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2-Fuzzy Ward Continuity and N_θ-ward Continuity in 2-Fuzzy Metric Space

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Abstract: In this paper 2-fuzzy ward continuous function in 2-fuzzy metric space is coined. Further N_θ-convergence, N_θ-sequentially continuous functions, N_θ-*ward* continuous functions are developed. Also some related theorems are concentrated using this concepts.

Keywords: 2-fuzzy ward sequentially continuous function, 2-fuzzy uniform continuous function, 2-fuzzy ward continuous functions N_θ convergence, N_θ sequentially continuous functions, N_θ ward continuous functions

1. INTRODUCTION

In 1965 L.A. Zadeh [13] introduced the concept of fuzzy sets. After the pioneering work of Zadeh there has been a great effort to obtain fuzzy analogues of classical theories. The concept of fuzzy metric space is one of such progressive development in the field and fuzzy topology. This has been investigated by many authors in different point of view.

In 1975, Kramosil and Michalek [7] introduced the concept of fuzzy metric space. George and Veeramani [5] modified the concept of fuzzy metric space. In 1963, Gahler [3, 4] generalized usual notion of metric space called 2-metric space. Using the notion of 2-metric space, S.Sharma [11] and S.Kumar [8] introduced fuzzy 2-metric spaces without knowing each other but Ha et al. in [6] shows that 2-metric need not be continuous functions. Further there is no easy relationship between results obtained in the two settings.

The concept of ward continuity of real functions and ward compactness of a subset of real line are introduced by Cakalli in [1]. The notion of statistical convergence was investigated by Steinhaus and Fast. Fridy and Orhan [2] introduced the idea of lacunary statistical convergence Using this idea Mursaleen and Mohiuddine [10] and Thillaigovindan, S.Anita Shanthi and Y.B.Jun [12] investigated lacunary statistical convergence in intuitionistic fuzzy normed spaces and intuitionistic fuzzy n- normed space respectively.

In this paper 2-fuzzy ward sequentially continuous function and 2-fuzzy uniform continuous function and 2-fuzzy ward continuous functions are coined. The equal lent condition regarding these concepts are developed. Further N_θ convergence, $N\theta$ sequentially continuous functions, N_θ *ward* continuous functions are developed. Also some related theorems are concentrated using this concepts.

2. PRELIMINARIES

DEFINITION - 2.1

A binary operation $*: [0,1] \times [0,1] \to [0,1]$ is called a t-norm if for all $a, b, c, d \in [0, 1]$ the following conditions are satisfied

(i) $a * 1 = a$

(ii) $a * b = b * a$

(iii) $a * b \leq c * d$ whenever $a \leq c,\ b \leq d$

(iv) $a * (b * c) = (a * b) * c$

DEFINITION - 2.2

The 3 tuple (X, M, *) is called a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set in $X^2 \times [0, \infty]$ satisfying the conditions.

(M1) $M(x, y, t) = 0$

(M2) $M(x, y, t) = 1, \forall t > 0$ if and only if $x = y$

(M3) $M(x, y, t) = M(y, x, t)$

(M4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$

(M5) $M(x, y, \cdot): [0, \infty] \to [0,1]$ is left continuous.

(M6) $\lim_{t \to \infty} M(x, y, t) = 1$

DEFINITION - 2.3

The 3 tuple $(X, M, *)$ is called a 2- fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set in $X^2 \times [0, \infty]$ satisfying the conditions.

(2-M1) $M(f,g,t) = 0$

(2-M2) $M(f,g,t) = 1, \forall t > 0$ if and only if $f = g$

(2-M3) $M(f,g,t) = M(g,f,t)$

(2-M4) $M(f,g,t) * M(g,h,s) \leq M(f,h,t+s)$

(2-M5) $M(f,g,\cdot)\colon [0,\infty] \to [0,1]$ is left continuous.

