IJCTA, 9(35), 2016, pp. 23-36

Optimal Control of the Temperature Field of a Complex Control System

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Abstract : The article is dedicated to the improvement of the stabilization system of temperature fields of complex control systems. During the synthesis of the control system, the function of the initial heating was obtained, some experiments were performed to analyze the distribution of the temperature fields, and the analysis of the results was carried out. To provide the maximal accuracy and adequacy of the simulation model, all calculations were made in the enlarged three-dimensional mathematic formulation. By means of the obtained regulator, the hardware and software system was created in the programming language Pascal allowing to simulate the behavior of the temperature fields. The practical results of the research data allow to draw a conclusion on the possibility to change the structures of heating elements of complex controlled objects, that is, to replace solid heating elements by sampling elements.

Keywords: Green function, thermal field, sampling interval, controlled object, analysis, synthesis.

1. INTRODUCTION

An electric tunnel kiln of conveyor type has some advantages and disadvantages. One of the main disadvantages of electric kilns of this type is their high energy consumption. This is connected to the necessity of the continuous supply of the electric power to the heating elements. In the tunnel kiln, heating elements are located along the whole length of the chamber.

Such big number of heating elements leads to a bigger consumption of electric power. And this, in its turn, influences significantly the final cost of the item. We shall consider the possibility to decrease the cost due to the use of sampling heating elements. The short-term switching off will help to save electric power, and as a result, it will decrease the cost of the item.

Setting of the problem

There are some methods of mathematical simulation to describe the behavior of dynamic control systems. One of the main methods of simulation is to study the system according to the set characteristics “input-output”. The data of characteristics were obtained at zero boundary conditions. This method can not take into account some states of the system; thus, the matrix of the system states is not a comprehensive characteristic. Differential equations of the system state describe the dynamic and statistic properties and determine the transmission matrix of the system, although this connection is not one-one. This is explained by the fact that the transmission matrix can correspond to various differential equations.

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Let us consider this statement on the equation system described by the scheme shown in Figure 1.

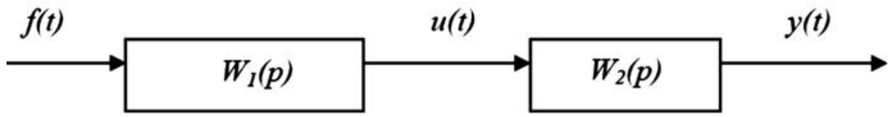

Figure 1: Control system

Where $f(t)$ is input action, (t) is control action, and $y(t)$ is output action.

$$\mathrm{W}_1(p) \quad = \quad \frac{p+a}{(p+b)(p+c)}$$

$$\mathrm{W}_2(p) \quad = \quad \frac{1}{p+a}$$

Then the transmission function of the control system will be the following:

$$\mathrm{W}(p) \quad = \quad \mathrm{W}_1(p) + \mathrm{W}_2(p) = \frac{p+a}{(p+b)(p+c)(p+a)}$$

It is easy to observe that when considering this system at the initial conditions, the inertial part $\mathrm{W}_2(p) = \dfrac{1}{p+a}$ will not be observed at any input actions. During the study of such systems, the notions of observability and controllability are usually introduced. For such cases, it is usual to represent the system in the terms of the state of spaces.

$$\begin{cases} \dot{x} = \mathrm{A}x + \mathrm{B}f \\ y = \mathrm{H}x \end{cases}$$

Where A, B, H are the matrices of the state of the system. During such consideration of the system, it can turn out that a part of input signals is absent. That means a part of differential equations or phase coordinates does not participate in the formation of the output signal.

According to the observability criterion, the system is observable if it is equivalent to the system of the type

$$\begin{cases} \dot{x}^1 = \mathrm{A}^n x^1 + \mathrm{B}^1 f \\ \dot{x}^2 = \mathrm{A}^n x^1 + \mathrm{A}^n x^2 + \mathrm{B}^2 f \\ y = \mathrm{H}^1 x^1 \end{cases}$$

Where dimension $x^1 = u_1^{\,1}$, and $x^2 - u_2^{\,1} = \mathrm{N} - u_1^{\,1}$.

In other words, the criterion of observability is the absence of the basis when the phase coordinates would be divided into two groups, and the coordinates of the group x^2 would not be included into the equation for the coordinates of the first group or into the algebraic relations for the output variables.

According to the controllability criterion, the system is controlled if it is equivalent to the system of the type

$$\begin{cases} \dot{x}^1 = \mathrm{A}^{11} x^1 + \mathrm{A}^{12} x^2 + \mathrm{B}^1 f \\ \dot{x}^1 = \mathrm{A}^{22} x^2 \\ y = \mathrm{H}^1 x^1 + \mathrm{H}^2 x^2 \end{cases}$$

Where dimension $x^1 = u_1^{\,1}$, and $x^2 - u_2^{\,1} = \mathrm{N} - u_1^{\,1}$.

In other words, the criterion of controllability is the absence of the basis when the system would be divided into two groups of equations, so the equations of the second group of equations would not be included into the phase coordinates of the first group or into the input signals.

The solving of these problems gives the possibility to search for the differential equations if the transmission functions (identification of the system) are known. However, the transmission function determines only one controlled and observed system. Thus, after restoration of the system according to the transmission function, one controlled and observed system will be obtained.

If we consider the possibility of formation of the observer for the space distributed non-linear objects, for such problems it is necessary to consider the equations of the type:

$$x = \mathrm{A1}x + \mathrm{A2}y + \mathrm{A3}z + \mathrm{Bu}(t) + \mathrm{D}f(t, x)$$

Where the phase vectors of the space coordinates *x, y, z; u* are control action at the disturbance *f(t,x)*; A1, A2, A3, B, D are matrices of the system state.

However, when considering such systems, there are some complexities, for example, the creation of the simulation of considered system. The impossibility to describe the adequate system is stipulated by the absence of the mechanisms of consideration of the observer from the point of view of the space distributed object. The second problem is the correspondence of the matrices of the system state to the infinite polynomials and that makes the matrices of the system state infinitely dimensional. And this makes the possibility of the mathematical analysis of such systems more complicated. If we consider the problem of controlling the temperature field of the tunnel kiln of conveyor type, we can do the following: it is necessary to consider a particular object as several points located at minimally possible distance from each other. At such sampling, the reaction of the observer can be determined as a sum of values of the system state at the fixed points of the space. Such approach to the solution of this problem does not erase the problem of the infinite matrices but it will bring the loss to the minimum at the synthesis of the observer. The problem of the infinitely dimensional matrices can be decreased by specifying the finite number of the space modes reflected in the matrices but this object will be approximated to the physical process.

