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CERTAIN INTEGRALS PROPERTIES ON GENERALIZED MULTIINDEX MITTAG-LEFFLER FUNCTION AND SRIVASTAVA POLYNOMIALS

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Abstract: The objective of the present paper is to establish certain (presumably) new and useful integral results involving the generalized Multiindex Mittag-Leffler function and the Srivastava polynomial. Next, we obtain certain new integrals and expansion formulas by the application of our theorems. Some interesting special cases of our main results are also considered.

Keywords: Multiindex Mittag-Leffler function Srivastava Polynomials.

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1. INTRODUCTION AND DEFINITIONS

In recent years, the fractional calculus has become one of the most popular research subject of mathematical analysis due to its applications in various fields of science as well as mathematics. A rich literature is available presenting the development and applications of the fractional calculus. (see, e.g., [1, 2, 3, 4, 5, 6, 7, 8] and the related references there in).

For the present study, we consider the following definitions and earlier works.

The Swedish mathematician Mittag-Leffler [9] introduced the function $E_\alpha(z)$ defined by

DEFINITION - 1

For $z \in \mathbb{C}; \alpha \geq 0$ $$E_\alpha(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n+1)} \tag{1.1}$$

A generalization of $E_\alpha(z)$ is given by Wiman [10], also known as Wiman's function, defined as:

DEFINITION - 2

For $z, \alpha, \beta \in \mathbb{C}; \Re(\alpha) > 0, \Re(\beta) > 0$

$$E_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n+\beta)} \tag{1.2}$$

Prabhakar [11] introduced the function $E^{\xi}_{\alpha,\beta}(z)$ defined as:

DEFINITION - 3

For $z, \alpha, \beta, \xi \in \mathbb{C}; \Re(\alpha) > 0, \Re(\beta) > 0, \Re(\xi) > 0$

$$E^{\xi}_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{(\xi)_n}{\Gamma(\alpha n+\beta)} \frac{z^n}{n!} \tag{1.3}$$

Further, Shukla and Prajapati [12] introduced the function $E^{\xi,\kappa}_{\alpha,\beta}(z)$ defined as:

DEFINITION - 4

For $z, \alpha, \beta, \xi \in \mathbb{C}; \Re(\alpha) > 0, \Re(\beta) > 0, \Re(\xi) > 0; \kappa \in (0,1) \cup \mathbb{N}$

$$E^{\xi,\kappa}_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{(\xi)_{\kappa n}}{\Gamma(\alpha n+\beta)} \frac{z^n}{n!} \tag{1.4}$$

The generalized multi-index Mittag-Leffler function[13] is defined as

DEFINITION - 5

For $\alpha_j, \beta_j, \xi, \kappa, z \in \mathbb{C}, \Re(\beta_j) > 0 (j = 1,2,\ldots.m); \Re\left(\sum_{j=1}^{m} \alpha_j\right) > \max\{0, R(\kappa) - 1\}$

$$E^{\xi,\kappa}_{(\alpha_j,\beta_j)_k}(z) = \sum_{n=0}^{\infty} \frac{(\xi)_{\kappa n}}{\prod_{j=1}^{k} \Gamma(\alpha_j n+\beta_j)} \frac{z^n}{n!} \tag{1.5}$$

Fox [14] and Wright [15] introduced and investigated the generalized Fox-Wright hypergeometric function ${}_p\Psi_q$ defined by

DEFINITION - 6

For $(p, q \in \mathbb{N}_0)$ with p numerator and q denominator parameters defined for $a_1, \ldots, a_p \in \mathbb{C}$ and $b_1, \ldots, b_q \in \mathbb{C}\backslash\mathbb{Z}_0^-$ $\alpha_1, \ldots, \alpha_p, \beta_1, \ldots, \beta_q \in \mathbb{R}^+$

$$ {}_p\Psi_q\left[\begin{matrix}(a_1,\alpha_1),\ldots,(a_p,\alpha_p);\\(b_1,\beta_1),\ldots,(b_q,\beta_q);\end{matrix}\quad z\right]=\sum_{n=0}^{\infty}\frac{\Gamma(a_1+\alpha_1 n)\ldots\Gamma(a_p+\alpha_p n)}{\Gamma(b_1+\beta_1 n)\ldots\Gamma(b_q+\beta_q n)}\frac{z^n}{n!}, \tag{1.6}$$

$$1+\sum_{j=1}^{q}\beta_j-\sum_{i=1}^{p}\alpha_i\geq 0 \tag{1.7}$$

For $\alpha_i=\beta_j=1$ $(i=1,\ldots,p; j=1,\ldots,q)$, Eq. (1.6) reduces immediately to the generalized hypergeometric function ${}_pF_q$ $(p,q\in\mathbb{N}_0)$ (see [16], Section 1.5):

$$ {}_pF_q\left[\begin{matrix}a_1,\ldots,a_p;\\b_1,\ldots,b_q;\end{matrix}\quad z\right]=\frac{\Gamma(a_1)\ldots\Gamma(a_p)}{\Gamma(b_1)\ldots\Gamma(b_q)}\,{}_p\Psi_q\left[\begin{matrix}(a_1,1),\ldots,(a_p,1);\\(b_1,1),\ldots,(b_q,1);\end{matrix}\quad z\right] \tag{1.8}$$

where $(\gamma)_n$ is the Pochhammer symbol defined by:

$$(\gamma)_n=\begin{cases}1 & (n=0;\gamma\in\mathbb{C}\setminus\{0\})\\ \gamma(\gamma+1)(\gamma+2)\ldots(\gamma+n-1) & (n=\mathbb{N};\gamma\in\mathbb{C})\end{cases}$$

$$=\frac{\Gamma(\gamma+n)}{\Gamma(\gamma)}\quad\left(n\in\mathbb{N};\gamma\in\mathbb{C}\setminus\mathbb{Z}^-_0\right) \tag{1.9}$$

and $\Gamma(\gamma)$ is the familiar Gamma function and $\mathbb{Z}_0^-$.

Srivastava polynomial [17] $S_n^m(x)$ is defined as follows: For $(n\in\mathbb{N}_0=\mathbb{N}\cup\{0\};\ m\in\mathbb{N})$

DEFINITION - 7

For $(n\in\mathbb{N}_0=\mathbb{N}\cup\{0\};\ m\in\mathbb{N})$

$$S_n^m(x):=\sum_{l=0}^{[n/m]}\frac{(-n)_{ml}}{l!}A_{n,l}x^l \tag{1.10}$$

Where $\mathbb{N}$ is the set of positive integer, the coefficients $A_{n,l}$ $(n,l\geq 0)$ are arbitrary constants, real or complex.

