, \\ &A_4 = -\sigma_1, && B_4 = -2\tau_4, \\ &A_5 = -\sigma_2, && B_5 = -2\tau_9. \end{aligned} \tag{6}$$

subject to the conditions

$$\begin{aligned} &3A_1 + 2A_2 > 0, \quad A_2 > 0, \quad A_3 > 0, \\ &3A_4 + 2A_5 > 0, \quad A_5 > 0, \\ &2B_1 + 2B_2 > 0, \quad B_2 > 0, \\ &B_5 > 0, \quad -B_2 < B_4 < B_2, \quad B_3 + B_4 + B_5 > 0. \end{aligned} \tag{7}$$

The displacement equations of motion are

$$(A_1 + A_2 - A_3) u_{p,pm} + (A_2 + A_3) u_{m,pp} + 2A_3 \varepsilon_{pkm} \phi_{p,k} = \rho \frac{\partial^2 u_m}{\partial t^2}, \tag{8}$$

$$2B_3 \phi_{p,mm} + 2(B_4 + B_5) \phi_{m,mp} - 4A_3 (r_p + \phi_p) = \rho j \frac{\partial^2 \phi_p}{\partial t^2}, \tag{9}$$

$$B_1 \phi_{pp,kk} \delta_{ij} + 2B_2 \phi_{(ij),kk} - A_4 \phi_{pp} \delta_{ij} - 2A_5 \phi_{ij} = \frac{1}{2} \rho j \frac{\partial^2 \phi_{(ij)}}{\partial t^2}, \tag{10}$$

where ρ is the average mass density, j is the micro-inertia. The macro displacement in the micro elastic continuum is denoted by u_k and the micro deformation by ϕ_{mn} for the linear theory, we have the macro-strain $e_{km} = e_{(k,m)}$, the macro rotation vector $r_k = \frac{1}{2}\varepsilon_{kmn}u_{n,m}$, the micro-strain $\phi_{(m,n)}$ and micro-rotation $\phi_p = \frac{1}{2}\varepsilon_{pkm}\phi_{km}$. The stress measures are the asymmetric stress (macro-stress) t_{mn}, the relative stress (micro-stress) σ_{km} and the stress moment t_{kmn} and also the couple stress tensor $m_{kp} = \varepsilon_{pnm}t_{kmn}$. The symbol ( ) appeared in suffix of a quantity indicate that the quantity is symmetric and [ ] shows the quantity is skew-symmetric. $\lambda, \mu, \sigma_1, \sigma_2, \sigma_5, \tau_3, \tau_4, \tau_7, \tau_9$ and τ_{10} are the ten elastic moduli. Further, ε_{pkm} is the permutation symbol and δ_{km} is the Kronecker delta.

The velocity of dilatation and distortion waves in micro-isotropic, micro-elastic medium are respectively given by Sree Lakshmi and Sambaiah [11]

$$C_L^2 = \frac{A_1 + 2A_2}{\rho} \tag{11}$$

and

$$C_T^2 = \frac{A_2 + A_3}{\rho} \tag{12}$$

3. PROBLEM FORMULATION AND ITS SOLUTION

We consider the two micro-isotropic, micro-elastic half-spaces namely medium-*a* and medium-*b* having contact at $x_2 = 0$ and the positive direction of x_2-axis is inside the medium-*b*. We suppose the motion of the *P*-wave in (x_1, x_2)-plane. In general, it should be anticipated