The observation of the system behavior can be made not only by means of specialized observers but also by creating the matrices of the system state. In the generalized variant, the value returned by the observer is a numerical solution of some function in this point of the space. However, there are some methods of measuring the system in the points different from the system of the observer. Thus, for example, the Green function also returns the numerical value of the function in this object. Of course, the creation of the observer gives a more accurate solution taking into account all disturbing actions; however, the Green function creates the same numerical values when it is expanded into a Fourier series. Therefore, the use of the Green function is reasonable for the research of the processes in the distributed systems of data processing.

2. METHOD

Let us consider the mathematical simulation of the object with distributed parameters. The problem shall be set to determine the sample spacing in the space three-dimensional controlled object represented by the following:

$$\frac{\partial \mathrm{Q}(x,y,z,t)}{\partial t} - a^2\left[\frac{\partial^2 \mathrm{Q}(x,y,z,t)}{\partial x^2} + \frac{\partial^2 \mathrm{Q}(x,y,z,t)}{\partial y^2} + \frac{\partial^2 \mathrm{Q}(x,y,z,t)}{\partial z^2}\right] = f(x, y, z, t) ;$$

$$\mathrm{Q}(x, y, z, 0) = \mathrm{Q}_0(x, y, z) ;$$

$$\mathrm{Q}(0, y, z, t) = q_1(y, z, t) ; \mathrm{Q}(l_1, y, z, t) = q_2(y, z, t) ; \mathrm{Q}(x, 0, z, t)\, q_3 = (y, z, t) ;$$

$$\mathrm{Q}(\mathrm{x}, l_2, z, t) = q_4(x, z, t) ; \mathrm{Q}(x, y, 0, t) = q_5(x, y, t) ; \mathrm{Q}(x, y, l_3, t)\, q_3 = q_6(x, y, t) ;$$

$$0 \le x \le l_1 ;\ 0 \le y \le l_2 ;\ 0 \le z \le l_3 ;\ t \ge 0 ;\ > 0 ;$$

We shall calculate the indicators of the sensors of the temperature field by means of the Green function Let us consider this function in the form of the infinite Fourier series:

$$\mathrm{G}(x, y, z, \rho, \nu, \vartheta, t) = \frac{8}{l_1 \cdot l_2 \cdot l_3} \cdot \sum_{k,m,n=1}^{\infty} B_{k,m,n}(\cdot) \cdot \exp\left[-a^2\pi^2 \cdot t \cdot \left(\frac{k^2}{l_1^2} + \frac{m^2}{l_2^2} + \frac{n^2}{l_3^2}\right)\right]$$

$$\mathrm{B}_{k,m,n}(\cdot) = \sin\left(\frac{k \cdot \pi \cdot x}{l_1}\right) \cdot \sin\left(\frac{m \cdot \pi \cdot y}{l_2}\right) \cdot \sin\left(\frac{m \cdot \pi \cdot z}{l_3}\right) \cdot \sin\left(\frac{k \cdot \pi \cdot \rho}{l_1}\right) \cdot \sin\left(\frac{k \cdot \pi \cdot \nu}{l_2}\right) \cdot \sin\left(\frac{n \cdot \pi \cdot \vartheta}{l_3}\right);$$

The transfer function will be the following:

$$\mathrm{W}(x, y, z, \rho, \nu, \vartheta, s) = \frac{8}{l_1 \cdot l_2 \cdot l_3} \cdot \sum_{k,m,n=1}^{\infty} \frac{\mathrm{B}_{k,m,n}(\cdot)}{s + a^2\pi^2 \left(\frac{k^2}{l_1^2} + \frac{m^2}{l_2^2} + \frac{n^2}{l_3^2}\right)}$$

Considering the objects with distributed parameters, the user often faces the impossibility of mathematical analysis of the object due to its dimension. Let us consider the points of the temperature impact to determine the boundaries of its impact with the specified error. For this, we shall consider the object with the geometric dimensions l_1, l_2, l_3. We shall introduce the variable N and accept it as a number of sampling points located on every axis of coordinates. Considering this object, not only space distribution shall be taken into account but also the interconnection of sampling heating elements. However, for the consideration of the system to determine the impact of the heating points we shall consider the heating element.

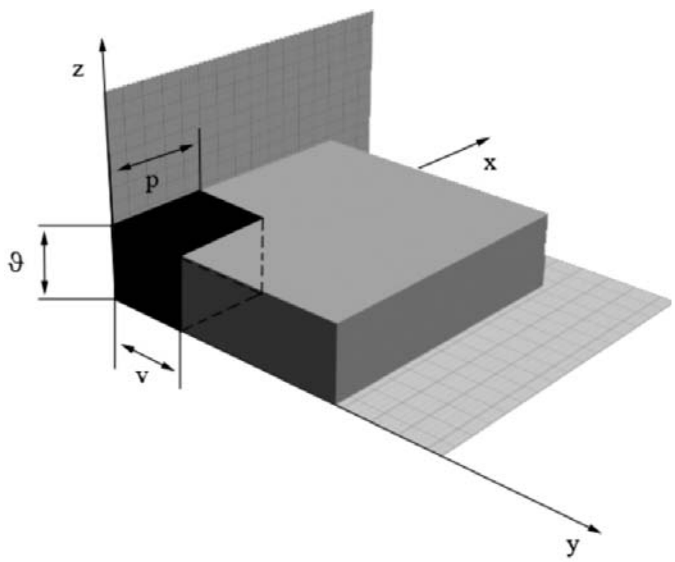

Figure 2: Three-dimensional controlled object

In such a case, the sample spacing for the controlled object (see Figure 2) shall be determined by the following:

1. $\rho_1 = \frac{l_1}{\mathrm{N}+1}$ sample spacing on the axis x
2. $\nu_1 = \frac{l_2}{\mathrm{N}+1}$ sample spacing on the axis y
3. $\vartheta_1 = \frac{l_3}{\mathrm{N}+1}$ sample spacing on the axis z

Let us record the generalized algorithm to determine the sample spacing of the controlled object with distributed parameters. This algorithm allows to calculate the sample spacing for the distributed controlled objects for which there is a solution in the form of the Green function.

