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Generalized Projective Synchronization of Six-Term Sundarapandian Chaotic Systems by Adaptive Control

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ABSTRACT

This paper investigates the generalized projective synchronization (GPS) of identical six-term Sundarapandian 3-D chaotic systems (2013) with unknown parameters via adaptive control. Sundarapandian chaotic system (2013) is a new six-term 3-D chaotic system which has two saddle-node focus equilibrium points. The maximal Lyapunov exponent (MLE) for the six-term Sundarapandian 3-D chaotic system was found as $L_1 = 3.2827$ and Lyapunov dimension as $D_L = 2.1668$. Generalized projective synchronization (GPS) of chaotic systems is a new type of chaos synchronization, which generalizes common types of synchronization such as complete synchronization, anti-synchronization, hybrid synchronization and projective synchronization. In this paper, we derive new results for the GPS of identical Sundarapandian 3-D chaotic systems with unknown system parameters. Lyapunov stability theory and adaptive control theory have been applied for deriving the new GPS results for Sundarapandian 3-D chaotic systems with unknown system parameters. MATLAB simulations have been shown to demonstrate the validity and effectiveness of the adaptive GPS results derived for the Sundarapandian chaotic systems.

Keywords: Chaos, chaotic systems, synchronization, generalized projective synchronization, Sundarapandian system.

1. INTRODUCTION

Chaos theory has been developed and extensively studied over the past four decades. A chaotic system is a nonlinear dissipative dynamical system that is highly sensitive to even small perturbations in its initial conditions. This sensitivity of chaotic systems is popularly referred to as the *butterfly effect* [1].

The Lyapunov exponent of a dynamical system is a quantitative measure that characterizes the rate of separation of infinitesimally close trajectories of the system. Thus, a chaotic system is also defined as a dynamical system having at least one positive Lyapunov exponent.

The synchronization of chaotic system was first studied by Fujisaka and Yemada [2] in 1983. This problem did not receive great attention until Pecora and Carroll [3-4] published their results on chaos synchronization in early 1990s. From then on, chaos synchronization has been rigorously studied in the last four decades. Chaos theory has been applied to a variety of fields such as lasers [5-6], oscillators [7-8], chemical reactions [9-10], biology [11-12], neural networks [13-15], ecology [16-17], robotics [18-20], etc.

Chaos synchronization problem can be stated as follows. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of the synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically. Chaos synchronization has applications in secure communications [21-23], cryptosystems [24-25], encryption [26-27] etc.

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The chaos synchronization problem has been paid great attention in the literature and a variety of impressive approaches have been proposed. A few important methods for the chaos synchronization problem can be listed as active control method [28-33], adaptive control method [34-40], sampled-data feedback control method [41-42], time-delay feedback approach [43], backstepping method [44-47], sliding mode control method [48-50], etc.

In this paper, we derive new results for the generalized projective synchronization (GPS) for Sundarapandian six-term novel 3-D chaotic systems ([51], 2013). Generalized projective synchronization [52-55] is a new type of synchronization of chaotic systems, which generalizes common types of synchronization such as complete synchronization [28-33], anti-synchronization [56-59], hybrid synchronization [50-63], projective synchronization [64], etc.

The rest of this paper is organized as follows. Section 2 contains a description and analysis of the Sundarapandian six-term 3-D chaotic system (2013). Section 3 contains the main results of this paper, *viz.* adaptive controller design for the GPS of identical Sundarapandian systems. Numerical simulations using MATLAB are shown to illustrate the main results derived in this paper. Section 4 has a summary of the main results derived in this paper.

2. SUNDARAPANDIAN SIX-TERM CHAOTIC SYSTEM

In this section, we describe the equations and properties of the Sundarapandian six-term novel 3-D chaotic system ([51], 2013).

Sundarapandian six-term chaotic system is modelled by the 3-D dynamics

$$\begin{aligned}\dot{x}_1 &= a(x_2 - x_1) + x_2 x_3 \\ \dot{x}_2 &= -b x_2 x_3 \\ \dot{x}_3 &= \exp(x_1 x_2) - c\end{aligned} \tag{1}$$

where x_1, x_2, x_3 are the state variables and a, b, c are positive, constant, parameters of the system.