(2-M6) $\lim_{t\to\infty} M(f,g,t) = 1$

3. FUZZY WARD CONTINUITY IN 2-FUZZY METRIC SPACE

DEFINITION - 3.1

A sequence $\{f_n\}$ of points in a 2-fuzzy metric space $(F(X),M,*)$ is said to be 2-fuzzy quasi Cauchy if $\lim_{n\to 0} M(f_{n+1},f_n,t) = 1$ for every $t \in (0,1)$ and for every natural number n.

DEFINITION - 3.2

A subspace of a 2-fuzzy metric space $(F(X),M,*)$ is said to be 2-fuzzy ward compact if any sequence has a 2-fuzzy quasi cauchy subsequence.

DEFINITION - 3.3

Let $(F(X),M_1,*)$ and $(F(Y),M_2,*)$ be 2-fuzzy metric spaces. A function ψ:F(X) → F(Y) is said to 2-fuzzy ward continuous if it preserve 2-fuzzy quasi cauchy condition.

$$\text{(ie)} \lim_{n\to\infty} M_2\,(\psi(f_{n+1}),\psi(f_n)\,,t) = 1 \text{ whenever } \lim_{n\to\infty} M(f_{n+1},f_n,t) = 1.$$

DEFINITION - 3.4

A function ψ on the subspace A of a 2-fuzzy metric space $(F(X),M_1,*)$ is said to be 2-fuzzy sequentially continuous at f_0 if for any sequence $\{f_n\}$ in A converging to f_0 then $\psi(f_n)$ converges to $\psi(f_0)$ in $(F(Y),M_2,*)$.

THEOREM - 3.5

If ψ: F(X) → F(Y) is a 2-fuzzy ward continuous on A the subspace of F(X) then it is 2-fuzzy sequentially continuous on A.

Proof

Assume $\{f_n\}$ is a convergent sequence in A.

(ie) $\lim_{n\to\infty} M_1\ (f_n, f_0, t\ ) = 1$ for t ∈ (0,1).

Construct a sequence $\{h_n\}$ as

$$h_n = \begin{cases} f_n\ if\ n = 2k - 1\ where\ k\ is\ a\ positive\ integer. \\ f_0\ if\ n\ is\ even \end{cases}$$

Then,

$$M_1\ (h_n, f_0, t\ ) \geq M_1(h_n, f_n, \tfrac{t}{2}) * M_1(f_n, f_0, \tfrac{t}{2})$$

If n is even, $M_1(h_n, f_0, t) \geq M_1(f_n, f_0, \tfrac{t}{2})$

If n is odd,

$$M_1(h_n, f_0, t) \geq\ M_1(f_0, f_n, \tfrac{t}{2}) * M_1(f_n, f_0, \tfrac{t}{2})$$

$$= M_1\ (f_0, f_n, \tfrac{t}{2})$$

In either case $\{h_n\}$ converges to f_0.

Given ψ is a 2 – fuzzy ward continuous function. The sequence $\psi(h_n)$ is defined as

$$\psi(h_n) = \begin{cases} \psi(f_n)\ if\ n = 2k - 1 \text{ where k is a positive integer} \\ \psi(f_0)\ if\ n\ is\ even \end{cases}$$

$$M_2\ (\psi(h_{n+1}), \psi(h_n), \text{t}) \geq M_2\ (\psi(h_{n+1}), \psi(f_{n+1}), \tfrac{t}{3}\ )$$

$$* M_2\ (\psi(f_{n+1}), \psi(f_n), \tfrac{t}{3}\ ) * M_2\ (\psi(f_n), \psi(f_n), \tfrac{t}{3}\ )$$

If n is odd,

$$M_2\ (\psi(h_{n+1}), \psi(h_n), t) \geq M_2\ (\psi(f_{n+1}), \psi(f_n), \tfrac{t}{3})$$

If n is even,

$$M_2\ (\psi(h_{n+1}), \psi(h_n), \text{t}) \geq M_2\ (\psi(f_0), \psi(f_{n+1}), \tfrac{t}{3}) *$$

$$M_2\ (\psi(f_{n+1}), \psi(f_n), \tfrac{t}{3}) * M_2\ (\psi(f_n), \psi(f_0), \tfrac{t}{3})$$

$$\geq M_2\ (\psi(f_{n+1}), \psi(f_n), \tfrac{t}{3})$$

Hence $\{\psi(h_n)\}$ is a 2- fuzzy Cauchy quasi sequence.

