

# International Journal of Control Theory and Applications

ISSN: 0974-5572

© International Science Press

Volume 10 • Number 34 • 2017

# **Controlling Mobile Robot Based on Multi-scroll Dynamic Chaotic** Systems

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*Abstract:* In this paper, based on the dynamic features of chaotic systems, we present a study of special trajectories attainment for mobile robots. The construction of trajectories is envisaged for field exploration missions or for the specific purpose of research where a fast scanning of the workspace of the robot is required. Control the behavior of the mobile robot is obtained byadding the non-linear chaotic equations to the kinematic equations of the mobile robot. This chaotic equations is inspired from Lorenzattractor, Chua attractor double-scroll, therefore with generationof multi-scroll Chua attractor, which are well known equationshaving a chaotic behavior. Simulations results show the effectiveness of the proposed control based on non-linear chaoticsystems applied to a wheeled mobile robot.

Keywords: Mobile robot, chaos, Chua circuit, Lorenz system, multi-scroll chaotic attractors, chaos control.

# **1. INTRODUCTION**

Mobile robotics, after decades of research and vital developments, remains an interesting research area following its increasing demands, and the relevance of its economic and technological impacts. Mobile robot has become a topic of great interest thanks to the continued growth of its applications in different activities. The devices of the fire-fighting and floor cleaning were developed by exploiting autonomous mobile robots as useful tools inactivities that put the integrity of the man in danger, such asmonitoring and exploration of terrains for explosives or dangerous materials and intrusion patrols at military installations, havedriven to the development of intelligent robotic systems [2]. Especially, robotic systems in their military missions should have avery important feature as the perception and target identification the positioning of the robot on the ground. However, the most interesting objective for those successful robot missions, is a good path planning.

The undeniable interest of mobile robotics is to have helped increase considerably our knowledge of the location and navigation of autonomous systems. The range of potentially raised by the simplest problems of wheeled mobile robots actually a subject of study and full form an excellent basis for the study of more complex mobile systems.

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Various control methods have been studied, including adaptive neural control [3] and fuzzy control design using genetic algorithm [4] to name a few. Among these control methods, many researchers have focused on the sliding mode control [5-7], simple neural network-based controllers were proposed for real-time fine control of mobile robots in [8,9]. Therefore, the unpredictability of thetrajectory is also a crucial factor for the success of mission forsuch autonomous mobile robot. To meet this challenge Sekiguchi andNakamura have suggested a strategy in 2001 to solve the problem ofpath planning based on chaotic systems [10].

Several other researches that interested in the chaotic trajectories of the mobile robot have been carried with other equations [11,12]. The main goal in the use of chaotic signals for autonomous mobile robot is to increase and benefit coverage areas resulting from its path of movements. Vast coverage areas are desirable for many applications of robots such as those dedicated for scanning unknown workspaces, for cleaning or patrolling [13].

In our work, we focus on the specific problem of terrain exploration in order to cover a terrain fast and with unpredictable way. In such missions additional features like quick scan of the entire work area are highly appropriate. Chaotic behavior, typical of a class of non-linear dynamical systems can guarantee an unpredictable robot motion that scans the entire workspace. Many classical chaotic systems can be used such as Lorenz attractor [14], Rössler chaotic system [15], Arneodo system [16],etc. Many new chaotic systems have been also discovered in the recent years like Sundarapandian systems [17, 18], Vaidyanathan systems[19]. Recently, chaos theory is found to have important applications in several areas such as science and engineering, robotics, chemistry, electrical circuits [20], etc. Some recent control methods are discussed in [21-35].

In this work, the chaotic behavior of Lorenz attractor and Chua attractor are imparted to the mobile robot's motion control. For the sake of clarity, we present the Lorenz chaotic system, the double-scroll chaotic system of Chua. Then, after modifying the mathematical equations, we generate anew behavior of multi-scroll dynamic system behaving as a multi-scroll chaotic attractor. These chaotic systems will be explored as input to control the mobile robot.