The system (1) exhibits a chaotic attractor for the values

$$a = 140, \quad b = 50, \quad c = 90 \tag{2}$$

Figure 1 shows the strange chaotic attractor of the system (1).

Sundarapandian system (1) is invariant under the coordinates transformation

$$(x_1, x_2, x_3) \to (-x_1, -x_2, x_3) \tag{3}$$

which persists for all values of the parameters. Thus, the system (1) has rotation symmetry about the x_3– axis. Hence, any non-trivial trajectory of the system (1) must have a twin trajectory.

For the parameter values in (2), the Sundarapandian system (1) has two equilibria

$$E_1 : (2.1213,\ 2.1213,\ 0) \ \text{and} \ E_2 : (-2.1213,\ -2.1213,\ 0).$$

Using Lyapunov stability theory, it can be shown that the equilibria E_1 and E_2 are saddle-nodes, which are unstable. Hence, E_1 and E_2 are both unstable equilibrium points.

The Lyapunov exponents of the Sundarapandian chaotic system (1) are calculated as

$$L_1 = 3.2827, \quad L_2 \approx 0, \quad L_3 = -19.6751 \tag{4}$$

Thus, the maximal Lyapunov exponent (MLE) of the Sundarapandian system (1) is given by

Also, the Lyapunov dimension of the Sundarapandian chaotic system (1) is calculated as

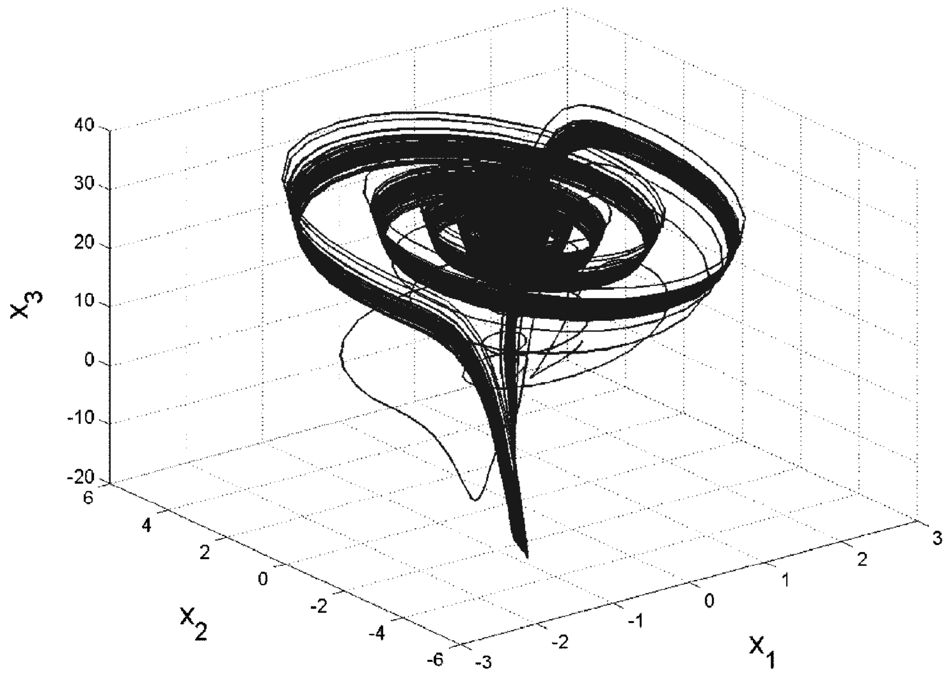

Figure 1: Strange Attractor of the Sundarapandian Chaotic System

$$D_L = j + \frac{\sum_{i=1}^{j} L_i}{|L_{j+1}|} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.1668, \tag{5}$$

which is fractional.

The dynamics of the Lyapunov exponents of the Sundarapandian chaotic system (1) is depicted in Figure 2.

