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A NOTE ON ASSET PRICE GAMBLES: SIMPLE MODELS WITH COMPLEX DYNAMICS

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ABSTRACT

The speed of price adjustment can play an important role in determining whether the asset price dynamics may evolve through a cycle of infinite period. We are able to derive two critical values of the speed of price adjustment k and k**: if the actual speed of price adjustment k is such that k>k* then the asset prices remain bounded but never repeat exhibiting chaotic dynamical behaviour. On the other hand, for values of speed of price adjustment k such that k<k**, the asset price dynamics converge on a stable equilibrium.*

Keywords: *Asset Price Dynamics, Fixed Points, Speed of Price Adjustment, Chaos*

EL Classifications: *G10, G11, G12*

1. INTRODUCTION

Chaotic behaviour can characterise many important facets of economics (Saari, 1996). We now know that complex and unpredictable behaviour is not only a product of complex systems with many degrees of freedom but can also be caused by simple and deterministic dynamic systems. Since the early 1980s a series of important papers highlighted the relevance of non-linear dynamic models exhibiting chaotic dynamical behaviour in economics (see Benhabib and Day, 1982; Day, 1982; Jensen and Urban, 1984; Hommes, 1991 and 1993; Chiarella, 1988). In this letter we draw attention to some very complex behaviour that occurs in simple models of asset prices. We posit that both buyers and sellers of an asset face a gamble regarding the future price of an asset: the price may go up, or decline. We derive the asset price dynamics based on the expected value of this gamble. This example is, in its context, structurally stable while the underlying dynamics are not straightforward. We find that the dynamics of this model are represented by a quadratic map of the type that is well recognised in the literature on chaos. Despite the fact that the asset price dynamics are completely deterministic, yet we show that these dynamics can evolve in a chaotic fashion under a set of usual parametric restrictions.

2. A SIMPLE MODEL

At date t the asset price can take two values-either a high or a low one. We define the high value as $(R^N + \Delta_t)$ whilst the probability of its occurrence is $(1-\lambda)$; and the low value is $(R^N - \Delta_t)$ and the corresponding probability is λ. One may presume that R^N is the long-run price and D represents a short-run deviation from the long-run price. It is further postulated that the probability of low price (λ) is positively related to the magnitude of deviation Δ (ignoring the time subscript):

$$\lambda = \eta\Delta \text{ where } \eta > 0 \text{ and } \Delta \neq 0 \quad (1a')$$

(1a') purports to indicate that the larger the magnitude of deviation from the long-run price, the larger is the probability the asset price will assume the lower of the two possible values. We expect to stamp out any irrational exuberance by the assumption in (1a').

We call V_t as the expected value of a gamble at date t, which is given as:

$$V_t = R^N + (1-\eta R^N)\Delta_t - \eta\Delta_t^2 \quad (1a)$$

We postulate that the demand for the asset at date t+1 (D_{t+1}) bears a positive relation with the expected value of the gamble at date t (V_t):

$$D_{t+1} = c + dV_t \quad (1b)$$

We postulate the supply of the asset at date t+1 (S_{t+1}) bears a negative relation with the value of the asset at date t (V_t):

$$S_{t+1} = a - bV_t \quad (1c)$$

The excess demand for this asset at date t+1 is X_{t+1}:

$$X_{t+1} = (c-a)+(d+b)V_t \quad (1d)$$

The asset price is assumed to display a finite pace of adjustment as described in

$$\Delta_{t+1} = kX_{t+1} \quad (2a)$$

$$= k(c-a)+k(d+b)V_t \quad (2b)$$

while k is the speed of price adjustment.

Thus, $\Delta_{t+1}>0$ if $k\,(a-c)/(d+b)<V_t$ (2c)

And $D_{t+1}<0$ if $k\,(a-c)/(d+b)>V_t$ (2d)

Based on these we propose the following lemma.

Lemma 1: The asset price dynamics is captured by the following difference equation:

$$\Delta_{t+1} = m - h\Delta_t + A\Delta_t^2 \quad (3a)$$

where

$$m = k(c-a)-k(d+b)R^N \quad (3b)$$

$$h = k(1-nR^N)\,(d+b) \quad (3c)$$

$$A = k(d+b)n \quad (3d)$$

Proof: Substitution of (1b) and (1c) into (2b) yields the result. QED.

Lemma 2: The above dynamics has two fixed points Δ^*, Δ^{**}:

$$\Delta^* = [(1+h)-SQRT\{(1+h)^2-4Am\}]/(2A) \quad (4a)$$

$$\Delta^{**} = [(1+h)+SQRT\{(1+h)^2\text{-}4Am\}]/(2A) \quad (4b)$$

Δ^{**} is always unstable. Δ^* is stable if

$$SQRT\{(1+h)^2\text{-}4Am\}<2 \quad (4c)$$

Proof: The derivation, being simple, is omitted. QED.