- At the first stage, it is necessary to insert the initial and boundary values of the system: allowable value of error $-\Delta$, temperature range or a particular value of the temperature field – T_{set}, space coordinates of the distributed non-linear controlled object – l_1, l_2, l_3, the coefficient of thermal conductivity of the material – a^2 (the coefficient of thermal conductivity has a size in the second power $m/\sec^2$. Such a variable was introduced by Tikhonov and Samarsky to facilitate the work with control systems). The initial value of sampling points N shall be recorded.
- For every N, we shall calculate: ρ_1, v_1, ϑ_1, τ_1, τ_2, t_m.

$$\rho_1 = \frac{l_1}{N+1}, v_1 = \frac{l_2}{N+1}, \vartheta_1 = \frac{l_3}{N+1},$$

$$\tau_1 = A[l_1, l_2, l_3, \alpha, \rho_1, v_1, \vartheta_1, T_{set}],$$

$$\tau_2 = B[l_1, l_2, l_3, \rho_1, v_1, \vartheta_1, T_{set}, \tau_1],$$

$$\tau_m = C[l_1, l_2, l_3, a\ \rho_1, v_1, \vartheta_1, \tau_1].$$

- Let us check:

$$D[l_1, l_2, l_3, \rho_1, v_1, \vartheta_1] = E[\Delta, l_1, l_2, l_3, a, T_{set}, \tau_1, t_m] ;$$

$$F[l_1, l_2, l_3, \rho_1, v_1, \vartheta_1] \geq T_{set}.$$

It is necessary to remember the values N of those moments when the above stated conditions are fulfilled. Also, in such cases the calculation of the sample spacings shall be made: $S_{x1} = p_1$, $S_{y1} = v_1$, $S_{z1} = \vartheta_1$.

- At: $D[l_1, l_2, l_3, \rho_1, v_1, \vartheta_1] = H[l_1, l_2, l_3, a, \tau_1, \tau_2]$; $F[l_1, l_2, l_3, \rho_1, v_1, \vartheta_1] \geq T_{set}$.

  The value N shall be fixed and sample spacings shall be determined: $S_{x2} = p_1$, $S_{y2} = v_1$, $S_{z2} = \vartheta_1$.
- On the base of the obtained pairs of values at the different conditions, it is necessary to chose the smallest value:

$$S_x = \min\{S_{x1} ; S_{x2}\}, \ S_y = \min\{S_{y1} ; S_{y2}\}, S_z = \min\{S_{z1} ; S_{z2}\}.$$

Such algorithm allows to calculate the sample spacing of the multidimensional controlled object. Thus, it makes it possible to decrease the number of operations for the calculation of the thermal field and, as a result, to decrease the mathematical operations during simulation. Let us consider the object from the point of view of a three-dimensional distributed controlled object. As in the case with the two-dimensional object, we shall consider the impact of the components of the Fourier series at the initial moment of time. At the initial conditions of the system of equals

$$n \to \infty, t = 0, \ \tau = 0, \text{ we obtain :}$$

$$\exp\left[-a^2\pi^2 \cdot t \cdot \left(\frac{k^2}{l_1^2} + \frac{m^2}{l_2^2} + \frac{n^2}{l_3^2}\right)\right] = 1$$

The value of the temperature field in the space distributed controlled object will be the following:

$$G(x, y, z, \rho, v, \vartheta, t) = \frac{8}{l_1 \cdot l_2 \cdot l_3} \cdot \sum_{k,m,n=1}^{\infty} \sin\left(\frac{k \cdot \pi \cdot x}{l_1}\right) \cdot \sin\left(\frac{k \cdot \pi \cdot y}{l_2}\right) \cdot \sin\left(\frac{k \cdot \pi \cdot z}{l_3}\right) \times$$

$$\times \sin\left(\frac{k \cdot \pi \cdot \rho}{l_1}\right) \sin\left(\frac{k \cdot \pi \cdot v}{l_2}\right) \sin\left(\frac{k \cdot \pi \cdot \vartheta}{l_3}\right) \cdot \exp\left[-a^2\pi^2 \cdot t \cdot \left(\frac{k^2}{l_1^2} + \frac{m^2}{l_2^2} + \frac{n^2}{l_3^2}\right)\right]$$

The impulsive source creates the heating pulse in the point $x = \frac{l_1}{4}; y = \frac{l_2}{4}; z = \frac{l_3}{4};$ then the value of the temperature field will be the following:

$$\mathrm{G}(x, y, z, r, v, \vartheta, t) = \frac{8}{l_1 \cdot l_2 \cdot l_3} \cdot \sum_{k,m,n=1}^{\infty} \sin\left(\frac{k \cdot \pi \cdot \frac{l_1}{4}}{l_1}\right) \cdot \sin\left(\frac{k \cdot \pi \cdot \frac{l_2}{4}}{l_2}\right) \cdot \sin\left(\frac{k \cdot \pi \cdot \frac{l_3}{4}}{l_3}\right) \times$$

$$\times \sin\left(\frac{k \cdot \pi \cdot \rho}{4}\right) \sin\left(\frac{k \cdot \pi \cdot v}{4}\right) \sin\left(\frac{k \cdot \pi \cdot \vartheta}{4}\right)$$

For this function, the amplitude of the terms of the Fourier series is the following:

$$\mathrm{A}_n = \frac{8}{l_1 \cdot l_2 \cdot l_3} \left| \sin\frac{\pi p}{4} \cdot \sin\frac{\pi v}{4} \cdot \sin\frac{\pi \vartheta}{4} \right|$$

Let us calculate the function when we take into account only first five terms. In such calculation, we shall establish the following conditions:

$$x = \frac{l_1}{4}; y = \frac{l_2}{4}; z = \frac{l_3}{4}, k = m = n = 1, 2, 3, 4, 5$$