The following formulas (see, e.g.[18], p. 77, Eqn. 3.1, 3.2 and 3.3) will be required in our present study:

$$\int_0^\infty\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-p-1}dx=\frac{\sqrt{\pi}}{2a(4ab+c)^{p+1/2}}\frac{\Gamma(p+1/2)}{\Gamma(p+1)} \tag{1.11}$$

$$(a > 0; b \geq 0; c + 4ab > 0; \Re(p) + 1/2 > 0).$$

$$\int_0^\infty \frac{1}{x^2}\left[\left(ax + \frac{b}{x}\right)^2 + c\right]^{-p-1} dx = \frac{\sqrt{\pi}}{2b(4ab+c)^{p+1/2}} \frac{\Gamma(p+1/2)}{\Gamma(p+1)} \quad (1.12$$

$$(a \geq 0; b > 0; c + 4ab > 0; \Re(p) + 1/2 > 0)$$

$$\int_0^\infty \left(a + \frac{b}{x^2}\right)\left[\left(ax + \frac{b}{x}\right)^2 + c\right]^{-p-1} dx = \frac{\sqrt{\pi}}{(4ab+c)^{p+1/2}} \frac{\Gamma(p+1/2)}{\Gamma(p+1)} \quad (1.13)$$

$$(a > 0; b > 0; c + 4ab > 0; \Re(p) + 1/2 > 0)$$

2. MAIN RESULTS

To derive our main results, the following Orr's relation connecting products of hypergeometric series is also needed [19], given in the following Lemma:

LEMMA - 1 If

$$(1-y)^{\sigma+\rho-\gamma}{}_2F_1(2\sigma, 2\rho; 2\gamma; y) := \sum_{r=0}^\infty a_r y^r \quad (2.1)$$

Then

$${}_2F_1\left(\sigma, \rho; \gamma + \frac{1}{2}; y\right){}_2F_1\left(\gamma - \sigma, \gamma - \rho; \gamma + \frac{1}{2}; y\right) = \sum_{r=0}^\infty \frac{(\gamma)_k}{\left(\gamma + \frac{1}{2}\right)_k} a_r y^r \quad (2.2)$$

Let X stands for $\left(ax + \frac{b}{x}\right)^2 + c$, then we have the following Theorems:

THEOREM - 1

Let $a > 0;\ b \geq 0;\ c + 4ab > 0;\ \Re(\lambda - r - \mu l + \delta n) + 1/2 > 0, -\frac{1}{2} < \sigma - \rho - \gamma < \frac{1}{2}, \lambda, \mu, \delta, \alpha_j, \beta_j, \xi, \kappa, z \in \mathbb{C}, \Re(\beta_j) > 0\ (j = 1,2, \ldots k); \Re\left(\sum_{j=1}^k \alpha_j\right) > \max\{0, \Re(\kappa) - 1\}$, $m, r \in \mathbb{N}$ and $a_r, A_{n,l}$ $(n, l \geq 0)$ are arbitrary (real or complex) constants, then

$$\int_0^\infty X^{-\lambda-1}{}_2F_1(\sigma, \rho; \gamma + 1/2; X){}_2F_1(\gamma - \sigma, \gamma - \rho; \gamma + 1/2; X)$$

$$\times S_n^m[yX^{\mu}]E_{(\alpha_j,\beta_j)_k}^{(\xi,\kappa)}[zX^{-\delta}]dx$$

$$= \frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}\Gamma(\xi)} \sum_{r=0}^{\infty} \sum_{l=0}^{[n/m]} a_r \frac{(\gamma)_r}{\left(\gamma+\frac{1}{2}\right)_r} \frac{(-n)_{ml}}{l!} A_{n,l} y^l (4ab+c)^{r+\mu l} \quad (2.3)$$

$$\times {}_2\Psi_{k+1}\left[\begin{matrix}(\xi,\kappa),(\lambda-r-\mu l+1/2,\delta);\\ (\beta_1,\alpha_1),(\beta_2,\alpha_2),\dots,(\beta_k,\alpha_k),(\lambda-r-\mu l+1,\delta);\end{matrix}\middle| \frac{z}{(4ab+c)^{\delta}}\right]$$

Proof. Denote left hand side of eqn. (2.3) by I and using the results (2.1), (1.10) and (1.5), in equation (2.3), we have

$$I = \int_0^{\infty} X^{-\lambda-1} \sum_{r=0}^{\infty} a_r \frac{(\gamma)_r}{\left(\gamma+\frac{1}{2}\right)_r} X^r \sum_{l=0}^{[n/m]} \frac{(-n)_{ml}}{l!} A_{n,l}[y^l X^{\mu l}] \quad (2.4)$$

$$\times \sum_{n=0}^{\infty} \frac{(\xi)_{\kappa n}}{\prod_{j=1}^{k}\Gamma(\alpha_j n+\beta_j)} \frac{z^n X^{-\delta n}}{n!} dx$$

Now interchanging the order of integration and summation, the above equation (2.4) reduces to

$$I = \sum_{r=0}^{\infty} a_r \frac{(\gamma)_r}{\left(\gamma+\frac{1}{2}\right)_r} \sum_{l=0}^{[n/m]} \frac{(-n)_{ml}}{l!} A_{n,l} y^l \sum_{n=0}^{\infty} \frac{(\xi)_{\kappa n}}{\prod_{j=1}^{k}\Gamma(\alpha_j n+\beta_j)} \frac{z^n}{n!} \quad (2.5)$$

$$\times \int_0^{\infty} X^{-(\lambda-r-\mu l+\delta n)-1} dx$$

Now using the formula given in eq.(1.11), the above equation (2.5), reduces to

$$I = \sum_{r=0}^{\infty} \sum_{l=0}^{[n/m]} a_r \frac{(\gamma)_r}{\left(\gamma+\frac{1}{2}\right)_r} \frac{(-n)_{ml}}{l!} A_{n,l} y^l \sum_{n=0}^{\infty} \frac{(\xi)_{\kappa n}}{\prod_{j=1}^{k}\Gamma(\alpha_j n+\beta_j)} \frac{z^n}{n!} \quad (2.6)$$

$$\times \frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda-r-\mu l+\delta n+1/2}} \frac{\Gamma(\lambda-r-\mu l+\delta n+1/2)}{\Gamma(\lambda-r-\mu l+\delta n+1)}$$

After little simplification, we have

$$I = \frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}\Gamma(\xi)} \sum_{r=0}^{\infty} \sum_{l=0}^{[n/m]} a_r \frac{(\gamma)_r}{\left(\gamma+\frac{1}{2}\right)_r} \frac{(-n)_{ml}}{l!} A_{n,l} y^l (4ab+c)^{r+\mu l} \quad (2.7)$$

$$\times \sum_{n=0}^{\infty} \frac{\Gamma(\xi+\kappa n)}{\prod_{j=1}^{k}\Gamma(\alpha_j n+\beta_j)} \frac{\Gamma(\lambda-r-\mu l+1/2+\delta n)}{\Gamma(\lambda-r-\mu l+1+\delta n)} \left(\frac{z}{(4ab+c)^{\delta}}\right)^n \frac{1}{n!}$$

Interpreting the above result (2.7) in the view of eqn. (1.6), we get the required result (2.3).

THEOREM - 2

Let $a \geq 0;\ b > 0;\ c + 4ab > 0;\ \Re(\lambda - r - \mu l + \delta n) + 1/2 > 0, -\frac{1}{2} < \sigma - \rho - \gamma < \frac{1}{2}, \lambda, \mu, \delta, \alpha_j, \beta_j, \gamma, \kappa, z \in \mathbb{C}, \Re(\beta_j) > 0\ (j = 1,2,\ldots k); \Re\left(\sum_{j=1}^{k} \alpha_j\right) > \max\{0, \Re(\kappa) - 1\}, m, r \in \mathbb{N}$ and $a_r, A_{n,l}$ $(n, l \geq 0)$ are arbitrary (real or complex) constants, then

$$\int_0^\infty \frac{1}{x^2} X^{-\lambda-1} {}_2F_1(\sigma, \rho; \gamma + 1/2; X) {}_2F_1(\gamma - \sigma, \gamma - \rho; \gamma + 1/2; X)$$

$$\times S_n^m[yX^\mu] E_{(\alpha_j,\beta_j)_k}^{(\xi,\kappa)}[zX^{-\delta}]dx$$

$$= \frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}\Gamma(\xi)} \sum_{r=0}^\infty \sum_{l=0}^{[n/m]} a_r \frac{(\gamma)_r}{\left(\gamma+\frac{1}{2}\right)_r} \frac{(-n)_{ml}}{l!} A_{n,l} y^l (4ab + c)^{r+\mu l} \quad (2.8)$$

$$\times {}_2\Psi_{k+1}\left[\begin{matrix}(\xi, \kappa), (\lambda - r - \mu l + 1/2, \delta); \\ (\beta_1, \alpha_1), (\beta_2, \alpha_2), \ldots, (\beta_k, \alpha_k), (\lambda - r - \mu l + 1, \delta);\end{matrix} \middle| \frac{z}{(4ab + c)^\delta}\right]$$