The rest of this paper is organized as follows: The kinematic model of the robot is introduced in the next section. Then, the proposed chaotic systems are given in section 3, which presents the generation of multi-scroll chaotic attractor. In Section4, our control method of mobile robot and simulation results are given. Our concluding remarks are contained in the final section.

## 2. MODEL DESCRIPTION

An electrically driven non-holonomic mobile robot can be modeled via kinematic and dynamic equations. A non-holonomic mobile robot consists of two active wheels and a passive supporting wheel. The two driving wheels are independently driven by two DC motors to realize the robot motion and orientation. Assume that the mobile robot is made up of a rigid frame equipped with non-deformable wheels as described in [21-23], considering the robot configurations wheeled according to its position (x,y)and the direction  $\theta$  in a two-dimensional environment.

The space of the robot configurations is then constituted by all the triples of values  $(x, y, \theta) \in R \times R \times [0, 2\pi]$ , as shown in Figure 1.

The kinematic model of the robot can be described as a differential system comprising of two control parameters v and  $\omega$  which represent respectively the values of linear and angular speeds as follows:

$$\begin{pmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{pmatrix} = \begin{pmatrix} \cos \theta(t) & 0 \\ \sin \theta(t) & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v(t) \\ \omega(t) \end{pmatrix}$$
(1)



Figure 1: Geometry of the mobile robot on the Cartesian plane

The two driving wheels with the same radius rare separated by 2R. The point C is the center of mass of the mobile robot.

# 3. CHAOTIC ATTRACTOR WITH MULTI-SCROLLS

In order to obtain a chaotic trajectory of the mobile robot, this is achieved by the use of a controller that guarantees chaotic motion. The chaotic models used to generate the robot path are presented as the Lorenz attractor and the circuit equations of Chua.

## 3.1. Lorenz attractor

In this subsection, we recall Lorenz attractor. The Lorenz systemhas become one of paradigms in the research of chaos [24]. Lorenzsystem is utilized for the investigation. The dynamical equation of Lorenz attractor is given by the following equations:

$$\begin{cases} \dot{X}_{1} = -10X_{1} + 10.X_{2} \\ \dot{X}_{2} = 28X_{1} - X_{2} - X_{1}.X_{3} \\ \dot{X}_{3} = -\frac{8}{3}X_{3} + X_{1}.X_{2} \end{cases}$$
(2)

For numerical simulations, we take the initial conditions  $X_1(0) = 1$ ,  $X_2(0) = 0$  and  $X_3(0) = 1$ .

The implementation of this dynamic system is presented infigure 2.

# 3.2. Chua attractor

Chua attractor, which was introduced by Leon Ong Chua in 1983, are simplest electric circuits operating in the mode of chaoticoscillations. Different dynamic systems have been inspired from Chuacircuit such as:



#### Figure 2: Lorenz attractor

$$\begin{cases} \dot{X} = \alpha (Y - X - f(X)) \\ \dot{Y} = X - Y + Z \\ \dot{Z} = -\beta Y \end{cases}$$
(3)

Where  $f(X) = bX + \frac{1}{2}(a-b)(|X+1| - |X-1|), \alpha = 9, \beta = \frac{100}{7}, a = -\frac{8}{7}, b = -\frac{5}{7}$ 



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The implementation of this dynamic system is achieved in figure 3 at initial conditions X(0)=1, Y(0)=0 and Z(0)=1.

## 3.3. Generation of Chua attractor with multi-scroll

In the work presented by Suresh Rasappan and SundarapandianVaidyanathan [25], theauthors present the chaotic Chua system to generate a n-scroll attractor.By choosing the value of parameter c = 1, 2, 3 and 5 have been obtained2-scroll, 3-scroll, 4-scroll and 6-scroll attractors respectively. Then the maximum of scrolls can be obtained is n = 6. In [26], theauthors presented a family of hyperchaotic multi-scroll attractor in  $R^n$ , n = 4, based on unstable dissipative systems. This class of systems is constructed by a switching control law changing the equilibrium point of an unstable dissipative system.