Now $M_2(\psi(h_{n+1}), \psi(h_n), \text{t}) = 1$, by construction of $\{h_n\}$

We get $\lim_{n\to\infty} M_2(\ (\psi(f_{n+1})\ , \psi(f_0), \text{t}) = 1$.

Thus $\psi(f_{n+1})$ converges to $\psi(f_0)$ and so ψ is 2-fuzzy sequentially continuous on A.

THEOREM 3.6

Let (F(X), M_1, ∗) and (F(Y), M_2, ∗) be 2-fuzzy metric spaces and A be 2-fuzzy ward compact subspace of F(X). If ψ:F(X)→F(Y) is 2-fuzzy ward continuous on A then ψ(A) is 2- fuzzyward compact.

Proof

Given A is 2 – fuzzy ward compact subspace of F(X) so there exists a subsequence $\{h_{n_k}\}$ of $\{h_n\}$ with $\lim_{n\to\infty} M_1(h_{n_{k+1}}, h_{n_k}, t) = 1$ for t ∈ (0,1). Define $\psi(h_{n_k}) = g_{n_k}$. Where $\{g_{n_k}\}$ is a subsequence of $\{\psi(h_n)\}$ with $\lim_{n_k\to\infty} M_2(g_{n_{k+1}}, g_{n_k}, t) = 1$ as M_2 is jointly continuous and so the requirements are satisfied.

DEFINITION - 3.7

A function ψ:F(X) → F(X) is 2-fuzzy uniformly continuous if for ∈ ϵ(0,1) there exists δ ∈ (0,1) such that $M_2(\psi(f_1), \psi(f_2)$, t) > 1- ∈. Whenever $M_1(f_1,f_2$, t) > 1-δ for every f_1,f_2 ∈ F(X).

THEOREM - 3.8

If : F(X) → F(Y) is 2-fuzzy uniformly continuous on F(X) then it is 2 – fuzzy ward continuous.

Proof

Let ψ be 2-fuzzy uniformly continuous on F(X). Let $\{f_n\}$ be a 2 - fuzzy quasi Cauchy sequence in F(X).

For given ∈ $\epsilon(0,1)$ there exist δ ∈ (0,1) Such that $M_2(\psi(f_1), \psi(f_2)$ t) > 1- ∈.

Whenever $M_1(f_1, f_2$, t) > 1-δ because ψ is 2-fuzzy uniformly continuous.

Since $\{f_n\}$ is 2-fuzzy quasi Cauchy, $M_1(f_{n+1}, f_n$, t) = 1.

Choose N depending on ∈ and δ.

Therefore $M_1(f_{n+1}, f_n$, t) > 1- δ for all n ≥ N.

Hence $M_2(\psi(f_{n+1}), \psi(f_n)$, t) > 1- ∈ for all n ≥ N and so ψ is 2-fuzzy ward continuous.

THEOREM - 3.9

The image of a 2 – fuzzy ward compact space under a 2-fuzzy uniform continuous function is 2-fuzzy ward compact.

Proof

Let: F(X) →F(Y) be 2-fuzzy uniform continuous function from 2-fuzzy metric space (F(X), M_1, ∗) to 2-fuzzy metric space (F(X), M_2, ∗)

Let A be 2-fuzzy ward compact subspace of F(X). It asserts to show that ψ (A) is 2-fuzzy ward compact.

Let $\{h_n\}$ be a sequence in ∅(A). So that h_n = ψ (g_n) where $\{g_n\}$ is a sequence in A. Clearly $\{g_n\}$ has a 2-fuzzy quasi cauchy subsequence. $\{g_{nk}\}$ and so, $\lim_{n_k \to \infty} M_1(g_{n_{k+1}}, g_{n_k}, t) = 1$.

Since ψ is 2-fuzzy uniform continuous for given ∈ $\epsilon(0,1)$ there exists δ ϵ (0,1).