THEOREM - 3

Let $a > 0;\ b > 0;\ c + 4ab > 0;\ \Re(\lambda - r - \mu l + \delta n) + 1/2 > 0, -\frac{1}{2} < \sigma - \rho - \gamma < \frac{1}{2}, \lambda, \mu, \delta, \alpha_j, \beta_j, \gamma, \kappa, z \in \mathbb{C}, \Re(\beta_j) > 0\ (j = 1,2,\ldots k); \Re\left(\sum_{j=1}^{k} \alpha_j\right) > \max\{0, \Re(\kappa) - 1\}, m, r \in \mathbb{N}$ and $a_r, A_{n,l}$ $(n, l \geq 0)$ are arbitrary (real or complex) constants, then

$$\int_0^\infty \left(a + \frac{b}{x^2}\right) X^{-\lambda-1} {}_2F_1(\sigma, \rho; \gamma + 1/2; X) {}_2F_1(\gamma - \sigma, \gamma - \rho; \gamma + 1/2; X)$$

$$\times S_n^m[yX^\mu] E_{(\alpha_j,\beta_j)_k}^{(\xi,\kappa)}[zX^{-\delta}]dx$$

$$= \frac{\sqrt{\pi}}{(4ab+c)^{\lambda+1/2}\Gamma(\xi)} \sum_{r=0}^\infty \sum_{l=0}^{[n/m]} a_r \frac{(\gamma)_r}{\left(\gamma+\frac{1}{2}\right)_r} \frac{(-n)_{ml}}{l!} A_{n,l} y^l (4ab + c)^{r+\mu l} \quad (2.9)$$

$$\times {}_2\Psi_{k+1}\left[\begin{matrix}(\xi, \kappa), (\lambda - r - \mu l + 1/2, \delta); \\ (\beta_1, \alpha_1), (\beta_2, \alpha_2), \ldots, (\beta_k, \alpha_k), (\lambda - r - \mu l + 1, \delta);\end{matrix} \middle| \frac{z}{(4ab + c)^\delta}\right]$$

Proof. The proof of Theorem 2 and 3 are similar to Theorem 1, so we skip the details.

3. SPECIAL CASES

When $m = 2$, $A_{n,l} = (-1)^l$, the Srivastava polynomial in eqn. (1.10), reduces to hermite polynomial $S_n^2(x) \rightarrow x^{n/2} H_n\left[\frac{1}{2\sqrt{x}}\right]$, Then the results in the eqn. (2.3), (2.8) and (2.9) reduce to the following results:

COROLLARY - 1

Let $a > 0$; $b \geq 0$; $c + 4ab > 0$; $\Re(\lambda - r - \mu l + \delta n) + 1/2 > 0, -\frac{1}{2} < \sigma - \rho - \gamma < \frac{1}{2}$, $\lambda, \mu, \delta, \alpha_j, \beta_j, \gamma, \kappa, z \in \mathbb{C}, \Re(\beta_j) > 0\ (j = 1,2,\ldots.k); \Re\left(\sum_{j=1}^{k} \alpha_j\right) > \max\{0, \Re(\kappa) - 1\}$, $r \in \mathbb{N}$ and a_r are arbitrary (real or complex) constant, then

$$\int_0^\infty X^{-\lambda-1}{}_2F_1(\sigma, \rho; \gamma + 1/2; X){}_2F_1(\gamma - \sigma, \gamma - \rho; \gamma + 1/2; X)$$

$$\times [yX^\mu]^{n/2} H_n\left[\frac{1}{2\sqrt{yX^\mu}}\right] E_{(\alpha_j,\beta_j)_k}^{(\xi,\kappa)}[zX^{-\delta}]dx$$

$$= \frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}\Gamma(\xi)} \sum_{r=0}^{\infty} \sum_{l=0}^{[n/2]} a_r \frac{(\gamma)_r}{\left(\gamma+\frac{1}{2}\right)_r} \frac{(-n)_{2l}}{l!} (-y)^l (4ab + c)^{r+\mu l} \quad (3.1)$$

$$\times {}_2\Psi_{k+1}\left[\begin{matrix}(\xi, \kappa), (\lambda - r - \mu l + 1/2, \delta); \\ (\beta_1, \alpha_1), (\beta_2, \alpha_2), \ldots, (\beta_k, \alpha_k), (\lambda - r - \mu l + 1, \delta);\end{matrix} \middle| \frac{z}{(4ab+c)^\delta}\right]$$

COROLLARY - 2

Let $a \geq 0$; $b > 0$; $c + 4ab > 0$; $\Re(\lambda - r - \mu l + \delta n) + 1/2 > 0, -\frac{1}{2} < \sigma - \rho - \gamma < \frac{1}{2}$, $\lambda, \mu, \delta, \alpha_j, \beta_j, \gamma, \kappa, z \in \mathbb{C}, \Re(\beta_j) > 0\ (j = 1,2,\ldots.k); \Re\left(\sum_{j=1}^{k} \alpha_j\right) > \max\{0, \Re(\kappa) - 1\}$, $r \in \mathbb{N}$ and a_r are arbitrary (real or complex) constants, then

$$\int_0^\infty \frac{1}{x^2} X^{-\lambda-1}{}_2F_1(\sigma, \rho; \gamma + 1/2; X){}_2F_1(\gamma - \sigma, \gamma - \rho; \gamma + 1/2; X)$$

$$\times [yX^\mu]^{n/2} H_n\left[\frac{1}{2\sqrt{yX^\mu}}\right] E_{(\alpha_j,\beta_j)_k}^{(\xi,\kappa)}[zX^{-\delta}]dx$$

$$= \frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}\Gamma(\xi)} \sum_{r=0}^{\infty} \sum_{l=0}^{[n/2]} a_r \frac{(\gamma)_r}{\left(\gamma+\frac{1}{2}\right)_r} \frac{(-n)_{2l}}{l!} (-y)^l (4ab+c)^{r+\mu l} \quad (3.2)$$

$$\times {}_2\Psi_{k+1}\left[\begin{matrix}(\xi,\kappa),(\lambda-r-\mu l+1/2,\delta);\\(\beta_1,\alpha_1),(\beta_2,\alpha_2),\ldots,(\beta_k,\alpha_k),(\lambda-r-\mu l+1,\delta);\end{matrix}\middle|\frac{z}{(4ab+c)^{\delta}}\right]$$

COROLLARY - 3

Let $a > 0;\ b > 0;\ c + 4ab > 0;\ \Re(\lambda - r - \mu l + \delta n) + 1/2 > 0, -\frac{1}{2} < \sigma - \rho - \gamma < \frac{1}{2}, \lambda, \mu, \delta, \alpha_j, \beta_j, \gamma, \kappa, z \in \mathbb{C}, \Re(\beta_j) > 0\ (j = 1,2,\ldots k); \Re\left(\sum_{j=1}^{k} \alpha_j\right) > \max\{0, \Re(\kappa) - 1\}$, $r \in \mathbb{N}$ and a_r are arbitrary (real or complex) constants, then

$$\int_0^{\infty} \left(a + \frac{b}{x^2}\right) X^{-\lambda-1} {}_2F_1(\sigma,\rho;\gamma+1/2;X)\, {}_2F_1(\gamma-\sigma,\gamma-\rho;\gamma+1/2;X)$$

$$\times [yX^{\mu}]^{n/2} H_n\left[\frac{1}{2\sqrt{yX^{\mu}}}\right] E_{(\alpha_j,\beta_j)_k}^{(\xi,\kappa)}[zX^{-\delta}]dx$$

$$= \frac{\sqrt{\pi}}{(4ab+c)^{\lambda+1/2}\Gamma(\xi)} \sum_{r=0}^{\infty} \sum_{l=0}^{[n/2]} a_r \frac{(\gamma)_r}{\left(\gamma+\frac{1}{2}\right)_r} \frac{(-n)_{2l}}{l!} (-y)^l (4ab+c)^{r+\mu l} \quad (3.3)$$

$$\times {}_2\Psi_{k+1}\left[\begin{matrix}(\xi,\kappa),(\lambda-r-\mu l+1/2,\delta);\\(\beta_1,\alpha_1),(\beta_2,\alpha_2),\ldots,(\beta_k,\alpha_k),(\lambda-r-\mu l+1,\delta);\end{matrix}\middle|\frac{z}{(4ab+c)^{\delta}}\right]$$