In our approach, in order to obtain more complex scrolls, we use a simple methodbased on a modification of the system of equations presented in [25]. This leads to a new dynamic system is described by equation (4), equation (5) and equation (6) then we generate a multi-scroll chaotic system with a variable number of scrolls and maybe too superior to 6. Figure 4 shows the implementation of the multi-scroll Chua attractor.

The new dynamical equation of multi-scroll of Chua Circuit is givenby:

$$\begin{cases} X_1 = \alpha(Y_1 - f(X_1)) \\ Y_1 = ((X_1 - Y_1 + Z_1) - g(Y_1)) \\ Z_1 = -\beta Y_1 \end{cases}$$
(4)

Where  $f(X_1)$  is given by:

$$f(X_{1}) = \begin{cases} \frac{b}{2a}(X_{1} - 2ac)ifX_{1} \ge 2ac\\ -b\sin(\frac{\pi X_{1}}{2a} + d)if - 2ac \le X_{1} \le 2ac\\ \frac{b}{2a}(X_{1} + 2ac)ifX_{1} \le 2ac \end{cases}$$
(5)

And  $g(Y_1)$  is given by:

$$g(Y_{1}) = \begin{cases} \frac{b}{2a}(Y_{1} - 2ac)ifY_{1} \ge 2bc \\ -b\sin(\frac{\pi Y_{1}}{2b} + d)if - 2bc \le Y_{1} \le 2bc \\ \frac{b}{2a}(Y_{1} + 2bc)else \end{cases}$$
(6)

Where a,b, c and d are positive real constants.



Figure 4: Multi-scroll Chua attractor with c =3

Figure 4 shows the generation of multi-scroll Chuasystem implemented with a value of parameter c = 3, using the twoequations: equation (9) and equation (10).





Figure 5 shows the generation of more scroll Chuasystem implemented with c = 5.

#### 4. CONTROL OF MOBILE ROBOT USING CHAOTIC ATTRACTOR

In this section, the following proposed controlsystem will be applied to control the movement of the robot. Weadopt the chaos approach for controlling the trajectories of robotto different pseudo-periodic orbits .A dynamical system is called chaotic if it satisfies the three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [27]. Chaos theory investigates the qualitative and numerical study of unstable aperiodicbehavior in deterministic nonlinear dynamical systems.

The proposed controller generates an unpredictable path by giving a chaotic behavior of the system with two independent active wheels of mobile robot.

#### 4.1. Using Lorenz attractor

By using the dynamic equation introduced in equation (2), we will find robot equation of motion as follows:

$$\begin{aligned} \dot{X}_{1} &= -10X_{1} + 10.X_{2} \\ \dot{X}_{2} &= 28X_{1} - X_{2} - X_{1}.X_{3} \\ \dot{X}_{3} &= -\frac{8}{3}X_{3} + X_{1}.X_{2} \\ \dot{x} &= v\cos(\dot{X}_{1}) \\ \dot{y} &= v\sin(\dot{X}_{1}) \end{aligned}$$
(7)

Figure 6 shows the implementation result of robotmotion using the dynamic equation described inequation (7) in regard to the benefit coverage areasresulting from its path of movements, at initial conditions:



$$X_1(0) = 1, X_2(0) = 0, X_3(0) = 1, x(o) = 1, y(0) = 0, v = 3.$$



This scheme (Figure 6) does not require a mapping of the workspace or trajectory planning. The sensitivity to initial conditions makes it extremely unpredictable robot. The feature of chaotic systems is that its chaotic orbits have tobe dense. This means that, the trajectory of a dynamical system isdense, if it comes arbitrarily close to any point in the domain.