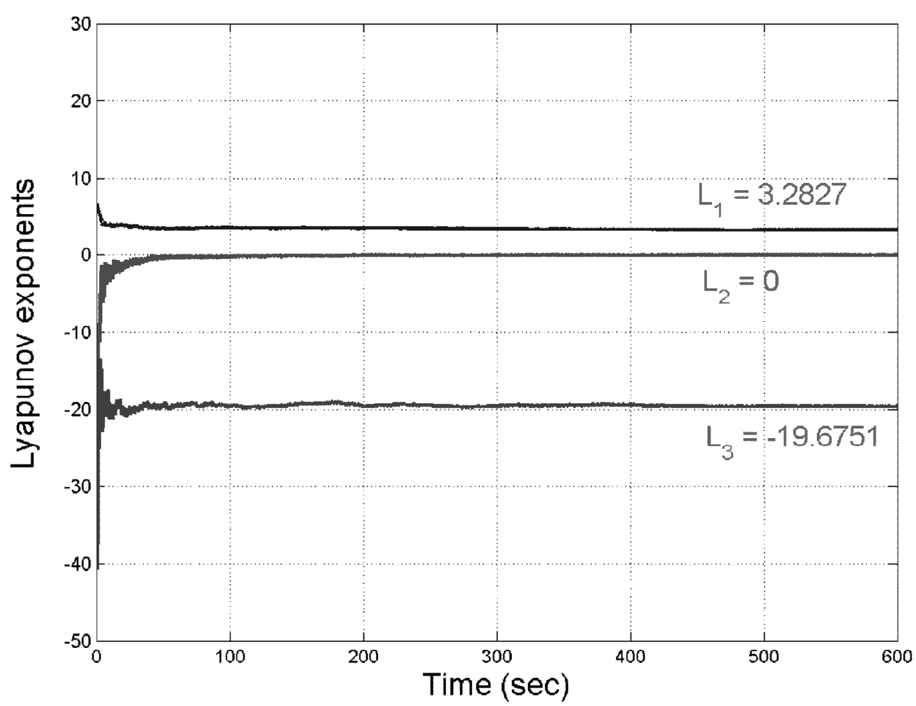

Figure 2: Dynamics of the Lyapunov Exponents

3. GPS OF IDENTICAL SUNDARAPANDIAN SIX-TERM SYSTEMS

In this section, we devise an adaptive controller to achieve generalized projective synchronization (GPS) of identical Sundarapandian six-term chaotic systems (2013).

As the master system, we consider the six-term Sundarapandian system given by

$$\begin{aligned}\dot{x}_1 &= a(x_2 - x_1) + x_2 x_3 \\ \dot{x}_2 &= -b x_2 x_3 \\ \dot{x}_3 &= \exp(x_1 x_2) - c\end{aligned} \tag{6}$$

where x_1, x_2, x_3 are state variables and a, b, c are constant, unknown, parameters of the system.

As the slave system, we consider the controlled Sundarapandian system given by

$$\begin{aligned}\dot{y}_1 &= a(y_2 - y_1) + y_2 y_3 + u_1 \\ \dot{y}_2 &= -b y_2 y_3 + u_2 \\ \dot{y}_3 &= \exp(y_1 y_2) - c + u_3\end{aligned} \tag{7}$$

where y_1, y_2, y_3 are state variables and u_1, u_2, u_3 are adaptive controls to be designed using estimates $A(t), B(t), C(t)$ for the unknown parameters $a, b, c,$ respectively.

The generalized projective synchronization (GPS) error between the Sundarapandian systems (6) and (7) is defined by

$$\begin{aligned}e_1(t) &= y_1(t) - \eta_1 x_1(t) \\ e_2(t) &= y_2(t) - \eta_2 x_1(t) \\ e_3(t) &= y_3(t) - \eta_3 x_1(t)\end{aligned} \tag{8}$$

where the GPS scales η_1, η_2, η_3 are real constants.

The GPS error dynamics is obtained as

$$\begin{aligned}\dot{e}_1(t) &= a(y_2 - \eta_1 x_2 - e_1) + y_2 y_3 - \eta_1 x_2 x_3 + u_1 \\ \dot{e}_2(t) &= -b(y_2 y_3 - \eta_2 x_2 x_3) + u_2 \\ \dot{e}_3(t) &= \exp(y_1 y_2) - \eta_3 \exp(x_1 x_2) + c(\eta_3 - 1) + u_3\end{aligned} \tag{9}$$

We consider an adaptive controller defined by

$$\begin{aligned}u_1(t) &= -A(t)(y_2 - \eta_1 x_2 - e_1) - y_2 y_3 + \eta_1 x_2 x_3 - k_1 e_1 \\ u_2(t) &= B(t)(y_2 y_3 - \eta_2 x_2 x_3) - k_2 e_2 \\ u_3(t) &= -\exp(y_1 y_2) + \eta_3 \exp(x_1 x_2) - C(t)(\eta_3 - 1) - k_3 e_3\end{aligned} \tag{10}$$

where the gains k_1, k_2, k_3 are positive constants.