By setting $S_n^2(x) \to L_n^{(\alpha')}$ in which $m = 2, A_{n,l} = \binom{n+\alpha'}{n}\frac{1}{\alpha'+1}$, for the case of Lagurre polynomial, Then the results in the eqn. (2.3), (2.8) and (2.9) reduce to the following results:

COROLLARY - 4

Let $a > 0;\ b \geq 0;\ c + 4ab > 0;\ \Re(\lambda - r - \mu l + \delta n) + 1/2 > 0, -\frac{1}{2} < \sigma - \rho - \gamma < \frac{1}{2}, \lambda, \mu, \delta, \alpha_j, \beta_j, \gamma, \kappa, z \in \mathbb{C}, \Re(\beta_j) > 0\ (j = 1,2,\ldots k); \Re\left(\sum_{j=1}^{k} \alpha_j\right) > \max\{0, \Re(\kappa) - 1\}$, $r \in \mathbb{N}$ and a_r are arbitrary (real or complex) constants, then

$$\int_0^{\infty} X^{-\lambda-1} {}_2F_1(\sigma,\rho;\gamma+1/2;X)\, {}_2F_1(\gamma-\sigma,\gamma-\rho;\gamma+1/2;X)$$

$$\times L_n^{(\alpha')}[yX^{\mu}] E_{(\alpha_j,\beta_j)_k}^{(\xi,\kappa)}[zX^{-\delta}]dx$$

$$= \frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+\frac{1}{2}}\Gamma(\xi)} \sum_{r=0}^{\infty} \sum_{l=0}^{\left[\frac{n}{2}\right]}$$

$$a_r \frac{(\gamma)_r}{\left(\gamma+\frac{1}{2}\right)_r} \frac{(-n)_{2l}}{l!} \binom{n+\alpha'}{n} \frac{1}{\alpha'+1} y^l (4ab+c)^{r+\mu l} \tag{3.4}$$

$$\times {}_2\Psi_{k+1}\left[\begin{matrix}(\xi,\kappa),(\lambda-r-\mu l+1/2,\delta); \\ (\beta_1,\alpha_1),(\beta_2,\alpha_2),\ldots,(\beta_k,\alpha_k),(\lambda-r-\mu l+1,\delta);\end{matrix} \middle| \frac{z}{(4ab+c)^{\delta}}\right]$$

COROLLARY - 5

Let $a \geq 0;\ b > 0;\ c + 4ab > 0;\ \Re(\lambda - r - \mu l + \delta n) + 1/2 > 0, -\frac{1}{2} < \sigma - \rho - \gamma < \frac{1}{2}, \lambda, \mu, \delta, \alpha_j, \beta_j, \gamma, \kappa, z \in \mathbb{C}, \Re(\beta_j) > 0\ (j = 1,2,\ldots k); \Re\left(\sum_{j=1}^{k} \alpha_j\right) > \max\{0, \Re(\kappa) - 1\}$, $r \in \mathbb{N}$ and a_r are arbitrary (real or complex) constants, then

$$\int_0^{\infty} \frac{1}{x^2} X^{-\lambda-1} {}_2F_1(\sigma,\rho;\gamma+1/2;X) {}_2F_1(\gamma-\sigma,\gamma-\rho;\gamma+1/2;X)$$

$$\times L_n^{(\alpha')}[yX^{\mu}] E_{(\alpha_j,\beta_j)_k}^{(\xi,\kappa)}[zX^{-\delta}]dx$$

$$= \frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+\frac{1}{2}}\Gamma(\xi)} \sum_{r=0}^{\infty} \sum_{l=0}^{\left[\frac{n}{2}\right]}$$

$$a_r \frac{(\gamma)_r}{\left(\gamma+\frac{1}{2}\right)_r} \frac{(-n)_{2l}}{l!} \binom{n+\alpha'}{n} \frac{1}{\alpha'+1} y^l (4ab+c)^{r+\mu l} \tag{3.5}$$

$$\times {}_2\Psi_{k+1}\left[\begin{matrix}(\xi,\kappa),(\lambda-r-\mu l+1/2,\delta); \\ (\beta_1,\alpha_1),(\beta_2,\alpha_2),\ldots,(\beta_k,\alpha_k),(\lambda-r-\mu l+1,\delta);\end{matrix} \middle| \frac{z}{(4ab+c)^{\delta}}\right]$$

COROLLARY - 6

Let $a > 0;\ b > 0;\ c + 4ab > 0;\ \Re(\lambda - r - \mu l + \delta n) + 1/2 > 0, -\frac{1}{2} < \sigma - \rho - \gamma < \frac{1}{2}, \lambda, \mu, \delta, \alpha_j, \beta_j, \gamma, \kappa, z \in \mathbb{C}, \Re(\beta_j) > 0\ (j = 1,2,\ldots k); \Re\left(\sum_{j=1}^{k} \alpha_j\right) > \max\{0, \Re(\kappa) - 1\}$, $r \in \mathbb{N}$ and a_r are arbitrary (real or complex) constants, then

$$\int_0^{\infty} \left(a + \frac{b}{x^2}\right) X^{-\lambda-1} {}_2F_1(\sigma,\rho;\gamma+1/2;X) {}_2F_1(\gamma-\sigma,\gamma-\rho;\gamma+1/2;X)$$

$$\times L_n^{(\alpha')}[yX^{\mu}] E_{(\alpha_j,\beta_j)_k}^{(\xi,\kappa)}[zX^{-\delta}]dx$$

$$= \frac{\sqrt{\pi}}{(4ab+c)^{\lambda+\frac{1}{2}}\Gamma(\xi)} \sum_{r=0}^{\infty} \sum_{l=0}^{\left[\frac{n}{2}\right]}$$

$$a_r \frac{(\gamma)_r}{\left(\gamma+\frac{1}{2}\right)_r} \frac{(-n)_{2l}}{l!} \binom{n+\alpha'}{n} \frac{1}{\alpha'+1} y^l (4ab+c)^{r+\mu l} \tag{3.6}$$

$$\times {}_2\Psi_{k+1}\left[\begin{matrix}(\xi,\kappa),(\lambda-r-\mu l+1/2,\delta); \\ (\beta_1,\alpha_1),(\beta_2,\alpha_2),\ldots,(\beta_k,\alpha_k),(\lambda-r-\mu l+1,\delta);\end{matrix} \middle| \frac{z}{(4ab+c)^{\delta}}\right]$$

If we put $\sigma = \gamma$, in the main theorems, the value of the a_r comes out to be $\frac{\rho_r}{r!}$, Then the results in the eqn. (2.3), (2.8) and (2.9) reduce to the following results:.