## 4.2. Using double-scroll Chua attractor

Now, by using the dynamic equation introduced in equation (3) we will find robot equation of motion as follows:

$$\begin{cases} \dot{X}_{1} = \alpha(Y_{1} - f(X_{1})) \\ \dot{Y}_{1} = X_{1} - Y_{1} + Z_{1} \\ \dot{Z}_{1} = -\beta Y_{1} \\ \dot{x} = v \cos(n\dot{X}_{1}) \\ \dot{y} = v \sin(n\dot{X}_{1}) \end{cases}$$
(8)

Where

$$f(X_1) = bX_1 + \frac{1}{2}(a-b)(|X_1+1| - |X_1-1|), \alpha = 9, \beta = \frac{100}{7}, a = -\frac{8}{7}, b = -\frac{5}{7} \text{ and } n = 50*pi$$

We used the time delay of the first state on themodel of Chua attractor; these states are used by combining with themodel of robot.



Figure 7: Behavior of robot controller with double-scroll Chua attractor with15000 iteration

Figure 7 shows very satisfactory results in regard to the fast scanning of the robots workspace with unpredictable way.



Figure 8: Behavior of robot controller with double-scroll Chua attractor with 25000 iterations





Figure 9 shows the zooming behavior of robot control with Double-scroll Chua attractor. It is clear that the scrolls presented in this behavior contain a large number of orbits.

## 4.3. Using multi-scroll Chua attractor

The integrated system of the multi-scroll Chua circuit equation [17] as a controller of the mobile robot will be as follows:

$$\begin{aligned} \dot{X}_{1} &= \alpha (Y_{1} - f(X_{1})) \\ \dot{Y}_{1} &= X_{1} - Y_{1} + Z_{1} \\ \dot{Z}_{1} &= -\beta Y_{1} \\ \dot{x} &= v \cos(N\dot{X}_{1}, n) \\ \dot{y} &= v \sin(N\dot{X}_{1}, n - 1) \end{aligned}$$
(9)

Where

$$f(X_{1}) = \begin{cases} \frac{b}{2a}(X_{1} + 2ac)ifX_{1} \ge 2ac \\ -ba\sin(\frac{\pi X_{1}}{2cX_{1}} + d)if - 2ac \le X_{1} \le 2ac \\ \frac{b}{2a}(X_{1} + 2ac)ifX_{1} \le 2ac \end{cases}$$
(10)

The resultant trajectory of the mobile robot is controlled by Chua equations, at initial conditions:

$$X_1(0) = 1, Y_1(0) = 0, Z_1(0) = 1, x(o) = 1, y(0) = 1, v = 3, N = 50\pi$$



Figure 10: Behavior of robot controller with Multi-scroll Chua attractor

As shown in Figure 10, when we used chaotic attractor with multi-scroll, the number of orbits in trajectory of the robot is decreased. There is relationship between the number of scrolls and the number of orbits.

This approach by chaotic attractor multi-scroll guarantee not only to accomplish the path planning of robot but also can optimize energy and reduce the time to finish his tasks.

# 5. CONCLUSIONS

In this work, we defined an approach based on non-linear dynamic systems that may be involved in the realization of a navigation trajectory for an autonomous mobile robot. It is based on a technique of control using the chaos, used to monitor the dynamics of Lorenz attractor, double-scroll Chua attractor and multi-scroll Chua attractor. This proposed control and implementation of chaotic behavior on a mobile robot, implies a mobile robot with a controller that guarantees its chaotic motion with the minimum of orbits. This will make the most economical robot in energy consumption and reduce the time to finish its tasks. Some numerical simulation results are provided to show the effectiveness of the method proposed in this work.