Substituting the control law (10) into (9), we obtain the closed-loop error dynamics

$$\begin{aligned}\dot{e}_1(t) &= (a - A(t))(y_2 - \eta_1 x_2 - e_1) - k_1 e_1 \\ \dot{e}_2(t) &= -(b - B(t))(y_2 y_3 - \eta_2 x_2 x_3) - k_2 e_2 \\ \dot{e}_3(t) &= (c - C(t))(\eta_3 - 1) - k_3 e_3\end{aligned} \tag{11}$$

The parameter estimation error is defined by

$$\begin{aligned} e_a(t) &= a - A(t) \\ e_b(t) &= b - B(t) \\ e_c(t) &= c - C(t) \end{aligned} \tag{12}$$

Using (12), the error dynamics (11) can be simplified as

$$\begin{aligned} \dot{e}_1 &= e_a(y_2 - \eta_1 x_2 - e_1) - k_1 e_1 \\ \dot{e}_2 &= -e_b(y_2 y_3 - \eta_2 x_2 x_3) - k_2 e_2 \\ \dot{e}_3 &= e_c(\eta_3 - 1) - k_3 e_3 \end{aligned} \tag{13}$$

Differentiating (12) with respect to t, we get

$$\begin{aligned} \dot{e}_a &= -\dot{A} \\ \dot{e}_b &= -\dot{B} \\ \dot{e}_c &= -\dot{C} \end{aligned} \tag{14}$$

Next, we use Lyapunov stability theory to find an update law for the parameter estimates $A(t)$, $B(t)$ and $C(t)$.

We consider the Lyapunov function defined by

$$V(e_1, e_2, e_3, e_a, e_b, e_c) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2 + e_c^2\right), \tag{15}$$

which is a quadratic and positive definite function on R^6.

Taking the time-derivative of V along the trajectories of (13) and (14), we obtain

$$\begin{aligned} \dot{V} = &-k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_a\left[e_1(y_2 - \eta_1 x_2 - e_1) - \dot{A}\right] \\ &+ e_b\left[-e_2(y_2 y_3 - \eta_2 x_2 x_3) - \dot{B}\right] + e_c\left[e_3(\eta_3 - 1) - \dot{C}\right] \end{aligned} \tag{16}$$

In view of Eq. (16), the parameter estimates update law is defined as

$$\begin{aligned} \dot{A} &= e_1(y_2 - \eta_1 x_2 - e_1) + k_4 e_a \\ \dot{B} &= -e_2(y_2 y_3 - \eta_2 x_2 x_3) + k_5 e_b \\ \dot{C} &= e_3(\eta_3 - 1) + k_6 e_c \end{aligned} \tag{17}$$

where k_4, k_5, k_6 are positive constants.

Next, we state and prove the main result of this section.

Theorem 1. The identical Sundarapandian novel chaotic systems given by (6) and (7) with unknown parameters a, b, c are globally and exponentially generalized projective synchronized (GPS) by the adaptive controller (10) and the parameter estimates update law (17), where the gains k_i, $(i = 1, 2, \ldots, 6)$ are positive constants. Moreover, the parameter estimation errors $e_a(t)$, $e_b(t)$ and $e_c(t)$ globally and exponentially converge to zero for all initial conditions.

Proof. We use Lyapunov stability theory [65] to prove this result.

Consider the quadratic Lyapunov function V defined by Eq. (15).

By substituting the parameter estimates update law into the dynamics (16), we obtain

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_a^2 - k_5 e_b^2 - k_6 e_c^2, \tag{18}$$

which is a quadratic and negative definite function on R^6.

Thus, by Lyapunov stability theory, it follows that the GPS errors e_1, e_2, e_3 and the parameter estimation errors e_a, e_b, e_c are globally exponentially stable.

This completes the proof. n

NUMERICAL RESULTS

For numerical simulations, the classical fourth-order Runge-Kutta method is used to solve the identical Sundarapandian systems (6) and (7) with the adaptive control (10) and the parameter estimates update law (17).