We obtain

$$\mathrm{T}_1(x, y, z) = \frac{8}{l_1 \cdot l_2 \cdot l_3} \left(\sin\frac{\pi}{4} \sin\frac{\pi}{l_2} \sin\frac{\pi}{l_1} x \right) \cdot \left(\sin\frac{\pi}{l_1} \sin\frac{\pi}{4} y \sin\frac{\pi}{l_2} y \right) \cdot \left(\sin\frac{\pi}{4} \sin\frac{\pi}{l_2} \sin\frac{\pi}{l_3} z \right) \cdot$$

$$\mathrm{T}_2(x, y, z) = \frac{8}{l_1 \cdot l_2 \cdot l_3} \left(\sin\frac{2\pi}{4} \sin\frac{2\pi}{l_2} \sin\frac{2\pi}{l_1} x \right) \cdot \left(\sin\frac{2\pi}{l_1} \sin\frac{2\pi}{4} y \sin\frac{2\pi}{l_2} y \right) \cdot \left(\sin\frac{2\pi}{4} \sin\frac{2\pi}{l_2} \sin\frac{2\pi}{l_3} z \right) \cdot;$$

$$\mathrm{T}_3(x, y, z) = \frac{8}{l_1 \cdot l_2 \cdot l_3} \left(\sin\frac{3\pi}{4} \sin\frac{3\pi}{l_2} \sin\frac{3\pi}{l_1} x \right) \cdot \left(\sin\frac{3\pi}{l_1} \sin\frac{3\pi}{4} y \sin\frac{3\pi}{l_2} y \right) \cdot \left(\sin\frac{3\pi}{4} \sin\frac{3\pi}{l_2} \sin\frac{3\pi}{l_3} z \right) \cdot$$

$$\mathrm{T}_4(x, y, z) = \frac{8}{l_1 \cdot l_2 \cdot l_3} \left(\sin\frac{4\pi}{4} \sin\frac{4\pi}{l_2} \sin\frac{4\pi}{l_1} x \right) \cdot \left(\sin\frac{4\pi}{l_1} \sin\frac{4\pi}{4} y \sin\frac{4\pi}{l_2} y \right) \cdot \left(\sin\frac{4\pi}{4} \sin\frac{4\pi}{l_2} \sin\frac{4\pi}{l_3} z \right) \cdot$$

$$\mathrm{T}_5(x, y, z) = \frac{8}{l_1 \cdot l_2 \cdot l_3} \left(\sin\frac{5\pi}{4} \sin\frac{5\pi}{l_2} \sin\frac{5\pi}{l_1} x \right) \cdot \left(\sin\frac{5\pi}{l_1} \sin\frac{5\pi}{4} y \sin\frac{5\pi}{l_2} y \right) \cdot \left(\sin\frac{5\pi}{4} \sin\frac{5\pi}{l_2} \sin\frac{5\pi}{l_3} z \right) \cdot$$

The Green function $\mathrm{G}(x, y, z, \rho, v, \vartheta, t) \geq 0$ at any x, y, z, p, v, θ, t. Then, it is evident that the range of positive values of the function in the range of values x will decrease. Narrowing will lead the system to the solution domain near the point $\rho = \frac{l_1}{4}; \nu = \frac{l_2}{4}; \vartheta = \frac{l_3}{4}$. At ρ, v $\vartheta \to \infty$, we obtain:

$$\frac{8}{l_1 \cdot l_2 \cdot l_3} \cdot \sum_{k,m,n=1}^{\infty} \sin\left(\frac{k \cdot \pi \cdot \frac{l_1}{4}}{l_1}\right) \cdot \sin\left(\frac{k \cdot \pi \cdot \frac{l_2}{4}}{l_2}\right) \cdot \sin\left(\frac{k \cdot \pi \cdot \frac{l_3}{4}}{l_3}\right) \cdot \sin\left(\frac{k \cdot \pi \cdot \rho}{l_1}\right) \sin\left(\frac{k \cdot \pi \cdot v}{l_2}\right) \sin\left(\frac{k \cdot \pi \cdot \vartheta}{l_3}\right)$$

That represents

$$\delta(x - p, y - v, z - \vartheta) = \frac{8}{l_1 \cdot l_2 \cdot l_3} \cdot \sum_{k,m,n=1}^{\infty} \sin\left(\frac{k \cdot \pi \cdot x}{l_1}\right) \cdot \sin\left(\frac{k \cdot \pi \cdot y}{l_2}\right) \cdot \sin\left(\frac{k \cdot \pi \cdot z}{l_3}\right) \cdot \sin\left(\frac{k \cdot \pi \cdot \rho}{l_1}\right)$$

$$\times \sin\left(\frac{k \cdot \pi \cdot v}{l_2}\right) \sin\left(\frac{k \cdot \pi \cdot \vartheta}{l_3}\right)$$

Such δ – function represented in the form of the Fourier series is considered to be the generalized function. These functions have a wide application in the system with distributed parameters, in particular for their simulation.

Arguments for such functions are usually coordinates of the controlled object.

$$\delta(x-p,\, y-v,\, z-\vartheta) \;=\; \begin{cases} \infty, & at \quad y = v \\ 0, & at \quad y \neq v \\ \infty, & at \quad x = p \\ 0, & at \quad x \neq p \\ \infty, & at \quad z = \vartheta \\ 0, & at \quad z \neq \vartheta \end{cases}$$

When considering the function $f(x, y, z)$, which has a space distribution $x \in [0, l_1]$, $y \in [0, l_2]$, $z \in [0, l_3]$, the following equation is unequal:

$$\int_0^{l_1}\int_0^{l_2}\int_0^{l} f(x,y,z)\delta(x-p,y-v,z-\vartheta)dx\cdot dy\cdot dz \;=\; f(p,\, v,\, \vartheta)\,;$$

Or in the integral approximation

$$f(p,\, v,\, \vartheta,\, \tau) \;=\; \int_0^{l_1}\int_0^{l_2}\int_0^{l_3}\int_{t_0}^{t} f(x,y,z,t)\cdot\delta(y-v)\cdot\delta(x-p)\cdot\delta(z-\vartheta)\cdot\delta(t-\tau)\cdot dx\cdot dy\cdot dz\cdot dt$$

In this case:

$$\mathrm{T}(x,\, y,\, z,\, t) \;=\; \int_0^{t}\int_0^{l}\int_0^{l}\int_0^{l} G(x,y,z,t,p,v,\vartheta,\tau)\delta(p-p_0)\delta(v-v_0)\delta(\tau-\tau_0)\delta(z-\vartheta)dpdvd\vartheta d\tau$$

$$= \mathrm{G}(x,\, y,\, z,\, t,\, p_0,\, v_0,\, \tau_0)$$

As a result,

$$\mathrm{G}(x,\, y,\, z,\, \rho,\, v,\, \vartheta,\, t) \;=\; \frac{8}{l_1\cdot l_2\cdot l_3}\cdot\sum_{k,m,n=1}^{\infty}\sin\left(\frac{k\cdot\pi\cdot x}{l_1}\right)\cdot\sin\left(\frac{k\cdot\pi\cdot y}{l_2}\right)\cdot\sin\left(\frac{k\cdot\pi\cdot z}{l_3}\right)$$

$$= \times\sin\left(\frac{k\cdot\pi\cdot\rho}{l_1}\right)\sin\left(\frac{k\cdot\pi\cdot v}{l_2}\right)\sin\left(\frac{k\cdot\pi\cdot\vartheta}{l_3}\right)\cdot\exp\left[-a^2\pi^2\cdot t\cdot\left(\frac{k^2}{l_1^2}+\frac{m^2}{l_2^2}+\frac{n^2}{l_3^2}\right)\right]$$