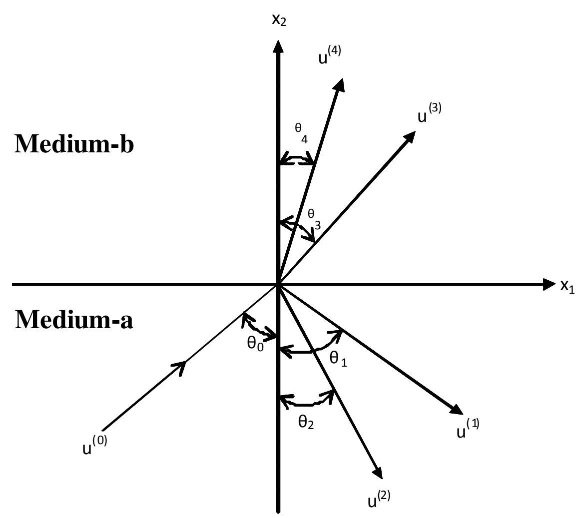

Figure 1

that P-wave incident on the surface $x_2 = 0$ of two micro-isotropic, micro-elastic solids will give rise to reflection and refraction of both P-waves and SV-waves. The complete geometry of the problem is shown in the Fig. 1.

The vectors of incident P-wave, reflected P-wave, reflected SV-wave, refracted P-wave and refracted SV-wave are respectively given by $(u_1^{(0)}, u_2^{(0)}, 0), (u_1^{(1)}, u_2^{(1)}, 0), (u_1^{(2)}, u_2^{(2)}, 0)$, $(u_1^{(3)}, u_2^{(3)}, 0)$ and $(u_1^{(4)}, u_2^{(4)}, 0)$ where $u_i^{(j)} (i = 1, 2,\ j = 0, 1, 2, 3, 4)$ are functions of x_1, x_2 and t. The displacement components with super suffix (0), (1) and (2) are pertaining to the medium-a and that of (3) and (4) refer to the medium-b.

The components of these vectors are given by

$$u_1^{(0)} = a_0 \sin\theta_0 \exp\left[ik_0\left(x_1 \sin\theta_0 + x_2 \cos\theta_0 - C_L^a t\right)\right]$$

$$u_2^{(0)} = a_0 \cos\theta_0 \exp\left[ik_0\left(x_1 \sin\theta_0 + x_2 \cos\theta_0 - C_L^a t\right)\right]$$

$$u_1^{(1)} = a_1 \sin\theta_1 \exp\left[ik_1\left(x_1 \sin\theta_1 - x_2 \cos\theta_1 - C_L^a t\right)\right]$$

$$u_2^{(1)} = -a_1 \cos\theta_1 \exp\left[ik_1\left(x_1 \sin\theta_1 - x_2 \cos\theta_1 - C_L^a t\right)\right]$$

$$u_1^{(2)} = a_2 \cos\theta_2 \exp\left[ik_2\left(x_1 \sin\theta_2 - x_2 \cos\theta_2 - C_T^a t\right)\right]$$

$$u_2^{(2)} = a_2 \sin\theta_2 \exp\left[ik_2\left(x_1 \sin\theta_2 - x_2 \cos\theta_2 - C_T^a t\right)\right]$$

$$u_1^{(3)} = a_3 \sin\theta_3 \exp\left[ik_3\left(x_1 \sin\theta_3 + x_2 \cos\theta_3 - C_L^b t\right)\right]$$

$$u_2^{(3)} = a_3 \cos\theta_3 \exp\left[ik_3\left(x_1 \sin\theta_3 + x_2 \cos\theta_3 - C_L^b t\right)\right]$$

$$u_1^{(4)} = -a_4 \cos\theta_4 \exp\left[ik_4\left(x_1 \sin\theta_4 + x_2 \cos\theta_4 - C_T^b t\right)\right]$$

$$u_2^{(4)} = a_4 \sin\theta_4 \exp\left[ik_4\left(x_1 \sin\theta_4 + x_2 \cos\theta_4 - C_T^b t\right)\right] \tag{13}$$

where $a_i (i = 0, 1, 2, 3, 4)$ are amplitude of respective waves, $\theta_i (i = 0, 1, 2, 3, 4)$ are the angles of incident P-wave, reflected P-wave, reflected SV-wave, refracted P-wave, refracted SV-wave, $i = \sqrt{-1}$, $k_i (i = 0, 1, 2, 3, 4)$ are wave numbers, C_T^a, C_T^b are velocities of distortion waves in medium-a and medium-b and C_L^a, C_L^b are the velocities of dilatation waves in these mediums.