For the Sundarapandian systems (6) and (7), the parameter values are taken as those which result in chaotic behaviour of the systems, *viz.*

$$a = 140, \quad b = 50, \quad c = 90 \tag{19}$$

We take the feedback gains as $k_i = 5$ for $i = 1, 2, \ldots, 6$.

We take the GPS scales as $\eta_1 = 0.6$, $\eta_2 = -2.3$ and $\eta_3 = 1.7$.

The initial values of the master system (6) are taken as

$$x_1(0) = 2.4, \; x_2(0) = -1.7, \; x_3(0) = 0.8 \tag{20}$$

The initial values of the slave system (7) are taken as

$$y_1(0) = 1.6, \; y_2(0) = 3.1, \; y_3(0) = -1.5 \tag{21}$$

The initial values of the parameter estimates are taken as

$$A(0) = 6, \; B(0) = 22, \; C(0) = 14. \tag{22}$$

Figure 3 depicts the GPS of the Sundarapandian chaotic systems (6) and (7). Figure 4 depicts the time-history of the GPS errors e_1, e_2, e_3.

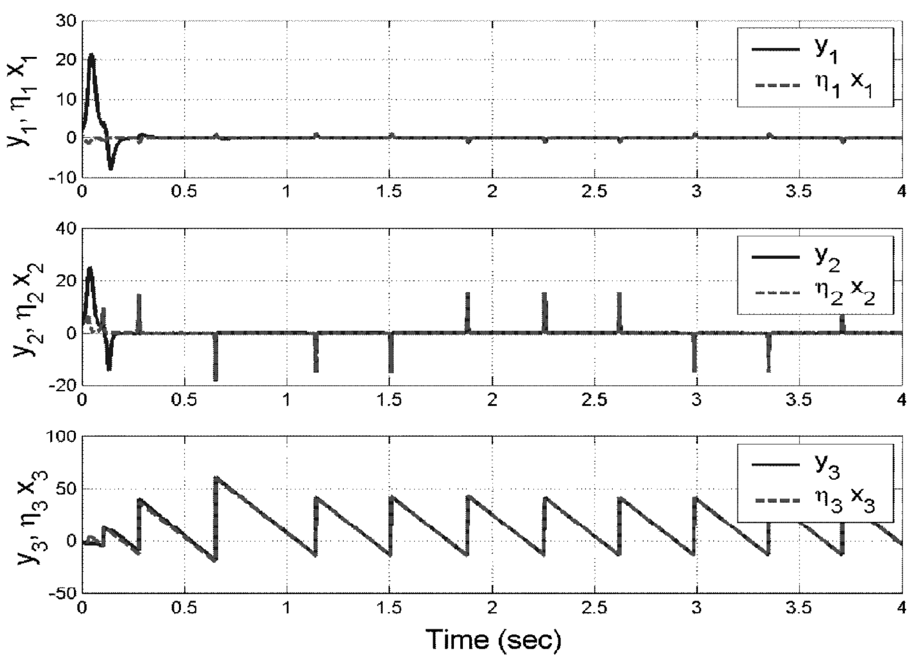

Figure 3: GPS of Identical Sundarapandian Chaotic Systems

Figure 5 depicts the time-history of the parameter estimates $A(t)$, $B(t)$. Figure 6 depicts the time-history of the parameter estimation errors e_a, e_b, e_c.