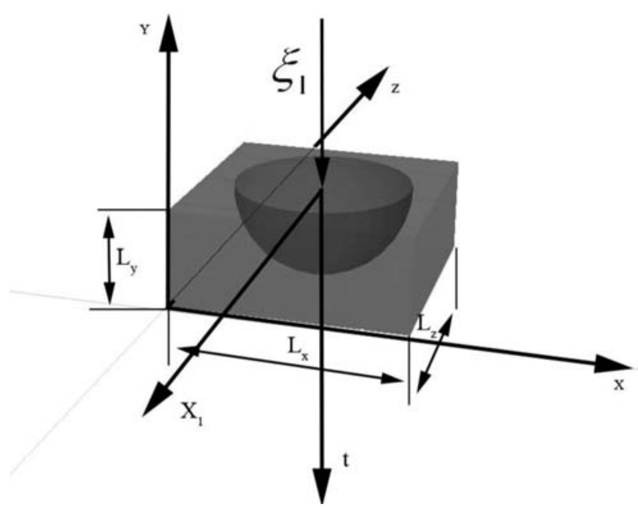

Figure 3: Graphic view of the initial heating function

This function shows the behavior of the system at the initial moment of time when the system receives the first impulse. Due to the fact that the system is in the state of rest, this impulse has the maximal amplitude. The distribution of the heat along the object is in different directions with the same speed, which is connected to the homogeneity of the material. In case of non-homogeneity of the material, the temperature process will be non-uniform, and this will lead to the various speed of heating of the material. Graphically, the function of initial heating will be the following (see Figure 3):

In some time, the action of the first heating pulse will be over, and to maintain the temperature, the second heating pulse occurs. The value of the temperature field will be a sum of two values. The function of the initial heating plus the function of the system behavior will be the following as time passes:

$$G(x_j, y_j, z_j, \rho, v, \vartheta, t,) = \times\sin\left(\frac{k\cdot\pi\cdot\rho_i}{l_1}\right)\cdot\sin\left(\frac{k\cdot\pi\cdot v_i}{l_2}\right)\cdot\sin\left(\frac{k\cdot\pi\cdot z_j}{l_3}\right)\cdot\sin\left(\frac{k\cdot\pi\cdot\vartheta_i}{l_3}\right)$$

$$\times\exp\left[-a^2\pi^2\cdot t\cdot\left(\frac{k^2}{l_1^2}+\frac{m^2}{l_2^2}+\frac{n^2}{l_3^2}\right)\right]\cdot\sum_p\sum_{k,m,n=1}^{\infty}\sin\left(\frac{k\cdot\pi\cdot x_j}{l_1}\right)\cdot\sin\left(\frac{k\cdot\pi\cdot y_j}{l_2}\right)$$

$$\times\sin\left(\frac{k\cdot\pi\cdot z_j}{l_3}\right)\cdot\sin\left(\frac{k\cdot\pi\cdot\rho_{z(p)}}{l_1}\right)\sin\left(\frac{k\cdot\pi\cdot v_{z(p)}}{l_2}\right)\times\sin\left(\frac{k\cdot\pi\cdot\vartheta_{z(p)}}{l_3}\right)$$

$$\times\exp\left[-a^2\pi^2\cdot(t-\tau)\cdot\left(\frac{k^2}{l_1^2}+\frac{m^2}{l_2^2}+\frac{n^2}{l_3^2}\right)\right]$$

On the base of the obtained function, we can calculate the value of the temperature in the point with the coordinates $x_1(x, y, z)$ at the moment of time τ passed since the moment of the switching on of the system t. Let us study the behavior of the temperature field in the framework Mathcad 14. The initial conditions of the system are the following: $k = 10$, $d = 9$, $\zeta_1 = x_1 = 1$, $T_{set} = 0.3$, $l_1 = l_2 = l_3 = 10$, $a^2 = 0.01$, $t = 3$, $x_1 = y_1 = z_1 = \xi 1 = p_1 = v_1 = \vartheta_1 = 1$, $t = 1.500$, $\xi_i, p_1, \vartheta_1 \in \{1, 2, 3, 4, 5, 7, 8, 9\}$.

When using the framework Mathcad 14, we obtain the values stated in Table 1.

Table 1

Results of the research of the thermal conductivity

Source No.	*d = 5*	*d = 6*	*d = 8*	*d = 9*	*d = 10*
1.	0.08	0.004	6.32	7.44	6.05
2.	0.06	0.003	4.70	5.53	4.50
3.	0.045	0.002	3.49	4.11	3.34
4.	0.034	0.001	2.60	3.06	2.49
5.	0.025	0.001	1.93	2.27	1.85
6.	0.018	0.0009	1.43	1.69	1.37
7.	0.014	0.0007	1.06	1.26	1.02
8.	0.010	0.0005	7.95	9.37	7.61
9.	0.007	0.0004	5.91	6.97	5.66
10.	0.005	0.0002	4.40	5.18	6.05

As it is seen from the results obtained, the same dependence of the heating elements on the temperatures in the heating points is observed as in the result of other studies.

3. PRACTICAL RESULTS

The tunnel kiln of conveyor type has some advantages and disadvantages. One of the main disadvantages of electric kilns of this type is the high cost of power supply because to heat one heating element the energy of 0.12% cost of one brick is used. The tunnel kiln for brick burning has a different curve of the temperature fields depending upon the type of brick.

The optimal decision, in our opinion, is the decrease in the cost due to the use of sampling heating elements. Power is saved and the cost of a brick is decreased due to the short-term switching on.