Further,

$$C_T^a = \sqrt{\frac{A_2^a + A_3^a}{\rho^a}}, \qquad C_T^b = \sqrt{\frac{A_2^b + A_3^b}{\rho^b}},$$
$$C_L^a = \sqrt{\frac{A_1^a + 2A_2^a}{\rho^a}}, \quad C_L^b = \sqrt{\frac{A_1^b + 2A_2^b}{\rho^b}}. \tag{14}$$

The boundary conditions at $x_2 = 0$ for the problem under consideration are given by

$$u_1^{(0)} + u_1^{(1)} + u_1^{(2)} = u_1^{(3)} + u_1^{(4)}$$
$$u_2^{(0)} + u_2^{(1)} + u_2^{(2)} = u_2^{(3)} + u_2^{(4)}$$
$$t_{21}^{(0)} + t_{21}^{(1)} + t_{21}^{(2)} = t_{21}^{(3)} + t_{21}^{(4)}$$
$$t_{22}^{(0)} + t_{22}^{(1)} + t_{22}^{(2)} = t_{22}^{(3)} + t_{22}^{(4)} \tag{15}$$

where

$$t_{21} = \frac{1}{2}\left[(A_2 - A_3)\frac{\partial u_2}{\partial x_1} + (A_2 + A_3)\frac{\partial u_1}{\partial x_2} - 2A_3\phi_3\right]$$
$$t_{22} = (A_1 + 2A_2)\frac{\partial u_2}{\partial x_2} + A_1\frac{\partial u_1}{\partial x_1}.$$

Substituting (13) into (15) we get a system of equations in a_0, a_1, a_2, a_3 and a_4. These equations must be valid for all x_1 and t. Thus, the exponentials must appear as factor of these equations. This reaches to the following conclusions.

$$k_0 \sin\theta_0 = k_1 \sin\theta_1 = k_2 \sin\theta_2 = k_3 \sin\theta_3 = k_4 \sin\theta_4 \tag{16}$$
$$k_0 C_L^a = k_1 C_L^a = k_2 C_T^a = k_3 C_L^b = k_4 C_T^b \tag{17}$$

With the help of (16) and (17) the system of equations obtained in the amplitudes a_0, a_1, a_2, a_3 and a_4 reduces to (18)

$$\begin{bmatrix} -\sin\theta_1 & -\cos\theta_2 & \sin\theta_3 & -\cos\theta_4 \\ \cos\theta_1 & -\sin\theta_2 & \cos\theta_3 & \sin\theta_4 \\ \sin 2\theta_1 & m^a(\cos 2\theta_2 + s^a) & \frac{C_L^a}{C_L^b}\frac{A_2^b}{A_2^a}\sin 2\theta_3 & -\frac{C_L^a}{C_T^b}\left(\frac{A_2^b}{A_2^a}\cos 2\theta_4 + s^b\right) \\ -(m^a)^2\cos 2\theta_2 & m^a \sin 2\theta_2 & \frac{C_L^a}{C_L^b}\frac{A_2^b}{A_2^a}(m^b)^2\cos 2\theta_4 & \frac{C_L^a}{C_T^b}\frac{A_2^b}{A_2^a}\sin 2\theta_4 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} \sin\theta_0 \\ \cos\theta_0 \\ \sin 2\theta_0 \\ (m^a)^2\cos 2\theta_2 \end{bmatrix} a_0 \tag{18}$$

where

$$m^a = \frac{c_L^a}{C_T^a}, \quad m^b = \frac{C_L^b}{C_T^b}, \quad s^a = \frac{A_3^a}{A_2^a}, \quad s^b = \frac{A_3^b}{A_2^a}.$$