COROLLARY - 7

Let $a > 0;\ b \geq 0;\ c + 4ab > 0;\ \Re(\lambda - r - \mu l + \delta n) + 1/2 > 0, -\frac{1}{2} < \sigma - \rho - \gamma < \frac{1}{2}, \lambda, \mu, \delta, \alpha_j, \beta_j, \gamma, \kappa, z \in \mathbb{C}, \Re(\beta_j) > 0\ (j = 1,2, \ldots k); \Re\left(\sum_{j=1}^{k} \alpha_j\right) > \max\{0, \Re(\kappa) - 1\}, m, r \in \mathbb{N}$ and $A_{n,l}$ $(n, l \geq 0)$ are arbitrary (real or complex) constants, then

$$\int_0^{\infty} X^{-\lambda-1} {}_2F_1(\sigma,\rho;\gamma+1/2;X) S_n^m[yX^{\mu}] E_{(\alpha_j,\beta_j)_k}^{(\xi,\kappa)}[zX^{-\delta}]dx$$

$$= \frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}\Gamma(\xi)} \sum_{r=0}^{\infty} \sum_{l=0}^{[n/m]} \frac{(\alpha)_r(\rho)_r}{\left(\alpha+\frac{1}{2}\right)_r r!} \frac{(-n)_{ml}}{l!} A_{n,l} y^l (4ab+c)^{r+\mu l} \tag{3.7}$$

$$\times {}_2\Psi_{k+1}\left[\begin{matrix}(\xi,\kappa),(\lambda-r-\mu l+1/2,\delta); \\ (\beta_1,\alpha_1),(\beta_2,\alpha_2),\ldots,(\beta_k,\alpha_k),(\lambda-r-\mu l+1,\delta);\end{matrix} \middle| \frac{z}{(4ab+c)^{\delta}}\right]$$

COROLLARY - 8

Let $a \geq 0;\ b > 0;\ c + 4ab > 0;\ \Re(\lambda - r - \mu l + \delta n) + 1/2 > 0, -\frac{1}{2} < \sigma - \rho - \gamma < \frac{1}{2}, \lambda, \mu, \delta, \alpha_j, \beta_j, \gamma, \kappa, z \in \mathbb{C}, \Re(\beta_j) > 0\ (j = 1,2, \ldots k); \Re\left(\sum_{j=1}^{k} \alpha_j\right) > \max\{0, \Re(\kappa) - 1\}, m, r \in \mathbb{N}$ and $A_{n,l}$ $(n, l \geq 0)$ are arbitrary (real or complex) constants, then

$$\int_0^{\infty} \frac{1}{x^2} X^{-\lambda-1} {}_2F_1(\sigma,\rho;\gamma+1/2;X) S_n^m[yX^{\mu}] E_{(\alpha_j,\beta_j)_k}^{(\xi,\kappa)}[zX^{-\delta}]dx$$

$$= \frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}\Gamma(\xi)} \sum_{r=0}^{\infty} \sum_{l=0}^{[n/m]} \frac{(\alpha)_r(\rho)_r}{\left(\alpha+\frac{1}{2}\right)_r r!} \frac{(-n)_{ml}}{l!} A_{n,l} y^l (4ab+c)^{r+\mu l} \quad (3.8)$$

$$\times {}_2\Psi_{k+1}\left[\begin{matrix}(\xi,\kappa),(\lambda-r-\mu l+1/2,\delta); \\ (\beta_1,\alpha_1),(\beta_2,\alpha_2),\ldots,(\beta_k,\alpha_k),(\lambda-r-\mu l+1,\delta);\end{matrix} \middle| \frac{z}{(4ab+c)^{\delta}}\right]$$

COROLLARY - 9

Let $a > 0;\ b > 0;\ c + 4ab > 0;\ \Re(\lambda - r - \mu l + \delta n) + 1/2 > 0, -\frac{1}{2} < \sigma - \rho - \gamma < \frac{1}{2}, \lambda, \mu, \delta, \alpha_j, \beta_j, \gamma, \kappa, z \in \mathbb{C}, \Re(\beta_j) > 0\ (j = 1,2,\ldots k); \Re\left(\sum_{j=1}^{k} \alpha_j\right) > \max\{0, \Re(\kappa) - 1\}$, $m, r \in \mathbb{N}$ and $A_{n,l}$ $(n, l \geq 0)$ are arbitrary (real or complex) constants, then

$$\int_0^{\infty} \left(a + \frac{b}{x^2}\right) X^{-\lambda-1} {}_2F_1(\sigma,\rho;\gamma+1/2;X) S_n^m[yX^{\mu}] E_{(\alpha_j,\beta_j)_k}^{(\xi,\kappa)}[zX^{-\delta}]dx$$

$$= \frac{\sqrt{\pi}}{(4ab+c)^{\lambda+1/2}\Gamma(\xi)} \sum_{r=0}^{\infty} \sum_{l=0}^{[n/m]} \frac{(\alpha)_r(\rho)_r}{\left(\alpha+\frac{1}{2}\right)_r r!} \frac{(-n)_{ml}}{l!} A_{n,l} y^l (4ab+c)^{r+\mu l} \quad (3.9)$$

$$\times {}_2\Psi_{k+1}\left[\begin{matrix}(\xi,\kappa),(\lambda-r-\mu l+1/2,\delta); \\ (\beta_1,\alpha_1),(\beta_2,\alpha_2),\ldots,(\beta_k,\alpha_k),(\lambda-r-\mu l+1,\delta);\end{matrix} \middle| \frac{z}{(4ab+c)^{\delta}}\right]$$

If we put $\rho = \sigma + \frac{1}{2}$ and $\sigma = -\eta$ (η is non negative integer), Then the results in the eqn. (2.3), (2.8) and (2.9) reduce to the following results:

COROLLARY - 10

Let $a > 0;\ b \geq 0;\ c + 4ab > 0;\ \Re(\lambda - r - \mu l + \delta n) + 1/2 > 0, -\frac{1}{2} < \sigma - \rho - \gamma < \frac{1}{2}, \lambda, \mu, \delta, \alpha_j, \beta_j, \gamma, \kappa, z \in \mathbb{C}, \Re(\beta_j) > 0\ (j = 1,2,\ldots k); \Re\left(\sum_{j=1}^{k} \alpha_j\right) > \max\{0, \Re(\kappa) - 1\}$, $m, r \in \mathbb{N}$ and $A_{n,l}$ $(n, l \geq 0)$ are arbitrary (real or complex) constants, then

$$\int_0^{\infty} X^{-\lambda-1}(1-X)^{\eta} S_n^m[yX^{\mu}] E_{(\alpha_j,\beta_j)_k}^{(\xi,\kappa)}[zX^{-\delta}]dx$$

$$= \frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}\Gamma(\xi)} \sum_{r=0}^{\infty} \sum_{l=0}^{[n/m]} \frac{(-\eta)_r}{r!} \frac{(-n)_{ml}}{l!} A_{n,l} y^l (4ab+c)^{r+\mu l} \quad (3.10)$$

$$\times {}_2\Psi_{k+1}\left[\begin{matrix}(\xi,\kappa),(\lambda-r-\mu l+1/2,\delta);\\(\beta_1,\alpha_1),(\beta_2,\alpha_2),\dots,(\beta_k,\alpha_k),(\lambda-r-\mu l+1,\delta);\end{matrix}\left|\frac{z}{(4ab+c)^{\delta}}\right.\right]$$