As one can see in the figures, the whole process of brick burning can be divided into 43 positions. Every position corresponds to its temperature mode. In connection with the fact that in all cases the mathematical simulation of the thermal process corresponds to

$$\frac{\partial T}{\partial t} = a\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right];$$

$$0 < x < L_x; 0 < y < L_y; 0 < z < L_z$$

such location of the heating elements can be found that can allow to create the same temperature curve at the pulse heating. Let us try to do this on the base of the Green function. Now we will try to stabilize the temperature in the set of segments of the temperature curve. For this, we will write a program in the framework Delphi to calculate such situation. We calculate the values of the temperature field in every one of the forty two sections.

For this, we will write the program in the framework Delphi, by means of which we can calculate the place and time of switching on of every heating element for every one of forty two positions. For this, the program shall input 129 variables that characterize the physical parameters of every section of the kiln (the length of a section, the necessary value of the temperature in the section, the number of heating elements).

By setting various parameters into the developed program, it is possible to obtain the calculation of the locations of the heating elements depending upon the set curve of the temperature process. The accuracy of the location of the place of installation of the heating element will depend upon the number of the located heating elements because the program will switch on only those heating elements that are necessary for the bringing of the controlled object to the required temperature mode.

This program allows to calculate the number and position of the point heating elements to obtain the set curve of the temperature field.

Let us estimate the efficiency of the replacement of the method of heating of the close heating elements from the constant element to the sampling point elements. For this we will conduct the research of the consumption of power resources for the preheating of the silicon carbide rod which in its turn will transfer the heat to the whole chamber. The calculation of the power of the silicon carbide rod is carried out according to the formula:

$$N = D. L. P. W$$

Where: N is power of the heater, W; D is a diameter of the working zone of the heater, cm; L is the length of the working zone of the heater, cm; P is pi character = 3.14; W is the average specific capacity (W/cm^2).

The specific capacity of the silicon carbide heating element is determined depending upon the atmosphere according to the following graphics.

So, knowing that silicon carbide rod has a diameter 25 × 400 × 1200 mm, R = 0.87 ohm + 10% at the temperature 1,070 °C, it can be calculated that such rod can have the capacity:

$$N = 2.5 \cdot 40 \cdot 3.16 \cdot 6 = 1{,}884 \text{ W}$$

It is also necessary to know the operating voltage supplied to the rods. It will be calculated from the Ohm's law according to the formula

$$U = \sqrt{N \times R}$$

Where: U is voltage; N is the power of the heater; R is resistance. It is noteworthy that in calculating the capacity, the resistance corresponding to the temperature mode was selected. For other cases with a higher temperature, the resistance also changes. Thus, for example, for the temperature of 1,400 °C, the resistance is by 20% higher and is equal to 1.04 Ohm. The specific resistance corresponds to the graphics shown in the Figure 4.

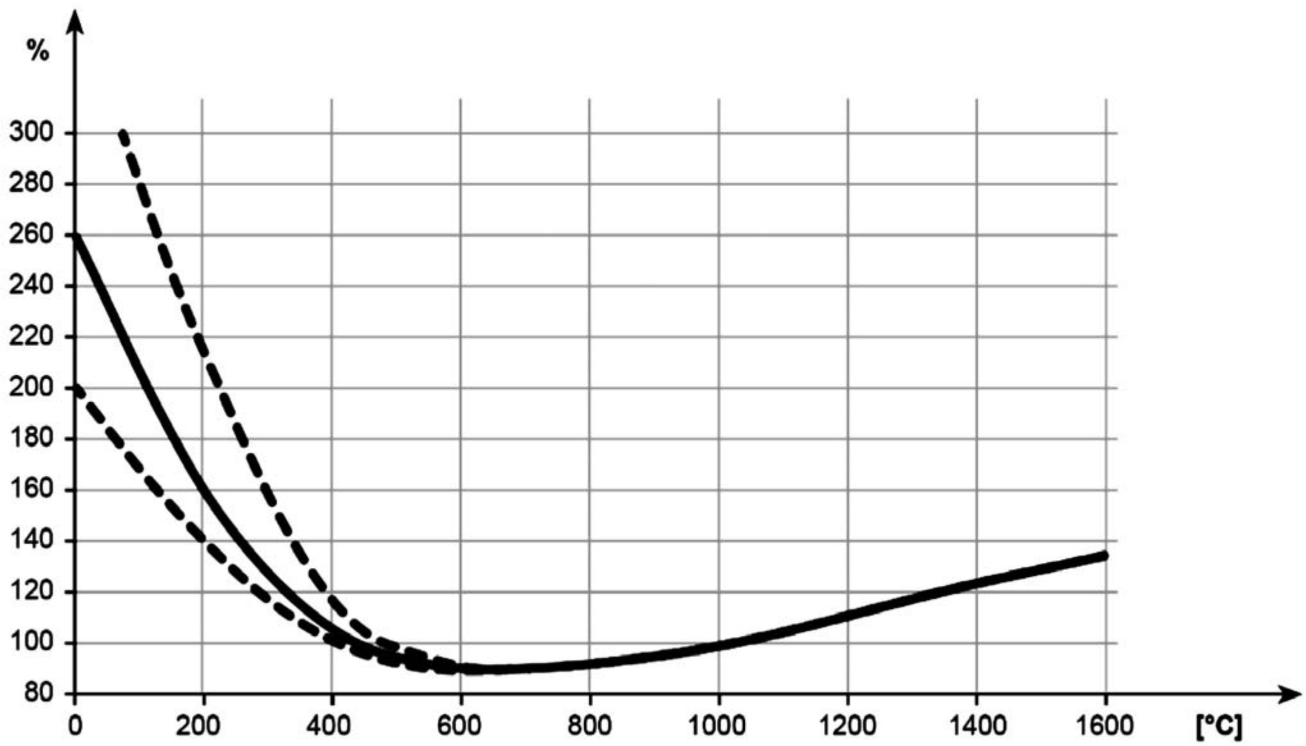

Figure 4: Dependence of the specific resistance on the temperature

Together with the internal resistance, the watt load on the element is also growing. The working load in the kiln is reached at 1,400 °C, but it can be decreased by using other types of atmospheres. The lower boundary of the heating of elements does not exist; however, the minimally set load for the total surface of the rod is reached at the temperature 900 °C. For other cases, the following formula can be used for the calculation:

$$W = D \cdot L \cdot P$$

Let us take the silicon carbide heating element with the following characteristics to estimate the efficiency:

Dimensions : 25 × 400 × 1,200 mm.