4. NUMERICAL CALCULATIONS

We assume the following values for non-dimensional quantities $\frac{\lambda^a}{\mu^a} = 0.3$, $\frac{\sigma_1^a}{\mu^a} = 0.1$, $\frac{\sigma_2^a}{\mu^a} = 0.2$, $\frac{\lambda^b}{\mu^b} = 0.4$, $\frac{\sigma_1^b}{\mu^b} = 0.2$, $\frac{\sigma_2^b}{\mu^b} = 0.3$, $\frac{\mu^b}{\mu^a} = 0.25$, $\frac{\rho^b}{\rho^a} = 1$, $\frac{\sigma_5^a}{\mu^a} = 0.01$ and $\frac{\sigma_5^b}{\mu^b} = 0.02$. The amplitude ratios $\frac{a_1}{a_0}$, $\frac{a_2}{a_0}$, $\frac{a_3}{a_0}$ and $\frac{a_4}{a_0}$ are computed for various angles of incidence with the assumed values of non-dimensional quantities and they are shown graphically in Fig. 2 to Fig. 5.

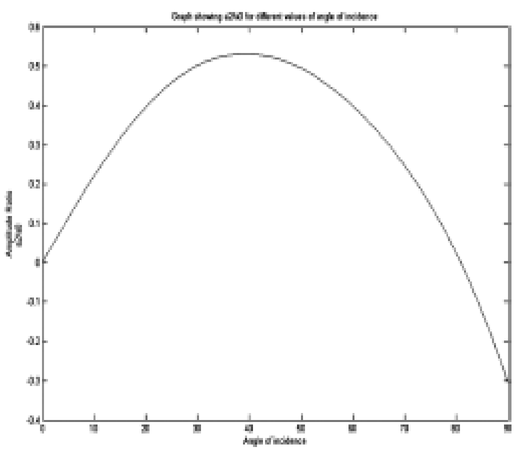

Figure 2

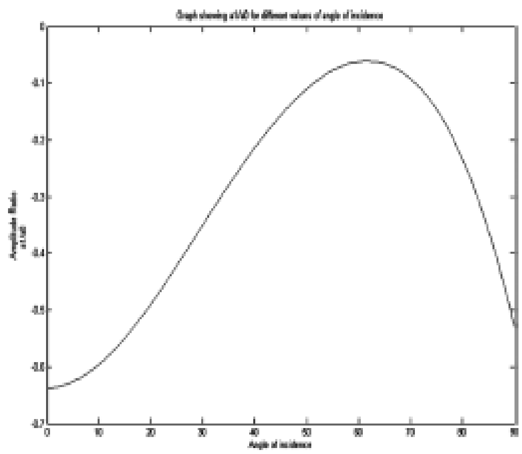

Figure 3

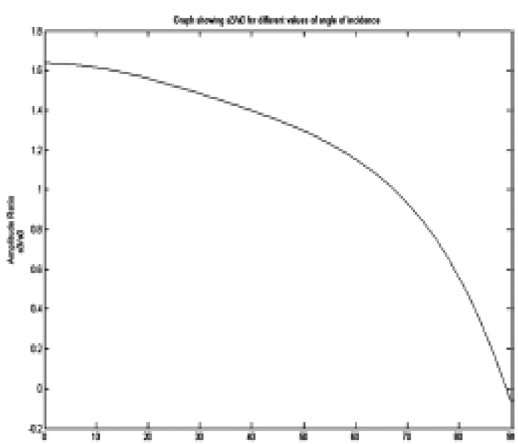

Figure 4

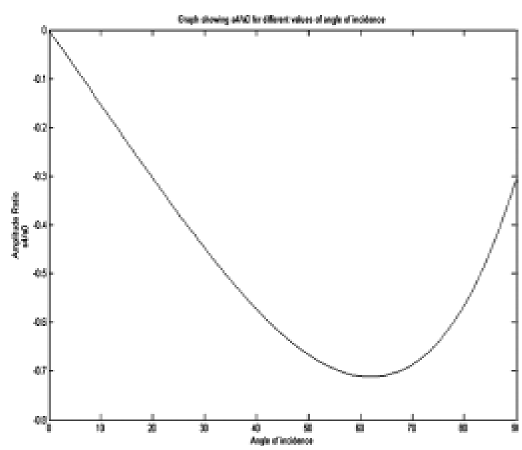

Figure 5