## REFERENCES

- M.J.M. Tavera, M.S. Dutra, E.Y.V. Diaz, and O. Lengerke, "Implementation of chaotic behaviour on a fire fighting robot," In Proc. Of the 20th Int. Congress of Mechanical Engineering, Gramado, Brazil, November 2009.
- [2] L.S. Martins-Filho and E.E.N. Macau, "Trajectory planning for surveillance missions of mobile robots," Studies in Computational Intelligence, Springer, Berlin, pp. 109-117, 2007.
- [3] Mohareri, O., Dhaouadi, R., Rad, A.B.: Indirect adaptive tracking control of a nonholonomic mobile robot via neural networks. Neurocomputing,88, 54-66, 2012.
- [4] Martnez, R., Castillo, O., Aguilar, L.T.: Optimization of interval type-2 fuzzy logic controllers for a perturbed autonomous wheeled mobile robot using genetic algorithms. Inform. Sci. 179(13), 2158-2174, 2009.
- [5] B. S. Park, S. J. Yoo, Y. H. Choi, and J. B. Park, "A new sliding surface based tracking control of nonholonomic mobile robots," Journalof Institute of Control, Robotics and Systems (in Korean), vol. 14, no. 8, pp. 842-847, 2008.
- [6] J. H. Lee, C. Lin, H. Lim, and J. M. Lee, "Sliding mode control for trajectory tracking of mobile robot in the RFID sensor space," International Journal of Control, Automation, and Systems, vol. 7, no. 3, pp, 429-435, 2009.
- [7] J.K. Lee, Y.H. Choi, and J.B. Park, "Sliding mode tracking control of mobile robots with approach angle in Cartesian coordinates", *International Journal of Control, Automation, and Systems*, 13(3), 1-7, Springer 2015.
- [8] Y. Li, L. Zhu and M. Sun, "Adaptive neural-network control of mobile robot formations including actuator dynamics", *Appl. Mech Mater*, 303-306,1768-1773, 2013.
- J. Ye, "Tracking control of a nonholonomic mobile robot using compound cosine function neural networks" Intel Serv Robotics, Springer 6:191-198 DOI 10.1007/s11370-013-0136-4, 2013.
- [10] Y. Nakamura, A. Sekiguchi, The Chaotic Mobile Robot, IEEE Trans. Robot. Autom., Vol. 17(6), pp. 898-904, 2001.
- [11] J. Palacin, J. A. Salse, I. Valganon, and X. Clua, "Building a mobile robot for a floor-cleaning operation in domestic environments", *IEEETransactions on Instrumentation and Measurement*, 53 (5), 1418-1424, 2004.
- [12] P. Sooraksa and K. Klomkarn, "No-CPU chaotic robots from classroom to commerce", IEEE Circuits and Systems Magazine, 10.1109/MCAS, pp. 46-53, 2010.
- [13] L.S. Martins-Filho and E.E.N. Macau,"Patrol mobile robots and chaotic trajectories", *Mathematical Problems in Engineering*, vol. 2007, Article ID61543, 2007.
- [14] E.N. Lorenz, "Deterministic nonperiodic flow," Journal of the Atmospheric Sciences, 20, 130-141, 1963.
- [15] O.E. Rössler, "An equation for continuous chaos," Physics Letters A, 57, 397-398, 1976.
- [16] A. Arneodo, P. Coullet and C. Tresser, "Possible new strange attractors with spiral structure", Communications in Mathematical Physics, 79, 573-579, 1981.