Such that $\lim_{n_k\to\infty} M_2(\psi(g_{n_{k+1}}), \psi(g_{n_k}), t) = 1$ provided $\lim_{n_k\to\infty} M_1(g_{n_{k+1}}, g_{n_k}, t) = 1$. For all $n_k >$ N where N depends on $\in$ and δ. Hence ψ(A) is 2-fuzzy ward compact.

THEOREM - 3.10 (2-fuzzy uniform limit theorem)

Let $\{\psi_n\}$ be a sequence of 2-fuzzy uniformly continuous functions defined on 2-fuzzy metric space (F(X), M_1, ∗) to (F(Y), M_2, ∗). If $\{\psi_n\}$ converges 2-fuzzy uniformly to ψ then ψ is 2-fuzzy uniformly continuous.

Proof

Given $\{\psi_n\}$ is 2-fuzzy uniformly convergent to ψ, choose $\in \epsilon$ (0,1) and a positive integer N($\in$) such that, M_2 (ψ_n(f), ψ(f), t) > 1- ϵ when n ≥ N and f ϵ A.

By 2-fuzzy uniform continuity, given $\in$ ϵ (0,1) there exists δ ϵ (0,1) Such that, M_2 ($\psi_N(f_1)$, ψ_N (f_2), t) > 1- ϵ for f_1,f_2 ϵ A.

Provided M_1 (f_1,f_2, t) > 1- δ.

$$M_2(\psi(f_1),\ \psi(f_2),\ t) \geq M_2(\psi\ (f_1),\ \psi_N(f_1), \tfrac{t}{3}) * M_2(\psi_N(f_1),\ \psi_N(f_2),\ \tfrac{t}{3}) * M_2(\psi_N(f_2), \psi_N(f_2), \tfrac{t}{3})$$

$$\geq (1 - \in) * (1 - \in) * (1 - \in)$$

$$\geq 1 - \in.$$

Provided M_1 (f_1,f_2, t) > 1- δ satisfying the requirement of ψ to be 2-fuzzy uniformly continuous.

THEOREM - 3.11

Let $\{\psi_n\}$ be a sequence of 2-fuzzy ward continuous functions on a subspace A of a 2-fuzzy metric space (F(X), M_1, ∗) to (F(Y), M_2, ∗). If $\{\psi_n\}$ is 2-fuzzy uniformly convergent to ψ then ψ is 2-fuzzy ward continuous.

Proof

Let $\{f_n\}$ be a 2-fuzzy quasi Cauchy sequence in A. Given $\{\psi_n\}$ is uniformly convergent to ψ, choose $\in \epsilon$ (0,1). So that a positive integer N($\in$) satisfying.

$$M_2\,(\psi_n(f_n), \psi(f_n), \text{t}) > 1 \text{ - } \in \text{ with n} \geq \text{N}.$$

Since, ϕ_N is 2-fuzzy ward continuous on A there exists a positive integer $n_0 \geq$ N.

Such that $M_2\,(\psi_N(f_{n+1}), \psi_N(f_n), \text{t}) > 1 \text{ - } \in$ for n $\geq n_0$.

$$M_2\,(\psi(f_{n+1}), \psi(f_n), \text{t}) \geq M_2\,(\psi(f_{n+1}), \psi_N(f_{n+1}), \tfrac{t}{3}) \ast$$

$$M_2(\psi_N(f_{n+1}), \psi_N(f_n),\tfrac{t}{3}) \ast M_2\,(\psi_N(f_n), \psi(f_n), \tfrac{t}{3})$$

$$\geq (1 \text{ - } \in) \ast (1 \text{ - } \in) \ast (1 \text{ - } \in)$$

$$= 1 \text{ - } \in \text{ for n} \geq N_0$$

Satisfying the requirement that ψ is 2-fuzzy ward continuous.

4. N_θ-WARD CONTINUITY IN 2-FUZZY METRIC SPACE

DEFINITION - 4.1

Let (F(X), M, ∗) be a 2 – fuzzy metric space. A sequence $\{f_n\}$ is said to be strongly lacunary convergent to f_0 if for $\in\ \epsilon$ (0,1), t ϵ (0,1).