The comparative graphs of amplitude ratio of classical and micro-isotropic, micro-elastic are shown in Fig. 6 to Fig. 9. The amplitude ratio $\frac{a_1}{a_0}$ of classical case is less than micro-isotropic, micro-elastic up to angle 34.5 degrees of incidence and it is reversed for the angle of incidence greater than 34.5 degrees. The amplitude ratios $\frac{a_2}{a_0}, \frac{a_3}{a_0}$ of classical are greater than that of micro-isotropic, micro-elastic material. Whereas for the amplitude ratio $\frac{a_4}{a_0}$ it is reversed.

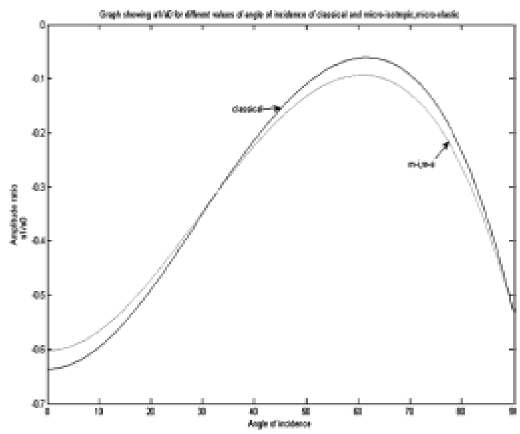

Figure 6

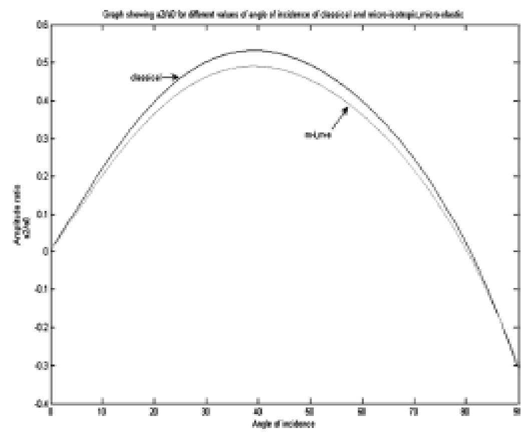

Figure 7

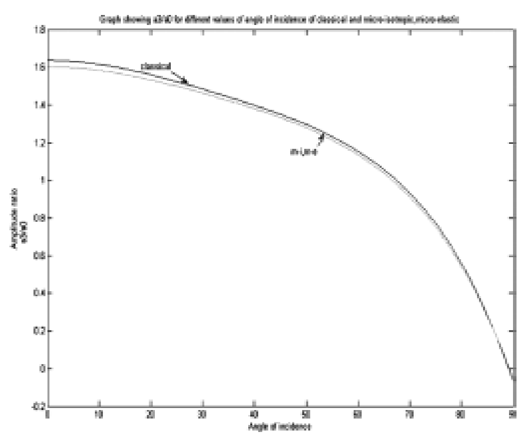

Figure 8

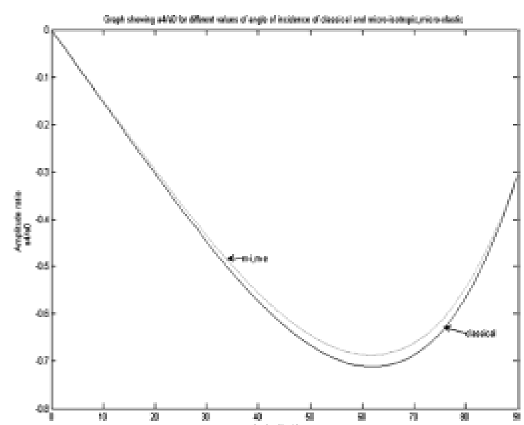

Figure 9

The amplitude ratios $\frac{a_1}{a_0}, \frac{a_2}{a_0}, \frac{a_3}{a_0}$ and $\frac{a_4}{a_0}$ are computed for various angles of incidence with the following three sets of values and shown in Fig. 10 to Fig. 13. It is observed that all the graphs are parabolic.