COROLLARY - 11

Let $a \geq 0;\ b > 0;\ c + 4ab > 0;\ \Re(\lambda - r - \mu l + \delta n) + 1/2 > 0, -\frac{1}{2} < \sigma - \rho - \gamma < \frac{1}{2}, \lambda, \mu, \delta, \alpha_j, \beta_j, \gamma, \kappa, z \in \mathbb{C}, \Re(\beta_j) > 0\ (j = 1,2,\dots k); \Re\left(\sum_{j=1}^{k}\alpha_j\right) > \max\{0, \Re(\kappa) - 1\}$, $m, r \in \mathbb{N}$ and $A_{n,l}$ $(n, l \geq 0)$ are arbitrary (real or complex) constants, then

$$\int_0^\infty \frac{1}{x^2} X^{-\lambda-1}(1-X)^{\eta} S_n^m[yX^{\mu}] E_{(\alpha_j,\beta_j)_k}^{(\xi,\kappa)}[zX^{-\delta}]dx$$

$$= \frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}\Gamma(\xi)} \sum_{r=0}^{\infty} \sum_{l=0}^{[n/m]} \frac{(-\eta)_r}{r!} \frac{(-n)_{ml}}{l!} A_{n,l} y^l (4ab+c)^{r+\mu l} \quad (3.11)$$

$$\times {}_2\Psi_{k+1}\left[\begin{matrix}(\xi,\kappa),(\lambda-r-\mu l+1/2,\delta);\\(\beta_1,\alpha_1),(\beta_2,\alpha_2),\dots,(\beta_k,\alpha_k),(\lambda-r-\mu l+1,\delta);\end{matrix}\left|\frac{z}{(4ab+c)^{\delta}}\right.\right]$$

COROLLARY - 12

Let $a > 0;\ b > 0;\ c + 4ab > 0;\ \Re(\lambda - r - \mu l + \delta n) + 1/2 > 0, -\frac{1}{2} < \sigma - \rho - \gamma < \frac{1}{2}, \lambda, \mu, \delta, \alpha_j, \beta_j, \gamma, \kappa, z \in \mathbb{C}, \Re(\beta_j) > 0\ (j = 1,2,\dots k); \Re\left(\sum_{j=1}^{k}\alpha_j\right) > \max\{0, \Re(\kappa) - 1\}$, $m, r \in \mathbb{N}$ and $A_{n,l}$ $(n, l \geq 0)$ are arbitrary (real or complex) constants, then

$$\int_0^\infty \left(a + \frac{b}{x^2}\right) X^{-\lambda-1}(1-X)^{\eta} S_n^m[yX^{\mu}] E_{(\alpha_j,\beta_j)_k}^{(\xi,\kappa)}[zX^{-\delta}]dx$$

$$= \frac{\sqrt{\pi}}{(4ab+c)^{\lambda+1/2}\Gamma(\xi)} \sum_{r=0}^{\infty} \sum_{l=0}^{[n/m]} \frac{(-\eta)_r}{r!} \frac{(-n)_{ml}}{l!} A_{n,l} y^l (4ab+c)^{r+\mu l} \quad (3.12)$$

$$\times {}_2\Psi_{k+1}\left[\begin{matrix}(\xi,\kappa),(\lambda-r-\mu l+1/2,\delta);\\(\beta_1,\alpha_1),(\beta_2,\alpha_2),\dots,(\beta_k,\alpha_k),(\lambda-r-\mu l+1,\delta);\end{matrix}\left|\frac{z}{(4ab+c)^{\delta}}\right.\right]$$

If we put $k = 1$ in results (2.3), (2.8) and (2.9), Then the results in the eqn. (2.3), (2.8) and (2.9) reduce to the following results:

COROLLARY - 13

Let $a > 0;\ b \geq 0;\ c + 4ab > 0;\ \Re(\lambda - r - \mu l + \delta n) + 1/2 > 0, -\frac{1}{2} < \sigma - \rho - \gamma < \frac{1}{2}, \lambda, \mu, \delta, \alpha, \beta, \xi, \kappa, z \in \mathbb{C}, \Re(\alpha) > 0, \Re(\beta) > 0, \Re(\xi) > 0, \kappa \in (0,1) \cup \mathbb{N}$, $m, r \in \mathbb{N}$ and $a_r, A_{n,l}$ $(n, l \geq 0)$ are arbitrary (real or complex) constants, then

$$\int_0^\infty X^{-\lambda-1}{}_2F_1(\sigma, \rho; \gamma + 1/2; X)_2F_1(\gamma - \sigma, \gamma - \rho; \gamma + 1/2; X)$$

$$\times S_n^m[yX^\mu]E_{(\alpha,\beta)}^{(\xi,\kappa)}[zX^{-\delta}]dx$$

$$= \frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}\Gamma(\xi)} \sum_{r=0}^\infty \sum_{l=0}^{[n/m]} a_r \frac{(\gamma)_r}{\left(\gamma+\frac{1}{2}\right)_r} \frac{(-n)_{ml}}{l!} A_{n,l} y^l (4ab + c)^{r+\mu l} \quad (3.13)$$

$$\times_2 \Psi_2 \left[\begin{matrix} (\xi, \kappa), (\lambda - r - \mu l + 1/2, \delta); \\ (\beta, \alpha), (\lambda - r - \mu l + 1, \delta); \end{matrix} \middle| \frac{z}{(4ab+c)^\delta} \right]$$

COROLLARY - 14

Let $a > 0;\ b \geq 0;\ c + 4ab > 0;\ \Re(\lambda - r - \mu l + \delta n) + 1/2 > 0, -\frac{1}{2} < \sigma - \rho - \gamma < \frac{1}{2}, \lambda, \mu, \delta, \alpha, \beta, \xi, \kappa, z \in \mathbb{C}, \Re(\alpha) > 0, \Re(\beta) > 0, \Re(\xi) > 0, \kappa \in (0,1) \cup \mathbb{N}$, $m, r \in \mathbb{N}$ and $a_r, A_{n,l}$ $(n, l \geq 0)$ are arbitrary (real or complex) constants, then

$$\int_0^\infty \frac{1}{x^2} X^{-\lambda-1}{}_2F_1(\sigma, \rho; \gamma + 1/2; X)_2F_1(\gamma - \sigma, \gamma - \rho; \gamma + 1/2; X)$$

$$\times S_n^m[yX^\mu]E_{(\alpha,\beta)}^{(\xi,\kappa)}[zX^{-\delta}]dx$$

$$= \frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}\Gamma(\xi)} \sum_{r=0}^\infty \sum_{l=0}^{[n/m]} a_r \frac{(\gamma)_r}{\left(\gamma+\frac{1}{2}\right)_r} \frac{(-n)_{ml}}{l!} A_{n,l} y^l (4ab + c)^{r+\mu l} \quad (3.14)$$

$$\times_2 \Psi_2 \left[\begin{matrix} (\xi, \kappa), (\lambda - r - \mu l + 1/2, \delta); \\ (\beta, \alpha), (\lambda - r - \mu l + 1, \delta); \end{matrix} \middle| \frac{z}{(4ab+c)^\delta} \right]$$

COROLLARY - 15

Let $a > 0;\ b \geq 0;\ c + 4ab > 0;\ \Re(\lambda - r - \mu l + \delta n) + 1/2 > 0, -\frac{1}{2} < \sigma - \rho - \gamma < \frac{1}{2}, \lambda, \mu, \delta, \alpha, \beta, \xi, \kappa, z \in \mathbb{C}, \Re(\alpha) > 0, \Re(\beta) > 0, \Re(\xi) > 0, \kappa \in (0,1) \cup \mathbb{N}$, $m, r \in \mathbb{N}$ and $a_r, A_{n,l}$ $(n, l \geq 0)$ are arbitrary (real or complex) constants, then

$$\int_0^\infty \left(a+\frac{b}{x^2}\right) X^{-\lambda-1} {}_2F_1(\sigma,\rho;\gamma+1/2;X) {}_2F_1(\gamma-\sigma,\gamma-\rho;\gamma+1/2;X)$$

$$\times S_n^m[yX^\mu] E_{(\alpha,\beta)}^{(\xi,\kappa)}[zX^{-\delta}]dx$$

$$= \frac{\sqrt{\pi}}{(4ab+c)^{\lambda+1/2}\Gamma(\xi)} \sum_{r=0}^{\infty} \sum_{l=0}^{[n/m]} a_r \frac{(\gamma)_r}{\left(\gamma+\frac{1}{2}\right)_r} \frac{(-n)_{ml}}{l!} A_{n,l} y^l (4ab+c)^{r+\mu l} \quad (3.15)$$

$$\times {}_2\Psi_2 \left[\begin{matrix} (\xi,\kappa), (\lambda-r-\mu l+1/2,\delta); \\ (\beta,\alpha), (\lambda-r-\mu l+1,\delta); \end{matrix} \middle| \frac{z}{(4ab+c)^\delta} \right]$$