$\lim \frac{1}{h_l}\{k \in K : M(f_k, f_0, t) \leq 1 - \in\} = 0$ .Where K is the subset of natural numbers and $\{k_l\}$ is an increasing sequence of positive integers. Such that k_0= 0. Where h_l is defined as $h_l = h_l$ - $h_{l_{-1}} \rightarrow \infty$ as $\iota \rightarrow \infty$ and I_l= (k_{l-1}, k_l]. It is also said to N_θ -convergent and denoted by N_θ limf_0 = L.

DEFINITION - 4.2

A subspace A of a 2-fuzzy metric space is said to be N_θ-sequentially compact if any sequence in A has N_θ-convergent sequence with N_θ-limit in A.

DEFINITION 4.3

A function ψ on a subspace A of a 2-fuzzy metric space is said to be strongly lacunary sequentially continuous or N_θ-sequentially continuous to f_0. If $\psi(f_n)$ is N_θ - convergent to $\psi(f_0)$. Whenever $\{\psi_n\}$ is N_θ- convergent to f_0 in A.

DEFINITION - 4.4

If ψ is strongly lacunary sequentialy continuous at every point of the subspace A of the 2-fuzzy metric spac (F(X), M, $*$) then it is said to be strongly lacunary sequentially continuous on A.

THEOREM - 4.5

Uniform limit of N_θ-sequentially continuous functions is N_θ-sequentially continuous.

Proof

Let $\{\psi_n\}$ be uniformly convergent sequence in A a subspace of 2-fuzzy metric space (F(X), M, $*$) with uniform limit ψ.

Let $\{f_n\}$ be N_θ-convergent sequence in A converging to f.

Then, $\lim \frac{1}{h_l}\{ k \in I_l: M_1 (f_n, f, t) < 1- \in\} = 0.$

By uniform convergence of $\{\psi_n\}$ for $\in \epsilon$ (0,1), M_2 (ψ_n(f), ψ(f),t) > 1- ∈, ∀ n ≥ N,

By N_θ- sequentially continuous of $\{\psi_n\}$ on A there exists N_2. Such that, $\lim \frac{1}{h_l}\{ k \in I_l: M_2 (\psi_n(f_k), \psi_n(f), t) > 1- \in \} = 0 \ \forall \ n \geq N_2$

So, $M_2 (\psi_n(f_k), \psi_n(f), t) > 1- \in$ for $n \geq N_2$.

Choose $N_0 = \max\{N_1, N_2\}$.

$$M_2 (\psi_n(f_k), \psi_n(f), t) \geq M_2 (\psi_n(f_n), \psi (f_n), \tfrac{t}{3}) * M_2 (\psi_n(f_n), \psi (f), \tfrac{t}{3}) * M_2 (\psi_n(f_n), \psi (f), \tfrac{t}{3})$$

$$> (1- \in) * (1- \in) * (1- \in).$$
$$= 1- \in$$

Thus, $\lim \frac{1}{h_l}\{ k \in I_l: M_2(\psi(f_n), \psi(f), t) < 1 - \in\} = 0$ and hence ψ is N_θ-sequentially continuous.

DEFINITION - 4.6

A sequence $\{f_n\}$ in a subspace A of a 2-fuzzy metric space (F(X), M, $*$) is to be N_θ-quasi Cauchy if $\{\Delta f_n\}$ is N_θ-convergent. Where $\Delta f_n = f_{n+1} - f_n$

(i,e,) $\lim \frac{1}{h_l}\{ k \in I_l: M(f_{n+1}, f_n, t) < 1 - \in\} = 0$.

A subspace A of a 2-fuzzy metric space (F(X), M, $*$) is said to be N_θ-ward compact if any sequence in A has an N_θ-quasi Cauchy subsequence.

A function ψ from 2-fuzzy metric space [F(X), M_1, $*$) to [F(Y), M_2, $*$) is said N_θ- ward continuous if it preserves. N_θ- quasi Cauchy sequence.