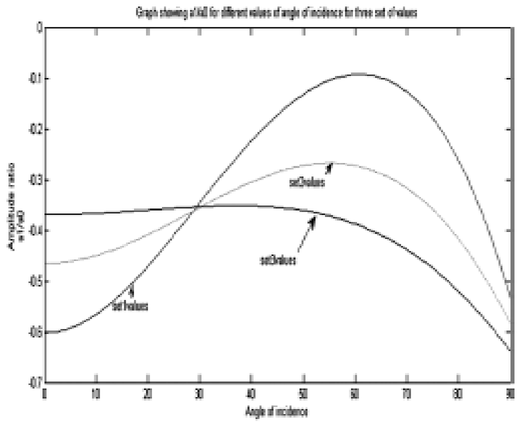

Figure 10

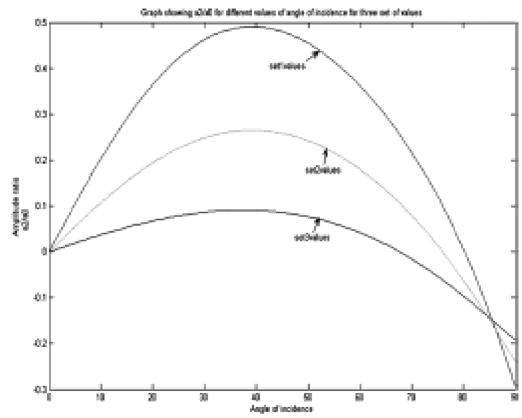

Figure 11

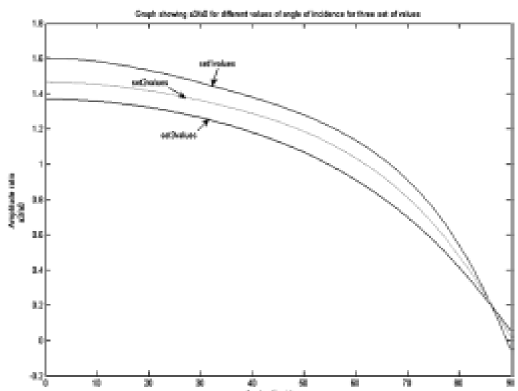

Figure 12

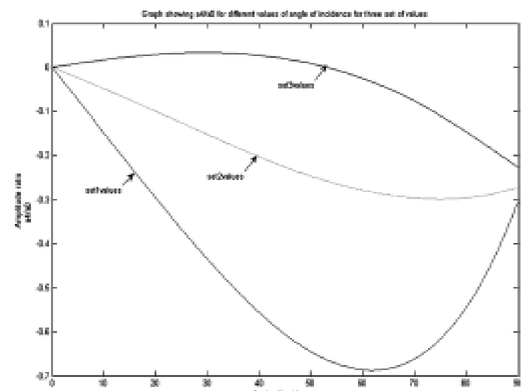

Figure 13

	$\frac{\lambda^a}{\mu^a}$	$\frac{\sigma_1^a}{\mu^a}$	$\frac{\sigma_2^a}{\mu^a}$	$\frac{\lambda^b}{\mu^b}$	$\frac{\sigma_1^b}{\mu^b}$	$\frac{\sigma_2^b}{\mu^b}$	$\frac{\mu^b}{\mu^a}$	$\frac{\rho^b}{\rho^a}$	$\frac{\sigma_5^a}{\mu^a}$	$\frac{\sigma_5^b}{\mu^b}$
Set-I	0.3	0.1	0.2	0.4	0.2	0.3	0.25	1	0.01	0.02
Set-II	0.5	0.15	0.2	0.5	0.25	0.35	0.5	1.2	0.02	0.03
Set-III	0.8	0.2	0.15	0.6	0.3	0.4	0.75	1.4	0.03	0.04

The Fig. 14 to Fig. 17 show the graphs of $\frac{a_1}{a_0}, \frac{a_2}{a_0}, \frac{a_3}{a_0}$ and $\frac{a_4}{a_0}$ for various angles of incidence when the incident medium and reflected medium are interchanged. It is observed that the variation of $\frac{a_1}{a_0}$ is reversed when the angle of incidence is greater than 66.5 degrees. The amplitude ratio $\frac{a_2}{a_0}, \frac{a_3}{a_0}$ and $\frac{a_4}{a_0}$ are greater than the corresponding amplitude ratios when the media are interchanged.