Capacity at the temperature of 1,070 °C (0.87 Ohm) : 1,884 W

Capacity at the temperature of 1,400 °C (1.1 Ohm) : 1,800 W

Operating mode: Continuous.

Operating mode : Pulse (calculated).

The result of the calculation of the heater capacity at the output to the set temperature mode is shown in the Table 2.

Table 2

Result of the power calculation of the heater

Current temperature, °C	*Rated power according to the technical documentation W/cm²*	*Power of heater when applying the sampling heating method W/cm²*
0	1	8
100	1	8
200	1	8
300	2	8
400	2	8
500	3	8
600	3	8
700	4	8
800	5	8
900	6	8
1,000	6	8
1,100	6	8
1,200	6	7
1,300	5	7
1,400	5	7
1,400	5	7

$$\tau_1 = \frac{1}{a^2\pi^2\left(\frac{1}{l_1}+\frac{1}{l_2}+\frac{1}{l_3}\right)}\ln\left(\frac{8\sin\frac{\pi}{l_1}\rho_1\sin\frac{\pi}{l_2}\nu_1\sin\frac{\pi}{l_3}\vartheta_1\sum_{i=1}^{N}\sin\frac{\pi}{l_1}\rho_i\sin\frac{\pi}{l_2}\nu_i\sin\frac{\pi}{l_3}\vartheta_i}{l_1 l_2 l_3 T_{zad}}\right)$$

We do not observe the advantage in the capacity; however, it should be mentioned that at pulse heating the working time of heating element is saved. Let us calculate the time of switching on of the heating elements. The switch-on time will be calculated according to the formula:

When stabilizing the temperature fields according to the above stated data, we obtained the results shown in Table 3.

Table 3

Time of switching on of heating elements depending upon their number

d = 8	*d = 7*	*d = 6*	*d = 5*
Switched on = 4.43	Switched on = 2.01	Switched on = 1.8	Switched on = 1.3
Switched on = 4.33	Switched on = 2.03	Switched on = 1.8	Switched on = 1.3
Switched on = 4.27	Switched on = 2.01	Switched on = 1.8	Switched on = 1.3
Switched on = 4.26	Switched on = 2.01	Switched on = 1.8	Switched on = 1.4
Switched on = 4.23	Switched on = 2.01	Switched on = 1.8	Switched on = 1.3
Switched on = 4.23	Switched on = 2.01	Switched on = 1.8	
Switched on = 4.23	Switched on = 2.01		
Switched on = 4.23			

On the base of the obtained results, the dependence of the number of heating elements and the time of switching on is observed. For example, in case of using 7 heating elements, the silicon carbide rod will heat up to the temperature of 1,400 °C when the heating elements are switched on every 10 seconds. Consequently, in comparison with the average working time of the heating element of 3,000 hours in a continuous mode, in case of using this method of heating the working time of the pulse heater will decrease to 1,500 hours. As a result, the energy will be saved. However, to heat the rod up to the temperature 1,400 °C, the pulse heater shall spend more energy in the pulse mode then in the continuous mode.

Total average capacity at the continuous heating is $3{,}000 \cdot 5 = 15{,}000$ W. At the pulse heating, the capacity is $1{,}200 \cdot 8 = 12{,}000$ W. That is 80% of the spent resources.

$$\frac{1{,}200 \cdot 100}{15{,}000} = 80\%$$

Then the profit from the use of this method is 20 percent for the ideal kiln.

4. DISCUSSION

The relevance of the conducted research is determined by the complexity of the realization of the control system of objects with distributed parameters. Controlled values of such systems depend not only upon the time but also upon the distribution along the space area taken by the object. In this regard, the class of the controlling impacts is extended, first of all, due to the possibility to include them into the four-dimensional control described by the functions of several variables of time and space coordinates.

The peculiarities of the systems with distributed parameters require the creation of the apparatus for their analysis on the base of the mathematical means non-traditional for the classical control theory. There are different forms of the description of the simulation of systems with distributed parameters: in the form of the differential equations in the partial derivatives; structural representation of the systems with the distributed parameters that supports on the fundamental solution of the boundary problem; representation of the distributed objects in the form of complex transfer factors according to their own vector functions of the object operator.

Approximation methods are most often used to analyze the controlled objects described by the non-linear equations in the partial derivatives. However, it should be mentioned that an approximation method of the distributed systems by the specially selected concentrated system has not been developed up to date; at the same time, in many problems the approximation process is not stable regarding the errors of the intermediate calculations. Recently, due to the indisputable relevance and high demand in the technical solutions in practice, many authors have developed the models of the considered systems and methods of synthesis. At the same time, many works were stopped at the stages of system simulations, supposing the further parametric synthesis, the application of which is connected to the solution of some problems. The offered method compares favorably by the fact that it is performed completely, the control algorithms are obtained.

The scientific value of this work is the development of the theoretical basis of the analysis and synthesis of non-linear distributed control systems.

The practical value of this work is that the developed method of the calculation of the installation of heating elements depending upon the value of the temperature field allows to consider the possibility of the installation of sectional pulse heaters in the electric tunnel kilns of conveyor type. The analysis of the results of the developed program complex of stabilization of the temperature fields showed:

1. The possibility to bring the kiln to the necessary temperature range due to the use of sampling heating elements.
2. The possibility of stabilization of the temperature field within tolerance. The dependence of the temperature mode from the length of the section was considered.

5. CONCLUSION

The represented method considers the possibility of the replacement of solid heating elements by the pulse ones. The novelty and technical peculiarity of this article is the following:

1. The use of an innovative approach to the heating of hexagonal silicon carbide structures is a relevant problem because the rods made of this alloy are used for the burning of ceramics, bricks and other items.
2. This method is not intended for hexagonal silicon carbide structures exclusively and has a general form that can be easily adapted for other alloys.
3. The offered method will allow to decrease the final cost of the item by saving the energy resources of the enterprise.
4. This method together with the hardware and software system for the stabilization of the temperature field of tunnel kilns of conveyor type will allow to solve a wide range of problems necessary for the modern industry [1, 18, 20].

Thus, the developed method can be generalized for the class of systems for which there is a fundamental solution (the Green's function). At the same time, the complication of the expression of the Green function will definitely lead to the increase in the costs for the calculation process. However, if the costs that are now related to the low efficiency factor of heating elements are compared, the use of the mathematic simulation for the calculation of the location of heating elements is reasonable [6-20].