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- [17] V. Sundarapandian and I. Pehlivan, "Analysis, control, synchronization and circuit design of a novel chaotic system,"*Mathematical and Computer Modelling*, 55, 1904-1915, 2012.
- [18] V. Sundarapandian,"Analysis and anti-synchronization of a novel chaotic system via active and adaptivecontrollers,"*Journal of Engineering Science and Technology Review*, 6, 45-52, 2013.
- [19] S. Vaidyanathan, "A new six-term 3-D chaotic system with an exponential nonlinearity," *Far East Journal of Mathematical Sciences*, 79, 135-143, 2013.
- [20] S. Vaidyanathan, "Anti-synchronization of novel coupled Van der Pol conservative chaotic systems via adaptive control method", *International Journal of ChemTech Research*, 9 (2), 106-123, 2016.
- [21] M. Defoort, J. Palos, A. Kokosy, T. Floquet, W. Perruquetti and D. Boulinguez, "Experimental motion planning and control for an autonomous nonholonomic mobile robot," In ICRA'07,pages 2221-2226, 2007.
- [22] J. Ye, "Hybrid trigonometric compound function neural networks for tracking control of a nonholonomic mobile robot," Intel Serv Robotics, Springer 6:191-198, DOI 10.1007/s11370-014-0155-9, 2014.
- [23] K. Bouallegue and A.Chaari, "Survey and implementation on DSP of algorithms of robot paths generation and of numeric control for mobile robot," *Journal of Applied Sciences*, 7 (13), 1854-1863, 2007.
- [24] S. Nasr, K. Bouallegue and H. Mekki, "Hyperchaos set by fractal processes system," 8th Chaos Conference Proceedings, Henri Poincare Institute, Paris France, 561-570, 26-29 May 2015.
- [25] S. Rasappan and S. Vaidyanathan, "Hybrid synchronization fn-scroll chaotic Chua circuits using adaptative backstepping control design with recursive feedback", *Malaysian Journal of Mathematical Sciences*, 7(2), 219-246, 2013.
- [26] L.J. Ontan-Garcaa, E. Jimnez-Lpezb, E. Campos-Cantnb, M. Basin, "A family of hyperchaotic multi-scroll attractors in Rn", *Applied Mathematics and Computation*, 233, 522-533, 2014.
- [27] S.Vaidyanathan, "Global chaos synchronization of Duffing double-well chaotic oscillators via integral sliding mode control,"*International Journal of ChemTech Research*,8 (11), 141-151, 2015.
- [28] A.T. Azar and S. Vaidyanathan, Chaos Modeling and Control Systems Design, Springer, Berlin, 2015.
- [29] S. Vaidyanathan, "A novel 3-D conservative chaotic system with sinusoidal nonlinearity and its adaptive control", International Journal of Control Theory and Applications, 9 (1), 115-132, 2016.
- [30] S. Vaidyanathan and S. Pakiriswamy, "A five-term 3-D novel conservative chaotic system and its generalized projective synchronization via adaptive control method", *International Journal of Control Theory and Applications*, 9 (1), 61-78, 2016.
- [31] V.T. Pham, S. Jafari, C. Volos, A. Giakoumis, S. Vaidyanathan and T. Kapitaniak, "A chaotic system with equilibria located on the rounded square loop and its circuit implementation," *IEEE Transactions on Circuits and Systems-II: Express Briefs*, 63 (9), 2016.
- [32] S. Vaidyanathan and S. Sampath, "Anti-synchronisation of identical chaotic systems via novel sliding control and its application to a novel chaotic system," *International Journal of Modelling, Identification and Control*, 27 (1), 3-13, 2017.
- [33] S. Vaidyanathan, K. Madhavan and B.A. Idowu, "Backstepping control design for the adaptive stabilization and synchronization of the Pandey jerk chaotic system with unknown parameters," *International Journal of Control Theory and Applications*, 9 (1), 299-319, 2016.
- [34] R.K. Goyal, S. Kaushal and S. Vaidyanathan, "Fuzzy AHP for control of data transmission by network selection in heterogeneous wireless networks," *International Journal of Control Theory and Applications*, 9 (1), 133-140, 2016.
- [35] C.K. Volos, D. Prousalis, I.M. Kyprianidis, I. Stouboulos, S. Vaidyanathan and V.T. Pham, "Synchronization and antisynchronization of coupled Hindmarsh-Rose neuron models," *International Journal of Control Theory and Applications*, 9 (1), 101-114, 2016.