If we put $k = \kappa = 1$ in results (2.3), (2.8) and (2.9), Then the results in the eqn. (2.3), (2.8) and (2.9) reduce to the following results:

COROLLARY - 16

Let $a > 0;\ b \geq 0;\ c + 4ab > 0;\ \Re(\lambda - r - \mu l + \delta n) + 1/2 > 0, -\frac{1}{2} < \sigma - \rho - \gamma < \frac{1}{2}, \lambda, \mu, \delta, \alpha, \beta, \xi, z \in \mathbb{C}, \Re(\alpha) > 0, \Re(\beta) > 0, \Re(\xi) > 0$, $m, r \in \mathbb{N}$ and $a_r, A_{n,l}\ (n, l \geq 0)$ are arbitrary (real or complex) constants, then

$$\int_0^\infty X^{-\lambda-1} {}_2F_1(\sigma,\rho;\gamma+1/2;X) {}_2F_1(\gamma-\sigma,\gamma-\rho;\gamma+1/2;X)$$

$$\times S_n^m[yX^\mu] E_{(\alpha,\beta)}^{\xi}[zX^{-\delta}]dx$$

$$= \frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}\Gamma(\xi)} \sum_{r=0}^{\infty} \sum_{l=0}^{[n/m]} a_r \frac{(\gamma)_r}{\left(\gamma+\frac{1}{2}\right)_r} \frac{(-n)_{ml}}{l!} A_{n,l} y^l (4ab+c)^{r+\mu l} \quad (3.16)$$

$$\times {}_2\Psi_2 \left[\begin{matrix} (\xi,1), (\lambda-r-\mu l+1/2,\delta); \\ (\beta,\alpha), (\lambda-r-\mu l+1,\delta); \end{matrix} \middle| \frac{z}{(4ab+c)^\delta} \right]$$

COROLLARY - 17

Let $a > 0;\ b \geq 0;\ c + 4ab > 0;\ \Re(\lambda - r - \mu l + \delta n) + 1/2 > 0, -\frac{1}{2} < \sigma - \rho - \gamma < \frac{1}{2}, \lambda, \mu, \delta, \alpha, \beta, \xi, z \in \mathbb{C}, \Re(\alpha) > 0, \Re(\beta) > 0, \Re(\xi) > 0$, $m, r \in \mathbb{N}$ and $a_r, A_{n,l}\ (n, l \geq 0)$ are arbitrary (real or complex) constants, then

$$\int_0^\infty \frac{1}{x^2} X^{-\lambda-1} {}_2F_1(\sigma,\rho;\gamma+1/2;X) {}_2F_1(\gamma-\sigma,\gamma-\rho;\gamma+1/2;X)$$

$$\times S_n^m[yX^\mu]E_{(\alpha,\beta)}^{\xi}[zX^{-\delta}]dx$$

$$= \frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}\Gamma(\xi)} \sum_{r=0}^{\infty} \sum_{l=0}^{[n/m]} a_r \frac{(\gamma)_r}{\left(\gamma+\frac{1}{2}\right)_r} \frac{(-n)_{ml}}{l!} A_{n,l} y^l (4ab+c)^{r+\mu l} \quad (3.17)$$

$$\times_2 \Psi_2 \left[\begin{matrix}(\xi,1),(\lambda-r-\mu l+1/2,\delta); \\ (\beta,\alpha),(\lambda-r-\mu l+1,\delta);\end{matrix} \middle| \frac{z}{(4ab+c)^\delta}\right]$$

COROLLARY - 18

Let $a > 0;\ b \geq 0;\ c + 4ab > 0;\ \Re(\lambda - r - \mu l + \delta n) + 1/2 > 0, -\frac{1}{2} < \sigma - \rho - \gamma < \frac{1}{2}, \lambda, \mu, \delta, \alpha, \beta, \xi, z \in \mathbb{C}, \Re(\alpha) > 0, \Re(\beta) > 0, \Re(\xi) > 0$, $m, r \in \mathbb{N}$ and $a_r, A_{n,l}$ $(n, l \geq 0)$ are arbitrary (real or complex) constants, then

$$\int_0^\infty \left(a + \frac{b}{x^2}\right) X^{-\lambda-1} {}_2F_1(\sigma, \rho; \gamma + 1/2; X) {}_2F_1(\gamma - \sigma, \gamma - \rho; \gamma + 1/2; X)$$

$$\times S_n^m[yX^\mu]E_{(\alpha,\beta)}^{\xi}[zX^{-\delta}]dx$$

$$= \frac{\sqrt{\pi}}{(4ab+c)^{\lambda+1/2}\Gamma(\xi)} \sum_{r=0}^{\infty} \sum_{l=0}^{[n/m]} a_r \frac{(\gamma)_r}{\left(\gamma+\frac{1}{2}\right)_r} \frac{(-n)_{ml}}{l!} A_{n,l} y^l (4ab+c)^{r+\mu l} \quad (3.18)$$

$$\times_2 \Psi_2 \left[\begin{matrix}(\xi,1),(\lambda-r-\mu l+1/2,\delta); \\ (\beta,\alpha),(\lambda-r-\mu l+1,\delta);\end{matrix} \middle| \frac{z}{(4ab+c)^\delta}\right]$$

If we put $k = \xi = \kappa = 1$ in results (2.3), (2.8) and (2.9), Then the results in the eqn. (2.3), (2.8) and (2.9) reduce to the following results:

COROLLARY - 19

Let $a > 0;\ b \geq 0;\ c + 4ab > 0;\ \Re(\lambda - r - \mu l + \delta n) + 1/2 > 0, -\frac{1}{2} < \sigma - \rho - \gamma < \frac{1}{2}, \lambda, \mu, \delta, \alpha, \beta, z \in \mathbb{C}, \Re(\alpha) > 0, \Re(\beta) > 0$, $m, r \in \mathbb{N}$ and $a_r, A_{n,l}$ $(n, l \geq 0)$ are arbitrary (real or complex) constants, then

$$\int_0^\infty X^{-\lambda-1} {}_2F_1(\sigma, \rho; \gamma + 1/2; X) {}_2F_1(\gamma - \sigma, \gamma - \rho; \gamma + 1/2; X)$$

$$\times S_n^m[yX^\mu]E_{(\alpha,\beta)}[zX^{-\delta}]dx$$

$$= \frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{\infty} \sum_{l=0}^{[n/m]} a_r \frac{(\gamma)_r}{\left(\gamma+\frac{1}{2}\right)_r} \frac{(-n)_{ml}}{l!} A_{n,l} y^l (4ab+c)^{r+\mu l} \quad (3.19)$$

$$\times {}_2\Psi_2 \left[\begin{matrix} (1,1), (\lambda - r - \mu l + 1/2, \delta); \\ (\beta, \alpha), (\lambda - r - \mu l + 1, \delta); \end{matrix} \Big| \frac{z}{(4ab+c)^{\delta}} \right]$$