If Δ^* is stable, then the asset price dynamics (3a) will drive prices to equilibrium if the initial price is close enough as dictated by (4c). If the price at any date t should go beyond Δ^{**}, then this unstable fixed point will cause the asset price to diverge to infinity. Therefore, for the asset prices to be bounded it is imperative that the following is true:

$$\Delta_t<\Delta^{**} = \Delta^{max} \text{ for } t=0, 1, 2, 3\ldots \text{ And} \quad (4d)$$

$$\Delta_t>h/A\text{-}\Delta^{**} = \Delta^{min} \text{ for } t=0, 1, 2, 3\ldots \quad (4e)$$

Thus, asset prices will be bounded if the initial price lies on the interval $[D^{min}, D^{max}]$ and

$$SQRT\{(1+h)^2\text{-}4Am\}<3 \quad (4f)$$

If the restrictions on the parameters and initial prices, equations (4d)-(4f) hold the asset price dynamics remain bounded between Δ^{min} and Δ^{max}. Following Feigenbaum (1978) we now apply the change of variable technique that will transform the non-linear price dynamics to the logistic equation of May (1976).

Lemma 3: The quadratic asset price dynamics (3a) is equivalent to the following logistic equation with an appropriate transformation of the variable Δ:

$$P_t = h(\Delta^{**}\text{-}\Delta_t)/M \quad (5a)$$

$$M = 1+ SQRT\{(1+h)^2\text{-}4Am\} \quad (5b)$$

$$P_t = BP_{t-1}(1\text{-}P_{t-1}) \quad (5c)$$

Proof: The derivation is omitted. QED.

For 1<M<3 the price converges to the stable equilibrium Δ^*. If M>3 then Δ^* becomes unstable and the asset prices converge to a stable two-period cycle. As M is increased further the stable period cycles of period n bifurcates into cycles of 2n. At M=3.57 the asset prices evolve through a cycle of infinite period. The asset prices are within the relevant bounds but they never repeat. For a higher order, the asset prices may look like a random process but they are fully deterministic. For values of M greater than 3.57 we can have even more complex behaviour.

Result 1: The asset prices evolve through a cycle of infinite period and hence never repeat themselves if

$$Am>[6.60\text{-}(1+h)^2]/4 \quad (6a)$$

As a result, it is not possible for agents to have self-fulfilling expectations.

Result 2: In order to place these results in a sharper focus we consider a special case when $R^N=1/\lambda$, then we know h=0, A=k(d+b) λ, m=(c-a)-k(d+b)(1/λ). Then the chaotic dynamics emerges at

$$(1+h)^2-4Am>6.60 \tag{6b}$$

The substitution of h, A and m will reduce (6b) to

$$1+4k(d+b)(d+b+\lambda a-\lambda c)>6.60 \tag{6c}$$

Thus, the chaotic dynamics emerges if the speed of price adjustment is beyond a threshold:

$$k>k^*=1.4/[(d+b)(\ d+b+\lambda a-\lambda c)] \tag{6d}$$

The price dynamics converges to the stable equilibrium if the speed of price adjustment k is such that

$$k<k^{**} = 0.75/[(d+b)(d+b+\lambda a-\lambda c)] \tag{6e}$$

3. CONCLUSION

It is now well recognised in economics science that chaos cannot be given a short shrift as an outcome of highly artificial models. Seemingly innocuous models can exhibit chaotic dynamical behaviour as confirmed in this paper. The source of the chaotic behaviour in this paper is in the series of complicated decisions that economic agents make to buy, or sell, an asset on the basis of expected value of the proposed gamble involving unknown future (asset) prices. The resultant asset price dynamics, as a result, do not have sufficient refined properties that may eventually lead to a radical behaviour (Sarri, 1996). It is therefore the basic nature of economic problems that confront decision makers in the asset market, which triggers the chaotic dynamical behaviour in asset prices. The upshot is that the speed of price adjustment can be a critical factor in determining whether the asset price dynamics evolve through a cycle of infinite period. We are able to derive two critical values of the speed of price adjustment k* and k**: if the actual speed of price adjustment k is such that k>k* then the asset prices remain bounded but never repeat. Thus the price dynamics exhibit chaotic dynamical behaviour for k>k*. Asset prices can show time behaviour that is seemingly random but is purely deterministic. In this case, agents fail to make long-run predictions even though they act in a deterministic world. Time profiles that start very close together will separate exponentially. On the other hand, for values of speed of price adjustment k such that k<k**, the asset price dynamics converge on a stable equilibrium.

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