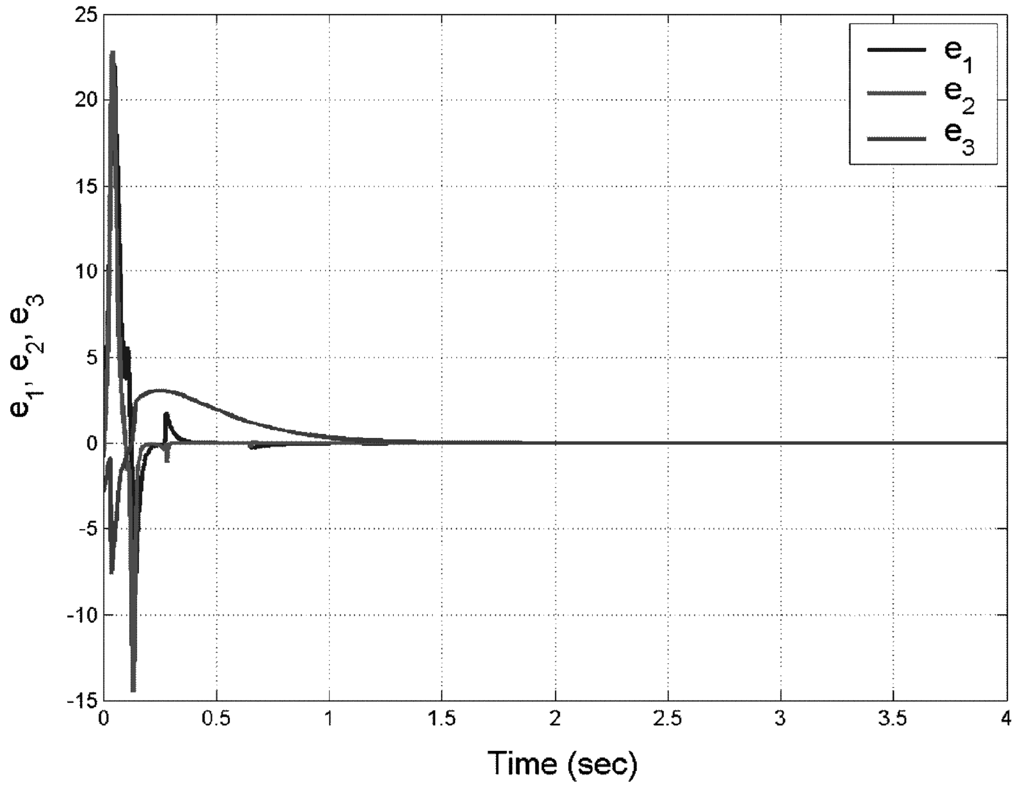

Figure 4: Time History of GPS Errors e_1, e_2, e_3

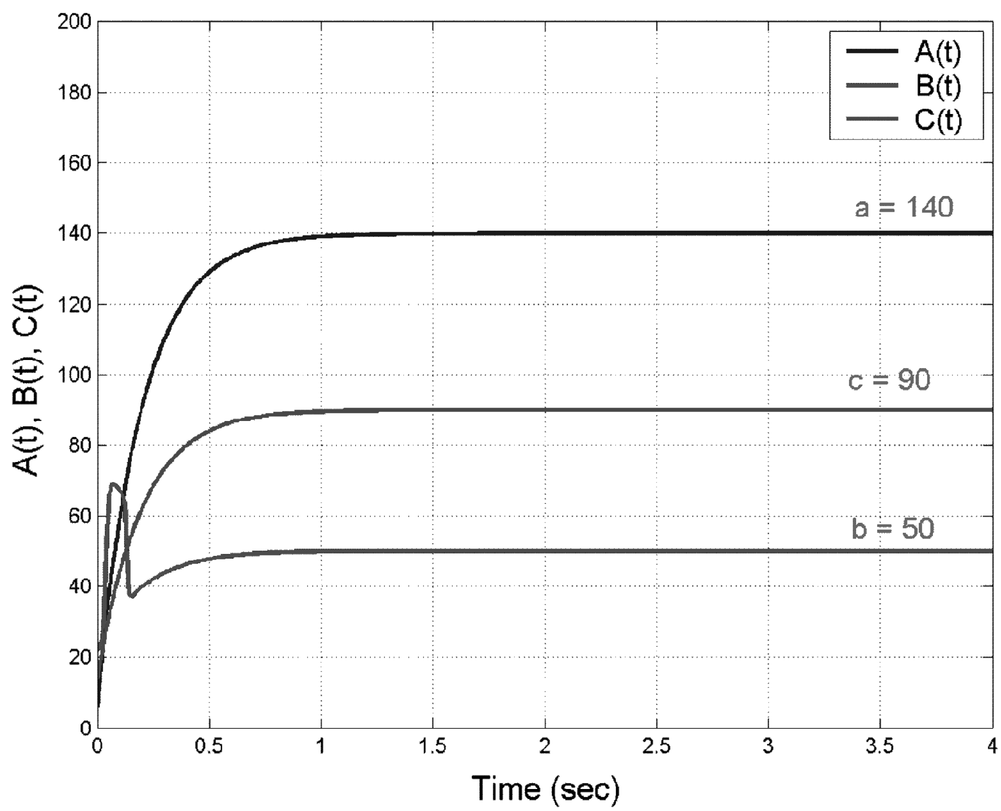

Figure 5: Time History of Parameter Estimates $A(t)$, $B(t)$, $C(t)$

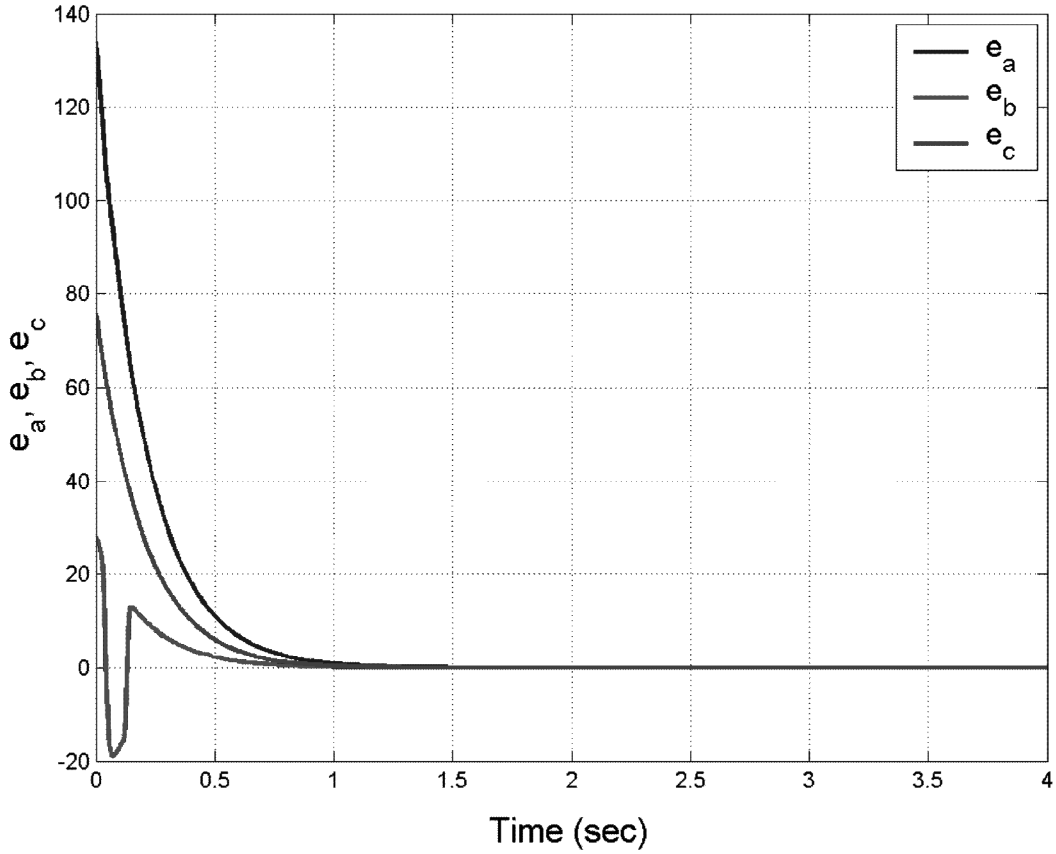

Figure 6: Time History of Parameter Estimates e_a, e_b, e_c

4. CONCLUSIONS

Generalized projective synchronization is a general type of synchronization, which generalizes common types of synchronization such as complete synchronization (CS), anti-synchronization (AS), hybrid synchronization (HS), projective synchronization (PS), etc. Sundarapandian chaotic system (2013) is a new six-term 3-D chaotic system which has two saddle-node focus equilibrium points. The maximal Lyapunov exponent (MLE) for the six-term Sundarapandian 3-D chaotic system was found as $L_1 = 3.2827$ and Lyapunov dimension as $D_L = 2.1668$. In this paper, we have derived new results for the generalized projective synchronization (GPS) of identical Sundarapandian six-term chaotic systems (2013) with unknown parameters. Main results were established using adaptive control theory and Lyapunov stability theory. MATLAB simulations were shown to demonstrate the main results derived in this paper.

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