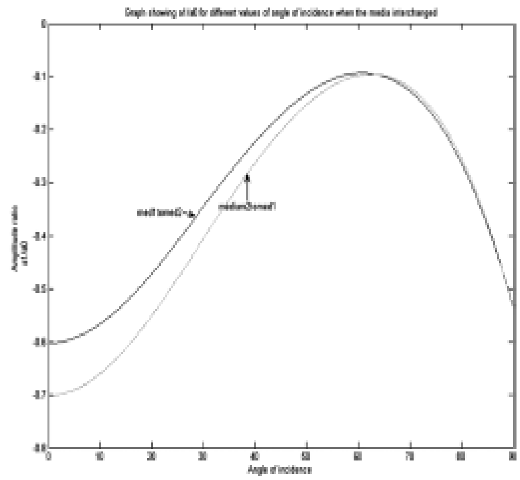

Figure 14

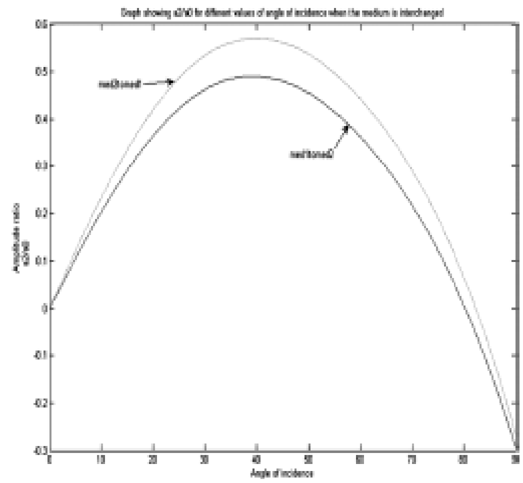

Figure 15

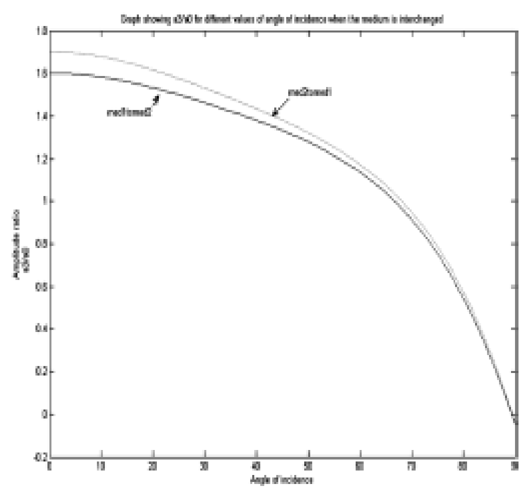

Figure 16

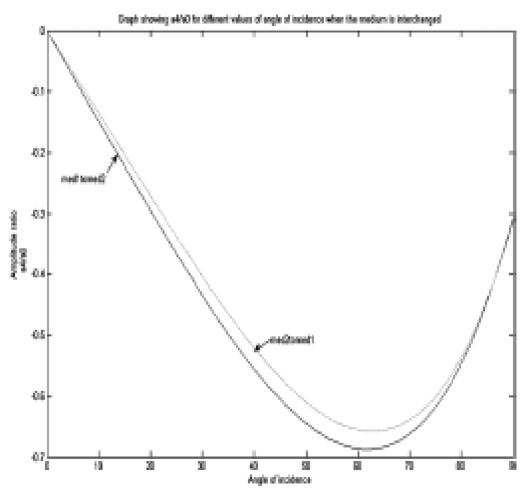

Figure 17

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