COROLLARY - 20

Let $a > 0;\ b \geq 0;\ c + 4ab > 0;\ \Re(\lambda - r - \mu l + \delta n) + 1/2 > 0, -\frac{1}{2} < \sigma - \rho - \gamma < \frac{1}{2}, \lambda, \mu, \delta, \alpha, \beta, z \in \mathbb{C}, \Re(\alpha) > 0, \Re(\beta) > 0, m, r \in \mathbb{N}$ and $a_r, A_{n,l}$ $(n, l \geq 0)$ are arbitrary (real or complex) constants, then

$$\int_0^{\infty} \frac{1}{x^2} X^{-\lambda-1} {}_2F_1(\sigma, \rho; \gamma + 1/2; X) {}_2F_1(\gamma - \sigma, \gamma - \rho; \gamma + 1/2; X)$$

$$\times S_n^m[yX^{\mu}] E_{(\alpha,\beta)}[zX^{-\delta}] dx$$

$$= \frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{\infty} \sum_{l=0}^{[n/m]} a_r \frac{(\gamma)_r}{\left(\gamma+\frac{1}{2}\right)_r} \frac{(-n)_{ml}}{l!} A_{n,l} y^l (4ab+c)^{r+\mu l} \quad (3.20)$$

$$\times {}_2\Psi_2 \left[\begin{matrix} (1,1), (\lambda - r - \mu l + 1/2, \delta); \\ (\beta, \alpha), (\lambda - r - \mu l + 1, \delta); \end{matrix} \Big| \frac{z}{(4ab+c)^{\delta}} \right]$$

COROLLARY - 21

Let $a > 0;\ b \geq 0;\ c + 4ab > 0;\ \Re(\lambda - r - \mu l + \delta n) + 1/2 > 0, -\frac{1}{2} < \sigma - \rho - \gamma < \frac{1}{2}, \lambda, \mu, \delta, \alpha, \beta, z \in \mathbb{C}, \Re(\alpha) > 0, \Re(\beta) > 0, m, r \in \mathbb{N}$ and $a_r, A_{n,l}$ $(n, l \geq 0)$ are arbitrary (real or complex) constants, then

$$\int_0^{\infty} \left(a + \frac{b}{x^2}\right) X^{-\lambda-1} {}_2F_1(\sigma, \rho; \gamma + 1/2; X) {}_2F_1(\gamma - \sigma, \gamma - \rho; \gamma + 1/2; X)$$

$$\times S_n^m[yX^{\mu}] E_{(\alpha,\beta)}[zX^{-\delta}] dx$$

$$= \frac{\sqrt{\pi}}{(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{\infty} \sum_{l=0}^{[n/m]} a_r \frac{(\gamma)_r}{\left(\gamma+\frac{1}{2}\right)_r} \frac{(-n)_{ml}}{l!} A_{n,l} y^l (4ab+c)^{r+\mu l} \quad (3.21)$$

$$\times {}_2\Psi_2 \left[\begin{matrix} (1,1), (\lambda - r - \mu l + 1/2, \delta); \\ (\beta, \alpha), (\lambda - r - \mu l + 1, \delta); \end{matrix} \Big| \frac{z}{(4ab+c)^{\delta}} \right]$$

If we put $k = \xi = \beta = \kappa = 1$ in results (2.3), (2.8) and (2.9), Then the results in the eqn. (2.3), (2.8) and (2.9) reduce to the following results:

COROLLARY - 22

Let $a > 0;\ b \geq 0;\ c + 4ab > 0;\ \Re(\lambda - r - \mu l + \delta n) + 1/2 > 0, -\frac{1}{2} < \sigma - \rho - \gamma < \frac{1}{2}, \lambda, \mu, \delta, \alpha, z \in \mathbb{C}, \alpha \geq 0, m, r \in \mathbb{N}$ and $a_r, A_{n,l}$ $(n, l \geq 0)$ are arbitrary (real or complex) constants, then

$$\int_0^\infty X^{-\lambda-1}{}_2F_1(\sigma,\rho;\gamma+1/2;X)_2F_1(\gamma-\sigma,\gamma-\rho;\gamma+1/2;X)$$

$$\times S_n^m[yX^\mu]E_\alpha[zX^{-\delta}]dx$$

$$= \frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}} \sum_{r=0}^\infty \sum_{l=0}^{[n/m]} a_r \frac{(\gamma)_r}{\left(\gamma+\frac{1}{2}\right)_r} \frac{(-n)_{ml}}{l!} A_{n,l} y^l (4ab+c)^{r+\mu l} \quad (3.22)$$

$$\times {}_2\Psi_2\left[\begin{matrix}(1,1),(\lambda-r-\mu l+1/2,\delta);\\(1,\alpha),(\lambda-r-\mu l+1,\delta);\end{matrix}\middle|\frac{z}{(4ab+c)^\delta}\right]$$

COROLLARY - 23

Let $a > 0;\ b \geq 0;\ c + 4ab > 0;\ \Re(\lambda - r - \mu l + \delta n) + 1/2 > 0, -\frac{1}{2} < \sigma - \rho - \gamma < \frac{1}{2}, \lambda, \mu, \delta, \alpha, z \in \mathbb{C}, \alpha \geq 0, m, r \in \mathbb{N}$ and $a_r, A_{n,l}$ $(n, l \geq 0)$ are arbitrary (real or complex) constants, then

$$\int_0^\infty \frac{1}{x^2} X^{-\lambda-1}{}_2F_1(\sigma,\rho;\gamma+1/2;X)_2F_1(\gamma-\sigma,\gamma-\rho;\gamma+1/2;X)$$

$$\times S_n^m[yX^\mu]E_\alpha[zX^{-\delta}]dx$$

$$= \frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}} \sum_{r=0}^\infty \sum_{l=0}^{[n/m]} a_r \frac{(\gamma)_r}{\left(\gamma+\frac{1}{2}\right)_r} \frac{(-n)_{ml}}{l!} A_{n,l} y^l (4ab+c)^{r+\mu l} \quad (3.23)$$

$$\times {}_2\Psi_2\left[\begin{matrix}(1,1),(\lambda-r-\mu l+1/2,\delta);\\(1,\alpha),(\lambda-r-\mu l+1,\delta);\end{matrix}\middle|\frac{z}{(4ab+c)^\delta}\right]$$

COROLLARY - 24

Let $a > 0;\ b \geq 0;\ c + 4ab > 0;\ \Re(\lambda - r - \mu l + \delta n) + 1/2 > 0, -\frac{1}{2} < \sigma - \rho - \gamma < \frac{1}{2}, \lambda, \mu, \delta, \alpha, z \in \mathbb{C}, \alpha \geq 0$, $m, r \in \mathbb{N}$ and $a_r, A_{n,l}$ $(n, l \geq 0)$ are arbitrary (real or complex) constants, then

$$\int_0^\infty \left(a + \frac{b}{x^2}\right) X^{-\lambda-1} {}_2F_1(\sigma, \rho; \gamma + 1/2; X) {}_2F_1(\gamma - \sigma, \gamma - \rho; \gamma + 1/2; X)$$

$$\times S_n^m[yX^\mu] E_\alpha[zX^{-\delta}] dx$$

$$= \frac{\sqrt{\pi}}{(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{\infty} \sum_{l=0}^{[n/m]} a_r \frac{(\gamma)_r}{\left(\gamma+\frac{1}{2}\right)_r} \frac{(-n)_{ml}}{l!} A_{n,l} y^l (4ab + c)^{r+\mu l} \quad (3.24)$$

$$\times_2 \Psi_2 \left[\begin{matrix} (1,1), (\lambda - r - \mu l + 1/2, \delta); \\ (1, \alpha), (\lambda - r - \mu l + 1, \delta); \end{matrix} \quad \middle| \frac{z}{(4ab+c)^\delta}\right]$$

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