It should be mentioned that it is useful to take into account the selection of the sampling parameters of the controlled impacts for the systems, boundary problem of which contains the non-zero boundary conditions. The fundamental solution (Green function) of the boundary problem of such systems has a form which is different from that considered in the work. Also the possibility to extend the working zone of the object should be studied, that is, the zone in the limits of which the required value of the object function can be reached to the specified accuracy. But this is the subject of the further research.

6. ACKNOWLEDGEMENTS

We would like to mention the people without whose participation this research would not be performed. First of all, this is Pershin Ivan Mitrofanovich (Doctor of Technical Sciences, Professor, Full Member of the Russian Academy of Natural Sciences, Honorary Figure of Russian Higher Professional Education of the Russian Federation), Kolesnikov Anatoly Arkadyevich (Honoured Master of Sciences and Engineering of the Russian Federation, Doctor of Technical Sciences, Professor, Full Member of the Academy of Electrotechnical Sciences and Academy of Navigation and Control of Motion, corresponding member of the Russian Academy of Natural Sciences, Soros Foundation laureate (four times) in Sciences), and Chernyshev Aleksandr Borisovich (Doctor of Technical Sciences, Professor) who developed the basis for the synthesis of the distributed regulators on the base of the Green function.

7. REFERENCES

1. Ilyushin, Y., Pervukhin, D., Afanasieva, O., Klavdiev, A., & Kolesnichenko, S. (2014). Designing of Distributed Control System with Pulse Control. Middle-East Journal of Scientific Research, 21(3), 436-439.
2. Ilyushin, Y., Pervukhin, D., Afanasieva, O., Klavdiev, A., & Kolesnichenko, S. (2014). The Methods of the Synthesis of the Nonlinear Regulators for the Distributed One-Dimension Control Objects. Modern Applied Science, 9(2), 42-61.
3. Ilyushin, Y., Pervukhin, D., Afanasieva, O., Klavdiev, A., & Kolesnichenko, S. (2015). The Methods of the Synthesis of the Nonlinear Regulators for the Distributed One-Dimension Control Objects. Journal of Theoretical and Applied Information Technology, 82(1), 54-67.
4. Chernyshev, A. (2009). Adaptatsiya chastotnogo kriteriya absolyutnoy ustoychivosti k sistemam s raspredelennymi parametrami [Adaptation of Absolute Stability Frequency Criterion to Systems with Distributed Parameters]. Mechatronics, Automation, Control, 7, 13-18.

5. Chernyshev, A. (2010). Interpretatsiya kriteriya absolyutnoy ustoychivosti dlya nelineynykh raspredelennykh system [Interpretation of Absolute Stability Criterion for Nonlinear Distributed Systems]. Automation. Modern technologies, 2, 28-32.
6. Chernyshev, A., Antonov, V., & Shurakov, D. (2010). Sistema stabilizatsii temperaturnogo polya v protsesse utilizatsii tepla pri kontaktnoy svarke [System of Temperature Field Stabilization Criterion in the Process of Heat Utilization in Contact Welding]. Scientific and technical sheets SPbGPU, 6(113), 151-155.
7. Ilyushin, Y. (2011). Proektirovanie sistemy upravleniya temperaturnymi polyami tunnel'nykh pechey konveyernogo tipa [Designing of Temperature Field Control System of Tunnel Kilns of Conveyor Type]. Scientific and technical sheets SPbGPU, 3(126), 67-72.
8. Kolesnikov, A. (2009). Nonlinear Oscillations Control. Energy Invariants. Journal of Computer and Systems Sciences International, 48(2), 185-198.
9. Kolesnikov, A., Zarembo, Ya., & Zarembo, V. (2007). Discharge of a Copper-Magnesium Galvanic Cell in the Presence of a Weak Electromagnetic Field. Russian Journal of Physical Chemistry A, 81(7), 1178-1180.
10. Kolesnikov, A., Zarembo, Ya., Puchkov, L., & Zarembo, V. (2007). Zinc Electrochemical Reduction on a Steel Cathode in a Weak Electromagnetic Field. Russian Journal of Physical Chemistry A, 81(10), 1715-1717.
11. Pershin, I. (2007). Analiz i sintez sistem s raspredelennymi parametrami [Analysis and Synthesis of Systems with Distributed Parameters]. Pyatigorsk: RIA-KMV.
12. Pleshivtseva, Y., & Rapoport, E. (2009). The Successive Parameterization Method of Control Actions in Boundary Value Optimal Control Problems for Distributed Parameter Systems. Journal of Computer and Systems Sciences International, 48(3), 351-362.
13. Rapoport, E. (1996). Alternance Properties of Optimal Solutions and Computational Algorithms in Problems of Semi-Infinite Optimization of Controlled Systems. Journal of Computer and Systems Sciences International, 35(4), 581-591.
14. Rapoport, E. (2006). Structural Parametric Synthesis of Automatic Control Systems with Distributed Parameters. Journal of Computer and Systems Sciences International, 45(4), 553-566.
15. Rapoport, E., & Pleshivtseva, Y. (2010). Combined Optimization of Metal Hot Forming Line with Induction Pre-Heating. In IEEE 26th Convention of Electrical and Electronics Engineers, Israel, Eilat.
16. Rapoport, E., & Pleshivtseva, Yu. (2006). Models and Methods of Semi-Infinite Optimization Inverse Heat-Conduction Problems. Heat Transfer Research, 37(3), 221-231.
17. Tikhonov, A., & Samarskiy, A. (1965). Uravneniya matematicheskoy fiziki [Equations of Mathematical Physics]. Moscow: Science.
18. Zarembo, V., & Kolesnikov, A. (2006). Background Resonant Acoustic Control of Heterophase Processes. Theoretical Foundations of Chemical Engineering, 40(5), 483-495.
19. Zarembo, V., Kolesnikov, A., & Ivanov, E. (2006). Fonovoe elektromagnitno-akusticheskoe upravlenie strukturnymi *i* plasticheskimi svoystvami metallicheskikh materialov [Background Electromagnetic-Acoustic Control of Structural and Plastic Properties of Metals]. Izvestiya RAN. Seriya fizicheskaya, 70(8), 1239-1243.
20. Zarembo, V., Kiseleva, O., Kolesnikov, A., Burnos, N., & Suvorov, K. (2004). Structuring of Inorganic Materials in Weak RF Electromagnetic Fields. Inorganic Materials, 40(1), 86-91.