THEOREM - 4.7

N_θ-ward continuous image of N_θ- compact subspace of a 2-fuzzy metric space (F(X), M, $*$) is N_θ- ward compact.

Proof

Let ψ be a N_θ- ward continuous map on A and assume A is N_θ- ward compact. Let $\{ \psi(f_n)\}$ be a sequence in $\emptyset$(A) where $\{f_n\}$ is a sequence in A.

There exists a N_θ- quasi Cauchy subsequence $\{f_{nk}\}$ of $\{f_n\}$.

(i,e,) $\lim \frac{1}{h_l}\{ k \in I_l: M_1(f_{n_{k+1}}, f_{n_k}, t) < 1 - \in\} = 0$.

Let $g_{n_k} = \psi(f_{n_k})$. Then $\{ \psi(f_{n_k})\}$ is N_θ- quasi Cauchy as ψ is N_θ-ward continuous.

Such that $\lim \frac{1}{h_l}\{ k \in I_l: M_2(g_{n_{k+1}}, g_{n_k}, t) < 1 - \in\} = 0$. Which satisfies the requirement.

THEOREM - 4.8

Every N_θ-ward continuous function is N_θ-sequentially continuous.

Proof

Let ψ be N_θ-ward continuous on a subspace of a 2-fuzzy metric space (F(X), M, $*$).

Assume ψ is N_θ- ward continuous and $\{f_n\}$ is a N_θ-convergent sequence.

Then $\lim \frac{1}{h_l}\{ k \in I_l: M_1(f_n, f_0, t) < 1 - \in\} = 0$.

Let $h_n = \begin{cases} f_n \; if\; n = 2k-1\; for\; a\; Positive\; integer\; k \\ \qquad\qquad f_0\; if\; n\; is\; ever \end{cases}$

By theorem 1,

$M_1(h_n, f_0, t) < 1 - \in$. Which implies $\{h_n\}$ is N_θ- quasi Cauchy.

Define $g_n = \begin{cases} \psi(f_n)\; if\; n = 2k-1` \\ \quad f_0\; if\; n\; is\; even \end{cases}$

In accordance to theorem 3.16, $\{g_n\}$ is N_θ- quasi Cauchy because,

$$M_2\,(g_{n+1}, g_n, t) > 1 - \in$$

Thus $\lim \frac{1}{h_l}\{ k \in I_l: M_2\,(\psi(f_k), \psi(f_0), t) < 1- \in\} = 0$.

Which implies $\{ \psi(f_k)\}$ is N_θ- convergent to $\psi(f_0)$.

THEOREM - 4.9

Uniform limit of N_θ- ward continuous function is N_θ- ward continuous.

Proof

Let $\{f_n\}$ be a $N_\theta$quasi Cauchy sequence and $\in \epsilon$ (0,1).

$M_2(\psi_n(f), \psi(f), t) > 1-\in\}$ for $n \geq N$ as $\{\psi_n\}$ is uniformly convergent to ψ.

Choose a positive integer $n_1 \geq N$. Such that , $M_2\,(\psi_N\,(f_{n+1}), \psi_N\,(f_N), t\,) > 1- \in$ since ϕ_N is N_θ- ward continuous.

Then, $M_2\,(\psi_N\,(f_{n+1}), \psi(f_n), t\,) \geq\; M_2\,(\psi(f_{n+1}), \psi_N(f_{n+1}), \frac{t}{3}\,) *$

$$M_2\,(\psi_N(f_{n+1}), \psi_N(f_n), \frac{t}{3}\,) * M_2(\psi_N(f_n), \psi(f_n), \frac{t}{3}\,)$$

$$> (1-\in\,) * (1-\in\,) * (1-\in\,)$$
$$= (1-\in\,).$$

Thus, $\lim \frac{1}{h_l}\{ k \in I_l: M_2\,(\psi(f_{n+1}), \psi(f_n), t\,) < 1 - \in\} = 0$

Hence, ψ preserves N_θ-quasi Cauchy sequences and so ψ is N_θ-ward continuous.

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